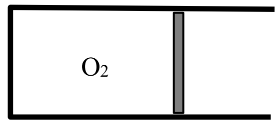
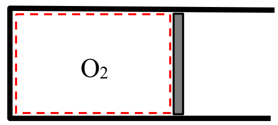
Two kilograms of oxygen gas fills the cylinder of a piston-cylinder assembly. The initial volume and pressure are 2 m^3 and 1 bar (abs), respectively. Heat transfer to the oxygen occurs at a constant pressure until the volume is doubled. Determine the heat transfer for the process, in kJ. Kinetic and potential energy effects can be ignored.



SOLUTION:
Apply the First Law of Thermodynamics to the oxygen, which is the system of interest as shown in the figure below.



$$\Delta E_{\text{sys}} = Q_{\substack{\text{into}\\ \text{sys}}} + W_{\substack{\text{on}\\ \text{sys}}} \, , \tag{1}$$

where,

$$\Delta E_{\text{sys}} = \Delta U_{\text{sys}} + \underbrace{\Delta KE_{\text{sys}} + \Delta PE_{\text{sys}}}_{=0,\text{ assume negligible}} \, , \tag{2}$$

and,

$$W_{\text{by } O_2} = \int_{V_1}^{V_2} p\,dV \overset{p=\text{const.}}{=} p\left(V_2 - V_1\right). \tag{3}$$

Substitute and simplify,

$$\Delta U = Q_{\text{into}} - p\left(V_2 - V_1\right), \tag{4}$$

$$m\left(u_2 - u_1\right) = Q_{\text{into}} - p\left(V_2 - V_1\right)$$

where $W_{\text{by O2}} = -W_{\text{on O2}}$ and $\Delta U = m\Delta u$. The subscript "sys" has also been dropped for convenience. Re-arrange Eq. (4) to solve for the energy added to the system via heat,

$$Q_{\text{into}} = m\left(u_2 - u_1\right) + p\left(V_2 - V_1\right) = m\left(u_2 - u_1\right) + mp\left(v_2 - v_1\right), \tag{5}$$

$$Q_{\text{into}} = m\left[\left(u_2 + pv_2\right) - \left(u_1 + pv_1\right)\right], \tag{6}$$

$$\underline{\therefore Q_{\text{into}} = m\left(h_2 - h_1\right).} \tag{7}$$

The change in specific enthalpy may be found from the Ideal Gas Tables for oxygen. However, in order to use these tables, we must first calculate the temperatures at the initial and final states since for an ideal gas the specific enthalpy is a function of temperature. We can find these temperatures using the Ideal Gas Law,

$$T = \frac{pV}{mR} \, , \tag{8}$$

Using the given parameters,

p = 1 bar (abs) = 100000 Pa (abs),
V_1 = 2 m^3 and V_2 = 4 m^3,
m = 2 kg,
R_{O2} = 0.25983 kJ/(kg.K) (found from a gas properties table),
=> T_1 = 384.87 K and T_2 = 769.74 K.

Using the Ideal Gas Tables for O_2, interpolating, and noting that the molecular mass of O_2 is 31.998 kg/kmol,

$h_1 = h(T_1)$ = 11260.07 kJ/kmol = 351.90 kJ/kg,
$h_2 = h(T_2)$ = 23507.22 kJ/kmol = 734.65 kJ/kg.

Substituting into Eq. (7) and solving for the energy via heat transfer,

$\boxed{Q_{\text{into}} = 765 \text{ kJ}}$.

Temp. [K]	h [kJ/kmol]	u [kJ/kmol]	s° [kJ/kmol/K]
380	11114	7955	212.2
390	11414	8172	213.0
760	23181	16862	234.1
770	23516	17114	234.5

If we had assumed that the oxygen could be treated as a perfect gas, i.e., an ideal gas with constant specific heats, then,

$$h_2 - h_1 = c_p\left(T_2 - T_1\right). \quad (9)$$

Substituting into Eq. (7) gives,

$$Q_{into} = mc_p(T_2 - T_1). \quad (10)$$

Using the given value for mass and the previously calculated temperature values along with,

c_p = 1.003 kJ/(kg.K) (found from a property table for O_2 at a temperature of 600 K),

=> Q_{into} = 772 kJ.

This result is approximately 1% larger than the more accurate value found using the Ideal Gas Tables. Thus, a perfect gas assumption still provides a reasonable estimation of the heat transfer.

Following is a Python code for the calculations.

```
# COE_42.py

import numpy as np
# Import a class created for reading the Ideal Gas Tables.
from IdealGasTable import IdealGasTable

# Import the Ideal Gas Table data for diatomic oxygen.
IGT = IdealGasTable("./IdealGasTables.xlsx", "O2")

p = 1*100  # kPa, pressure, given
V_1 = 2  # m^3, initial volume, given
V_2 = 4  # m^3, final volume, given
m = 2  # kg, mass, given
MW_O2 = 2*15.999  # kmol/kg, molar mass of diatomic oxygen
print("MW_O2 = %.3f kmol/kg" % MW_O2)
R_O2 = 8.314/MW_O2  # gas constant for diatomic oxygen
print("R_O2 = %.5f kJ/(kg.K)" % R_O2)

T_1 = p*V_1/m/R_O2  # K, initial temperature
T_2 = p*V_2/m/R_O2  # K, final temperature
print("(T_1, T_2) = (%.2f, %.2f) K" % (T_1, T_2))

h_1 = IGT.GetTableValue('h', T=T_1) # kJ/kmol, initial specific enthalpy
h_2 = IGT.GetTableValue('h', T=T_2) # kJ/kmol, final specific enthalpy
print("(h_1, h_2) = (%.2f, %.2f) kJ/kmol)" % (h_1, h_2))
h_1 = h_1/MW_O2  # convert to kJ/kg
h_2 = h_2/MW_O2  # convert to kJ/kg
print("(h_1, h_2) = (%.2f, %.2f) kJ/kg" % (h_1, h_2))

# Calculate the heat transfer.
Q_in = m*(h_2 - h_1)
print("Q_in = %.2f kJ" % Q_in)

# Assuming oxygen is a perfect gas.
cp = 1.003  # kJ/(kg.K), found from a property table
Q_in = m*cp*(T_2 - T_1)
print("Q_in = %.2f kJ (perfect gas)" % Q_in)
```

```
≫ python3 ./COE_42.py
MW_O2 = 31.998 kmol/kg
R_O2 = 0.25983 kJ/(kg.K)
(T_1, T_2) = (384.87, 769.74) K
(h_1, h_2) = (11260.07, 23507.22) kJ/kmol)
(h_1, h_2) = (351.90, 734.65) kJ/kg
Q_in = 765.49 kJ
Q_in = 772.05 kJ (perfect gas)
```