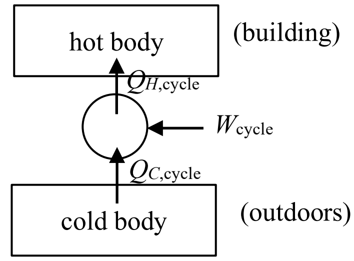
A heat pump with a coefficient of performance of 3.5 provides energy at an average rate of 70,000 kJ/h to maintain a building at 20 °C on a day when the outside temperature is – 5 °C. If electricity costs $0.085/(kW.h),

a. determine the actual daily operating cost and the minimum theoretical daily operation cost.
b. Compare the results of part (a) with the daily cost of electrical resistance heating.

SOLUTION:



To determine the actual operating cost of the heat pump, first determine the power required to operate it. The power may be found using the heat pump's coefficient of performance,

$$\text{COP}_{\text{hp}} = \frac{\dot{Q}_{H,\text{cycle}}}{\dot{W}_{\text{cycle}}} \Rightarrow \dot{W}_{\text{cycle}} = \frac{\dot{Q}_{H,\text{cycle}}}{\text{COP}_{\text{hp}}}, \tag{1}$$

where,

$\dot{Q}_{H,\text{cycle}}$ = 70000 kJ/h,

COP_{hp} = 3.5,

$\Rightarrow$ $\dot{W}_{\text{cycle}}$ = 20000 kJ/h.

The operating cost is found by multiplying the power by the cost per kilowatt-hour and making appropriate unit conversions,

$$\text{cost}_{\text{actual}} = \left(20000\ \tfrac{\text{kJ}}{\text{h}}\right)\left(\$0.085\ \tfrac{1}{\text{kW-h}}\right)\left(\tfrac{1\ \text{kW}}{1\ \text{kJ/s}}\right)\left(\tfrac{\text{h}}{3600\ \text{s}}\right)\left(\tfrac{24\ \text{h}}{1\ \text{day}}\right), \tag{2}$$

$\boxed{\text{cost}_{\text{actual}} = \$11.33/\text{day}}$

The minimum theoretical daily cost can be found by first calculating the power required to operate the heat pump if it was operating in an internally reversible manner (i.e., no losses). Under these conditions,

$$\left.\frac{\dot{Q}_{H,\text{cycle}}}{\dot{Q}_{C,\text{cycle}}}\right|_{\text{rev}} = \frac{T_H}{T_C}, \tag{3}$$

which may be substituted into the expression for the heat pump's coefficient of performance,

$$\text{COP}_{\text{hp}} = \frac{\dot{Q}_{H,\text{cycle}}}{\dot{W}_{\text{cycle}}} = \frac{\dot{Q}_{H,\text{cycle}}}{\dot{Q}_{H,\text{cycle}} - \dot{Q}_{C,\text{cycle}}} \Rightarrow \text{COP}_{\text{hp,rev}} = \frac{T_H}{T_H - T_C}. \tag{4}$$

The temperatures of the thermal reservoirs are,

T_H = 20 °C = 293 K,

T_C = -5 °C = 268 K,

$\Rightarrow$ $\text{COP}_{\text{hp,rev}}$ = 11.72.

Note that this COP is the best performance that can be achieved by any heat pump operating between the given thermal reservoirs. The current heat pump has a COP_{hp} = 3.5, which is quite a bit smaller than the ideal one suggesting that there is room for improvement in the design or choice of the current heat pump.

Using the $\text{COP}_{\text{hp,rev}}$ = 11.72 and Eq. (1), the amount of power required to operate the ideal heat pump is,

$$\dot{W}_{\text{cycle,rev}} = 5973\ \text{kJ/h}, \tag{5}$$

and the minimum daily operating cost is,

$$\text{cost}_{\text{min}} = \left(5973\ \tfrac{\text{kJ}}{\text{h}}\right)\left(\$0.085\ \tfrac{1}{\text{kW-h}}\right)\left(\tfrac{1\ \text{kW}}{1\ \text{kJ/s}}\right)\left(\tfrac{\text{h}}{3600\ \text{s}}\right)\left(\tfrac{24\ \text{h}}{1\ \text{day}}\right), \tag{6}$$

$\boxed{\text{cost}_{\text{min}} = \$3.38/\text{day}}$

The conversion efficiency from electrical power to heat is approximately 100%. Hence, if an electric resistance heater is used instead of a heat pump, the power required to supply the necessary heat is,

$$\dot{W}_{\text{cycle,elec heater}} = \dot{Q}_{H,\text{cycle}} = 70000 \text{ kJ/h}. \quad (7)$$

The daily cost for operating the electrical resistance heater is then,

$$\text{cost}_{\text{elec heater}} = \left(70000 \ \tfrac{\text{kJ}}{\text{h}}\right)\left(\$0.085 \ \tfrac{1}{\text{kW-h}}\right)\left(\tfrac{1 \text{ kW}}{1 \text{ kJ/s}}\right)\left(\tfrac{\text{h}}{3600 \text{ s}}\right)\left(\tfrac{24 \text{ h}}{1 \text{ day}}\right), \quad (8)$$

$$\boxed{\text{cost}_{\text{elec heater}} = \$39.67/\text{day}}$$

Clearly the given heat pump provides significant daily cost savings as compared to an electric resistance heater (3.5x cheaper to operate). In electrical resistance heating, the electric power is converted directly into heat. However, in a heat pump, the work that goes into operating the cycle is used to move energy from the cold reservoir into the hot reservoir. There is no conversion of work into heat. The work is used to move energy from one location to another (via the refrigerant used in the heat pump system), which is a much more efficient operation.