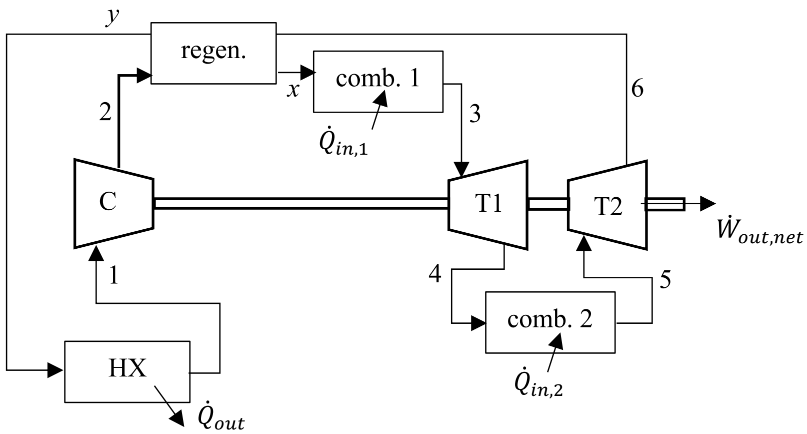
Air enters the compressor of a cold air-standard Brayton cycle with regeneration and reheat at 100 kPa (abs), 300 K, with a mass flow rate of 6 kg/s. The compressor pressure ratio is 10 and the inlet temperature for each turbine stage is 1400 K. The pressure ratios across each turbine stage are equal. The turbine stages and compressor each have isentropic efficiencies of 80% and the regenerator effectiveness is 80%. For a specific heat ratio of 1.4, calculated:

a. the thermal efficiency of the cycle,
b. the back work ratio, and
c. the net power developed by the cycle.

SOLUTION:



Since we're assuming a cold air-standard analysis, state the properties of the air in the analysis:
R = 0.287 kJ/(kg.K) and $c_{p@300\text{ K}}$ = 1.005 kJ/(kg.K).

The net power from the cycle is found by applying the 1st Law to a CV surrounding the compressor and turbines (assuming SSSF, adiabatic operation, negligible KE and PE),

$$\dot{W}_{out,net} = \dot{m}(h_1 - h_2 + h_3 - h_4 + h_5 - h_6), \tag{1}$$

and, since we're performing a cold air-standard analysis, meaning the air is a perfect gas,

$$\dot{W}_{out,net} = \dot{m}c_p(T_1 - T_2 + T_3 - T_4 + T_5 - T_6). \tag{2}$$

The power into the compressor is found by applying the 1st Law to a CV surrounding just the compressor (assuming SSSF, adiabatic operation, negligible KE and PE),

$$\dot{W}_{in} = \dot{m}(h_2 - h_1), \tag{3}$$

$$\dot{W}_{in} = \dot{m}c_p(T_2 - T_1) \quad \text{(assuming perfect gas behavior).} \tag{4}$$

The back work ratio (bwr) is,

$$bwr = \frac{\dot{W}_{in}}{\dot{W}_{out}} = \frac{\dot{W}_{in}}{\dot{W}_{out,net} + \dot{W}_{in}}. \tag{5}$$

The rate at which heat is added into the two combustors is found by applying the 1st Law to CVs surrounding each combustor (assuming SSSF, passive devices, negligible KE and PE),

$$\dot{Q}_{in} = \dot{Q}_{in,1} + \dot{Q}_{in,2} = \dot{m}(h_3 - h_x) + \dot{m}(h_5 - h_4), \tag{6}$$

$$\dot{Q}_{in} = \dot{m}c_p(T_3 - T_x + T_5 - T_4) \quad \text{(assuming perfect gas behavior).} \tag{7}$$

The thermal efficiency of the cycle is,

$$\eta = \frac{\dot{W}_{out,net}}{\dot{Q}_{in}}. \tag{8}$$

Now find the temperatures at the various states.

State 1:
$\dot{m} = 6$ kg/s, $p_1 = 100$ kPa (abs), $T_1 = 300$ K (given)

State 2s: (assuming the process is isentropic and involves a perfect gas)
$$\frac{T_{2s}}{T_1} = \left(\frac{p_{2s}}{p_1}\right)^{\frac{k-1}{k}} \Rightarrow T_{2s} = 579.21 \text{ K using } p_{2s}/p_1 = p_2/p_1 = 10. \tag{9}$$

State 2:
$$\eta_{comp,isen} = \frac{w_{in,isen}}{w_{in}} = \frac{h_{2s}-h_1}{h_2-h_1} = \frac{c_p(T_{2s}-T_1)}{c_p(T_2-T_1)} \Rightarrow T_2 = T_1 + \frac{T_{2s}-T_1}{\eta_{comp,isen}}, \tag{10}$$
=> $T_2 = 649.01$ K.

State 3:
$T_3 = 1400$ K (given)

State 4s:
$$\frac{T_{4s}}{T_3} = \left(\frac{p_{4s}}{p_3}\right)^{\frac{k-1}{k}} \Rightarrow T_{4s} = 1007.56 \text{ K using } p_3/p_{4s} = 3.162. \tag{11}$$
Note that since we're told that the pressure drops across both turbine stages are equal,
$$\frac{p_2}{p_1} = \frac{p_3}{p_6} = \left(\frac{p_3}{p_4}\right)\underbrace{\left(\frac{p_4}{p_5}\right)}_{=1}\underbrace{\left(\frac{p_5}{p_6}\right)}_{=p_3/p_4} = \left(\frac{p_3}{p_4}\right)^2 \Rightarrow \frac{p_3}{p_4} = \sqrt{\frac{p_2}{p_1}} \Rightarrow \frac{p_3}{p_4} = \frac{p_5}{p_6} = 3.162. \tag{12}$$

State 4:
$$\eta_{turb,isen} = \frac{w_{out}}{w_{out,isen}} = \frac{h_3-h_4}{h_3-h_{4s}} = \frac{c_p(T_3-T_4)}{c_p(T_3-T_{4s})} \Rightarrow T_4 = T_3 - \eta_{turb,isen}(T_3 - T_{4s}), \tag{13}$$
=> $T_4 = 1086.05$ K.

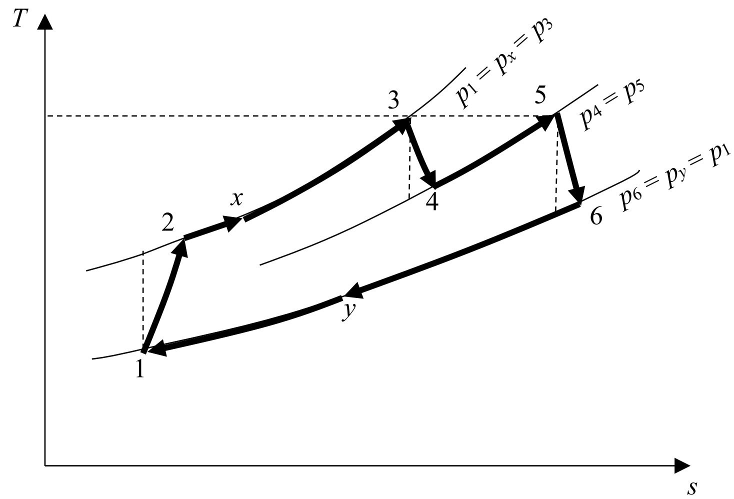
State 5:
$T_5 = 1400$ K (given)

State 6s:
$$\frac{T_{6s}}{T_5} = \left(\frac{p_{6s}}{p_5}\right)^{\frac{k-1}{k}} \Rightarrow T_{6s} = 1007.56 \text{ K using } p_{6s}/p_5 = 3.162 \text{ (Eq. (12))} \tag{14}$$

State 6:
$$\eta_{turb,isen} = \frac{w_{out}}{w_{out,isen}} = \frac{h_5-h_6}{h_5-h_{6s}} = \frac{c_p(T_5-T_6)}{c_p(T_5-T_{6s})} \Rightarrow T_6 = T_5 - \eta_{turb,isen}(T_5 - T_{6s}), \tag{15}$$
=> $T_6 = 1086.05$ K.

State x:
$$\eta_{reg} = \frac{h_x-h_2}{h_6-h_2} = \frac{c_p(T_x-T_2)}{c_p(T_6-T_2)} \Rightarrow T_x = T_2 + \eta_{reg}(T_6 - T_2). \tag{16}$$
=> $T_x = 998.64$ K using $\eta_{reg} = 0.80$ (given) and the previously calculated values.

Using the calculated temperatures and Eqs. (2), (4), (5), (7), and (8),
$\boxed{\dot{W}_{out,net} = 1680 \text{ kW}}$,
$\boxed{\dot{W}_{in} = 2100 \text{ kW}}$,
$\boxed{\dot{Q}_{in} = 4300 \text{ kW}}$,
$\boxed{bwr = 0.556 = 55.6\%}$,
$\boxed{\eta = 0.390 = 39.0\%}$.


T
s
1
2
x
3
4
5
6
y
$p_1 = p_x = p_3$
$p_4 = p_5$
$p_6 = p_y = p_1$