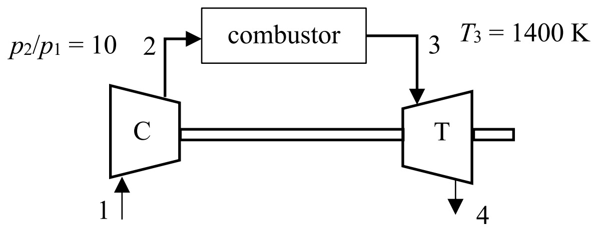
Air enters the compressor of an ideal cold air-standard Brayton cycle at 100 kPa (abs) and 300 K, with a mass flow rate of 6 kg/s. The compressor pressure ratio is 10 and the turbine inlet temperature is 1400 K. For a specific heat ratio of 1.4, calculate:

1. the thermal efficiency of the cycle,
2. the back work ratio, and
3. the net power developed.

SOLUTION:




$p_1 = 100$ kPa (abs)
$T_1 = 300$ K
$\dot{m} = 6$ kg/s

Calculate the thermal efficiency for the Brayton cycle,

$$\eta = 1 - \left(\frac{p_2}{p_1}\right)^{\frac{1-k}{k}}, \tag{1}$$

where,

$k = 1.4$,
$p_2/p_1 = 10$,
⇨ $\boxed{\eta = 0.482}$.

The back work ratio (bwr) is,

$$bwr \equiv \frac{\dot{W}_{into\ comp}}{\dot{W}_{by\ turb}}, \tag{2}$$

where,

$$\dot{W}_{into\ comp} = \dot{m}(h_2 - h_1) = \dot{m}c_p(T_2 - T_1), \tag{3}$$

$$\dot{W}_{by\ turb} = \dot{m}(h_3 - h_4) = \dot{m}c_p(T_3 - T_4), \tag{4}$$

so that Eq. (2) becomes,

$$bwr = \frac{T_2 - T_1}{T_3 - T_4}. \tag{5}$$

Note that Eqs. (3) and (4) were derived by applying the 1st Law to CVs that surround the compressor and turbine, respectively, and assuming steady flow, one inlet and one outlet, adiabatic conditions, and neglecting changes in kinetic and potential energies. In addition, the air is assumed to be a perfect gas (constant specific heats).

The temperature ratios $T_2/T_1$ and $T_4/T_3$ may be found by noting that the flow through the compressor and turbine are assumed to be adiabatic an reversible => isentropic. Since the air is also assumed to be a perfect gas, the temperature and pressure ratios are related by,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}. \tag{6}$$

$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{k-1}{k}}. \tag{7}$$

For the given values,

$p_2/p_1 = 10$, $k = 1.4$ => $T_2/T_1 = 1.9307$ ,
$p_4/p_3 = 1/10$, $k = 1.4$ => $T_4/T_3 = 0.51795$,

(Since the pressure remains constant in the combustor, $p_3 = p_2$. In addition, the pressure at 4 will be the same as the pressure at 1, i.e., $p_4 = p_1$, since both are either open to the atmosphere or are connected via another heat exchanger.)

Given that $T_1 = 300$ K and $T_3 = 1400$ K,

⇨ $T_2 = 579.2$ K,
⇨ $T_4 = 725.1$ K.

Substituting these temperature values into Eq. (5) gives,

$\boxed{bwr = 0.414}$.

The net power developed is,

$$\dot{W}_{by,net} = \dot{W}_{by\ turb} - \dot{W}_{into\ comp} = \dot{W}_{by\ turb}\left(1 - \frac{\dot{W}_{into\ comp}}{\dot{W}_{by\ turb}}\right) = \dot{W}_{by\ turb}(1 - bwr), \quad (8)$$

$$\dot{W}_{by,net} = \dot{m}c_p(T_3 - T_4)(1 - bwr), \quad (9)$$

where Eq. (4) has been used to derive Eq. (9). Using the given values,

$c_p$ = 1.005 kJ/(kg.K), (value for air at 300 K, since it's a cold air-standard analysis)

$\dot{m}$ = 6 kg/s,

⇨ $\boxed{\dot{W}_{by,net} = 2390\ \text{kW}}$.