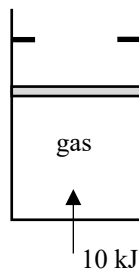


A gas contained within the piston/cylinder assembly shown in the figure is subject to the following processes:

1. Ten kilojoules of energy is added to the gas via heat transfer while the piston slowly moves upward until it just hits the stops in the cylinder. During this process, the gas pressure, which is initially at a pressure of 1 bar (abs) and volume of  $1 \text{ m}^3$ , varies as,  
$$p = c/V^{1.3},$$
where  $c$  is a constant and  $V$  is the gas volume. The final gas volume is  $2 \text{ m}^3$ .
2. After hitting the stops, an additional 10 kJ of energy is added via heat transfer. During this process the absolute gas pressure increases to 5 bar (abs).



1. Determine the gas pressure at the end of process #1.
2. Determine the work done by the gas on the piston during process #1.
3. Determine the change in the gas's internal energy during process #1.
4. Determine the work done by the gas on the piston during process #2.
5. Determine the change in the gas's internal energy during process #2.

SOLUTION:

Let the system be the gas contained within the cylinder. The work done by the gas is,

$$W_{by\ gas} = \int_{V_1}^{V_2} p dV. \quad (1)$$

Making use of the pressure relationship given in the problem statement,

$$W_{by\ gas} = \int_{V_1}^{V_2} c V^{-1.3} dV, \quad (2)$$

$$W_{by\ gas} = c \int_{V_1}^{V_2} V^{-1.3} dV, \quad (3)$$

$$W_{by\ gas} = \frac{c}{-0.3} (V^{-0.3} |_{V_1}^{V_2}), \quad (4)$$

$$W_{by\ gas} = \frac{c}{-0.3} (V_2^{-0.3} - V_1^{-0.3}). \quad (5)$$

The constant  $c$  may be found from the initial conditions,

$$p = c/V^{1.3} \Rightarrow c = pV^{1.3} = p_1 V_1^{1.3}. \quad (6)$$

Using the given data,

$$p_1 = 1\ \text{bar (abs)} = 100\ \text{kPa (abs)},$$

$$V_1 = 1\ \text{m}^3,$$

$$\Rightarrow c = 100\ (\text{kPa}\cdot\text{m}^3)^{1.3}$$

$$V_2 = 2\ \text{m}^3,$$

$$\Rightarrow \boxed{p_2 = 40.6\ \text{kPa (abs)}},$$

$$\Rightarrow \boxed{W_{by\ gas} = 62.6\ \text{kJ}}.$$

The change in the gas's internal energy may be found from the 1<sup>st</sup> Law,

$$\Delta E_{gas} = \Delta U_{gas} + \Delta KE_{gas} + \Delta PE_{gas} = Q_{added\ to\ gas} - W_{by\ gas}, \quad (7)$$

where  $\Delta KE_{gas} = 0$  (quasi-steady process) and  $\Delta PE_{gas} \approx 0$  (small elevation change of a gas). Using the given data and the work value calculated previously,

$$Q_{added\ to\ gas} = 10\ \text{kJ},$$

$$W_{by\ gas} = 62.6\ \text{kJ},$$

$$\Rightarrow \boxed{\Delta U_{gas} = -52.6\ \text{kJ}}.$$

After hitting the cylinder stops, the gas volume no longer changes. Thus, there is no  $p dV$  work,

$$\Rightarrow \boxed{W_{by\ gas} = 0}.$$

The change in the gas's internal energy is again found from the 1<sup>st</sup> Law,

$$\Delta E_{gas} = \Delta U_{gas} + \Delta KE_{gas} + \Delta PE_{gas} = Q_{added\ to\ gas} - W_{by\ gas}, \quad (7)$$

where  $\Delta KE_{gas} = 0$  (quasi-steady process) and  $\Delta PE_{gas} = 0$  (no elevation change). Using the given data and the work value calculated previously,

$$Q_{added\ to\ gas} = 10\ \text{kJ},$$

$$W_{by\ gas} = 0,$$

$$\Rightarrow \boxed{\Delta U_{gas} = 10\ \text{kJ}}.$$