A gallon of milk at 68 °F is placed in a refrigerator. If energy is removed from the milk by heat transfer at a total rate of 0.08 Btu/s, how long would it take, in minutes, for the milk to cool to 40 °F? The specific heat and density of the milk are 0.94 Btu/(lb_m.°R) and 64 lb_m/ft^3, respectively.

SOLUTION:
The system consists of the milk as shown in the following figure.

Applying the rate form of the 1st Law to the system,

$$\frac{dE_{\text{sys}}}{dt} = \dot{Q}_{\substack{\text{into}\\\text{sys}}} + \dot{W}_{\substack{\text{on}\\\text{sys}}}, \tag{1}$$

where,

$$\frac{dE_{\text{sys}}}{dt} = \frac{dU_{\text{sys}}}{dt} = \frac{d(mu)_{\text{sys}}}{dt} = mc\frac{dT}{dt} \tag{2}$$

(Assuming KE, PE, *m*, and *c* remain constant. The milk is assumed to be an incompressible substance. The subscript "sys" is dropped for convenience.)

$$\dot{Q}_{\substack{\text{into}\\\text{sys}}} = \text{-0.08 Btu/s} \quad \text{(energy is being removed via heat transfer at a constant rate)} \tag{3}$$

$$\dot{W}_{\text{on sys}} = 0 \quad \text{(there is no work being done on the milk)} \tag{4}$$

Substitute and simplify,

$$mc\frac{dT}{dt} = \dot{Q}_{\substack{\text{into}\\\text{sys}}} \Rightarrow \frac{dT}{dt} = \frac{\dot{Q}_{\substack{\text{into}\\\text{sys}}}}{mc} \Rightarrow \int_{T_0}^{T} dT = \frac{\dot{Q}_{\substack{\text{into}\\\text{sys}}}}{mc}\int_0^t dt, \tag{5}$$

$$T = T_0 + \frac{\dot{Q}_{\substack{\text{into}\\\text{sys}}}}{mc}t, \tag{6}$$

$$\boxed{t = \frac{mc(T - T_0)}{\dot{Q}_{\substack{\text{into}\\\text{sys}}}}}. \tag{7}$$

Using the given data,

$\rho = 64\ \text{lb}_m/\text{ft}^3$
$V = (1\ \text{gal})(0.134\ \text{ft}^3/\text{gal}) = 0.134\ \text{ft}^3$
=> $m = \rho V = (64\ \text{lb}_m/\text{ft}^3)(0.134\ \text{ft}^3) = 8.58\ \text{lb}_m$
$c = 0.94\ \text{Btu}/(\text{lb}_m.\ °\text{R})$
$T_0 = 68\ °\text{F} = 528\ °\text{R}$
$T = 40\ °\text{F} = 500\ °\text{R}$
$\dot{Q}_{\substack{\text{into}\\\text{sys}}} = -0.08\ \text{Btu/s}$

$$t = \frac{(8.58\ \text{lb}_m)(0.94\ \text{Btu}/(\text{lb}_m \cdot °\text{R}))(500\ °\text{R} - 528\ °\text{R})}{(-0.08\ \text{Btu/s})}, \tag{8}$$

=> $\boxed{t = (2822\ \text{s})(\text{min}/60\ \text{s}) = 47\ \text{min}}$