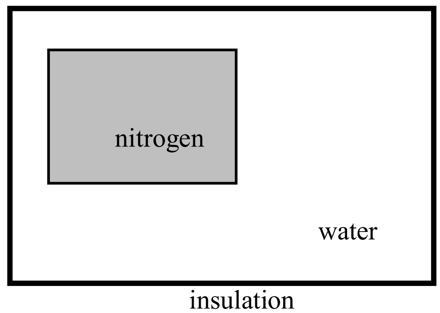
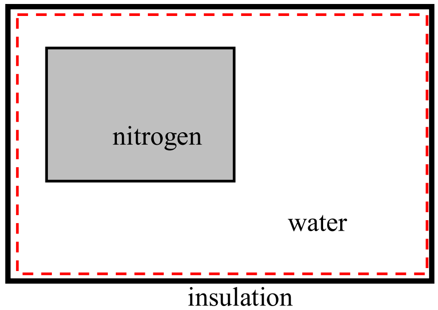
Ten kilograms of nitrogen (N_2) gas is contained in a closed, rigid tank surrounded by a 2.00 kg water bath. The initial temperature of the nitrogen and water are 50.0 °C and 20.0 °C, respectively. The entire unit is well insulated and the nitrogen and water interact until thermal equilibrium is achieved. The measured final temperature is 34.1 °C. The water can be modeled as an incompressible substance with a specific heat of 4.179 kJ/(kg.K) and the nitrogen is an ideal gas with constant specific heats. Determine the average value of nitrogen's specific heat at constant volume.



SOLUTION:
Apply the 1st Law to a system consisting of the nitrogen and water.



$$\Delta E_{sys} = Q_{into\ sys} - W_{by\ sys}, \tag{1}$$

where,

$$\Delta E_{sys} = \Delta U_{sys} + \Delta KE_{sys} + \Delta PE_{sys} = \Delta U_{sys}, \text{ (neglecting changes in KE and PE)}, \tag{2}$$

$$Q_{into\ sys} = 0 \quad \text{(since the tank is well insulated)}, \tag{3}$$

$$W_{by\ sys} = 0 \quad \text{(since the tank is rigid and there are no other sources of work)}. \tag{4}$$

Thus,

$$\Delta U_{sys} = 0. \tag{5}$$

The change in the system's internal energy may be written as,

$$\Delta U_{sys} = \Delta U_{N_2} + \Delta U_{H_2O} = \left(U_{f,N_2} - U_{i,N_2}\right) - \left(U_{f,H_2O} - U_{i,H_2O}\right), \tag{6}$$

where,

$$U_{f,N_2} - U_{i,N_2} = m_{N_2} c_{v,N_2}\left(T_{f,N_2} - T_{i,N_2}\right) \text{ (assuming perfect gas behavior)}, \tag{7}$$

$$U_{f,H_2O} - U_{i,H_2O} = m_{H_2O} c_{H_2O}\left(T_{f,H_2O} - T_{i,H_2O}\right) \text{ (assuming incompressible and constant specific heat)}. \tag{8}$$

Combining Eqs. (5) – (8) and also noting that at equilibrium,

$$T_{f,N_2} = T_{f,H_2O} = T_f, \tag{9}$$

we obtain,

$$m_{N_2} c_{v,N_2}\left(T_f - T_{i,N_2}\right) + m_{H_2O} c_{H_2O}\left(T_f - T_{i,H_2O}\right) = 0, \tag{10}$$

$$\boxed{c_{v,N_2} = -\frac{m_{H_2O} c_{H_2O}\left(T_f - T_{i,H_2O}\right)}{m_{N_2}\left(T_f - T_{i,N_2}\right)}}. \tag{11}$$

Using the given values,

$m_{H2O} = 2.00$ kg,
$m_{N2} = 10.0$ kg,
$c_{H2O} = 4.179$ kJ/(kg.K),
$T_f = 34.1$ °C,
$T_{i,H2O} = 20$ °C,
$T_{i,N2} = 50$ °C,
⇨ $\boxed{c_{v,N2} = 0.741 \text{ kJ/(kg.K)}}$.