A closed, rigid tank fitted with a fine-wire electric resistor is filled with Refrigerant 22 , initially at $-10^{\circ} \mathrm{C}$, a quality of $80 \%$, and a volume of $0.01 \mathrm{~m}^{3}$. A 12 V battery provides a 5 A current to the resistor for 5 min . If the final temperature of the refrigerant is $40^{\circ} \mathrm{C}$, determine the heat transfer, in kJ , from the refrigerant.


## SOLUTION:

The heat transferred from the refrigerant to the surroundings may be found using the First Law applied to the refrigerant (our system),

$$
\begin{equation*}
\Delta E_{R 22}=Q_{\substack{\text { into } \\ R 22}}+\underset{\substack{\text { on } \\ R 22}}{W_{R 22}} \Rightarrow \underset{\substack{\text { into } \\ R 22}}{ }=\Delta E_{R 22}-W_{\text {on }}, \tag{1}
\end{equation*}
$$

where,

$$
\begin{equation*}
\Delta E_{R 22}=\Delta U_{R 22}=U_{2}-U_{1}=m\left(u_{2}-u_{1}\right) \tag{2}
\end{equation*}
$$

assuming that other forms of energy change, e.g., kinetic and potential, are negligible. Note that since the container is closed,

the initial and final refrigerant masses will be the same.
Furthermore, the resistor wire is not considered to be part of the system.

The specific internal energy at state 1 is also found using the thermodynamic property tables,

$$
\begin{equation*}
u_{1}=x u_{v}+(1-x) u_{l} \tag{3}
\end{equation*}
$$

where, at $-10^{\circ} \mathrm{C}$ in the saturated liquid-vapor phase,

$$
\begin{aligned}
x & =0.80, \\
u_{v} & =223.02 \mathrm{~kJ} / \mathrm{kg}, \\
u_{l} & =33.27 \mathrm{~kJ} / \mathrm{kg}, \\
\Rightarrow & u_{1}=185.07 \mathrm{~kJ} / \mathrm{kg} .
\end{aligned}
$$

The specific volume at state 1 may be found in a similar manner,

$$
\begin{equation*}
v_{1}=x v_{v}+(1-x) v_{l}, \tag{4}
\end{equation*}
$$

where,

$$
\begin{aligned}
x & =0.80, \\
v_{v} & =0.0652 \mathrm{~m}^{3} / \mathrm{kg}, \\
v_{l} & =0.7606^{*} 10^{-3} \mathrm{~m}^{3} / \mathrm{kg}, \\
\Rightarrow & v_{1}=0.0523 \mathrm{~m}^{3} / \mathrm{kg} .
\end{aligned}
$$

The mass of the refrigerant may be found from the initial state,

$$
\begin{equation*}
m=\frac{V}{v_{1}}, \text { (The electrical wire volume is assumed negligible compared to the tank volume.) } \tag{5}
\end{equation*}
$$

where,

$$
\begin{aligned}
& V=0.01 \mathrm{~m}^{3} \\
& \Rightarrow m=0.191 \mathrm{~kg} .
\end{aligned}
$$

The specific internal energy at state 2 (after the 5 min ) is found using the thermodynamic property tables for Refrigerant 22 at a temperature of $40^{\circ} \mathrm{C}$ and a specific volume of, $v_{2}=v_{1}$ (since the container volume and refrigerant mass remain constant).

Using the two-phase liquid-vapor thermodynamic table, observe that at the final temperature of $T_{2}=40^{\circ} \mathrm{C}$, the saturated vapor specific volume is $0.0151 \mathrm{~m}^{3} / \mathrm{kg}$, which is smaller than the specific volume at state $2, v_{2}=0.0523$ $\mathrm{m}^{3} / \mathrm{kg}$. Hence, the refrigerant must be in a superheated vapor phase. Interpolating from the superheated vapor table using $T_{2}$ and $v_{2}$,

$$
u_{2}=250.33 \mathrm{~kJ} / \mathrm{kg} .
$$

Combining $m, u_{2}$, and $u_{1}$, Eq. (2) becomes,

$$
\Delta U=12.46 \mathrm{~kJ} / \mathrm{kg} .
$$

There is no work acting on the refrigerant since the container volume remains constant and because the electrical work goes into the wire, which is not part of the system,

$$
\begin{equation*}
\underset{\substack{\text { on } 22}}{ }=0 \tag{7}
\end{equation*}
$$

There is, however, heat that is transferred from the wire into the system. This heat may be found by applying the $1^{\text {st }}$ Law to the wire. Assuming steady conditions so that the change in total energy of the wire is zero, the total heat from the wire will equal the total (electrical) work done on the wire,

$$
\begin{equation*}
\underbrace{\Delta E_{\text {wiready }}}_{=0}=-\underset{\text { wire }}{Q_{\text {from }}}+\underset{\text { wire }}{W_{\text {on }}} \Rightarrow \underset{\text { wire }}{Q_{\text {from }}}=W_{\text {on }}, \tag{8}
\end{equation*}
$$

where the total work done on the wire is,

$$
\begin{equation*}
\underset{\substack{\text { on } \\ \text { wire }}}{ }=V I \Delta t \text { (assuming that neither the voltage nor current change over time } \Delta t \text { ), } \tag{9}
\end{equation*}
$$

with,

$$
\begin{aligned}
& V=12 \mathrm{~V} \\
& I=5 \mathrm{~A} \\
& \Delta t=5 \mathrm{~min}=300 \mathrm{~s}, \\
& \Rightarrow W_{\text {on wire }}=18 \mathrm{~kJ} \Rightarrow Q_{\text {from wire }}=18 \mathrm{~kJ} .
\end{aligned}
$$

Break the heat into the refrigerant into two heat components, one from the wire and one from the remainder of the surroundings,

$$
\begin{equation*}
Q_{\text {into R22 }}=Q_{\substack{\text { into R22, } \\ \text { from wire }}}+Q_{\substack{\text { into R22, } \\ \text { from elsewhere }}} \tag{10}
\end{equation*}
$$

Substituting the expressions for heat, work, and energy into Eq. (1),

$$
\begin{align*}
& \underset{\substack{\text { into R22, }_{\text {from elsewhere }}}}{ }=\Delta U-Q_{\substack{\text { into R22, } \\
\text { from wire }}},  \tag{11}\\
& \Rightarrow Q_{\substack{\text { into R22, } \\
\text { from elsewhere }}}=-5.54 \mathrm{kJ.}
\end{align*}
$$

Since we're interested in the heat from the refrigerant,

$$
\begin{equation*}
Q_{\substack{\text { from R22, } \\ \text { into elsewhere }}}=-Q_{\substack{\text { into R22, } \\ \text { from elsewhere }}}=5.54 \mathrm{~kJ} \tag{12}
\end{equation*}
$$

The process and states are shown schematically in the following $T-v$ plot.


SLVM Table for R22 (from Moran et al., $8^{\text {th }}$ ed., Wiley).
Tables in SI Units


SHV Table for R22 (from Moran et al., $8^{\text {th }}$ ed., Wiley)


SHV Table for R22 (from Moran et al., $8^{\text {th }}$ ed., Wiley)


