

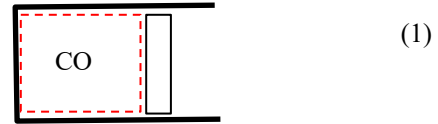
A piston-cylinder assembly contains carbon monoxide modeled as an ideal gas with a constant specific heat ratio of $k = 1.4$. The carbon monoxide undergoes a polytropic expansion with $n = k$ from an initial state, where the temperature is $200\text{ }^\circ\text{F}$ and pressure of 40 psia , to a final state where the volume is twice the initial volume. Determine:

- a. the final temperature, in $^\circ\text{F}$, and final pressure, in psia , and
- b. the work done by the gas and heat transfer into the gas, each in Btu/lb_m .

SOLUTION:

Treat the CO as an ideal gas. Hence,

$$pV = mRT \text{ or } pv = RT.$$



Since the process is polytropic,

$$pV^n = c \text{ or } p = \frac{c}{V^n} \Rightarrow \frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^n, \quad (1)$$

where $n = k = 1.4$, for this case, and c is a constant, which can be determined from the initial state. Note that the mass of CO remains constant, so combining Eqs. (1) and (2) gives,

$$\frac{p_2 V_2}{p_1 V_1} = \frac{T_2}{T_1} \Rightarrow \left(\frac{V_1}{V_2}\right)^n \frac{V_2}{V_1} = \frac{T_2}{T_1} \Rightarrow \left(\frac{V_1}{V_2}\right)^{n-1} = \frac{T_2}{T_1}. \quad (2)$$

Using the given parameters and Eqs. (2) and (3),

$$\begin{aligned} p_1 &= 40 \text{ psia} \\ T_1 &= 200 \text{ }^\circ\text{F} = 660 \text{ }^\circ\text{R} \\ n &= k = 1.4 \\ V_2 &= 2V_1 \\ \Rightarrow \boxed{p_2 = 15.16 \text{ psia}}, \quad \boxed{T_2 = 500.2 \text{ }^\circ\text{R} = 40.19 \text{ }^\circ\text{F}} \end{aligned}$$

The work done by the gas may be found using,

$$W_{\text{by gas}} = \int_{V_1}^{V_2} p dV = c \int_{V_1}^{V_2} V^{-n} dV = \frac{c}{1-n} (V_2^{1-n} - V_1^{1-n}) = \frac{p_1 V_1^n}{1-n} (V_2^{1-n} - V_1^{1-n}) \quad (n \neq 1), \quad (3)$$

$$W_{\text{by gas}} = \frac{p_1 m^n v_1^n}{1-n} m^{1-n} (v_2^{1-n} - v_1^{1-n}), \quad (4)$$

$$\frac{W_{\text{by gas}}}{m} = \frac{p_1 v_1^n}{1-n} (v_2^{1-n} - v_1^{1-n}). \quad (5)$$

where the initial conditions have been used to determine the constant c . The specific volume v_2 may be found using Eq. (1),

$$v = \frac{RT}{p}. \quad (6)$$

Using the given data,

$$\begin{aligned} M_{\text{CO}} &= 28.01 \text{ lb}_m/\text{lb}_m\text{mol} = 12.011 \text{ lb}_m/\text{lb}_m\text{mol} + 15.999 \text{ lb}_m/\text{lb}_m\text{mol}, \\ \bar{R}_u &= 1.986 \text{ Btu}/(\text{lb}_m\text{mol}\cdot^\circ\text{R}) \text{ (universal gas constant)}, \\ p_1 &= 40 \text{ psia} = 5760 \text{ lb}_m/(\text{ft}\cdot\text{s}^2), \\ T_1 &= 660 \text{ }^\circ\text{R}, \\ v_2 &= 2v_1, \\ n &= k = 1.4, \\ \Rightarrow R_{\text{CO}} &= 0.07090 \text{ Btu}/(\text{lb}_m\cdot^\circ\text{R}) = \bar{R}_u/M_{\text{CO}}, \quad v_1 = 8.124 \cdot 10^{-3} \text{ ft}^3/\text{lb}_m, \quad v_2 = 1.625 \cdot 10^{-2} \text{ ft}^3/\text{lb}_m, \\ \Rightarrow \boxed{W_{\text{by gas}}/m} &= \boxed{28.33 \text{ Btu}/\text{lb}_m}. \end{aligned}$$

The heat added to the gas may be found using the 1st Law applied to the gas,

$$\underbrace{\Delta E_{\text{sys}}}_{=\Delta u_{\text{sys}}} = Q_{\text{into sys}} - W_{\text{by sys}} \Rightarrow m\Delta u_{\text{sys}} = Q_{\text{into sys}} - W_{\text{by sys}} \Rightarrow \frac{Q_{\text{into sys}}}{m} = \Delta u_{\text{sys}} + \frac{W_{\text{by sys}}}{m}, \quad (7)$$

where kinetic and potential energies have been neglected. The change in specific internal energy of the gas, assuming ideal gas behavior, may be found either through ideal gas tables at the appropriate temperatures (e.g., Table A-23E of Moran et al., 7th ed.),

$$\bar{u}_1 = 3275.8 \text{ Btu}/\text{lb}_m\text{mol} \text{ (at } T_1 = 660 \text{ }^\circ\text{R}) \Rightarrow u_1 = 117.0 \text{ Btu}/\text{lb}_m$$

$$\bar{u}_2 = 2479.2 \text{ Btu}/\text{lb}_m\text{mol} \text{ (at } T_2 = 500 \text{ }^\circ\text{R}) \Rightarrow u_2 = 88.51 \text{ Btu}/\text{lb}_m$$

Note that $u = \bar{u}/M$ where $M_{\text{CO}} = 28.01 \text{ lb}_m/\text{lb}_m\text{mol}$. Hence, from Eq. (8) and the previously calculated value for specific work,

$$\boxed{Q_{\text{into gas}}/m = -0.16 \text{ Btu/lb}_m}$$

If we instead assume that the CO behaves as a perfect gas (an ideal gas with constant specific heats),

$$u_2 - u_1 = c_v(T_2 - T_1), \quad (9)$$

where $c_v = 0.178 \text{ Btu}/(\text{lb}_m \cdot ^\circ\text{R})$ (from Table A-20E in Moran et al., 7th ed., at a temperature of 100 °F).

Thus,

$$Q_{\text{into gas}}/m = -0.11 \text{ Btu/lb}_m.$$

We get approximately the same result using either method.

It can be shown that the heat transfer is, in fact, identically zero for a polytropic expansion of an ideal gas when $n = k$, as is the case here. Combining Eqs. (6) and (7) gives,

$$\frac{W_{\text{by gas}}}{m} = \frac{p_1 v_1^n}{1-n} (v_2^{1-n} - v_1^{1-n}) = \frac{(RT_1/v_1) v_1^n}{1-n} (v_2^{1-n} - v_1^{1-n}) = \frac{RT_1 v_1^{n-1}}{1-n} (v_2^{1-n} - v_1^{1-n}) = \frac{RT_1}{1-n} \left[\left(\frac{v_2}{v_1} \right)^{1-n} - 1 \right], \quad (10)$$

$$\frac{W_{\text{by gas}}}{m} = \frac{RT_1}{1-n} \left[\left(\frac{v_2}{v_1} \right)^{1-n} - 1 \right] = \frac{RT_1}{1-n} \left[\left(\frac{v_2}{v_1} \right) \left(\frac{v_2}{v_1} \right)^{-n} - 1 \right] = \frac{RT_1}{1-n} \left[\left(\frac{v_2}{v_1} \right) \left(\frac{p_2}{p_1} \right) - 1 \right], \quad (11)$$

where Eq. (2) has been used in the last step. Continuing to simplify,

$$\frac{W_{\text{by gas}}}{m} = \frac{RT_1}{1-n} \left[\left(\frac{RT_2}{RT_1} \right) - 1 \right] = \frac{R}{1-n} (T_2 - T_1). \quad (12)$$

Since we're told that $n = k$ for this polytropic process,

$$\frac{W_{\text{by gas}}}{m} = \frac{R}{1-k} (T_2 - T_1) = c_v (T_2 - T_1), \quad (13)$$

where the relationship between the specific heat at constant volume, the gas constant, and the specific heat ratio has been used for an ideal gas. Note that the right-hand side of this expression is the change in the specific internal energy, assuming a perfect gas (ideal gas with constant specific heats),

$$\frac{W_{\text{by gas}}}{m} = u_2 - u_1. \quad (14)$$

Thus, from the 1st Law, we must have $Q = 0$. Our previous answers were close to zero, but there is some numerical error. Plus, the $Q = 0$ result assumes constant specific heats, which isn't exactly true.