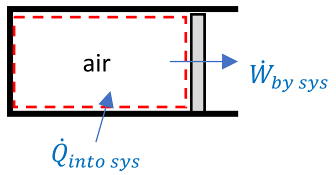
One kilogram of air in a piston-cylinder assembly undergoes a thermodynamic cycle consisting of three processes:

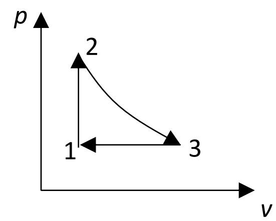
- Process 1-2: Constant specific volume
- Process 2-3: Constant-temperature expansion
- Process 3-1: Constant-pressure compression

At state 1, the temperature is 323 K, and the pressure is 1 bar (abs). At state 2, the pressure is 2 bar (abs). Employing the ideal gas equation of state,

a. Sketch the cycle on *p*-*v* coordinates.
b. Determine the temperature at state 2, in K.
c. Determine the specific volume at state 2, in m^3/kg.
d. Determine the heat transfer into the air per unit mass of air (i.e., the specific heat transfer) during process 1-2.
e. What is the change in the internal energy of the air over the entire cycle?

SOLUTION:





Use the Ideal Gas Law to determine the temperature at state 2,

$$pv = RT \;\; => \;\; T_2 = T_1\left(\frac{p_2}{p_1}\right), \tag{1}$$

since R is a constant and v is a constant in process 1-2. Using the given values,

T_1 = 323 K, p_2 = 2 bar (abs), and p_1 = 1 bar (abs) => $\boxed{T_2 = 646\text{ K}}$.

The specific volume may be found by applying the Ideal Gas Law,

$$pv = RT \;\; => \;\; v_2 = \frac{RT_2}{p_2}, \tag{2}$$

Using the given values of R = 0.287 kJ/(kg.K), T_2 = 646 K, and p_2 = 2 bar (abs) = 200 kPa (abs) => $\boxed{v_2 = 0.927\text{ m}^3/\text{kg}}$.

The heat transfer into the air per unit mass of air in process 1-2 is found using the First Law applied to the air,

$$\Delta e_{sys,12} = q_{into\ sys,12} - w_{by\ sys,12}, \tag{3}$$

where,

$$\Delta e_{sys,12} = \Delta u_{sys,12} + \Delta ke_{sys,12} + \Delta pe_{sys,12} = \Delta u_{sys,12} = u_2 - u_1 \text{ (neglecting changes in } ke \text{ and } pe\text{)}, \tag{4}$$

$$w_{by\ sys,12} = 0 \text{ (since the specific volume remains constant in process 1-2)}. \tag{5}$$

Thus,

$$q_{into\ sys,12} = u_2 - u_1. \tag{6}$$

Using the Ideal Gas Table for air (with interpolation),

$u(T_2 = 646\text{ K})$ = 470.12 kJ/kg,

$u(T_1 = 323\text{ K})$ = 230.56 kJ/kg,

=> $\boxed{q_{into\ sys,12} = 240\text{ kJ/kg}}$.

Temp. [K]	h [kJ/kg]	u [kJ/kg]
630	638.8	457.8
640	649.4	465.5
650	660.0	473.2

Temp. [K]	h [kJ/kg]	u [kJ/kg]
310	310.2	221.2
315	315.2	224.8
320	320.2	228.4
325	325.3	232.0

The internal energy of the air <u>over the cycle</u> (1-2-3-1) will be zero since the processes return to the same initial state.

Following is a Python code for the calculations.

```
# COE_57.py

import numpy as np
# Import a class created for reading the Ideal Gas Tables.
from IdealGasTable import IdealGasTable

# Import the Ideal Gas Table data for diatomic oxygen.
IGT = IdealGasTable("./IdealGasTables.xlsx", "Air")

R_air = 0.287  # kJ/(kg.K), given
T_1 = 323  # K, initial temperature, given
p_1 = 1*100  # kPa, initial pressure, given
p_2 = 2*100  # kPa, final pressure, given

T_2 = T_1*(p_2/p_1)  # calculate final temperature
print("T_2 = %.2f K" % T_2)

v_2 = R_air*T_2/p_2  # calculate final specific volume
print("v_2 = %.4f m^3/kg" % v_2)

u_1 = IGT.GetTableValue('u', T=T_1)  # kJ/kg, initial specific internal energy
u_2 = IGT.GetTableValue('u', T=T_2)  # kJ/kg, final specific internal energy
print("(u_2, u_1) = (%.2f, %.2f) kJ/kg" % (u_2, u_1))

q_in = u_2 - u_1  # calculate the specific heat transfer
print("q_in = %.1f kJ/kg" % q_in)
```

```
≫ python3 ./COE_57.py
T_2 = 646.00 K
v_2 = 0.9270 m^3/kg
(u_2, u_1) = (470.12, 230.56) kJ/kg
q_in = 239.6 kJ/kg
```