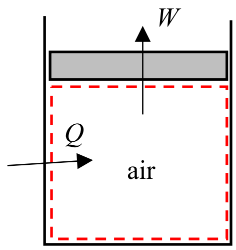
A piston-cylinder assembly contains air modeled as an ideal gas. The air undergoes a power cycle consisting of four processes in series:

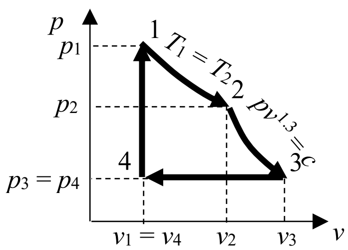
- Process 1 – 2: Constant-temperature expansion at 600 K from p_1 = 0.5 MPa (abs) to p_2 = 0.4 MPa (abs).
- Process 2 – 3: Polytropic expansion with n = 1.3 to p_3 = 0.3 MPa (abs).
- Process 3 – 4: Constant-pressure compression to $v_4 = v_1$.
- Process 4 – 1: Constant-volume heating.

1. Sketch the cycle on a *p*-*v* diagram.
2. Calculate the work and heat transfer for each process, in kJ/kg.

SOLUTION:



First, sketch the cycle on a p-v diagram.



To find the work and heat transfer, apply the 1st Law to every process and make use of the ideal gas relations.

Process 1 – 2:

$$\Delta E_{sys} = Q_{into\ sys} - W_{by\ sys}, \quad (1)$$

where

$$\Delta E_{sys} = \Delta U_{sys} + \Delta KE_{sys} + \Delta PE_{sys} = \Delta U_{sys} = m\Delta u_{sys} = m(u_2 - u_1), \quad (2)$$

(Neglecting changes in KE and PE.)

Since for an ideal gas, $u = u(T)$, and in Process 1 – 2 the temperature remains constant, we must have $u_2 = u_1$. Thus,

$$Q_{into\ sys} = W_{by\ sys}. \quad (3)$$

The work done by the system is boundary (or pressure) work,

$$W_{by\ sys} = \int_{V_1}^{V_2} p dV = m\int_{v_1}^{v_2} p dv \implies \frac{W_{by\ sys}}{m} = \int_{v_1}^{v_2} p dv \quad (4)$$

Since the air is treated as an ideal gas, we can use the ideal gas law to relate pressure and specific volume,

$$pv = RT, \quad (5)$$

where, for air, $R_{air} = 0.287$ kJ/(kg.K). For process 1 – 2, $T_1 = T_2 = 600$ K, so that Eq. (5) becomes,

$$p = \frac{RT}{v}, \quad (6)$$

where $T = 600$ K. Substituting Eq. (6) into Eq. (4) and evaluating gives,

$$\frac{W_{by\ sys}}{m} = \int_{v_1}^{v_2} \frac{RT}{v} dv = RT\int_{v_1}^{v_2} \frac{dv}{v} = RT\ln\left(\frac{v_2}{v_1}\right). \quad (7)$$

Using the given values,

$p_1 = 0.5$ MPa (abs),
$p_2 = 0.4$ MPa (abs),
$T = 600$ K,
$R_{air} = 0.287$ kJ/(kg.K),
⇨ $RT = 172.2$ kJ/kg,
⇨ $v_1 = 0.3444\ \text{m}^3/\text{kg}$, $v_2 = 0.4310\ \text{m}^3/\text{kg}$ (using Eq. (5)),
⇨ $\boxed{\frac{W_{by\ sys}}{m} = 38.6\ \text{kJ/kg}}$ (using Eq. (7)),
⇨ $\boxed{\frac{Q_{into\ sys}}{m} = 38.6\ \text{kJ/kg}}$ (using Eq. (3)),

Process 2 – 3:

A similar approach can be used to determine the work and heat for this process.

$$m(u_3 - u_2) = Q_{into\ sys} - W_{by\ sys} \Rightarrow u_3 - u_2 = \frac{Q_{into\ sys}}{m} - \frac{W_{by\ sys}}{m}. \tag{8}$$

We're given that this process is polytropic,

$$pv^n = c, \tag{9}$$

where the constant c may be found from state 2, i.e., $c = p_2 v_2^n$. Using the given pressure at state 3, $p_3 = 0.3$ MPa (abs), the specific volume at state 3 is,

$$p_3 v_3^n = p_2 v_2^n \Rightarrow v_3 = v_2 \left(\frac{p_2}{p_3}\right)^{1/n} \Rightarrow \underline{v_3 = 0.5378 \text{ m}^3/\text{kg}}. \tag{10}$$

where $v_2 = 0.4310$ m^3/kg (found previously), $p_2 = 0.4$ MPa (abs), $p_3 = 0.3$ MPa (abs), and $n = 1.3$. The temperature at state 3 may be found using the ideal gas law,

$$pv = RT \Rightarrow T = \frac{pv}{R}. \tag{11}$$

Using the values for state 3 and $R = 0.287$ kJ/(kg.K), $\underline{T_3 = 562.1 \text{ K}}$.

The boundary (pressure) work for this case is,

$$\frac{W_{by\ sys}}{m} = \int_{v_2}^{v_3} p\,dv = \int_{v_2}^{v_3} \frac{p_2 v_2^n}{v^n} dv = p_2 v_2^n \int_{v_2}^{v_3} \frac{dv}{v^n} = \frac{p_2 v_2^n}{1-n}\left(v_3^{1-n} - v_2^{1-n}\right). \tag{12}$$

Using the given values,

$p_2 = 0.4$ MPa (abs),

$v_2 = 0.4310$ m^3/kg (found previously),

$n = 1.3$,

$v_3 = 0.5378$ m^3/kg (found previously),

⇨ $\boxed{\frac{W_{by\ sys}}{m} = 36.9 \text{ kJ/kg}}$.

To find the heat transfer per unit mass, re-arrange the 1st Law expression given in Eq. (8),

$$\frac{Q_{into\ sys}}{m} = u_3 - u_2 + \frac{W_{by\ sys}}{m}. \tag{13}$$

The change in specific internal energy may be found using the ideal gas table for air with $T_2 = 600$ K (given) and $T_3 = 562.1$ K (found previously),

$u_2 = 434.8$ kJ/kg,

$u_3 = 406.0$ kJ/kg (using linear interpolation).

Thus, the heat transfer per unit mass is,

⇨ $\boxed{\frac{Q_{into\ sys}}{m} = 8.10 \text{ kJ/kg}}$.

Process 3 – 4:

The 1st Law for this process, after simplification and re-arranging, will look similar to Eq. (13),

$$\frac{Q_{into\ sys}}{m} = u_4 - u_3 + \frac{W_{by\ sys}}{m}. \tag{14}$$

The boundary work for this case is,

$$\frac{W_{by\ sys}}{m} = \int_{v_3}^{v_4} p\,dv = p_3 \int_{v_3}^{v_4} dv = p_3(v_4 - v_3) \quad \text{(since } p_3 = p_4\text{)}. \tag{15}$$

Using the given and previously calculated values,

$p_3 = 0.3$ MPa (abs),

$v_3 = 0.5378$ m^3/kg,

$v_4 = v_1 = 0.3444$ m^3/kg,

⇨ $\boxed{\frac{W_{by\ sys}}{m} = -58.0 \text{ kJ/kg}}$. (This is a compression process so work is done on the air.)

The temperature at state 4 is found using the ideal gas law (Eq. (11): T_4 = 360.0 K. The specific internal energy at state 4 is found using the ideal gas table for air: u_4 = 257.2 kJ/kg. Thus, from Eq. (14),

$$\boxed{\frac{Q_{into\ sys}}{m} = \text{-207 kJ/kg}}$$

Process 4 – 1:
Again, the 1st Law will simplify to,

$$\frac{Q_{into\ sys}}{m} = u_1 - u_4 + \frac{W_{by\ sys}}{m}\,. \tag{16}$$

For this case, since the specific volume remains constant,

$$\boxed{\frac{W_{by\ sys}}{m} = 0}.$$

The specific internal energy at state 1 (T_1 = 600 K) is u_1 = 434.8 kJ/kg (from the ideal gas table for air). Using this value and the previously determine value for u_4,

$$\boxed{\frac{Q_{into\ sys}}{m} = \text{178 kJ/kg}}$$

Summarizing,

Process	**($W_{by\ sys}/m$) [kJ/kg]**	**($Q_{into\ sys}/m$) [kJ/kg]**
1 – 2	38.63	38.63
2 – 3	36.91	8.096
3 – 4	-58.01	-206.8
4 – 1	0	177.6
Over whole cycle	17.53	17.53

The work and heat transfer over the whole cycle should be equal (and they are, to within numerical error) using the 1st Law since the beginning and end states are the same over a cycle and, thus, the change in internal energy should be zero, i.e.,

$$\Delta U_{sys,cycle} = 0 = Q_{into\ sys,cycle} - W_{by\ sys,cycle} \;\Rightarrow\; Q_{into\ sys,cycle} = W_{by\ sys,cycle}.$$