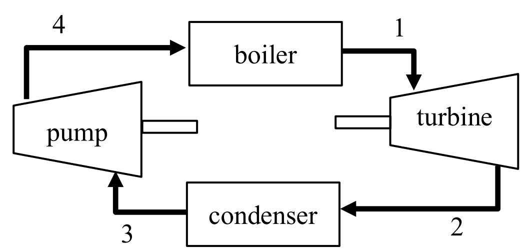
Consider the vapor power plant cycle shown in the figure. The working fluid is water. Water flows through the boiler and condenser at constant pressure and through the turbine and pump adiabatically. Kinetic and potential energy effects can be ignored. The process data are:

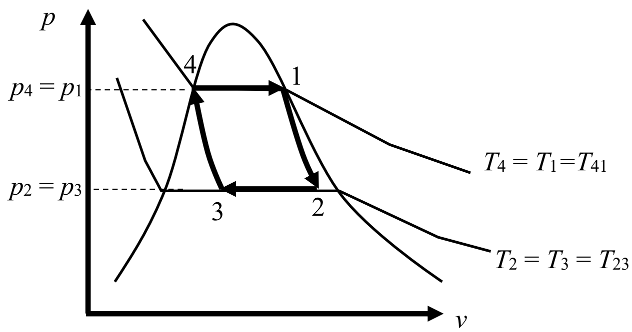
Process 4 – 1: constant pressure at 1 MPa (abs) from saturated liquid to saturated vapor,

Process 2 – 3: constant pressure at 20 kPa (abs) from $x_2 = 0.88$ to $x_3 = 0.18$.

a. Determine if the cycle is internally reversible, irreversible, or impossible.
b. Determine the thermal efficiency of the cycle.
c. Compare the thermal efficiency from (b) to the maximum possible efficiency.



SOLUTION:
First sketch the cycle on a p-v diagram for convenience.

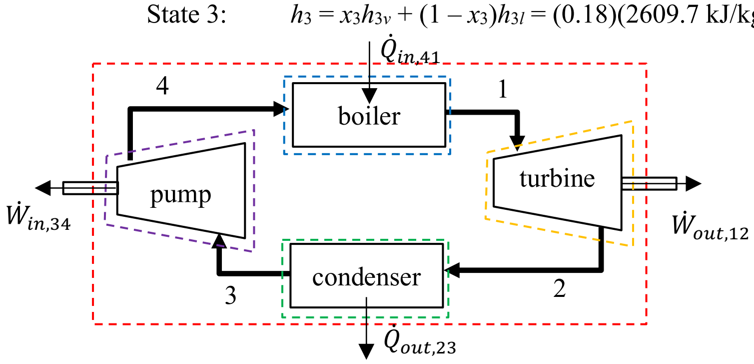


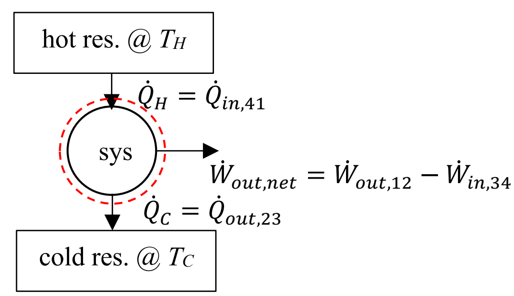
Using the thermodynamic property tables for water in a saturated state (e.g., Table A-3 in Moran et al., 7th ed.):

T_{41} = 179.9°C = 453.05 K @ 1 MPa = 10 bar (abs),
T_{23} = 60.06°C = 333.21 K @ 20 kPa = 0.2 bar (abs).

In addition, also using Table A-3 in Moran et al., 7th ed. at the given saturation temperature,

State 4: $h_4 = h_l$ = 762.81 kJ/kg (saturated liquid state)
State 1: $h_1 = h_v$ = 2778.1 kJ/kg (saturated vapor state)
State 2: $h_2 = x_2 h_{2v} + (1 - x_2) h_{2l}$ = (0.88)(2609.7 kJ/kg) + (1 – 0.88)(251.4 kJ/kg) = 2326.7 kJ/kg
State 3: $h_3 = x_3 h_{3v} + (1 - x_3) h_{3l}$ = (0.18)(2609.7 kJ/kg) + (1 – 0.18)(251.4 kJ/kg) = 675.89 kJ/kg





Applying the 1st Law to a control volume surrounding the boiler, assuming steady flow, negligible changes in kinetic and potential energy across the control volume, and no work other than pressure,

$$\dot{Q}_{in,41} = \dot{m}(h_1 - h_4) \Longrightarrow \frac{\dot{Q}_{in,41}}{\dot{m}} = h_1 - h_4. \tag{1}$$

Similarly, for the condenser,

$$\dot{Q}_{out,23} = \dot{m}(h_2 - h_3) \Longrightarrow \frac{\dot{Q}_{out,23}}{\dot{m}} = h_2 - h_3. \tag{2}$$

Apply the 1st Law to control volumes surrounding the turbine and pump, assuming steady flow, negligible changes in kinetic and potential energy across the control volume, and adiabatic conditions,

$$\dot{W}_{out,12} = \dot{m}(h_1 - h_2) \Longrightarrow \frac{\dot{W}_{out,12}}{\dot{m}} = h_1 - h_2. \tag{3}$$

$$\dot{W}_{in,34} = \dot{m}(h_4 - h_3) \Longrightarrow \frac{\dot{W}_{in,34}}{\dot{m}} = h_4 - h_3. \tag{4}$$

Substituting the specific enthalpy values found previously,

$$\frac{\dot{Q}_{in,41}}{\dot{m}} = 2015.29 \text{ kJ/kg}$$

$$\frac{\dot{Q}_{out,23}}{\dot{m}} = 1650.81 \text{ kJ/kg}$$

$\frac{\dot{W}_{out,12}}{\dot{m}}$ = 451.4 kJ/kg
$\frac{\dot{W}_{in,34}}{\dot{m}}$ = 86.92 kJ/kg

To determine if the cycle is internally reversible, irreversible, or impossible, consider the Clausius Inequality applied to the entire cycle (or, alternately, the Entropy Equation with $dS/dt = 0$ because the cycle is at steady state),

$$\int_b \frac{\delta \dot{Q}_{into,cycle}}{T} = -\dot{\sigma}, \tag{5}$$

where,

$$\int_b \frac{\delta \dot{Q}_{into,cycle}}{T} = -\dot{\sigma} = \dot{m}\left(\frac{\dot{Q}_{in,41}/\dot{m}}{T_H} - \frac{\dot{Q}_{out,23}/\dot{m}}{T_C}\right), \tag{6}$$

$$\frac{\dot{\sigma}}{\dot{m}} = -\left(\frac{2015.29\text{ kJ/kg}}{453.05\text{ K}} - \frac{1650.81\text{ kJ/kg}}{333.21\text{ K}}\right), \tag{7}$$

$$\frac{\dot{\sigma}}{\dot{m}} = 0.507\ \frac{\text{kJ}}{\text{kg.K}}. \tag{8}$$

Thus, we see that **the cycle is internally irreversible.** Note that in this analysis it is assumed that the temperatures at which the heat enters and leaves the CV are $T_H = T_{41}$ and $T_C = T_{23}$ since we're not given any information about their temperatures. In reality, this would not be the case since there must be some temperature difference between the hot/cold reservoir and the boiler/condenser to drive the heat transfer, i.e., $T_H > T_{41}$ and $T_C < T_{23}$. If there was a temperature difference between T_H and T_{41} and T_C an T_{23}, then this would create even more irreversibility due to the larger temperature gradient.

The thermal efficiency of the cycle is,

$$\eta = 1 - \frac{\dot{Q}_C}{\dot{Q}_H} = 1 - (1650.81\text{ kJ/kg})/(2015.29\text{ kJ/kg}) \tag{9}$$

$\Rightarrow$ $\boxed{\eta = 0.18 = 18\%}$.

We could have also calculated the efficiency using the work generated,

$$\eta = \frac{\dot{W}_{out,net}/\dot{m}}{\dot{Q}_H/\dot{m}} = \frac{(\dot{W}_{out,12} - \dot{W}_{in,34})/\dot{m}}{\dot{Q}_{in,41}/\dot{m}} = \frac{(451.4\text{ kJ/kg} - 86.92\text{ kJ/kg})}{2015.29\text{ kJ/kg}} = 0.18 \tag{10}$$

which is the same result as found previously.

The maximum possible efficiency is,

$$\eta_{max} = \eta_{int.rev.} = 1 - (333.21\text{ K})/(453.05\text{ K}), \tag{11}$$

$\Rightarrow$ $\boxed{\eta_{max} = 0.26 = 26\%}$.

Since the cycle is irreversible, the actual efficiency is less than the maximum possible efficiency.