A closed, piston-cylinder device contains air at an absolute temperature of T_1 = 300 K and specific volume of v_1 = 2 m^3/kg (State 1). The air undergoes a power cycle composed of the following four processes. Data at each state is included in the table below.

Process 1-2: Isothermal compression from v_1 = 2 m^3/kg to v_2 = 0.2 m^3/kg
Process 2-3: Heat transfer at constant volume until T_3 = 1500 K
Process 3-4: Isothermal expansion until v_4 = 2 m^3/kg
Process 4-1: Heat transfer at constant volume back to State 1

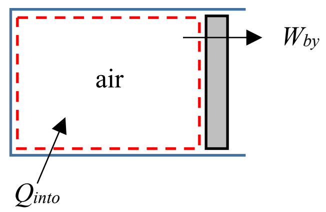
The molecular weight of air: 28.97 kg/kmol.

State	T [K]	v [m^3/kg]	u [kJ/kg]	h [kJ/kg]
1	300	2	214.1	300.1
2	300	0.2	214.1	300.1
3	1500	0.2	1205	1636
4	1500	2	1205	1636

a. Show the cycle on a p-v diagram. Label the axes and four states, show the lines of constant temperature, and indicate the process directions with arrows.
b. Calculate the specific work and specific heat transfer for each of the four processes in the cycle. Report your answers in kJ/kg.
c. Determine thermal efficiency of the cycle. Report your answer in %.

Identify the system, show mass/energy interactions (EFD), list any assumptions and basic equations, and provide your solution. There is no need to re-write the given and find.

SOLUTION:
The system of interest is the air inside the piston.



Determine the pressures using the ideal gas law,

$$p = \frac{R_{air} T}{v}, \tag{1}$$

where,

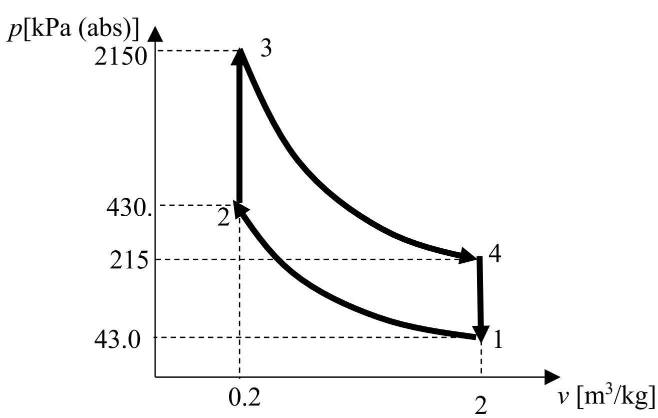
$$R_{air} = \frac{\bar{R}_u}{MW_{air}}. \tag{2}$$

Using the given values,

$\bar{R}_u = 8.314$ kJ/(kmol.K),
$MW_{air} = 28.97$ kg/kmol,
⇨ $R_{air} = 0.2870$ kJ/(kg.K).

$T_1 = 300$ K, $v_1 = 2$ m^3/kg => $p_1 = 43.0$ kPa,
$T_2 = 300$ K, $v_2 = 0.2$ m^3/kg => $p_2 = 430$ kPa,
$T_3 = 1500$ K, $v_3 = 0.2$ m^3/kg => $p_3 = 2150$ kPa,
$T_4 = 1500$ K, $v_4 = 2$ m^3/kg => $p_4 = 215$ kPa.

The cycle is shown in the following *p*-*v* plot.



The specific work, i.e., the work per unit mass, for each process is given by,

$$W_{by} = \int_{V_i}^{V_f} p dV = m \int_{v_i}^{v_f} p dv \Rightarrow w_{by} = W_{by}/m = \int_{v_i}^{v_f} p dv. \tag{3}$$

Thus,

$$w_{by,12} = \int_{v_1}^{v_2} p dv. \tag{4}$$

The specific heat transfer, i.e., the heat transfer per unit mass, for each process is found using the 1st Law,

$$\Delta E_{sys} = Q_{into} - W_{by} \Rightarrow m\Delta e_{sys} = mq_{into} - mw_{by} \Rightarrow \Delta e_{sys} = q_{into} - w_{by}, \quad (5)$$

where,

$$\Delta e_{sys} = \Delta u_{sys} + \Delta ke_{sys} + \Delta pe_{sys} = \Delta u_{sys} \quad \text{(since changes in } KE \text{ and } PE \text{ are assumed negligible).} \quad (6)$$

Thus,

$$q_{into} = \Delta u_{sys} + w_{by}. \quad (7)$$

For Process 1 – 2,

$$w_{by,12} = \int_{v_1}^{v_2} p dv = \int_{v_1}^{v_2} \frac{RT}{v} dv = RT \int_{v_1}^{v_2} \frac{dv}{v} = RT_{12} ln\left(\frac{v_2}{v_1}\right), \quad (8)$$

making use of the ideal gas law and knowing the process is isothermal. Using the given data, $\boxed{w_{by,12} = \text{-198 kJ/kg}}$. Evaluating Eq. (9), again using the given data, $\boxed{q_{into,12} = \text{-198 kJ/kg}}$.

For Process 2 – 3,

$$\boxed{w_{by,23} = \int_{v_2}^{v_3} p dv = 0}, \quad (9)$$

since the process occurs at constant volume. The corresponding heat transfer per unit mass is, $\boxed{q_{into,23} = 991 \text{ kJ/kg}}$.

For Process 3 – 4,

$$w_{by,34} = \int_{v_3}^{v_4} p dv = \int_{v_3}^{v_4} \frac{RT}{v} dv = RT \int_{v_3}^{v_4} \frac{dv}{v} = RT_{34} ln\left(\frac{v_4}{v_3}\right), \quad (10)$$

making use of the ideal gas law and knowing the process is isothermal. Using the given data, $\boxed{w_{by,34} = 991 \text{ kJ/kg}}$. The heat transfer per unit mass is $\boxed{q_{into,34} = 991 \text{ kJ/kg}}$.

For Process 4 – 1,

$$\boxed{w_{by,41} = \int_{v_4}^{v_1} p dv = 0}, \quad (11)$$

since the process occurs at constant volume. The corresponding heat transfer per unit mass is, $\boxed{q_{into,41} = \text{-991 kJ/kg}}$.

The thermal efficiency for the power cycle is,

$$\eta = \frac{W_{by,net,cycle}}{Q_{into,cycle}} = \frac{w_{by,net,cycle}}{q_{into,cycle}}. \quad (12)$$

In this case,

$$w_{by,net,cycle} = w_{by,12} + w_{by,23} + w_{by,34} + w_{by,41} = 793 \text{ kJ/kg} \quad (13)$$

The heat transfer into the cycle (not the net heat transfer, but only the heat that is actually put into the cycle) per unit mass is,

$$q_{into,cycle} = q_{into,23} + q_{into,34} = 1982 \text{ kJ/kg} \quad (14)$$

Thus, the thermal efficiency using Eq. (12) is,

$\eta = 0.400$ or $\boxed{\eta = 40.0\%}$.