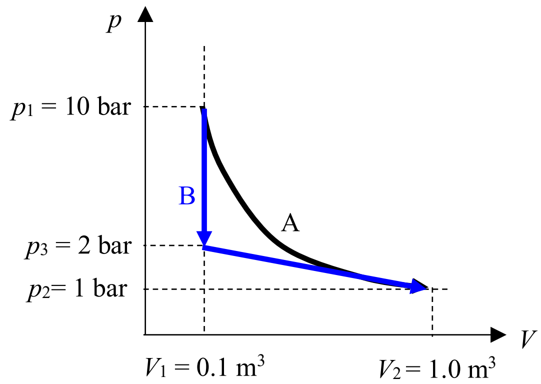
A gas contained within a piston-cylinder assembly undergoes two processes, A and B, between the same end states, 1 and 2, where at state 1 the pressure is 10 bar, the volume is 0.1 m^3, the internal energy is 400 kJ, and at state 2 the pressure is 1 bar, the volume is 1.0 m^3, and the internal energy is 200 kJ.

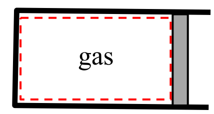
Process A: Process from 1 to 2 during which the pressure-volume relation is pV = constant.
Process B: Constant volume process from state 1 to a pressure of 2 bar, followed by a linear pressure-volume process to state 2.

Kinetic and potential energy effects can be ignored. For each of the processes A and B,

a. Sketch the process on a p-V diagram,
b. evaluate the work by the gas on the piston, in kJ, and
c. evaluate the heat transfer from the gas in kJ.

SOLUTION:
The processes are sketched on the plot shown below.





The work may be found by integrating the pdV work given the two processes described,

$$W_{\substack{\text{by gas}\\\text{on piston}}} = \int_1^2 p\,dV\,, \tag{1}$$

where for process A,

$$W_{\substack{\text{by gas}\\\text{on piston,}\\\text{path A}}} = c\int_{V=V_1}^{V=V_2} \frac{dV}{V} = c\ln\left(\frac{V_2}{V_1}\right), \tag{2}$$

noting that $pV = c \Rightarrow p = c/V$. The constant c may be found from the initial (or final) conditions,

$$p_1V_1 = c = p_2V_2\,, \tag{3}$$

$$W_{\substack{\text{by gas}\\\text{on piston,}\\\text{path A}}} = p_1V_1\ln\left(\frac{V_2}{V_1}\right). \tag{4}$$

Substituting the given numbers,

$p_1 \;= 10 \text{ bar} = 10*10^2 \text{ kPa}$

$V_1 \;= 0.1 \text{ m}^3$

$V_2 \;= 1.0 \text{ m}^3$

$$\Rightarrow \boxed{W_{\text{by gas on piston, path A}} = 230 \text{ kJ}} \tag{5}$$

The heat transferred from the gas may be found using the 1st Law of Thermodynamics,

$$\underbrace{\Delta E_{\text{gas}}}_{=\Delta U_{\text{gas}}} = Q_{\substack{\text{into}\\ \text{gas}}} - W_{\substack{\text{by}\\ \text{gas}}} \Rightarrow Q_{\substack{\text{into}\\ \text{gas}}} = \Delta U_{\text{gas}} + W_{\substack{\text{by}\\ \text{gas}}}\ , \tag{6}$$

where the total change of energy in the gas is due only to changes in internal energy (U). Using the given parameters,

U_1 = 400 kJ
U_2 = 200 kJ
$W_{\text{by gas}}$ = 230 kJ (from Eq. (5))

$$\Rightarrow \Delta U = \text{-200 kJ} \ \Rightarrow\ \boxed{Q_{\text{into gas}} = 30\text{ kJ}} \tag{7}$$

Thus, 30 kJ of heat is transferred into the gas (-30 kJ of heat is transferred from the gas).

For process B, there is no work done in the constant volume part of the process since the volume doesn't change. The work in the linear pressure-volume part of the process is,

$$W_{\substack{\text{by gas}\\ \text{on piston,}\\ \text{path B}}} = \int_{V=V_1}^{V=V_2} p\,dV = \int_{V=V_1}^{V=V_2}\left[\left(\frac{p_2-p_3}{V_2-V_1}\right)(V-V_1)+p_3\right]dV = \left[\left(\frac{p_2-p_3}{V_2-V_1}\right)\left(\tfrac{1}{2}V^2 - V_1V\right)+p_3V\right]_{V_1}^{V_2}, \tag{8}$$

$$W_{\substack{\text{by gas}\\ \text{on piston,}\\ \text{path B}}} = \left(\frac{p_2-p_3}{V_2-V_1}\right)\left(\tfrac{1}{2}V_2^2 - \tfrac{1}{2}V_1^2 - V_1V_2 + V_1^2\right) + p_3\left(V_2 - V_1\right), \tag{9}$$

$$W_{\substack{\text{by gas}\\ \text{on piston,}\\ \text{path B}}} = \tfrac{1}{2}\left(\frac{p_2-p_3}{V_2-V_1}\right)\left(V_2^2 - 2V_1V_2 + V_1^2\right) + p_3\left(V_2 - V_1\right), \tag{10}$$

where the pressure varies linearly with the volume,

$$p = \left(\frac{p_2-p_3}{V_2-V_1}\right)(V-V_1)+p_3\,,\ \text{(equation of a line)} \tag{11}$$

Substituting the given data,

p_2 = 1 bar = 1*10^2 kPa
p_3 = 2 bar = 2*10^2 kPa
V_1 = 0.1 m^3
V_2 = 1.0 m^3

$$\Rightarrow \boxed{W_{\substack{\text{by gas}\\ \text{on piston,}\\ \text{path B}}} = 135\text{ kJ}} \tag{12}$$

The heat transferred into the gas may be found using Eq. (6) with the following parameters,

U_1 = 400 kJ (Note that the internal energies are independent of the path. They're a property!)
U_2 = 200 kJ
$W_{\text{by gas}}$ = 135 kJ (from Eq. (12))

$$\Rightarrow\ \boxed{Q_{\text{into gas}} = \text{-65 kJ}} \tag{13}$$

Thus, 65 kJ of heat is transferred from the gas to the surroundings.