Four kilograms of a certain gas is contained within a piston-cylinder assembly. The gas undergoes a polytropic process where: $p V^{1.5}=$ constant. The initial pressure is 3 bars (abs), the initial volume is $0.1 \mathrm{~m}^{3}$, and the final volume is $0.2 \mathrm{~m}^{3}$. The change in the specific internal energy of the gas in the process is $\Delta u=-4.5 \mathrm{~kJ} / \mathrm{kg}$. There are no significant changes in the kinetic or potential energies of the gas. What is the net heat transfer for the process?


SOLUTION:
Apply the First Law to the system of gas as shown in the figure below,


$$
\begin{equation*}
\Delta E_{\mathrm{sys}}=Q_{\substack{\mathrm{added} \\ \text { to sys }}}+W_{\mathrm{on} \mathrm{sys}} \tag{1}
\end{equation*}
$$

where,

$$
\begin{equation*}
W_{o n ~ s y s}=\int_{V=V_{1}}^{V=V_{2}}-p d V=\int_{V=V_{1}}^{V=V_{2}}-\left(c V^{-1.5}\right) d V=2 c\left(V^{-0.5}\right)_{V_{1}}^{V_{2}}=2 \underbrace{p_{1} V_{1}^{1.5}}_{=c}\left(V_{2}^{-0.5}-V_{1}^{-0.5}\right), \tag{2}
\end{equation*}
$$

and,

$$
\begin{equation*}
\Delta E_{\text {sys }}=m_{\text {sys }} \Delta e_{\text {sys }}=m_{\text {sys }} \Delta u_{\text {sys }} \text {. (The kinetic and potential energy changes are negligible.) } \tag{2}
\end{equation*}
$$

Re-arranging Eq. (1) and substituting Eqs. (2) and (3) gives,

$$
\begin{equation*}
Q_{\text {added }}=m_{\text {sys }} \Delta u_{s y s}-2 p_{1} V_{1}^{1.5}\left(V_{2}^{-0.5}-V_{1}^{-0.5}\right) \tag{3}
\end{equation*}
$$

Using the given values:

$$
\begin{aligned}
m_{\text {sys }} & =4 \mathrm{~kg} \\
\Delta u_{\text {sys }} & =-4500 \mathrm{~J} / \mathrm{kg} \\
p_{1} & =3^{*} 10^{5} \mathrm{~Pa} \\
V_{1} & =0.1 \mathrm{~m}^{3} \\
V_{2} & =0.2 \mathrm{~m}^{3} \\
\Rightarrow & Q_{\text {added }}=-0.426 \mathrm{~kJ} \quad \text { (heat is leaving the system) }
\end{aligned}
$$

