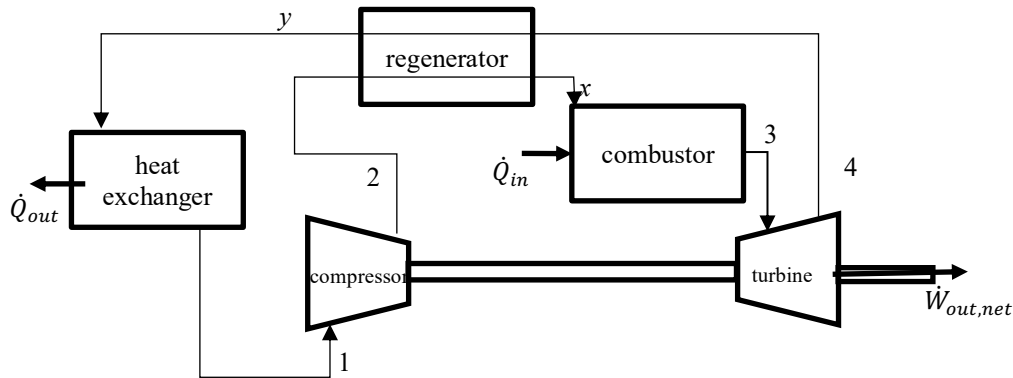


Air enters the compressor of a regenerative air-standard Brayton cycle with a volumetric flow rate of  $60 \text{ m}^3/\text{s}$  at  $0.8 \text{ bar (abs)}$  and  $280 \text{ K}$ . The compressor pressure ratio is  $20$  and the maximum cycle temperature is  $2100 \text{ K}$ . The compressor and turbine have isentropic efficiencies of  $92\%$  and  $95\%$ , respectively. For a regenerator effectiveness of  $85\%$ , determine:

- a. the net power developed,
- b. the rate of heat addition in the combustor,
- c. the thermal efficiency of the cycle.

SOLUTION:



To determine the net power developed, apply the 1<sup>st</sup> Law to a CV surrounding the compressor and turbine,  
 $\dot{W}_{out,net} = \dot{m}(h_1 + h_3 - h_2 - h_4)$  (assuming SSSF, adiabatic, and negligible KE and PE). (1)

The rate of heat transfer in the combustor is found by applying the 1<sup>st</sup> Law to a CV surrounding the combustor,  
 $\dot{Q}_{in} = \dot{m}(h_3 - h_x)$  (assuming SSSF, passive device, and negligible KE and PE). (2)

Now find the properties at the various states.

State 1:

$$\begin{aligned} \dot{V} &= 60 \text{ m}^3/\text{s}, p_1 = 0.8 \text{ bar (abs)} = 80 \text{ kPa (abs)}, T_1 = 280 \text{ K} \\ \Rightarrow h_1 &= 280.1 \text{ kJ/kg and } p_r(T_1) = 1.0889 \text{ (from the Ideal Gas Table for air)} \\ \text{Also, from the ideal gas law,} \\ \Rightarrow \rho_1 &= \frac{p_1}{RT_1} = 0.9955 \text{ kg/m}^3 \Rightarrow \dot{m} = \rho \dot{V} = 59.731 \text{ kg/s} \end{aligned} \quad (3)$$

State 3:

$$\begin{aligned} T_3 &= 2100 \text{ K}, \\ \Rightarrow h_3 &= 2377 \text{ kJ/kg and } p_r(T_3) = 2559 \text{ (from the Ideal Gas Table for air)} \end{aligned}$$

State 2:

$$\begin{aligned} p_2/p_1 = 20 = p_{2s}/p_1 \text{ and } \eta_{comp,isen} = 0.92 \text{ (given),} \\ \eta_{comp,isen} = \frac{w_{in,isen}}{w_{in}} = \frac{h_{2s} - h_1}{h_2 - h_1} \Rightarrow h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_{comp,isen}}. \end{aligned} \quad (4)$$

For an ideal gas undergoing an isentropic process,

$$\begin{aligned} \frac{p_{2s}}{p_1} = \frac{p_r(T_{2s})}{p_r(T_1)} \Rightarrow p_r(T_{2s}) = p_r(T_1) \left( \frac{p_{2s}}{p_1} \right), \\ \Rightarrow p_r(T_{2s}) = 21.778 \Rightarrow T_{2s} = 649.33 \text{ K}, h_{2s} = 659.29 \text{ kJ/kg (IGT),} \\ \Rightarrow h_2 = 692.26 \text{ kJ/kg.} \end{aligned} \quad (5)$$

State 4:

$$p_3/p_4 = 20 = p_3/p_{4s} (= p_2/p_1) \text{ and } \eta_{\text{turb,isen}} = 0.95 \text{ (given),}$$

$$\eta_{\text{turb,isen}} = \frac{w_{\text{out}}}{w_{\text{out,isen}}} = \frac{h_3 - h_4}{h_3 - h_{4s}} \Rightarrow h_4 = h_3 - \eta_{\text{turb,isen}}(h_3 - h_{4s}). \quad (6)$$

For an ideal gas undergoing an isentropic process,

$$\frac{p_{4s}}{p_3} = \frac{p_r(T_{4s})}{p_r(T_3)} \Rightarrow p_r(T_{4s}) = p_r(T_3) \left( \frac{p_{4s}}{p_3} \right), \quad (7)$$

$$\Rightarrow p_r(T_{4s}) = 127.95 \Rightarrow T_{4s} = 1029.19 \text{ K, } h_{4s} = 1079.57 \text{ kJ/kg (IGT),}$$

$$\Rightarrow h_4 = 1144.44 \text{ kJ/kg.}$$

State x:

From the definition of the regenerator effectiveness,

$$\eta_{\text{reg}} = \frac{h_x - h_2}{h_4 - h_2} \Rightarrow h_x = h_2 + \eta_{\text{reg}}(h_4 - h_2), \quad (8)$$

$$\text{Using the previously calculated specific enthalpy values and the given } \eta_{\text{reg}} = 0.85,$$

$$\Rightarrow h_x = 1076.61 \text{ kJ/kg.}$$

Using these state data, Eq. (1) gives,

$$\boxed{\dot{W}_{\text{out,net}} = 49.0 \text{ MW}},$$

and Eq. (2) gives,

$$\boxed{\dot{Q}_{\text{in}} = 77.7 \text{ MW}}.$$

The thermal efficiency for the cycle is,

$$\boxed{\eta_{\text{cycle}} = \frac{\dot{W}_{\text{out,net}}}{\dot{Q}_{\text{in}}} = 0.631 = 63.1\%}. \quad (9)$$

Note that as  $\eta_{\text{reg}}$  increases, then  $h_x$  approaches  $h_4$  and  $\dot{Q}_{\text{in}}$  decreases. As a result, the thermal efficiency for the cycle would increase. In the limit of  $\eta_{\text{reg}} = 100\%$ ,  $\dot{Q}_{\text{in,min}} = 73.6 \text{ MW}$  and  $\eta_{\text{cycle,max}} = 66.6\%$ . In contrast, without the regenerator ( $\eta_{\text{reg}} = 0$ ), then  $\dot{Q}_{\text{in,max}} = 100.6 \text{ MW}$  and  $\eta_{\text{cycle,min}} = 48.7\%$ . Thus, we observe that including a regenerator can substantially improve the cycle's thermal efficiency.

