An air compressor operates at steady-state with air entering at $p_{1}=15 \mathrm{psfa}$ and $T_{1}=60^{\circ} \mathrm{F}$. The air undergoes a polytropic process and exits at $p_{2}=75 \mathrm{psfa}$ and $T_{2}=294{ }^{\circ} \mathrm{F}$.
a. Evaluate the exponent for the polytropic process.
b. Sketch the process on a $p-v$ diagram.

SOLUTION:


Since the process is polytropic,

$$
\begin{equation*}
p_{1} v_{1}^{n}=c=p_{2} v_{2}^{n} \Rightarrow \frac{p_{2}}{p_{1}}=\left(\frac{v_{1}}{v_{2}}\right)^{n} \tag{1}
\end{equation*}
$$

where $c$ is a constant and $n$ is currently unknown. With the air treated as an ideal gas,

$$
\begin{equation*}
p v=R T \Rightarrow \frac{v_{1}}{v_{2}}=\frac{T_{1} p_{2}}{T_{2} p_{1}} \tag{2}
\end{equation*}
$$

and combining with Eq. (1),

$$
\begin{equation*}
\frac{p_{2}}{p_{1}}=\left(\frac{T_{1} p_{2}}{T_{2} p_{1}}\right)^{n} \Rightarrow \ln \left(\frac{p_{2}}{p_{1}}\right)=n \ln \left(\frac{T_{1} p_{2}}{T_{2} p_{1}}\right) \Rightarrow n=1.30, \tag{3}
\end{equation*}
$$

using the given values of,
$p_{1}=15 \mathrm{psfa}, p_{2}=75 \mathrm{psfa}, T_{1}=60^{\circ} \mathrm{F}=519.67^{\circ} \mathrm{R}, T_{2}=294^{\circ} \mathrm{F}=753.67^{\circ} \mathrm{R}$.
The specific volumes at the inlet and outlet may be found using Eq. (2) with $R_{\text {air }}=53.3533 \mathrm{ft} \cdot \mathrm{Ib}_{\mathrm{f}} /\left(\mathrm{lb}_{\mathrm{m}} .{ }^{\circ} \mathrm{R}\right)$, $v_{1}=1850 \mathrm{ft}^{3} / \mathrm{lb}_{\mathrm{m}}$ and $v_{2}=536 \mathrm{ft}^{3} / \mathrm{lb}_{\mathrm{m}}$.


