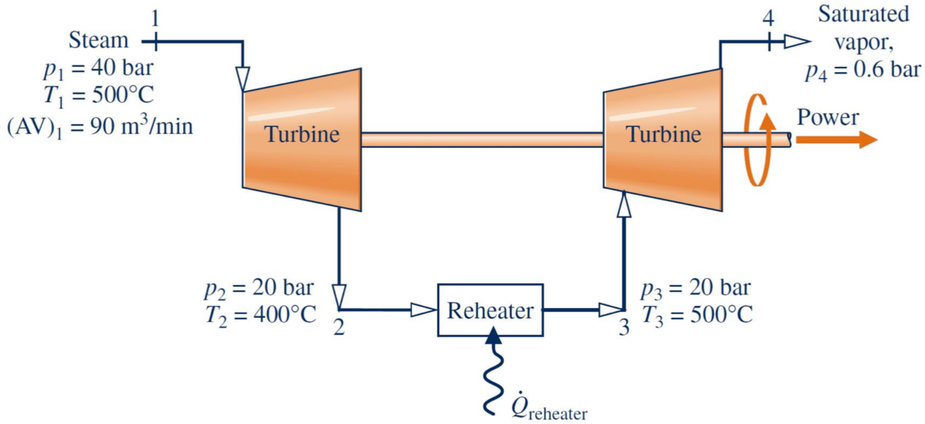
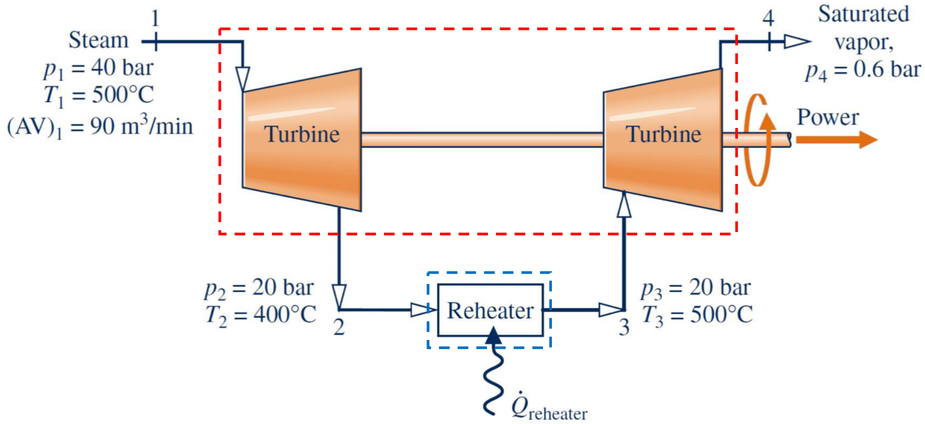
Steam enters the first-stage turbine shown in the figure below at 40 bar (abs) and 500 ◦C with a volumetric flow rate of 90 $m^3$/min. Steam exits the turbine at 20 bar (abs) and 400 ◦C. The steam is then reheated at constant pressure to 500 ◦C before entering the second-stage turbine. Steam leaves the second stage as saturated vapor at 0.6 bar (abs). For operation at steady state, and ignoring stray heat transfer and kinetic and potential energy effects, determine the:

a. mass flow rate of the steam, in kg/h.
b. total power produced by the two stages of the turbine, in kW.
c. rate of heat transfer to the steam flowing through the reheater, in kW



SOLUTION:




First determine the mass flow rate using the conditions at state 1,

$p_1$ = 40 bar (abs), $T_1$ = 500 °C => $v_1$ = 0.08644 m³/kg (from the SHV table for water).

The mass flow rate is,

$\dot{m}_1 = \frac{(AV)_1}{v_1}$ => $\boxed{\dot{m}_1 = 62500 \text{ kg/h}}$, (1)

where $(AV)_1$ = 90 m³/min.

Now apply Conservation of Mass to a control volume surrounding each component separately (not shown in the figure),

$\frac{dM_{CV}}{dt} = \sum_{in} \dot{m} - \sum_{out} \dot{m}$, (2)

where,

$\frac{dM_{CV}}{dt} = 0$ (assume the system operate at steady state). (3)

Since each device has a single inlet and single outlet,

$\dot{m}_{in} = \dot{m}_{out}$. (4)

Thus,

$\dot{m}_1 = \dot{m}_2 = \dot{m}_3 = \dot{m}_4 = \dot{m}$. (5)

The mass flow rate of the steam is the same throughout the entire system.

Now apply the First Law to the red control volume surrounding both turbines as shown in the previous figure,

$\frac{dE_{CV}}{dt} = \dot{Q}_{into} - \dot{W}_{by} + \sum_{in} \dot{m}\left(h + \frac{1}{2}V^2 + gz\right) - \sum_{out} \dot{m}\left(h + \frac{1}{2}V^2 + gz\right)$, (6)

where,

$\frac{dE_{CV}}{dt} = 0$ (assume steady state operation), (7)

$\dot{Q}_{into} = 0$ (assume adiabatic operation), (8)

$\dot{W}_{by} = ?$ (This is the parameter we're trying to find.), (9)

$\sum_{in} \dot{m}\left(h + \frac{1}{2}V^2 + gz\right) = \dot{m}h_1 + \dot{m}h_3$ (assume the contributions of *ke* and *pe* are small compared to *h*), (10)

$\sum_{out} \dot{m}\left(h + \frac{1}{2}V^2 + gz\right) = \dot{m}h_2 + \dot{m}h_4$ (assume the contributions of *ke* and *pe* are small compared to *h*). (11)

Substitute and solve for the power,

$\dot{W}_{by} = \dot{m}(h_1 + h_3 - h_2 - h_4)$. (12)

Use the property tables for water to determine the specific enthalpies at the various states,

$p_1$ = 40 bar (abs), $T_1$ = 500 °C => $h_1$ = 3446.0 kJ/kg (from the SHV table for water),

$p_2$ = 20 bar (abs), $T_2$ = 400 °C => $h_2$ = 3248.3 kJ/kg (from the SHV table for water),

$p_3$ = 20 bar (abs), $T_3$ = 500 °C => $h_3$ = 3468.2 kJ/kg (from the SHV table for water),

$p_4$ = 0.6 bar (abs), sat. vapor => $h_4$ = 2652.9 kJ/kg (from the SLVM table for water).

Using the previously determined values,

$\boxed{\dot{W}_{by} = 17.6\text{ kW}}$.

To determine the heat transfer into the reheater, apply the First Law to the blue control volume shown in the previous figure,

$$\frac{dE_{CV}}{dt} = \dot{Q}_{into} - \dot{W}_{by} + \sum_{in} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) - \sum_{out} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right), \quad (13)$$

where,

$\frac{dE_{CV}}{dt} = 0$ (assume steady state operation), (14)

$\dot{Q}_{into} = ?$ (This is the parameter we're trying to find.), (15)

$\dot{W}_{by} = 0$ (A reheater is a passive device.), (16)

$\sum_{in} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) = \dot{m}h_2$ (assume the contributions of *ke* and *pe* are small compared to *h*), (17)

$\sum_{out} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) = \dot{m}h_3$ (assume the contributions of *ke* and *pe* are small compared to *h*). (18)

Substitute and solve for the heat transfer,

$$\dot{Q}_{into} = \dot{m}(h_3 - h_2). \quad (19)$$

Using the previously determined values,

$\boxed{\dot{Q}_{into} = 3.82\text{ kW}}$.

The processes are sketched in the following *p*-*v* plot.