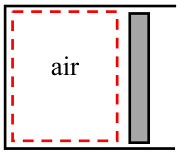
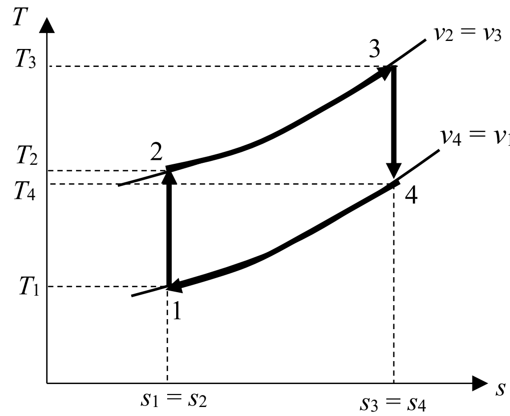
An air-standard Otto cycle has a compression ratio of 10. At the beginning of compression, the pressure is 100 kPa (abs) and temperature is 27 ºC. The mass of air is 5 g and the maximum temperature in the cycle is 727 ºC. Determine:

a. the heat rejection, in kJ,
b. the net work, in kJ,
c. the thermal efficiency of the cycle,
d. the mean effective pressure, in kPa (abs), and
e. sketch the process on a *T*-*s* plot, clearly indicating states, paths, and lines of constant specific volume.

SOLUTION:





Note: T_1 = 27 ºC = 300 K and T_3 = 727 ºC = 1000 K.

Assume air is an ideal gas so that,

$$\frac{V_2}{V_1} = \frac{v_r(T_2)}{v_r(T_1)} \Rightarrow v_r(T_2) = v_r(T_1)\frac{V_2}{V_1} \quad (1)$$

where,

V_2/V_1 = 1/10 = 0.1 (given),

$v_r(T_1 = 300\text{ K})$ = 621.2 (using the Ideal Gas Table for air),

⇨ $v_r(T_2)$ = 62.12.

Using the Ideal Gas Table for air,

T_2 = 730 K and u_2 = 536.1 kJ/kg.

In addition, u_1 = 214.1 kJ/kg.

Similarly,

$$\frac{V_4}{V_3} = \frac{v_r(T_4)}{v_r(T_3)} \Rightarrow v_r(T_4) = v_r(T_3)\frac{V_4}{V_3} \quad (2)$$

where,

V_4/V_3 = 10/1 = 10 (given),

$v_r(T_3 = 1000\text{ K})$ = 25.17 (using the Ideal Gas Table for air),

⇨ $v_r(T_4)$ = 251.7.

Using the Ideal Gas Table for air,

T_4 = 430 K and u_4 = 308.0 kJ/kg.

In addition, u_3 = 758.9 kJ/kg.

Apply the 1st Law to the system (i.e., the air), for process 2-3,

$$\Delta E_{sys,23} = Q_{into\ sys,23} - W_{by\ sys,23}, \quad (3)$$

where,

$$\Delta E_{sys,23} = \Delta U_{sys,23} = m(u_3 - u_2) \text{ (Neglecting changes in kinetic and potential energies.)}, \quad (4)$$

$$W_{by\ sys,23} = 0 \text{ (Constant volume process.)} \quad (5)$$

Substitute and simplify,

$$Q_{into\ sys,23} = m(u_3 - u_2). \quad (6)$$

Using the previously determined and given values,

$Q_{into\ sys,23}$ = 1.114 kJ.

Similarly, for process 4-1,

$$\Delta E_{sys,41} = Q_{into\ sys,41} - W_{by\ sys,41}, \quad (7)$$

where,

$$\Delta E_{sys,41} = \Delta U_{sys,41} = m(u_1 - u_4) \text{ (Neglecting changes in kinetic and potential energies.)}, \quad (8)$$

$$W_{by\ sys,41} = 0 \text{ (Constant volume process.)} \quad (9)$$

Substitute and simplify,

$$Q_{into\ sys,41} = m(u_1 - u_4). \quad (10)$$

Using the previously determined and given values,

$Q_{into\ sys,41}$ = -0.4695 kJ. Thus, 0.470 kJ of energy is rejected via heat transfer from the system.

The net work for the cycle may be found by applying the 1st Law to the system over the entire cycle,

$$\Delta E_{sys,cycle} = Q_{into\ sys,cycle} - W_{by\ sys,cycle}, \quad (11)$$

where,

$$\Delta E_{sys,cycle} = 0 \text{ (The net change in properties over a cycle is zero.)} \quad (12)$$

$Q_{into\ sys,cycle} = Q_{into\ sys,23} + Q_{into\ sys,41}$ (No heat is added in processes 1-2 and 3-4.) (13)

Substitute and simplify,

$W_{by\ sys,cycle} = Q_{into\ sys,23} + Q_{into\ sys,41}$. (14)

Using the previously calculated values,

$\boxed{W_{by\ sys,cycle} = 0.645\ \text{kJ}}$.

Alternately, we could have found the net work by applying the 1st Law to the compression and power strokes of the cycle separately,

$m(u_2 - u_1) = -W_{by\ sys,12}$, (15)

$m(u_4 - u_3) = -W_{by\ sys,34}$, (16)

⇨ $W_{by\ sys,12}$ = -1.61 kJ and $W_{by\ sys,34}$ = 2.2545 kJ,

⇨ $W_{by\ sys,cycle} = W_{by\ sys,12} + W_{by\ sys,34}$ = 0.645 kJ, which is the same answer found previously.

The thermal efficiency for the cycle is,

$\eta \equiv \frac{W_{by\ sys,cycle}}{Q_{into\ sys}}$, (17)

Using the previously calculated values,

$\boxed{\eta = 0.578}$.

The mean effective pressure is given by,

$mep \equiv \frac{W_{by\ sys,cycle}}{V_1 - V_2}$ => $mep \equiv \frac{W_{by\ sys,cycle}}{V_1(1 - V_2/V_1)}$ (18)

where,

$V_1 = \frac{mR_{air}T_1}{p_1}$, (19)

with,

R_{air} = 0.287 kJ/(kg.K), p_1 = 100 kPa (abs), and V_2/V_1 = (1/10) = 0.1,

⇨ $V_1 = 4.31*10^{-3}$ m^3 => $\boxed{mep = 166\ \text{kPa (abs)}}$