Four kilograms of a certain gas is contained within a piston-cylinder assembly. The gas undergoes a polytropic process where:  $pV^{1.5}$ =constant. The initial pressure is 3 bars (abs), the initial volume is 0.1 m<sup>3</sup>, and the final volume is 0.2 m<sup>3</sup>. The change in the specific internal energy of the gas in the process is  $\Delta u = -4.5$  kJ/kg. There are no significant changes in the kinetic or potential energies of the gas. What is the net heat transfer for the process?



## SOLUTION:

Apply the First Law to the system of gas as shown in the figure below,



where,

$$W_{on sys} = \int_{V=V_1}^{V=V_2} -pdV = \int_{V=V_1}^{V=V_2} -(cV^{-1.5})dV = 2c(V^{-0.5})_{V_1}^{V_2} = 2\underbrace{p_1V_1^{1.5}}_{=c}(V_2^{-0.5} - V_1^{-0.5}),$$
(2)

and,

 $\Delta E_{\rm sys} = m_{\rm sys} \Delta e_{\rm sys} = m_{\rm sys} \Delta u_{\rm sys} \,. \quad \text{(The kinetic and potential energy changes are negligible.)} \tag{2}$ 

Re-arranging Eq. (1) and substituting Eqs. (2) and (3) gives,

$$\frac{Q_{added}}{V_{c}} = m_{sys} \Delta u_{sys} - 2p_1 V_1^{1.5} (V_2^{-0.5} - V_1^{-0.5}).$$
(3)

Using the given values:

$$m_{sys} = 4 \text{ kg} \Delta u_{sys} = -4500 \text{ J/kg} p_1 = 3^{*}10^5 \text{ Pa} V_1 = 0.1 \text{ m}^3 V_2 = 0.2 \text{ m}^3 \Rightarrow Q_{added} = -0.426 \text{ kJ} \text{ (heat is leaving the system)}$$