Breaking the Valiant Load Balancing Barrier for Oblivious Reconfigurable Networks

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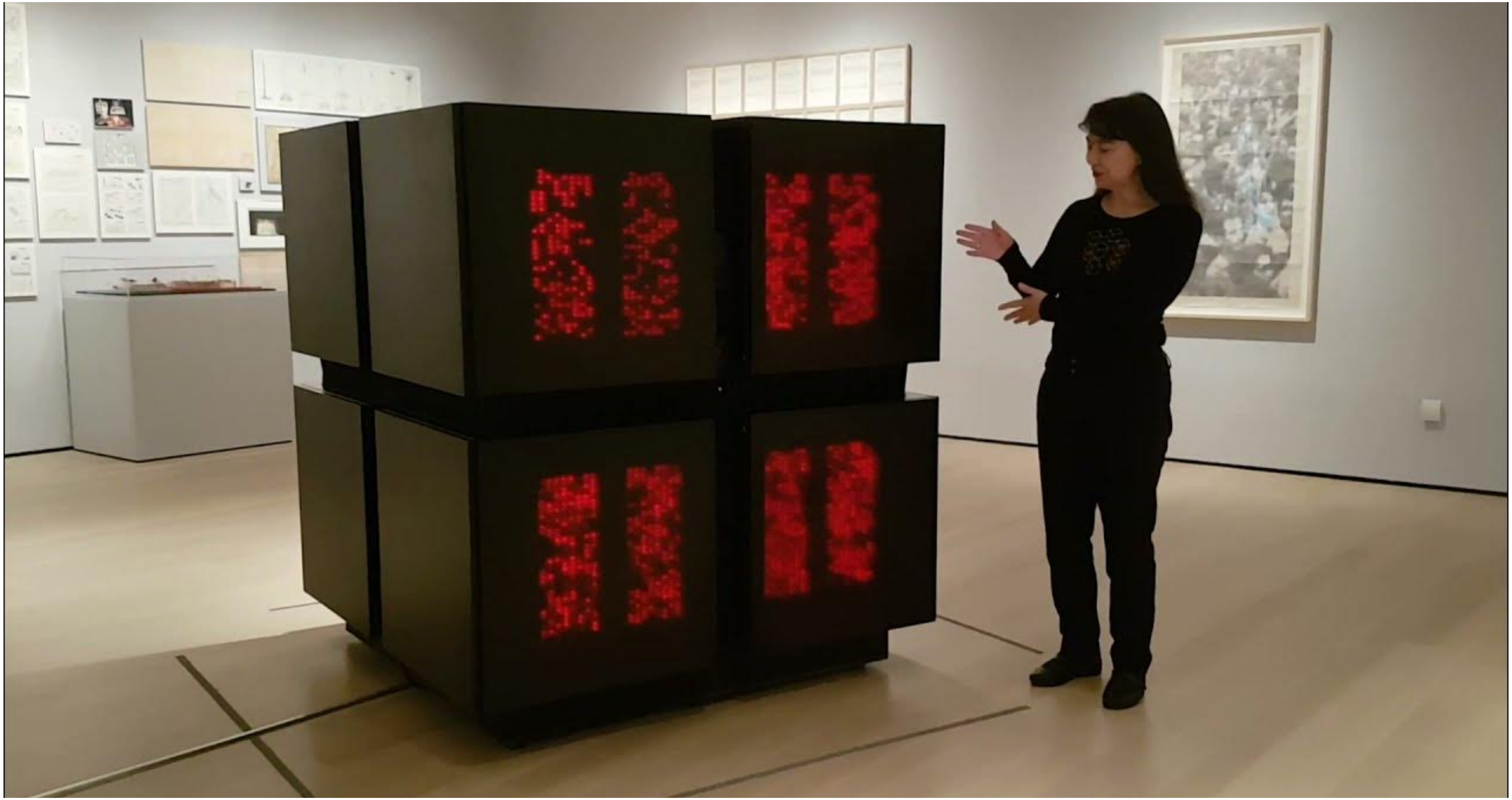
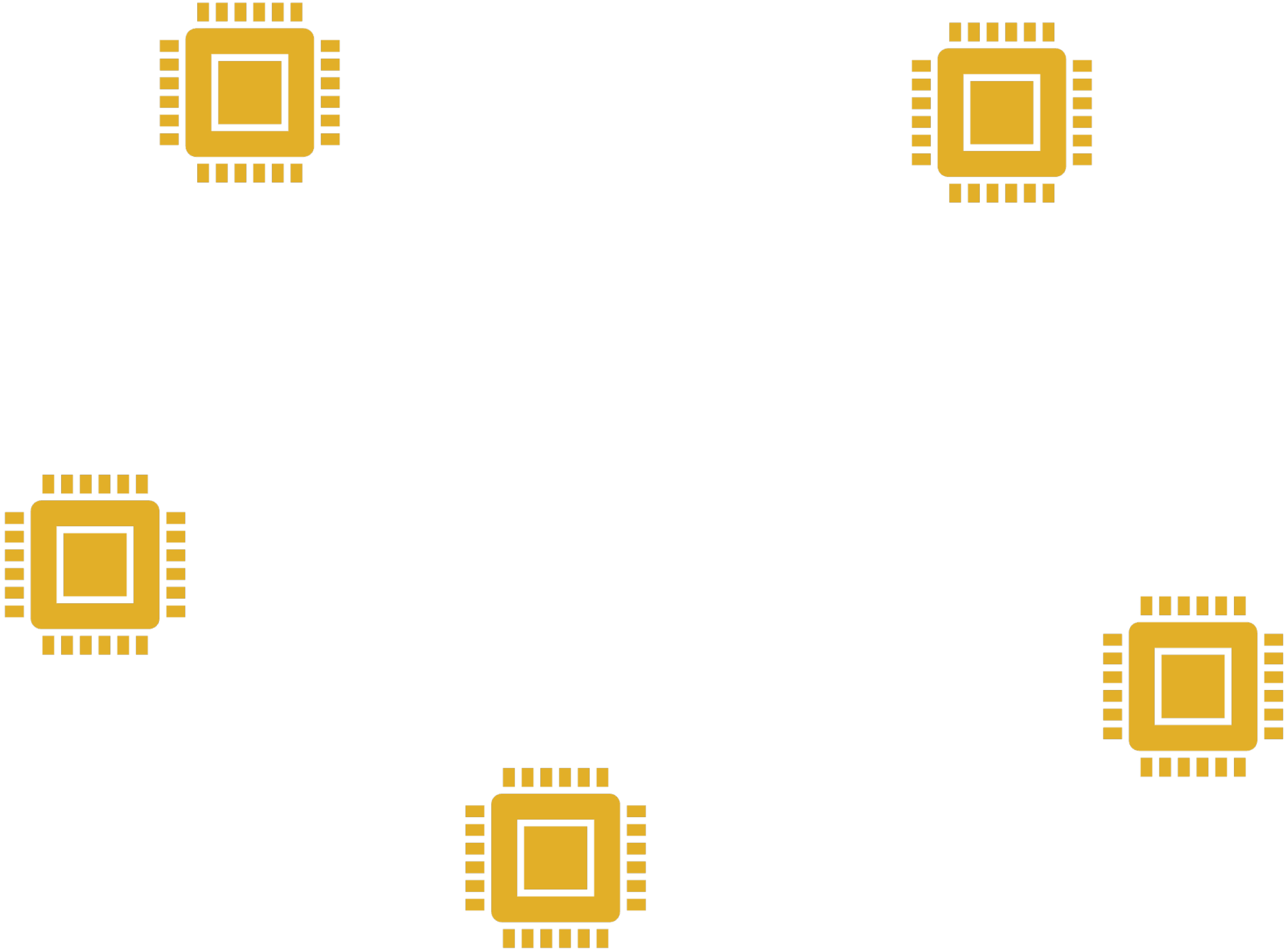
Thinking Machines Corporation

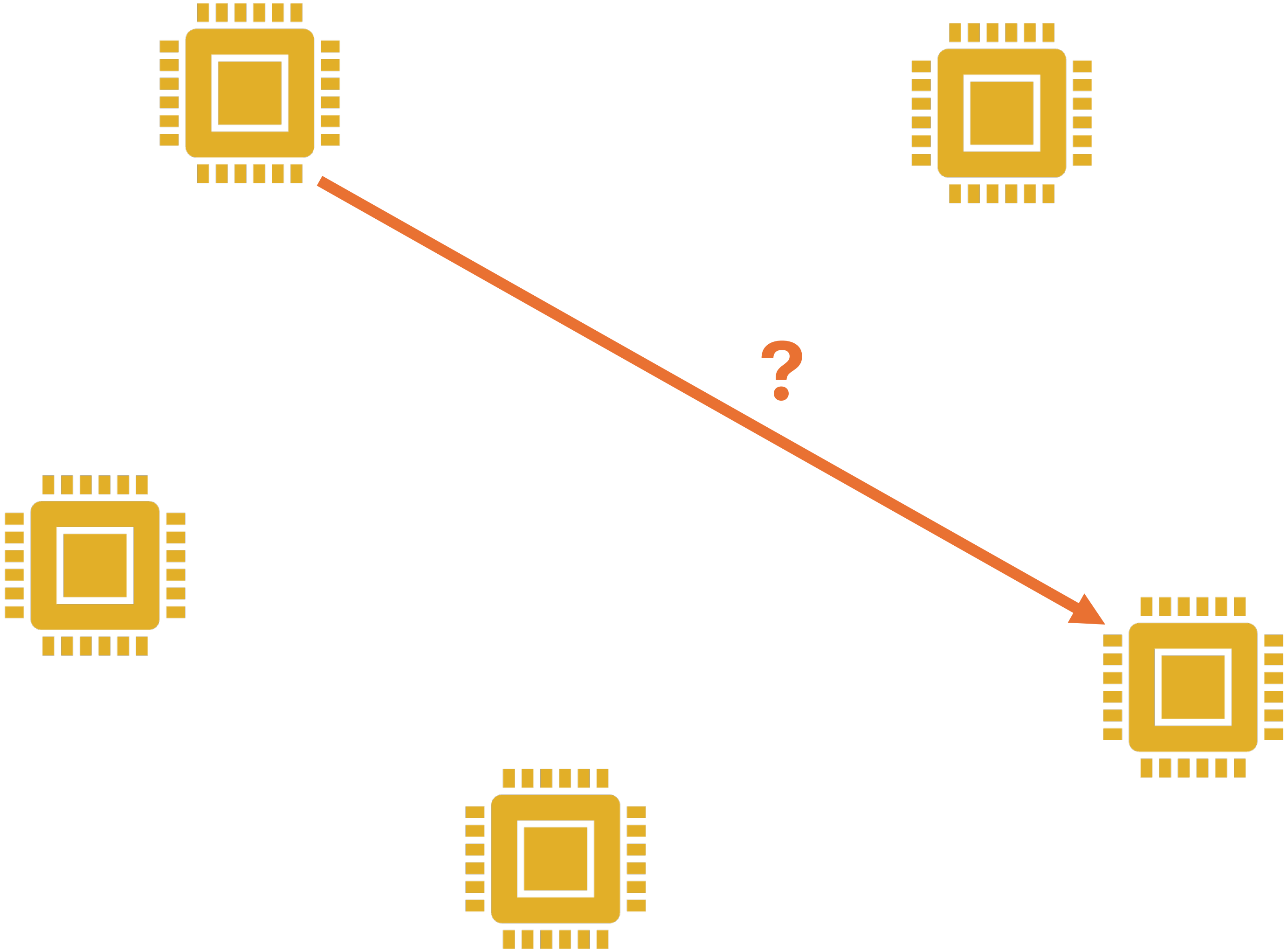
CRAY

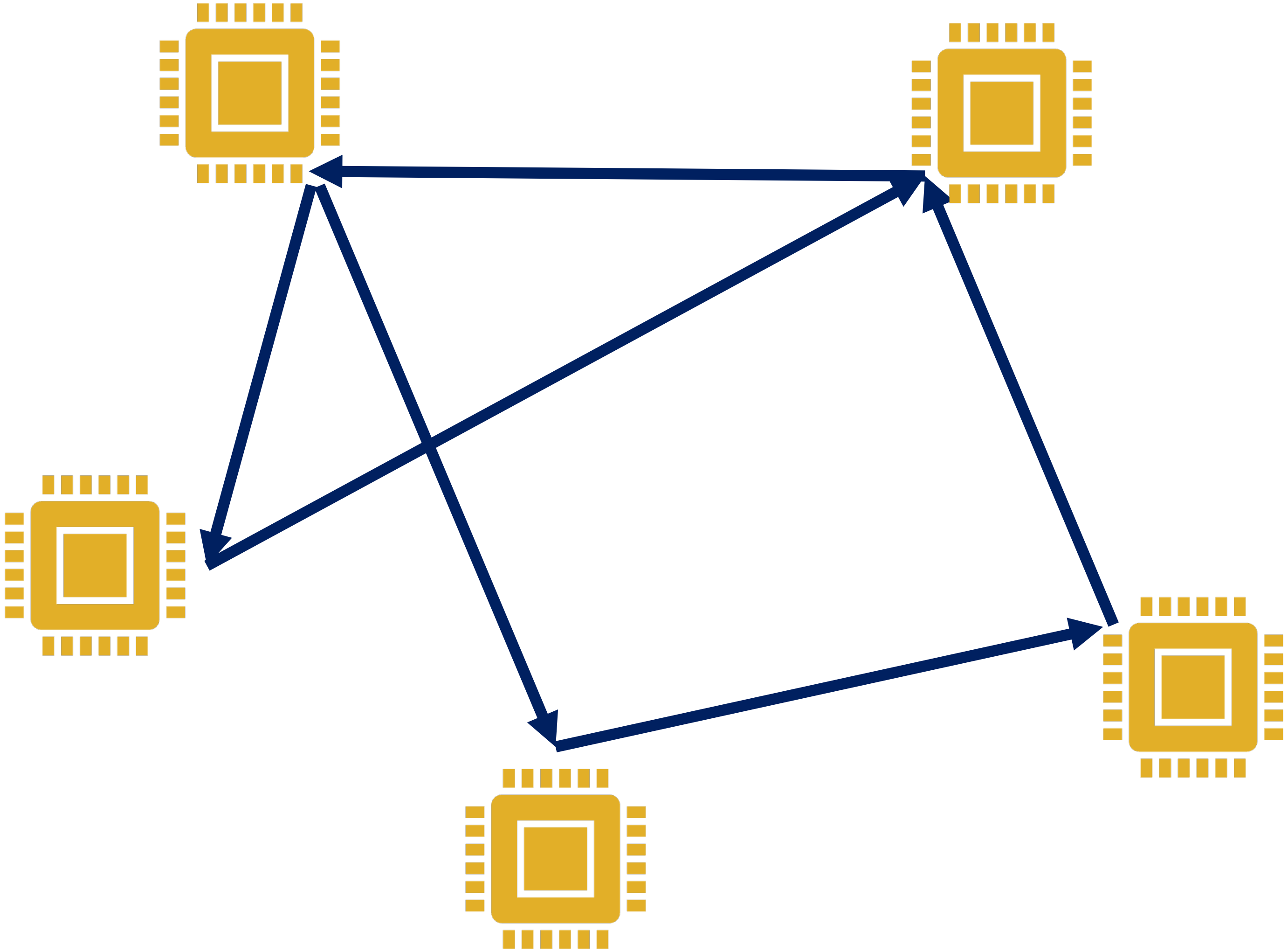
Supercomputer

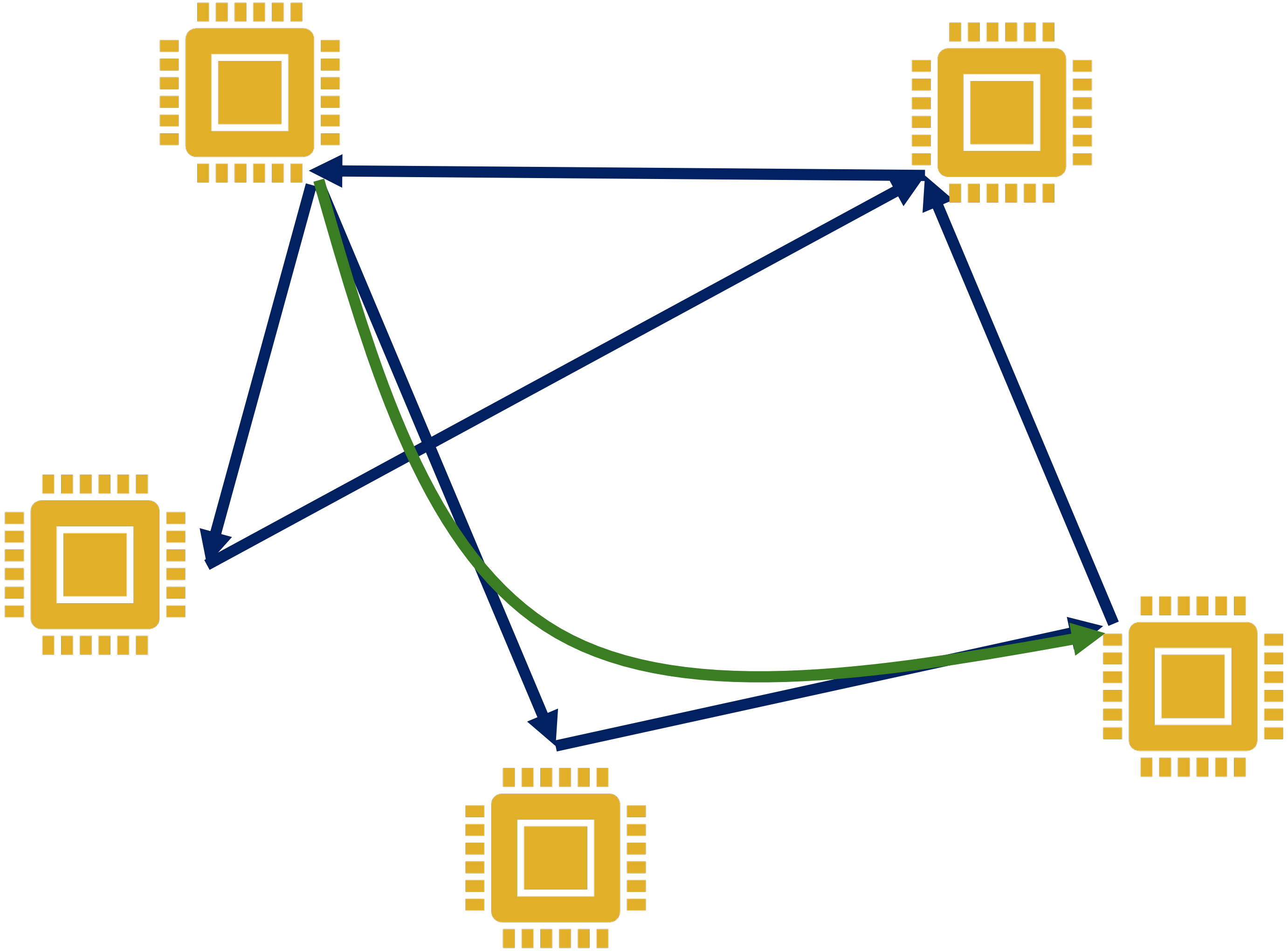


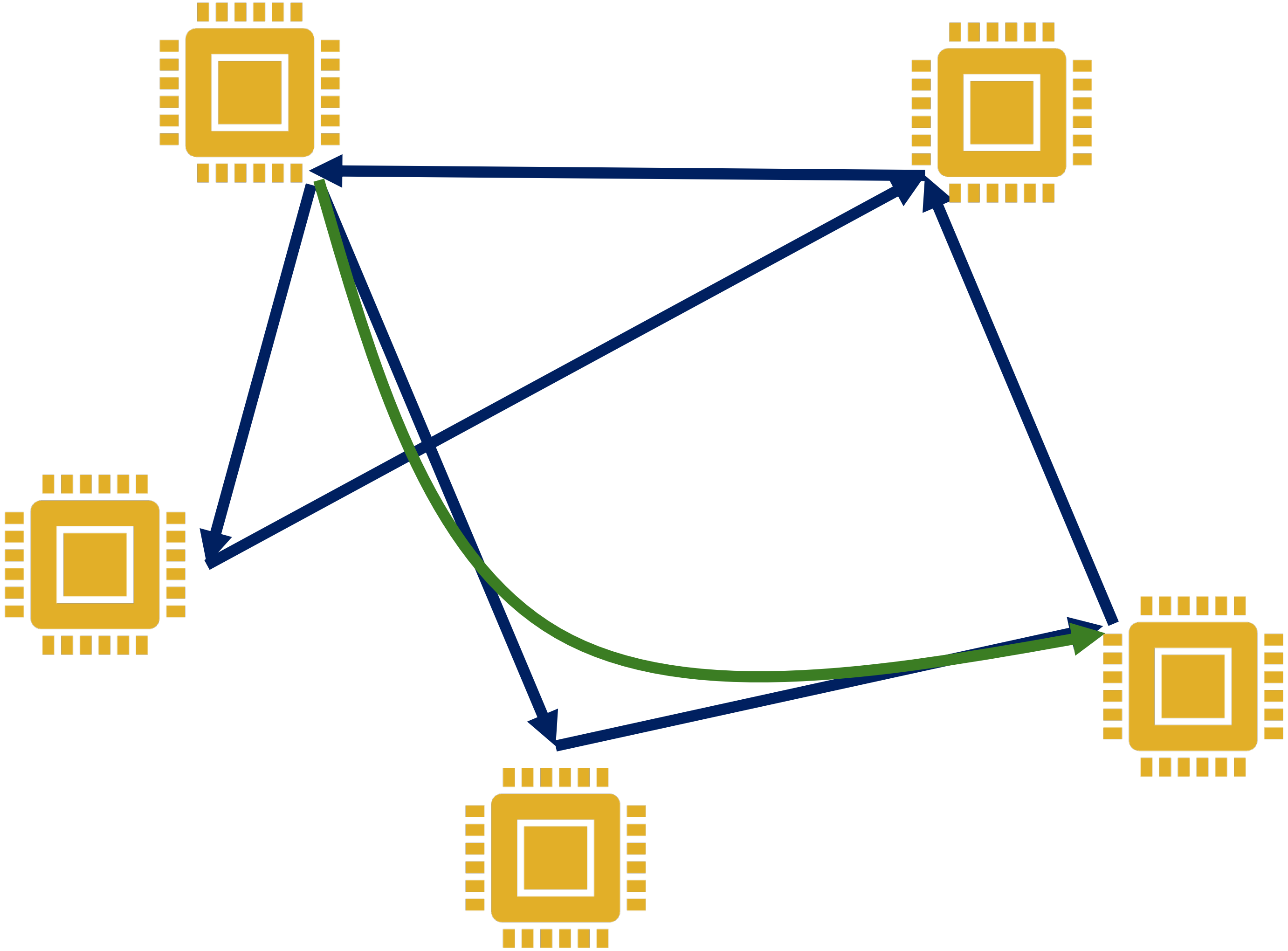
Thinking Machine Corporation CM-1 at the MoMA











Network Topology

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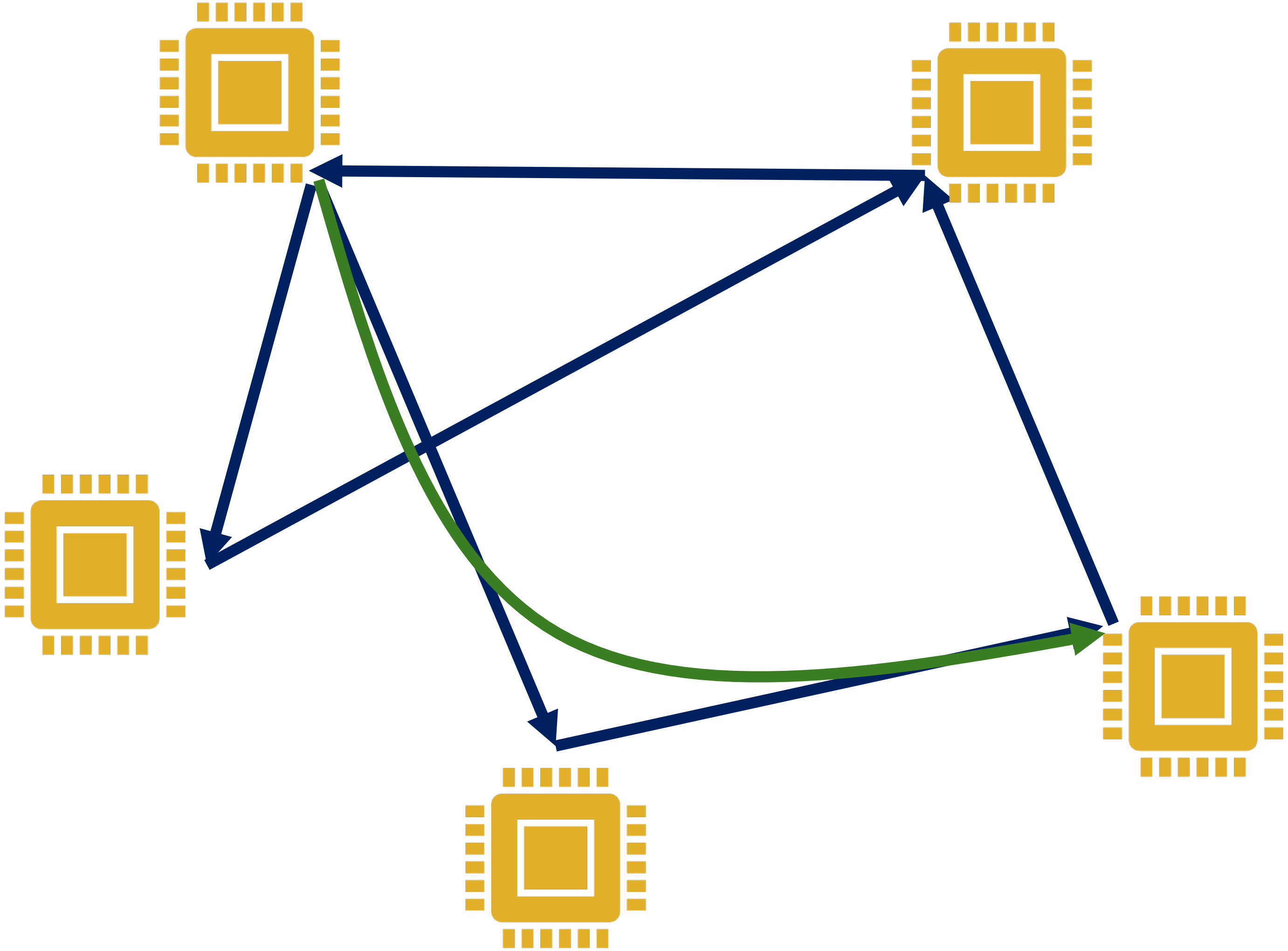
Routing Protocol



Network Topology

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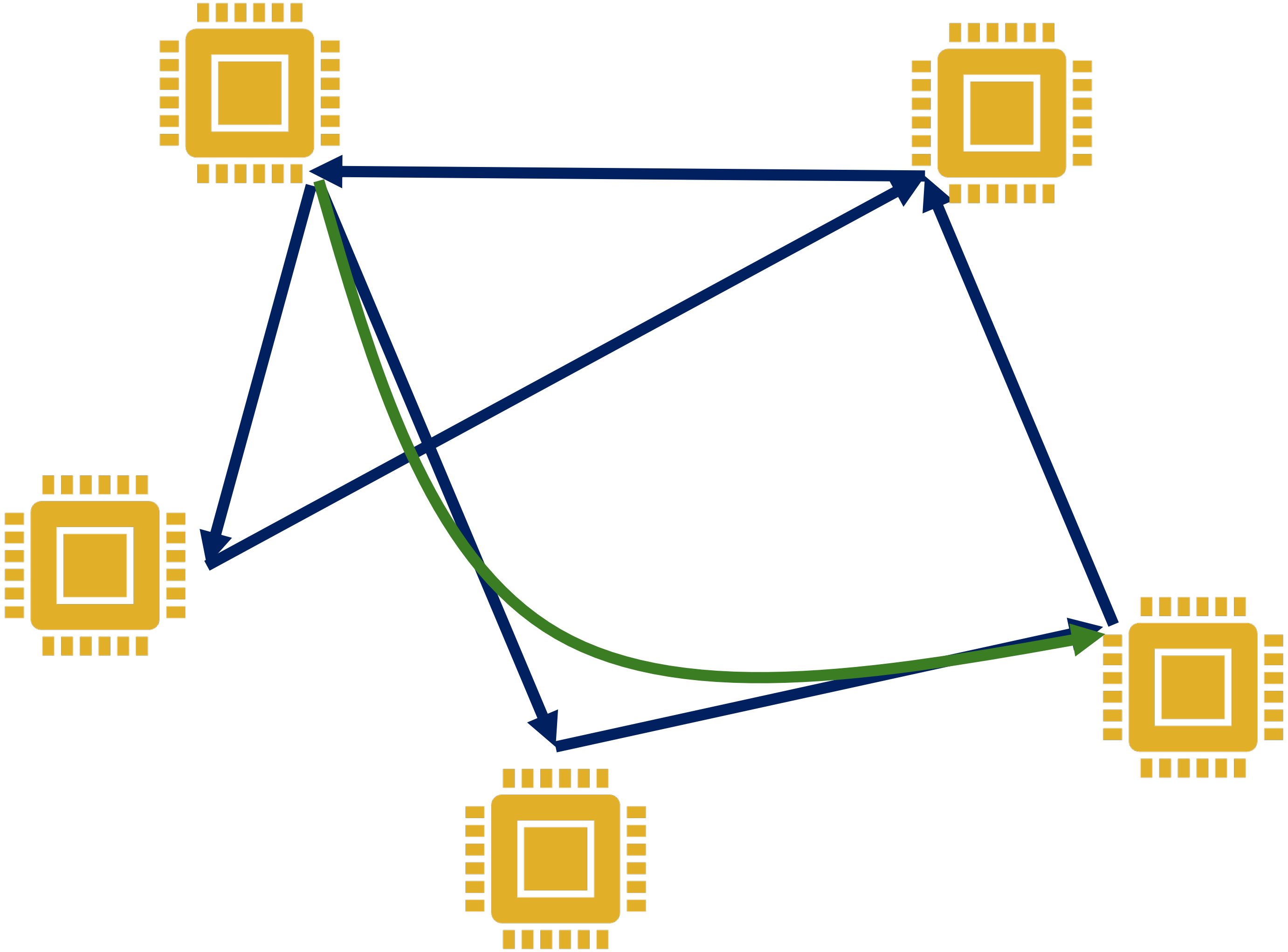
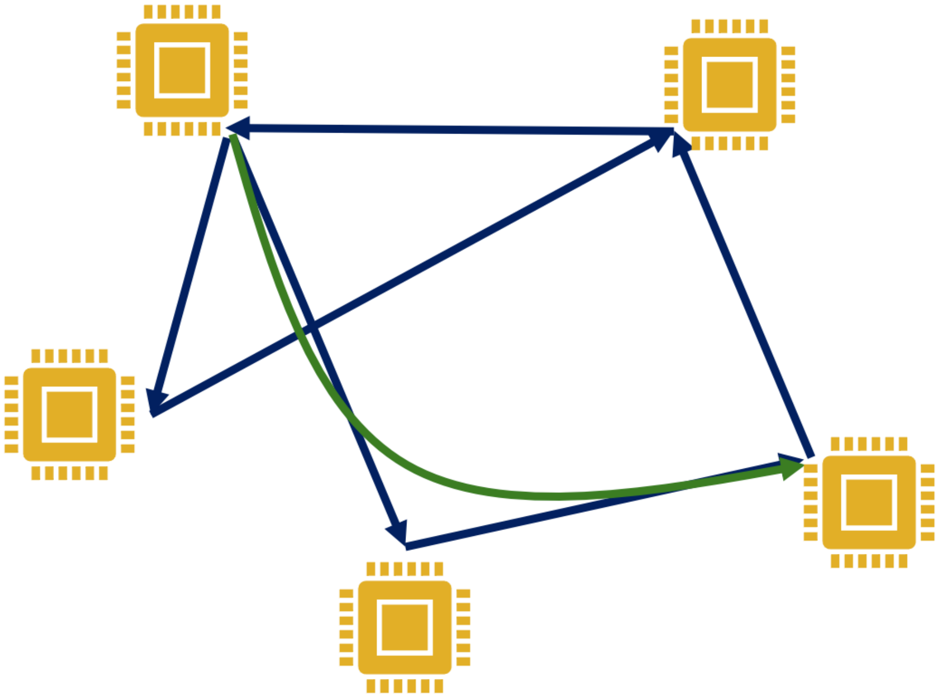
Network Design



Routing on “optimized topologies”

Network Topology

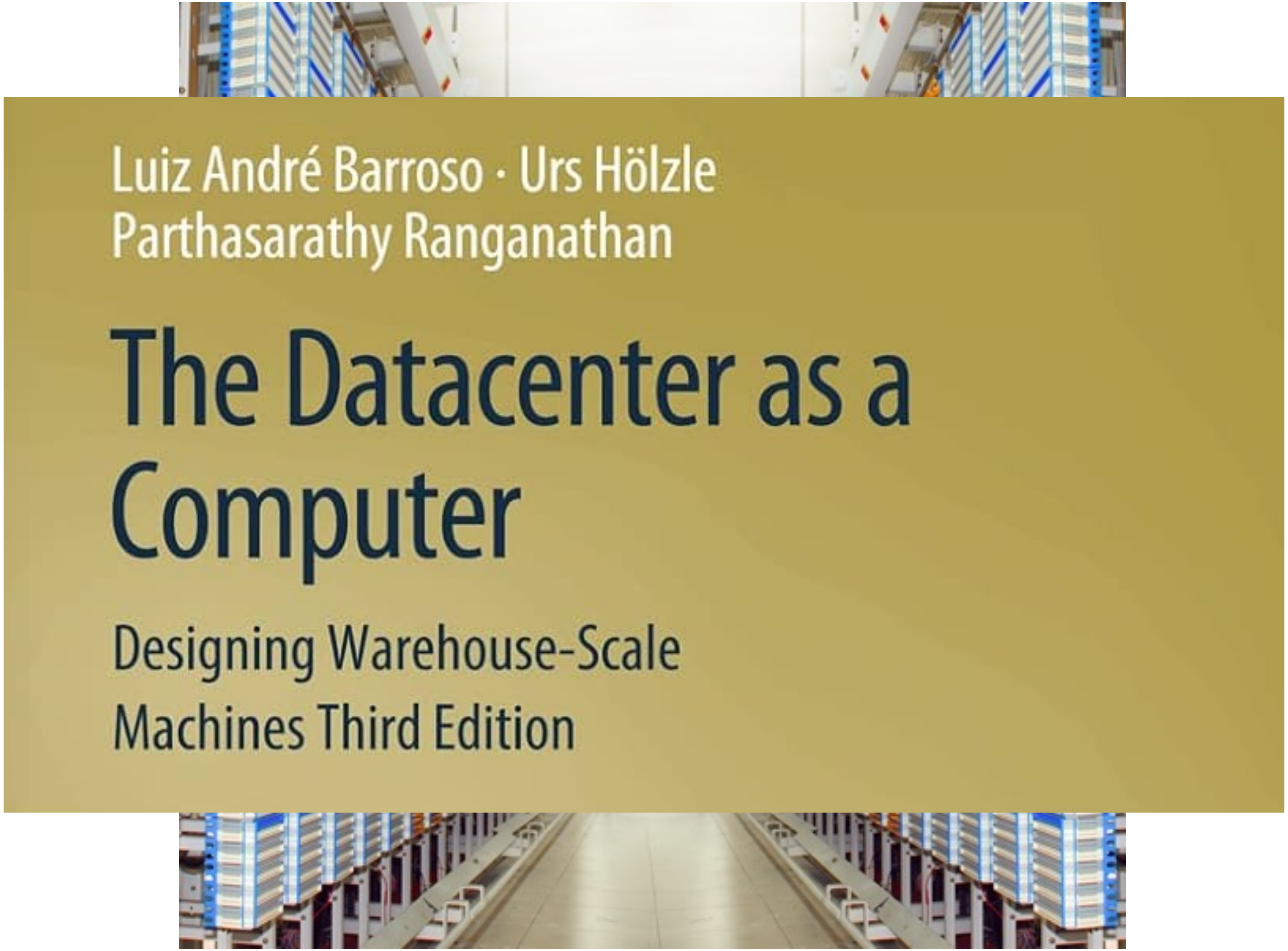
> *“The fundamental problem… is that of simulating arbitrary connection patterns among the processors via a fixed sparse network… For routing packets, the strategy will have to be based on only a minute fraction of the total information necessary to specify the complete communication pattern.”*
>
> *—Leslie Valiant and Gordon Brebner (1981)*



Goal: *Oblivious* Routing

All routing decisions made before traffic is seen

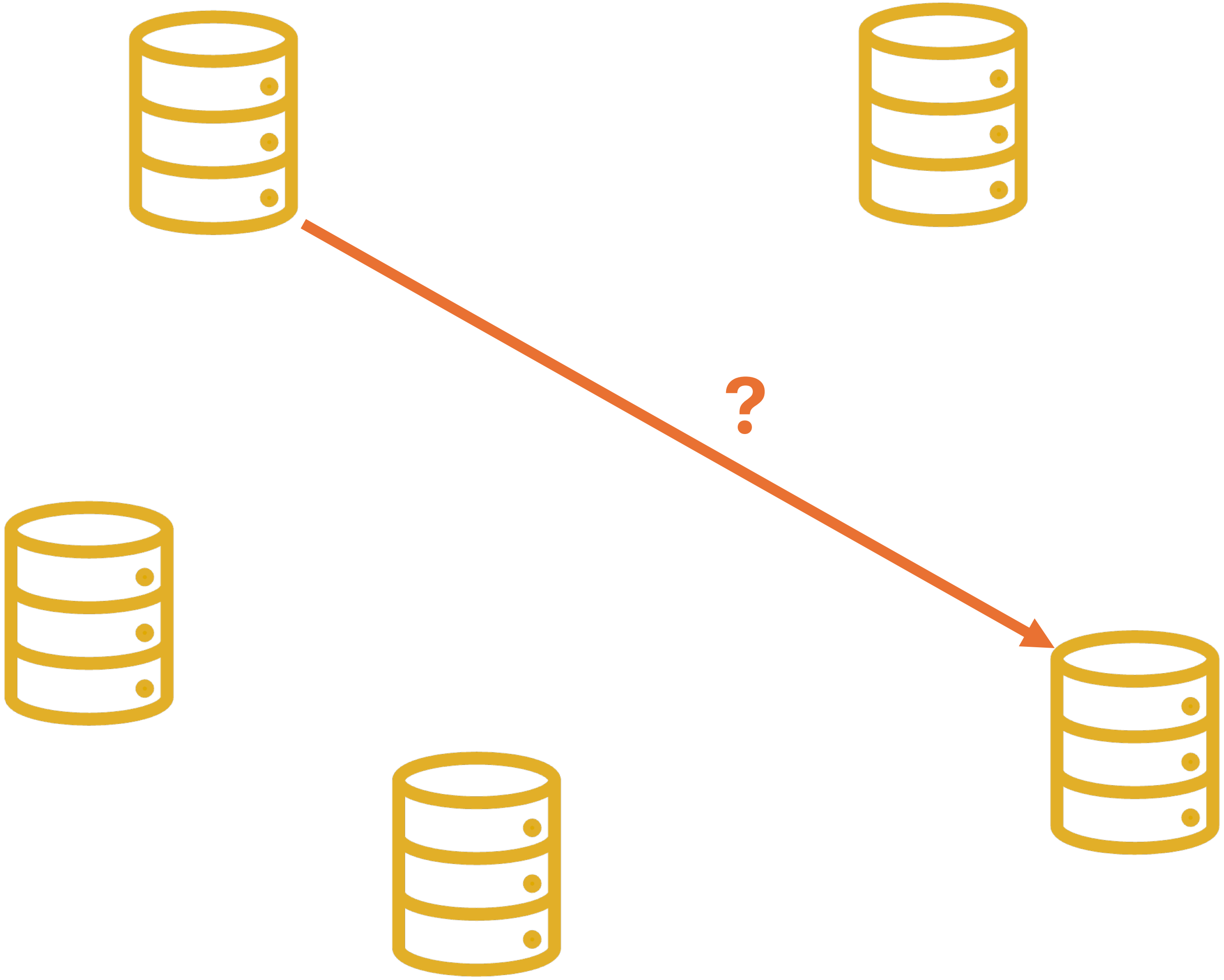
For every source-sink pair, define a distribution over routing paths

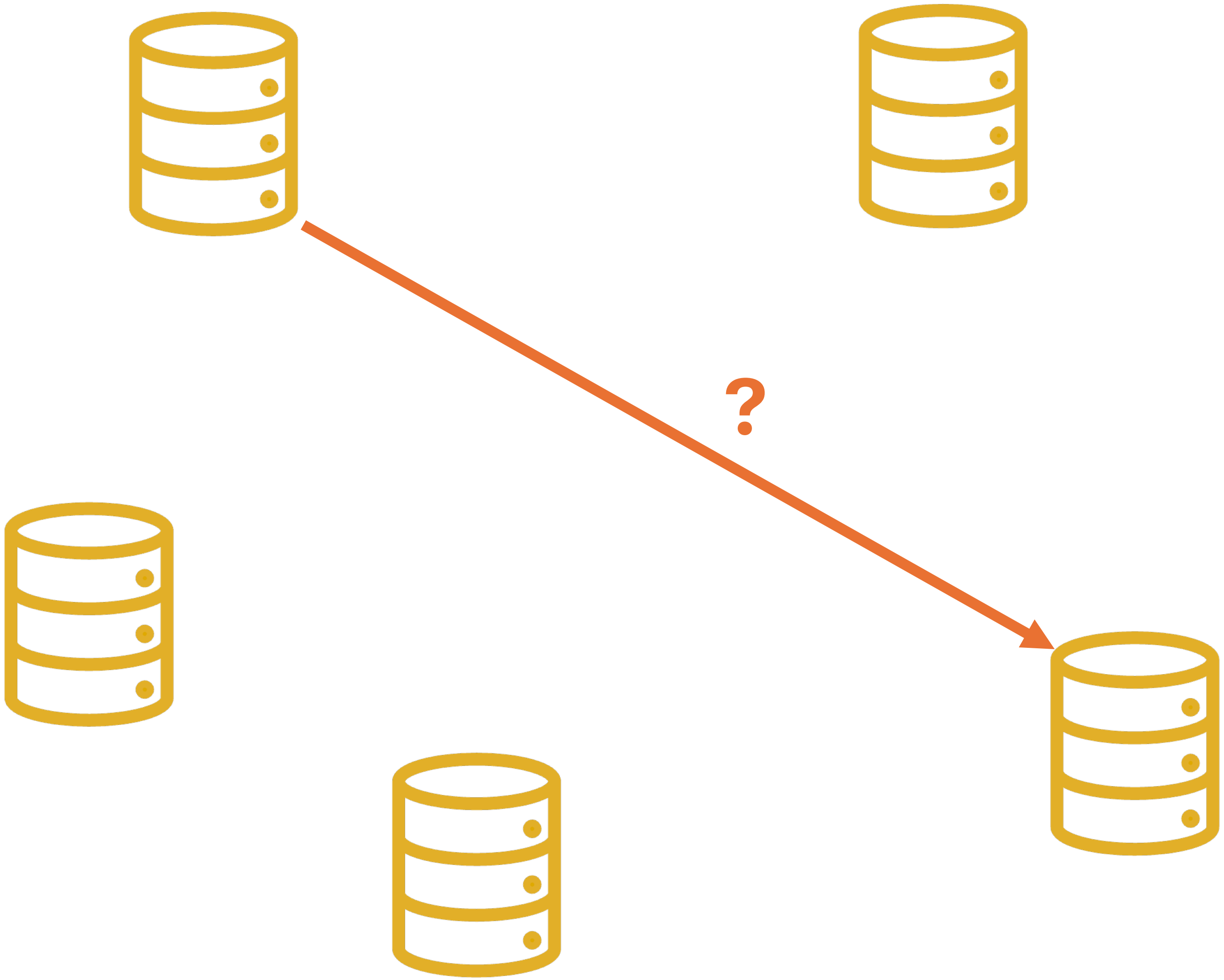


Luiz André Barroso · Urs Hölzle
Parthasarathy Ranganathan

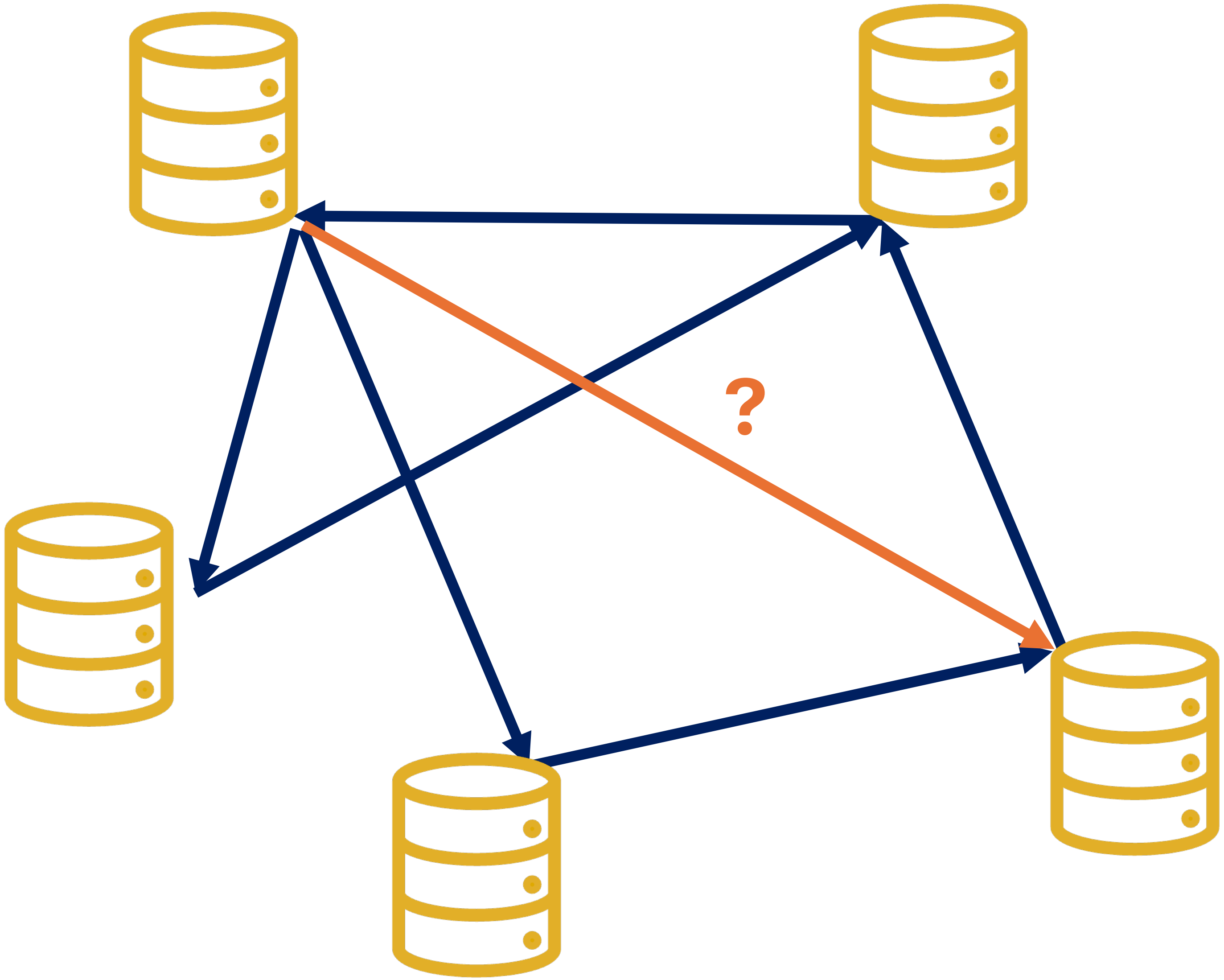
The Datacenter as a Computer

Designing Warehouse-Scale Machines Third Edition



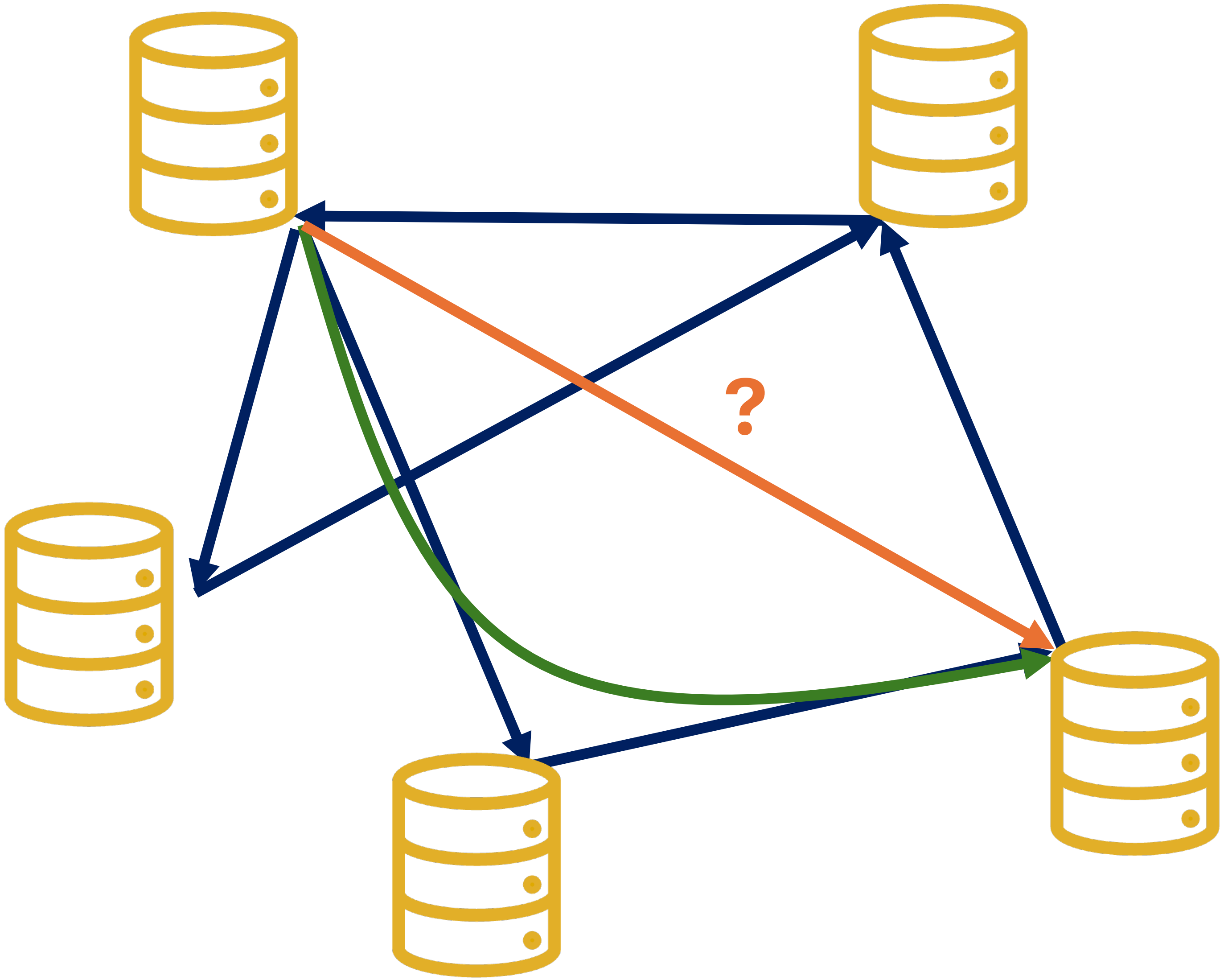


Network Design:



Network Design:

Network Topology



Network Design:

Network Topology

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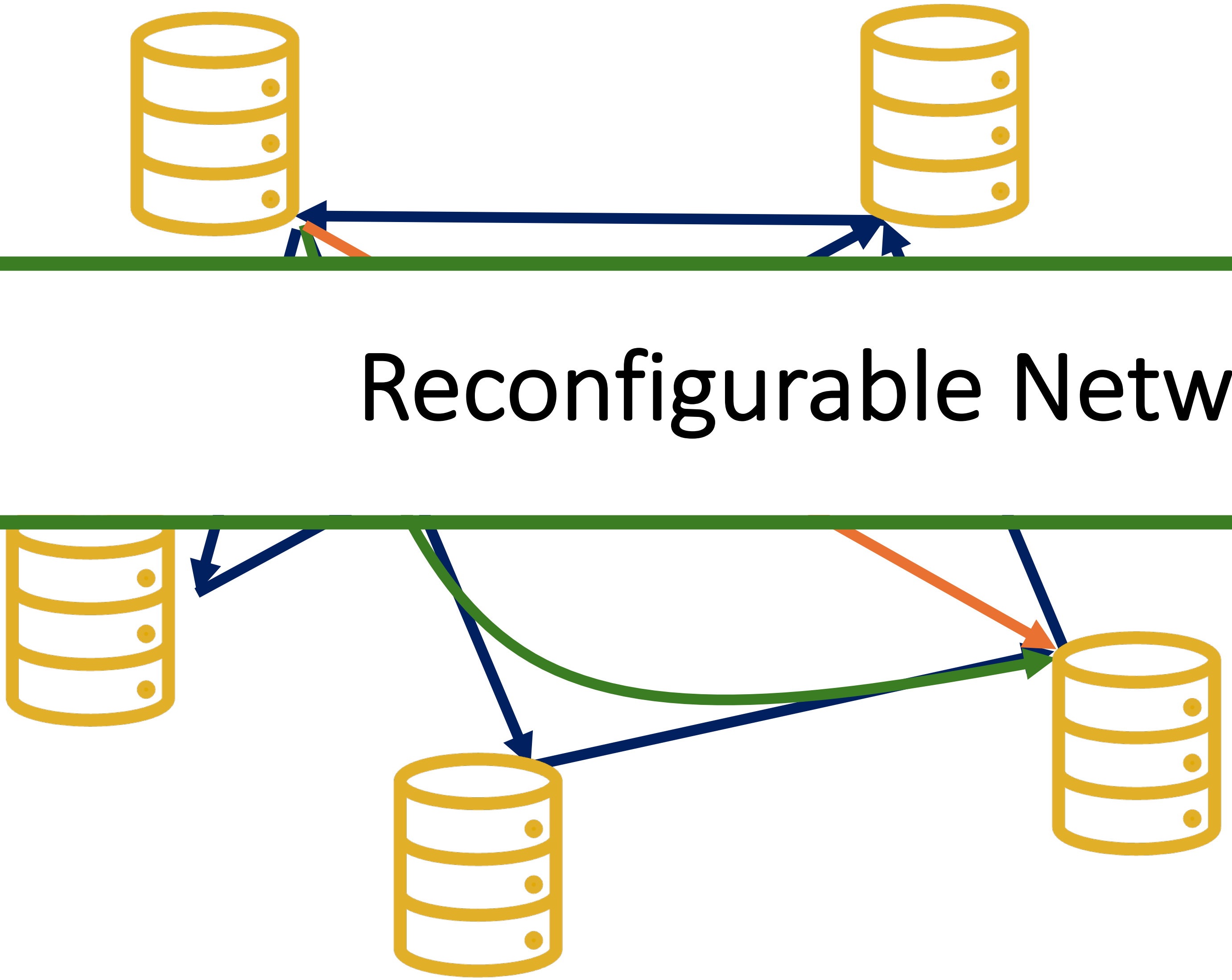
Routing Protocol

Reconfigurable Networks



Network Design:

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Routing Protocol

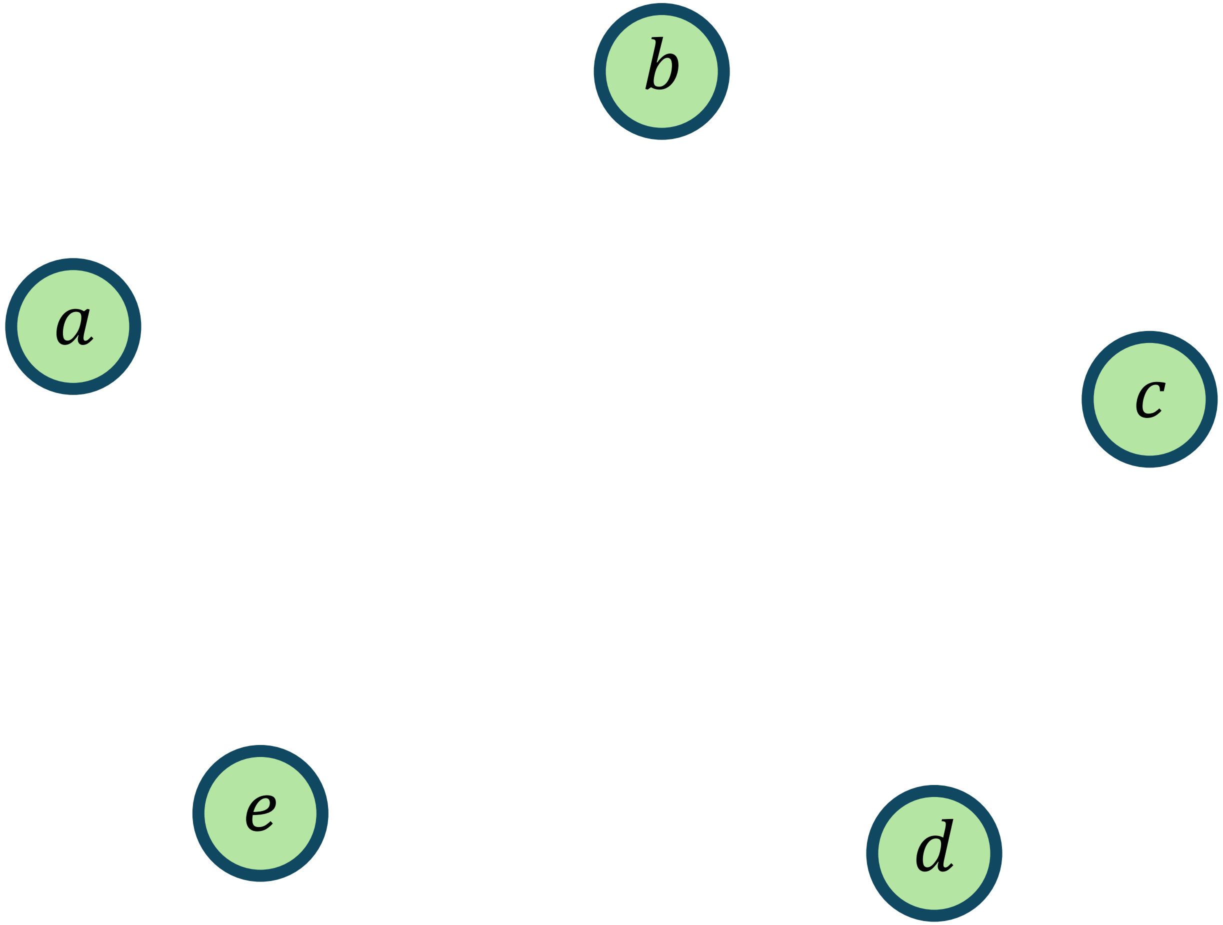
Reconfigurable Networks

- Edges can be reconfigured over time
- Edge set at each timestep may be arbitrary, with a small in/out-degree constraint d

Reconfigurable Networks

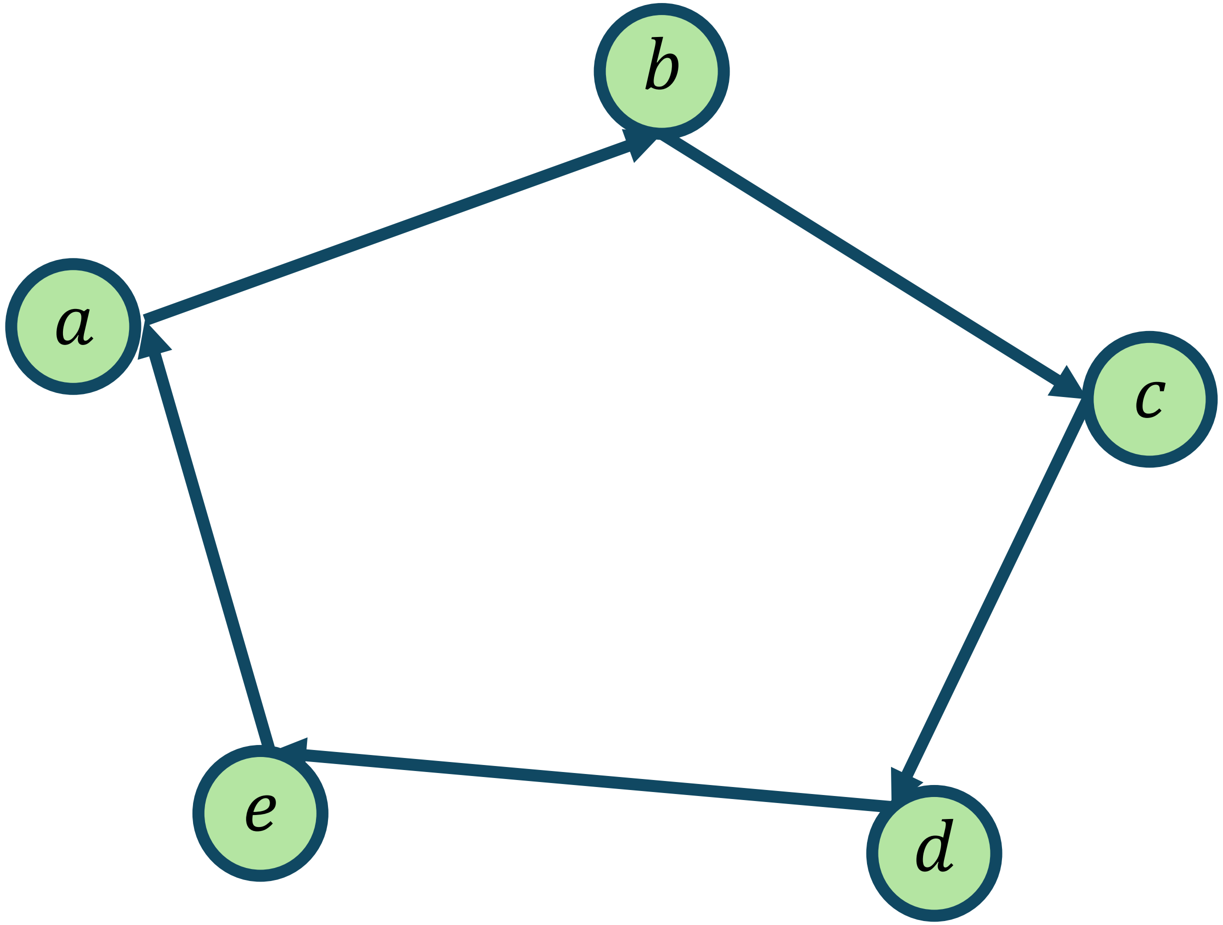
In/out-degree constraint: 1

Reconfigurable Networks



In/out-degree constraint: 1

Reconfigurable Networks



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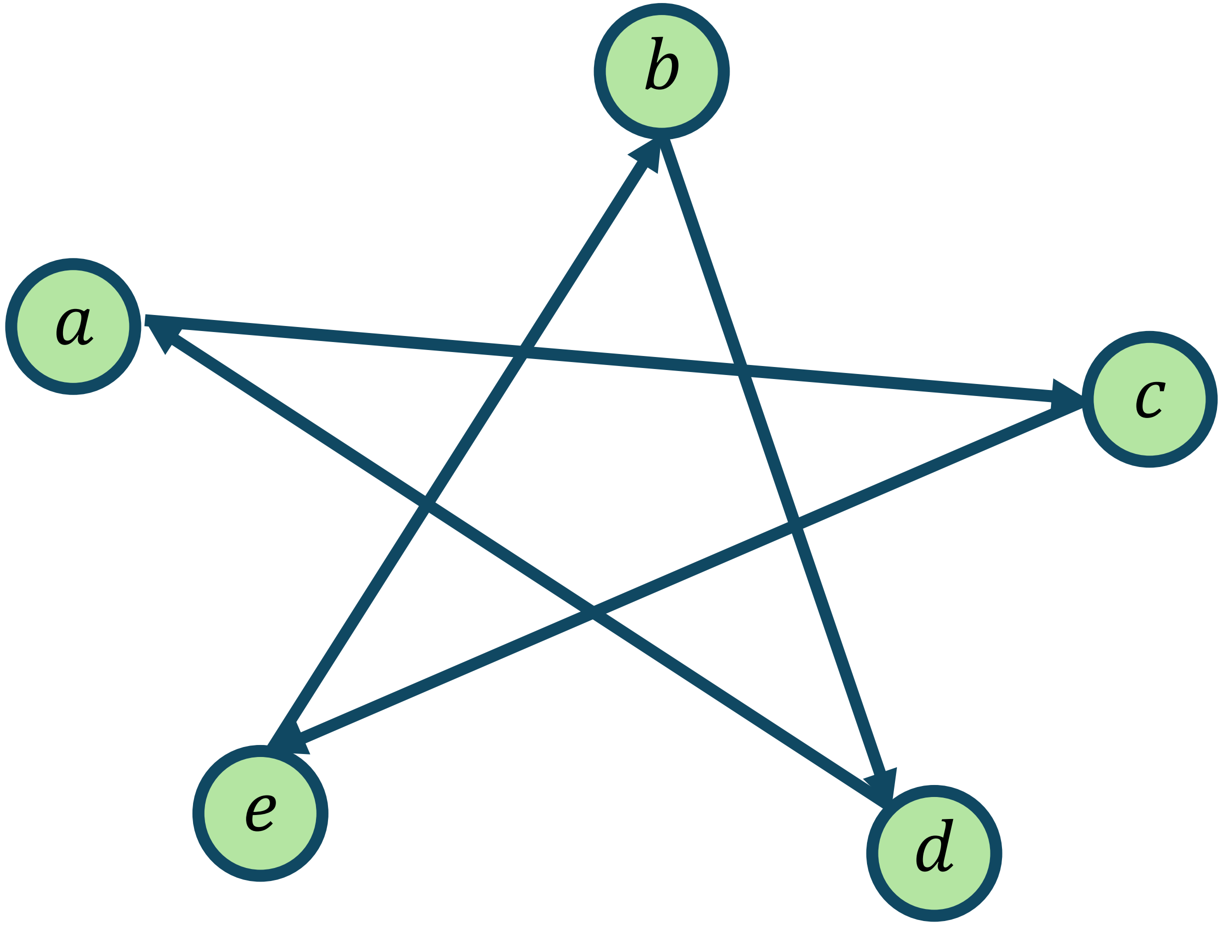
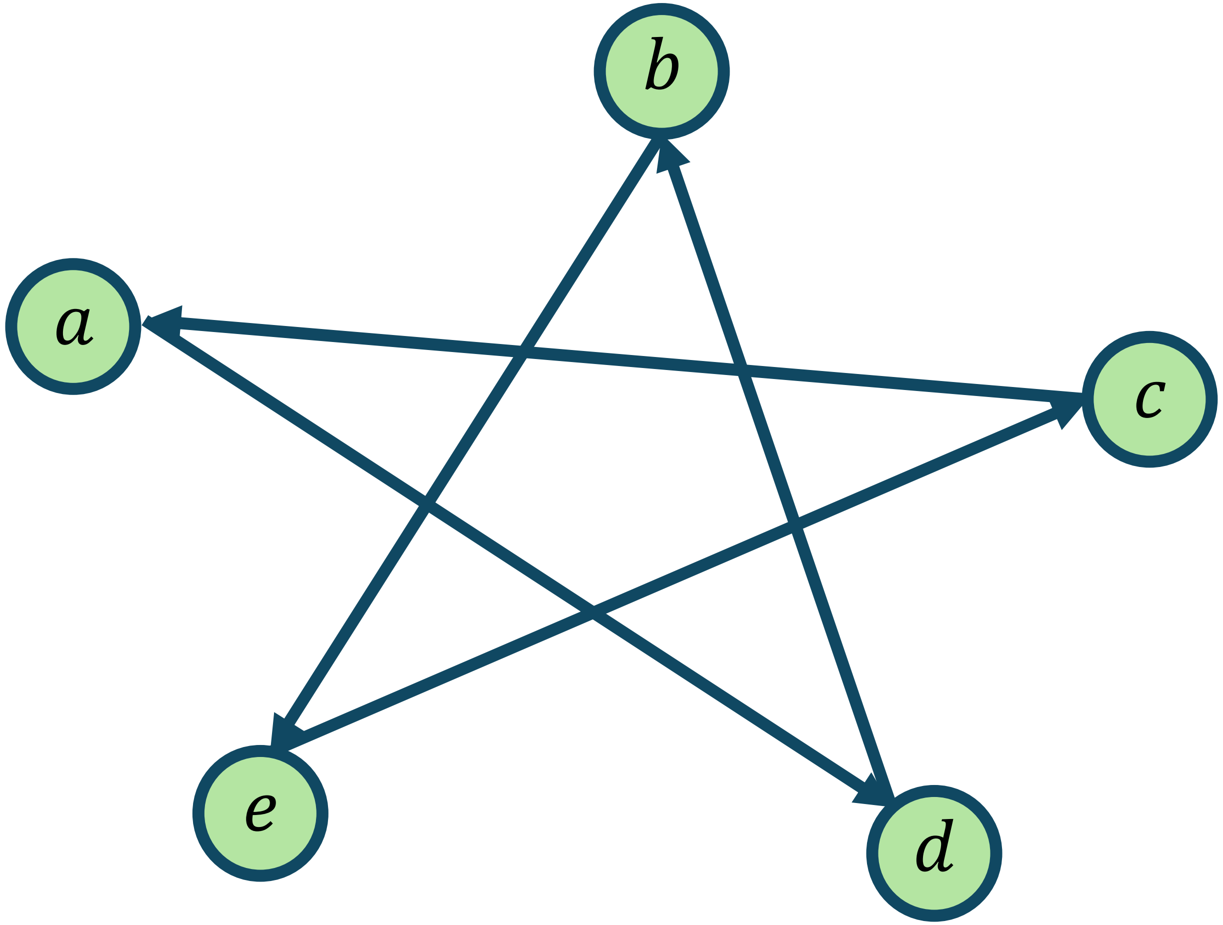
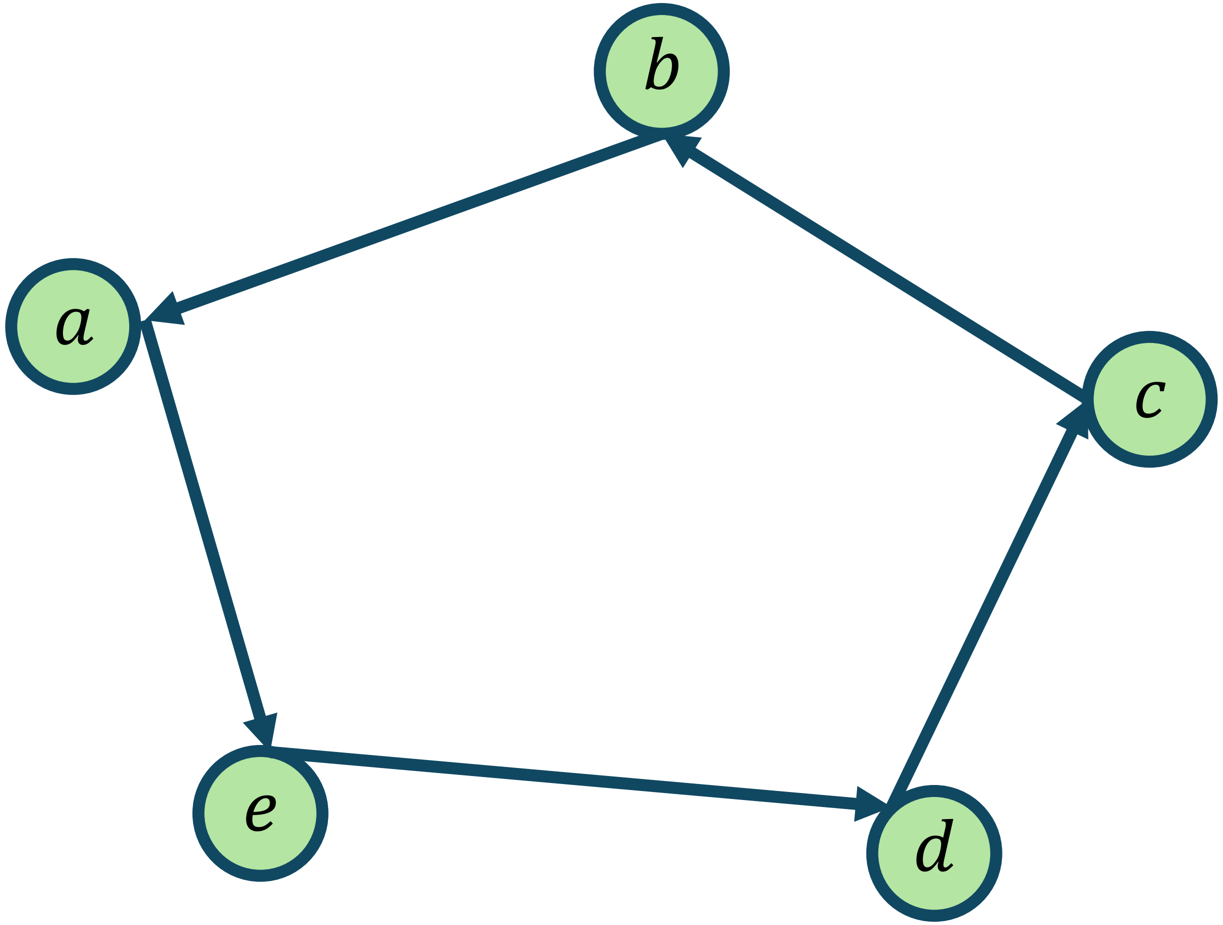
Reconfigurable Networks



Reconfigurable Networks



Reconfigurable Networks



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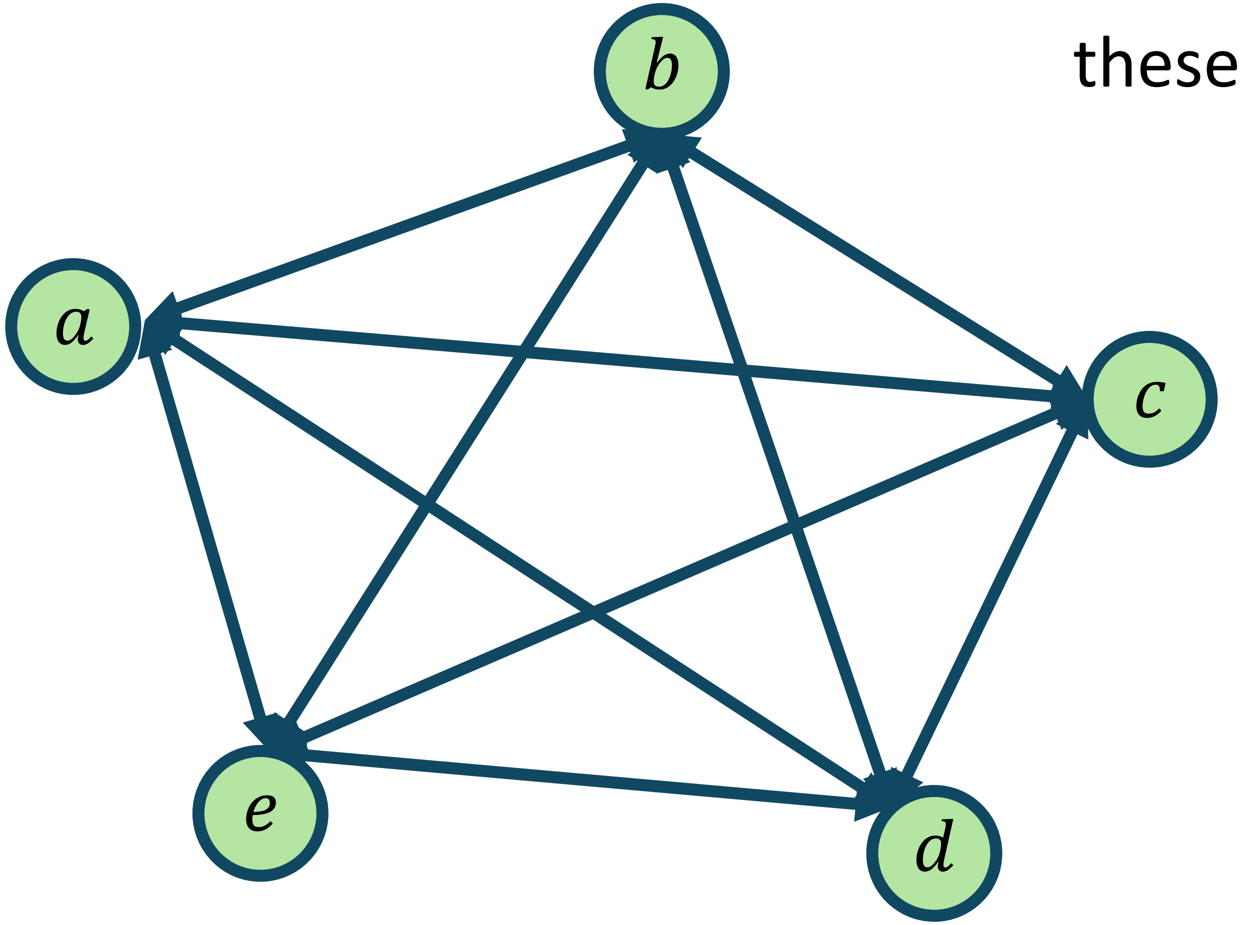
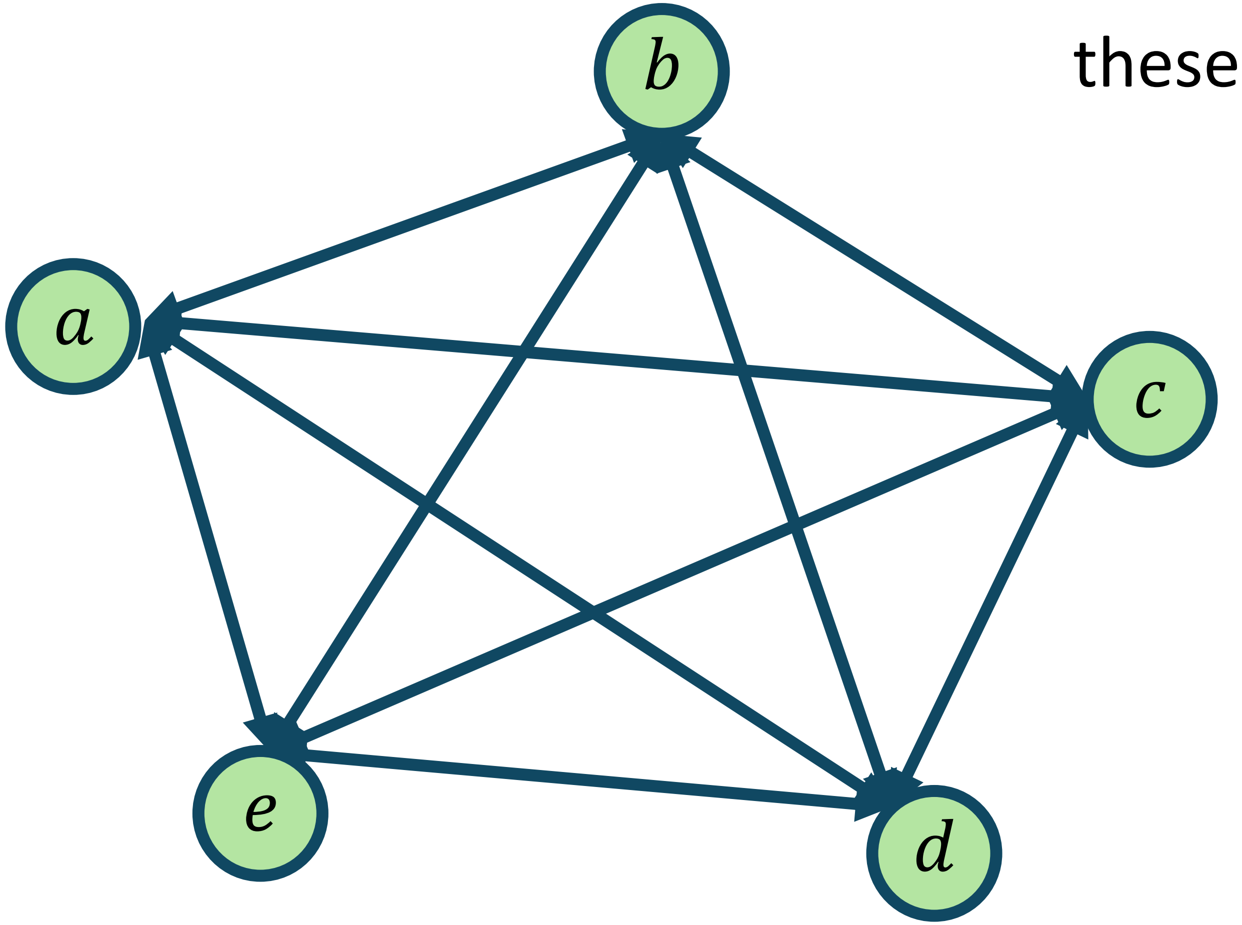


Periodically rotate through these connections

Reconfigurable Networks



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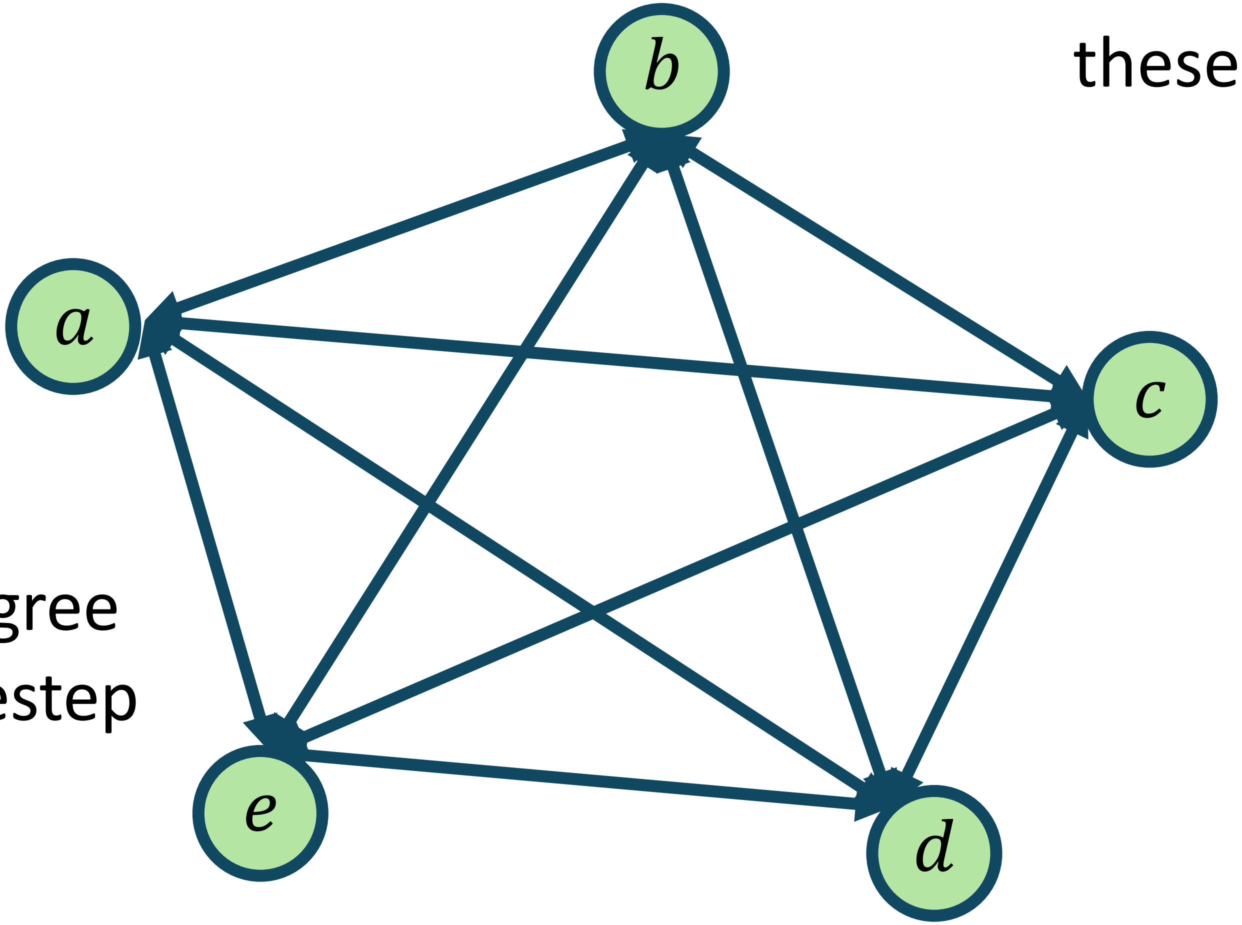
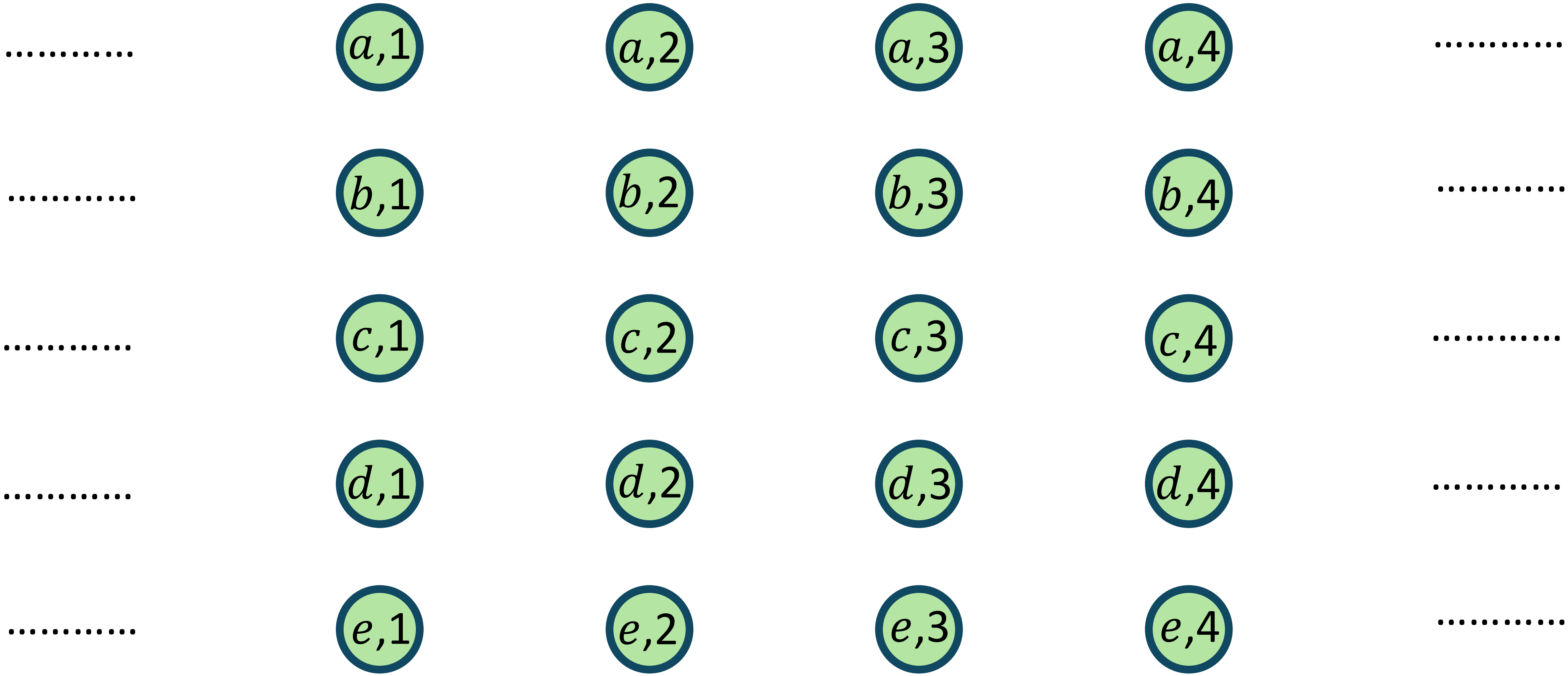


In- and out-degree 1 at every timestep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$a,3$
$a,4$
$b,1$
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$c,2$
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$d,2$
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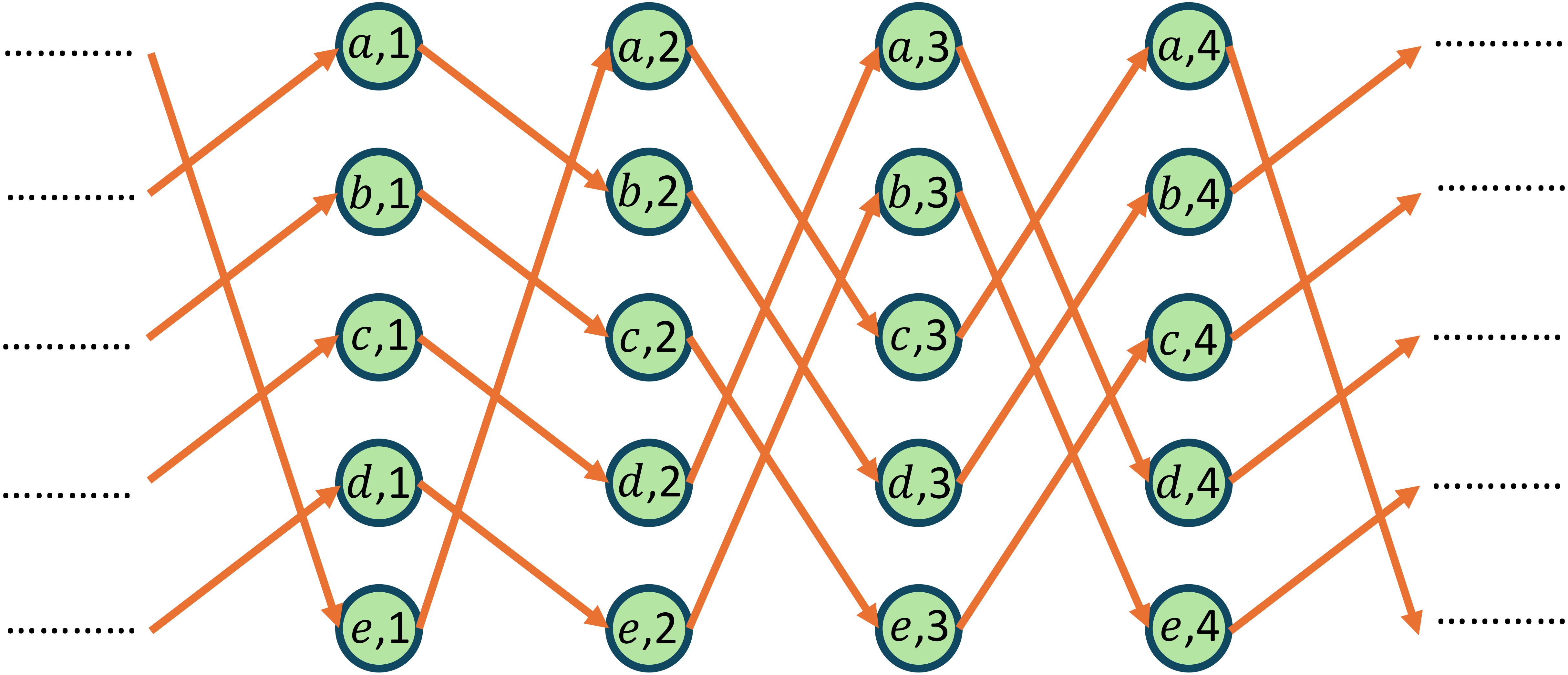
$e,4$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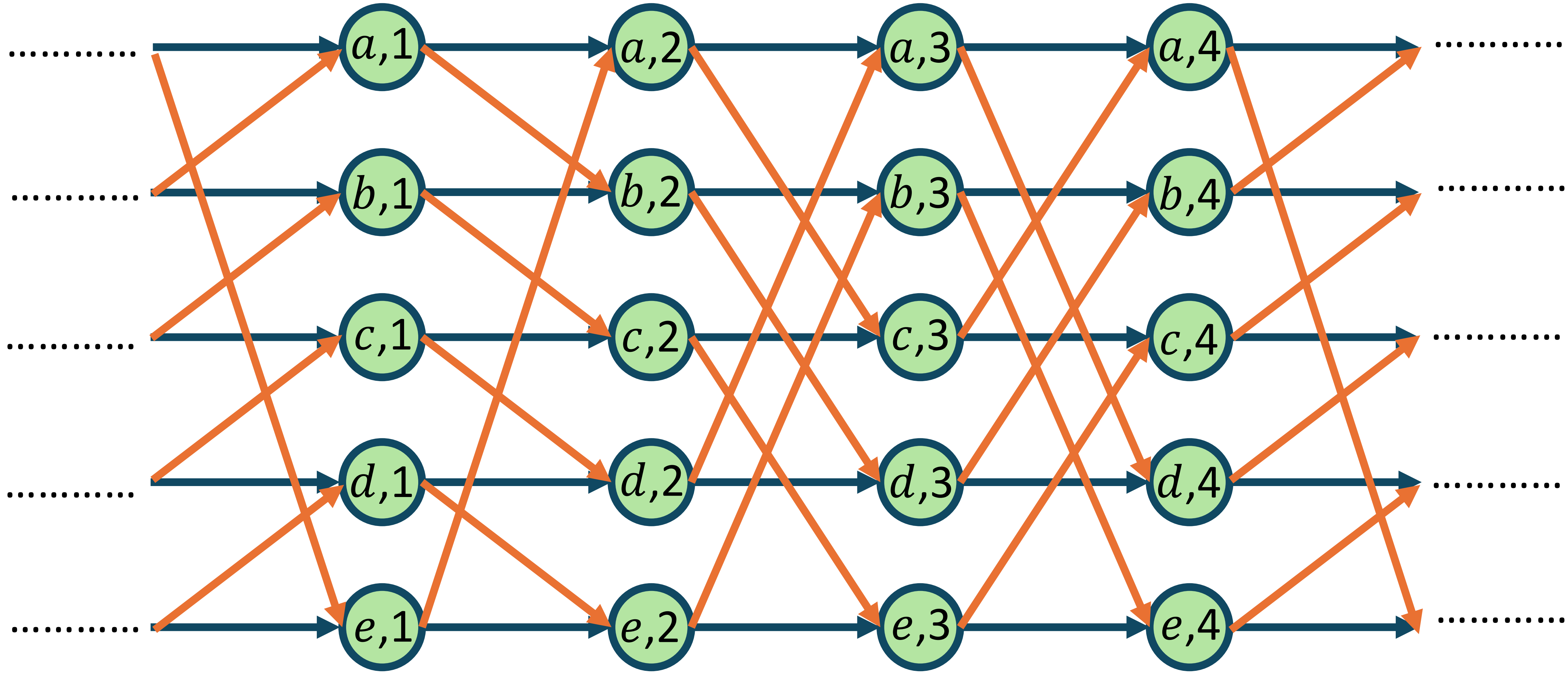
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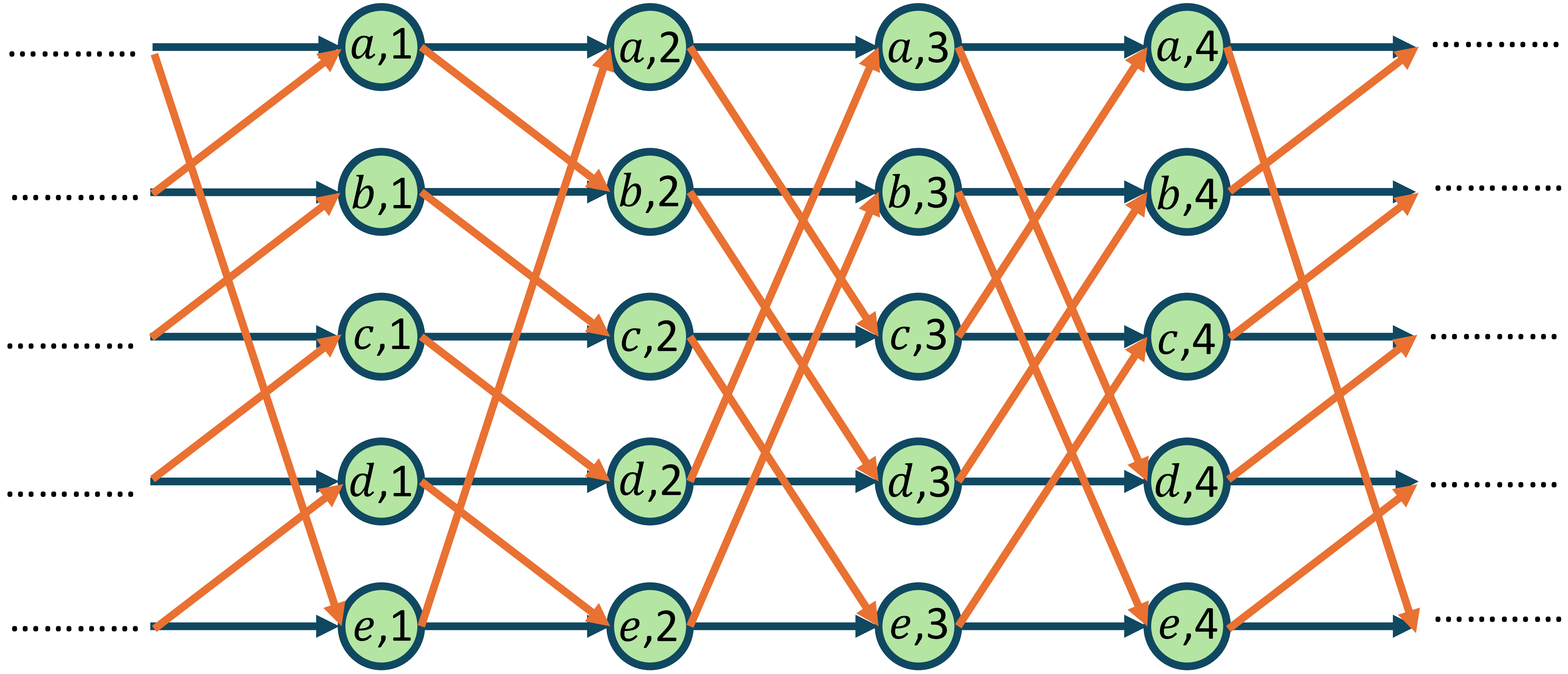
To route $a \rightarrow d$ starting at $t = 1$,



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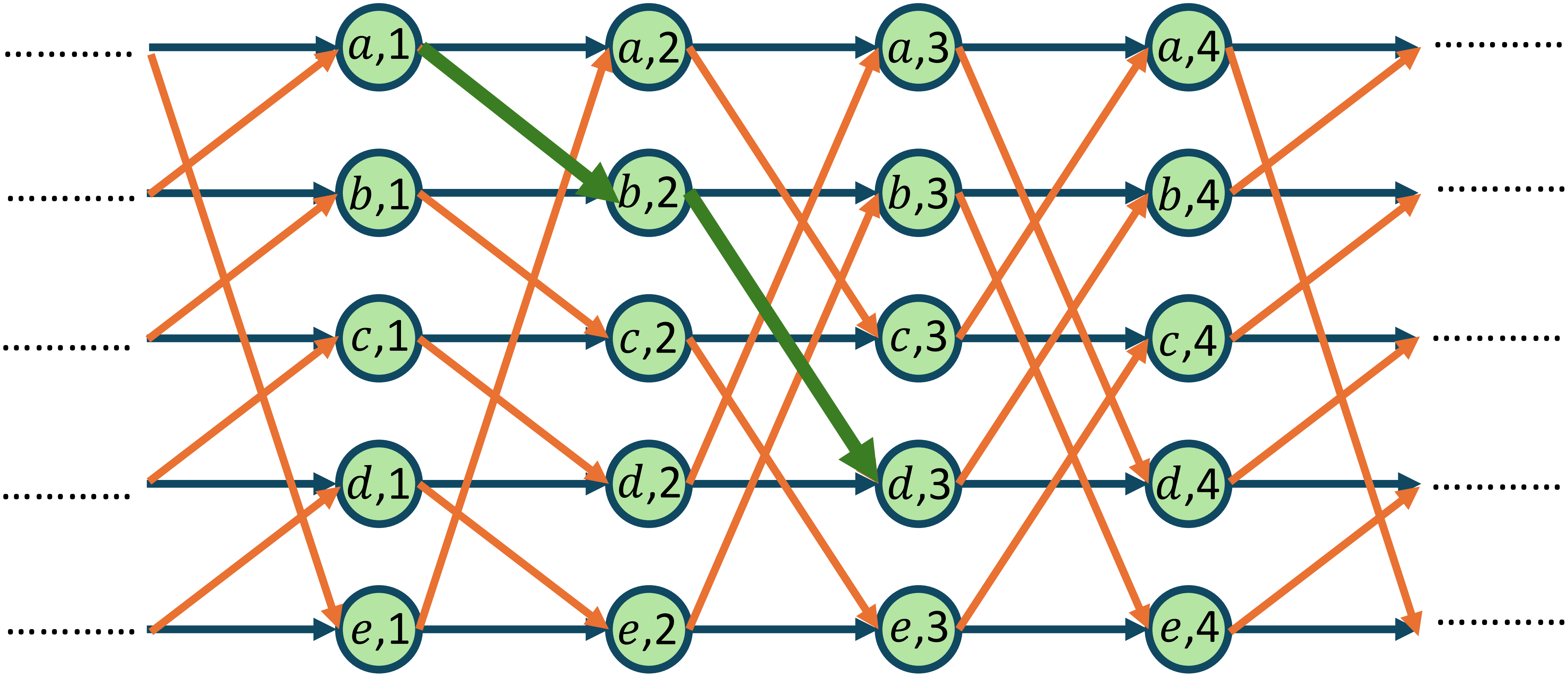

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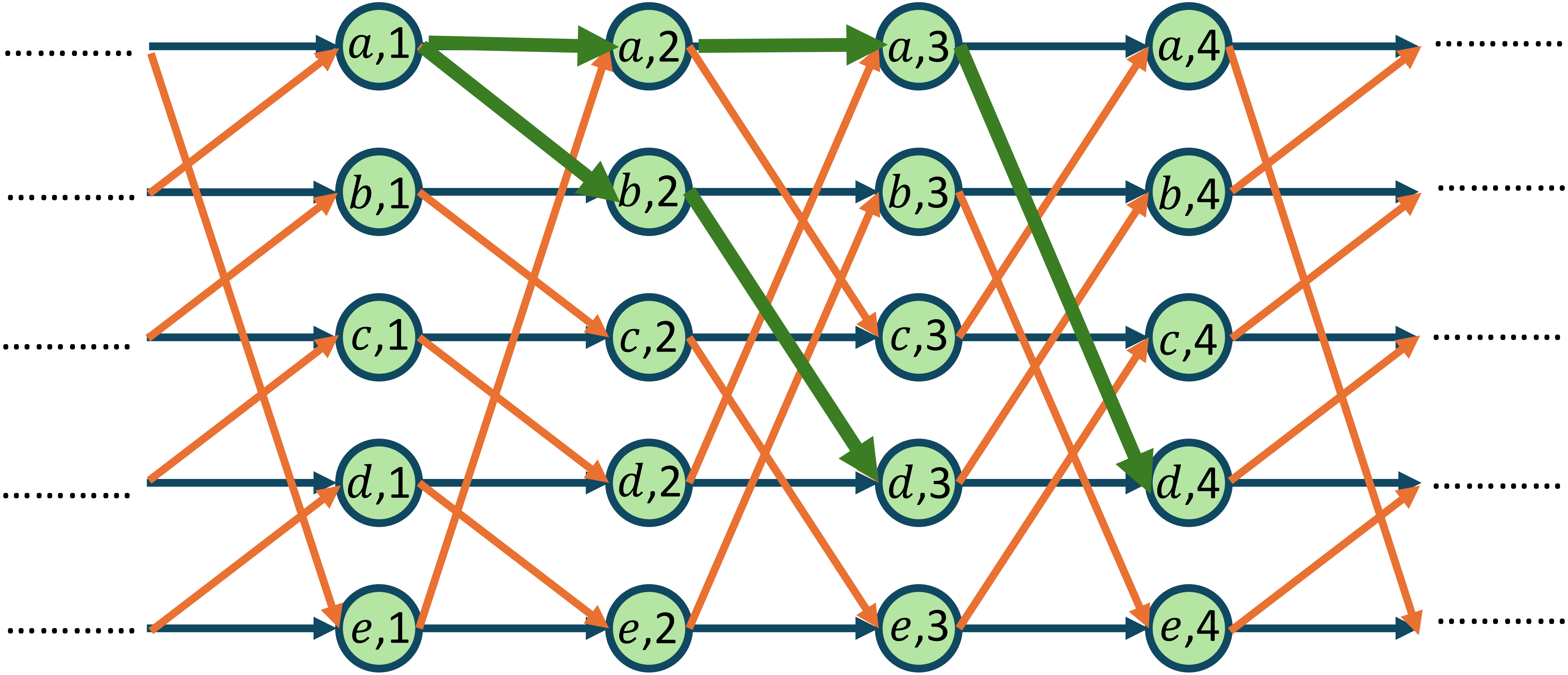
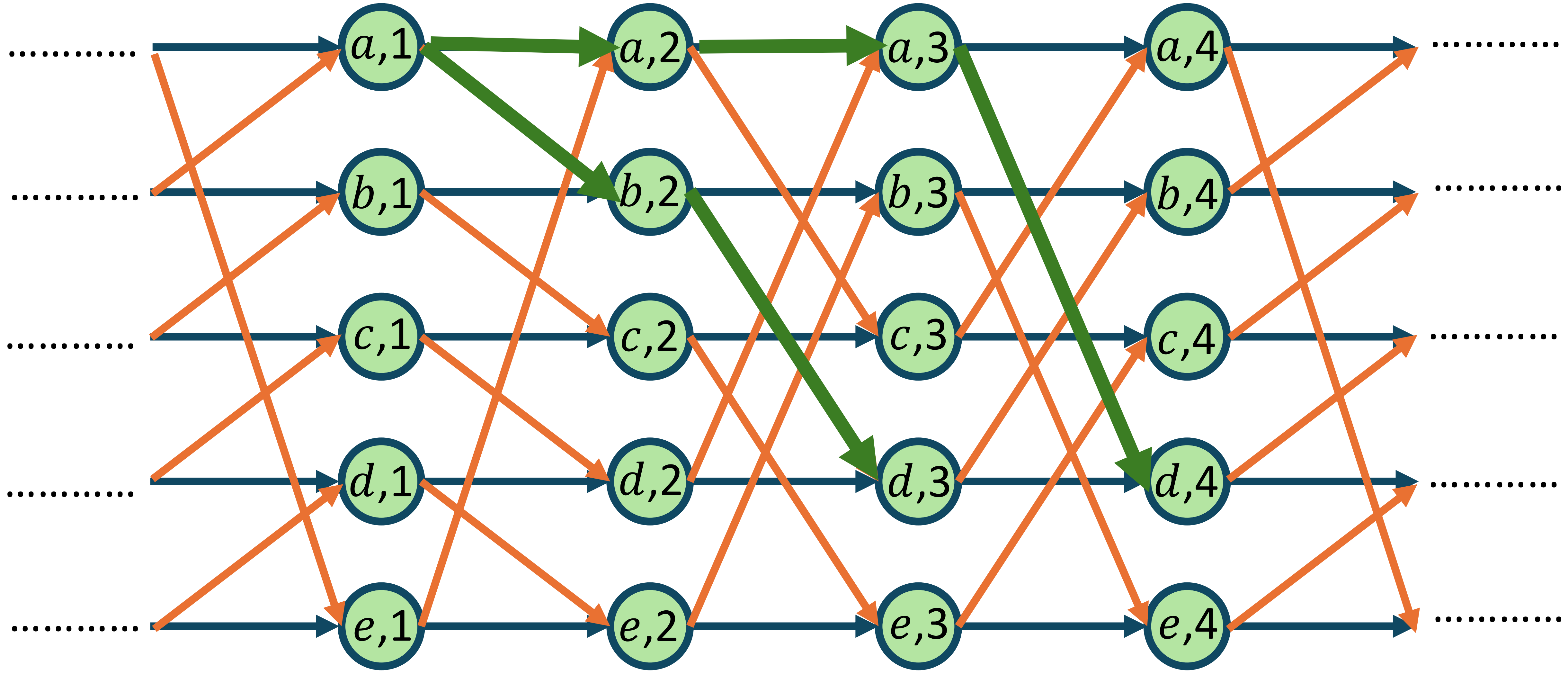


To route $a \rightarrow d$ starting at $t = 1$,

Build oblivious routing protocol with *bounded max latency* L



Congestion

At each timestep t we will receive arbitrary permutation demand D_{σ_t}

- $\forall a$, send 1 unit of flow from $a \rightarrow \sigma_t(a)$ starting at timestep t.

An oblivious routing protocol *guarantees* max congestion c if $\forall D_{\sigma_t}$ across all time, the max flow traversing any physical edge is $\leq c$.

Congestion on virtual edges is ignored

**If flow is balanced evenly across edges,

max congestion = average physical hop count

[Valiant & Brebner, STOC‘81]

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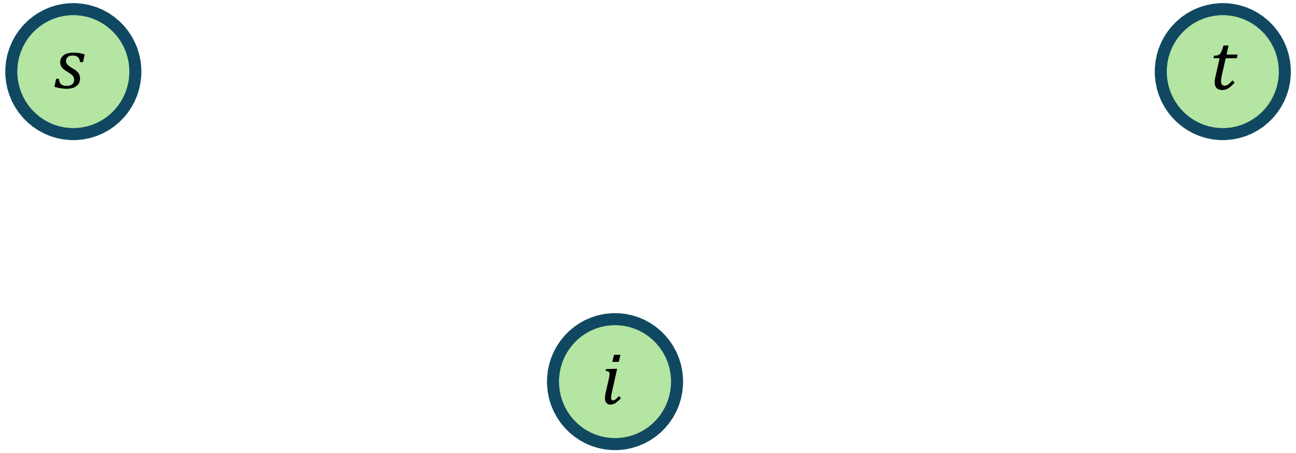

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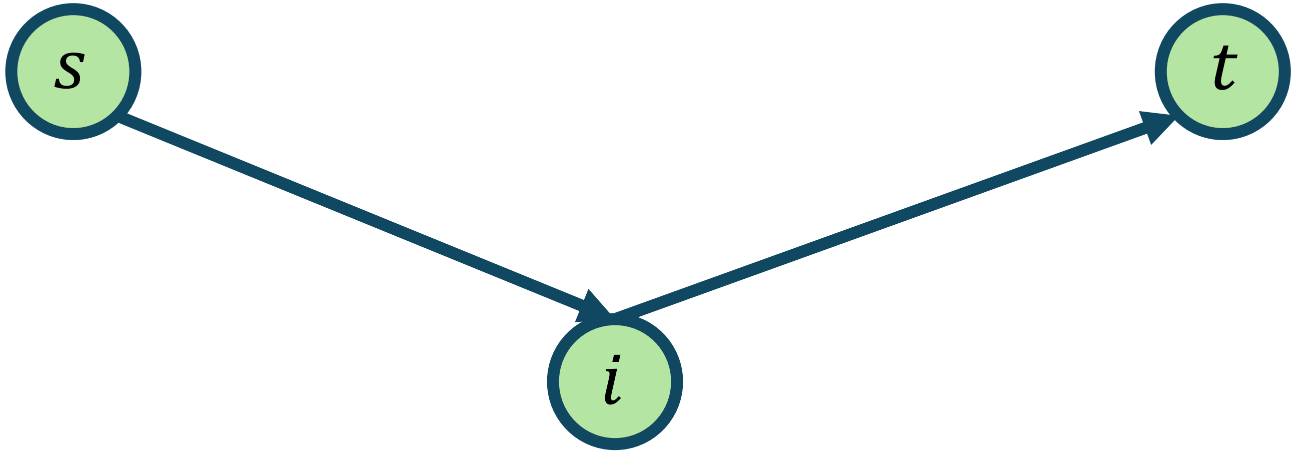

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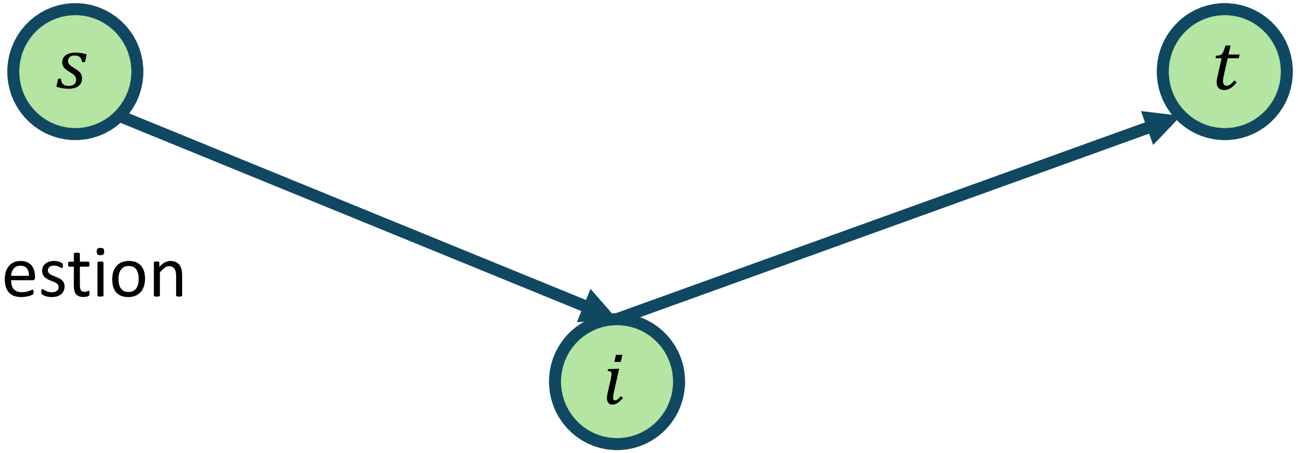

Doubles congestion
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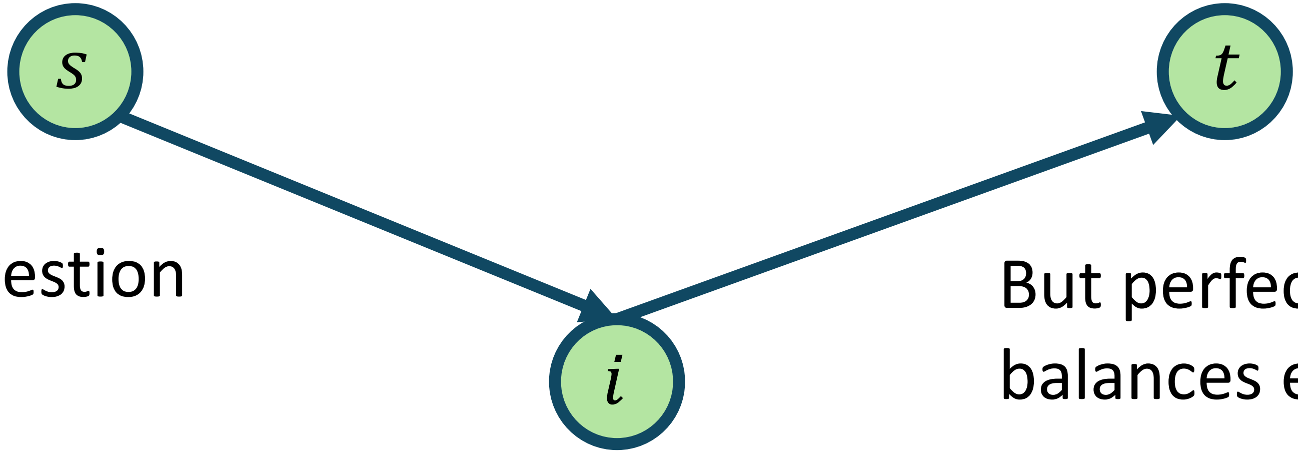

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But perfectly load balances edges

Valiant Load Balancing

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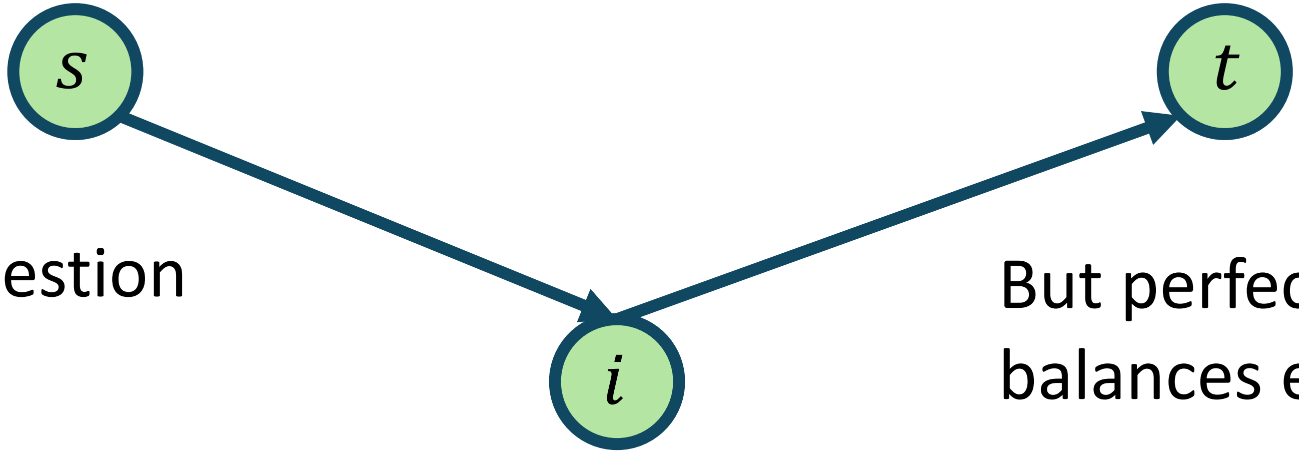

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VLB Factor 2 Overprovisioning is Optimal for:

- Static networks with fixed-capacity links
  [Shen & McKeown’05][KCML’05]
  [Babaioff & Chuang ’07]

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How to improve?

Räcke’s Hierarchical Tree Decomposition

[Räcke, STOC‘08]

- $\mathrm{O}(\log n)$-competitive and optimal oblivious routing protocol for *general networks*

- For optimized topologies in datacenters, even factor 2 overprovisioning is undesirable

We show that the *ability to randomize* a reconfigurable network allows oblivious routing protocols that break the “VLB Barrier”

Given a latency bound of $\tilde{O}\left(gN^{1/g}\right)$ for integer g:

Goal	Average Hop Count	Congestion	
Full Network Connectivity (lower bound)	g	$-$	Naïve counting

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Oblivious Routing (prob. 1)	$2g$	$2g$	[AWSKWA'22] (uses VLB)

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Probability that the congestion bound is violated is *negligible* in the network size

High-Level Overview

- Instead of routing to uniform random node i
  - Take a single physical hop to random neighbor
  - Then route on a shortest path to destination
  - Use randomness of connection schedule to prove load is effectively balanced
- Analysis relies on a complicated tail bound
  - Bilinear form on an orbit of a permutation group action
  - Negative association + suitable decomposition and conditioning

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Thank you!

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