Advanced VLSI Design (ECE 695KR)

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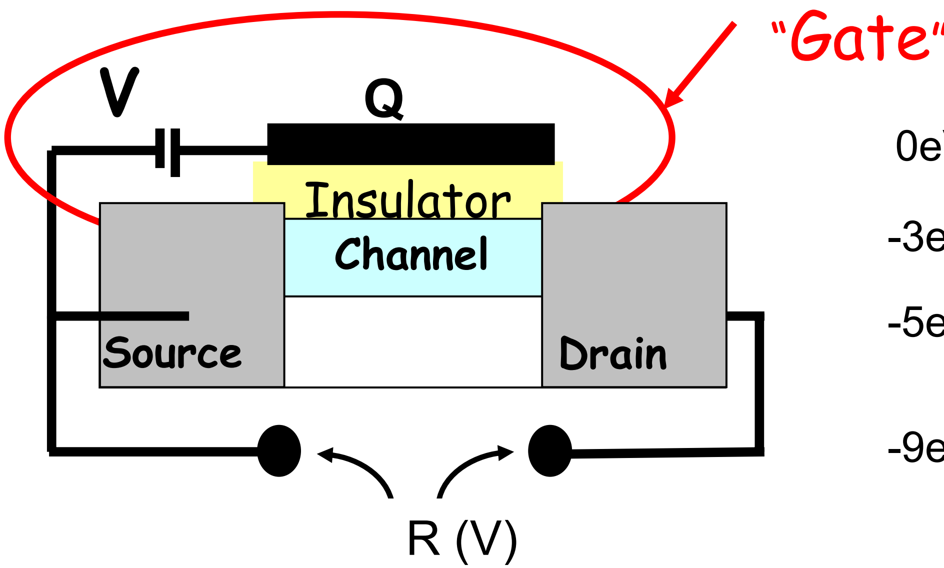
Outline

- Transistors – channel having few energy states
- Energy band diagram
- Current flow & I-V Characteristics
- Subthreshold Leakage
- Generalization to larger transistors

Acknowledgement: Professor Supriyo Datta

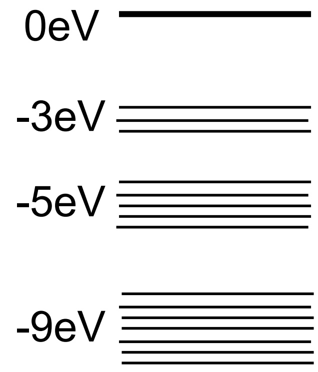
Transistors



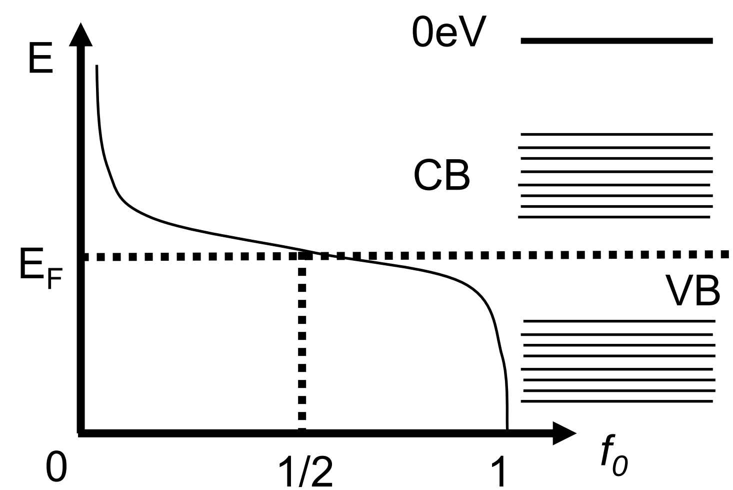
Field Effect Transistor

How to understand I-V characteristics

- Band diagram

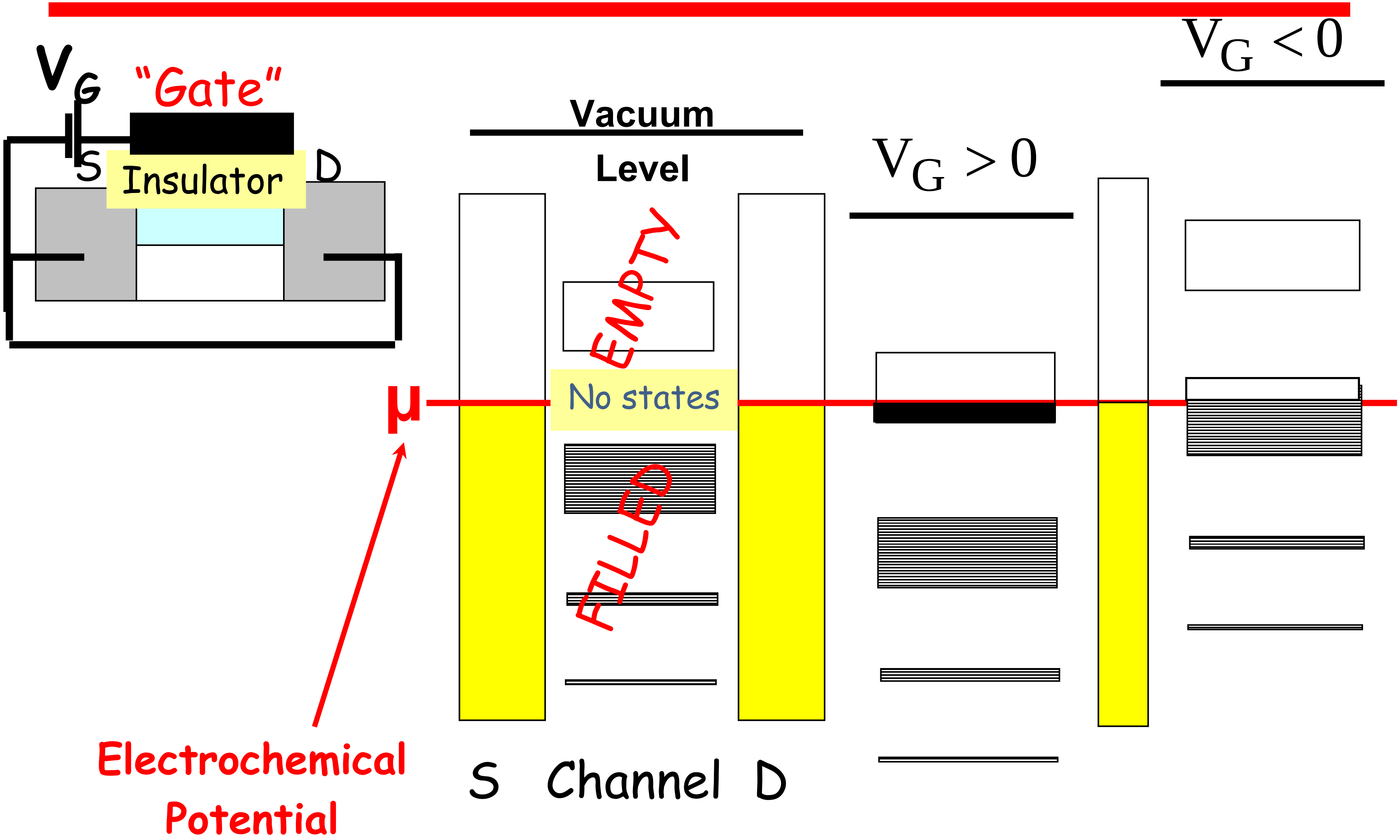


- Attributes of a good switch
  - Good gate control on channel
  - Less control of drain on channel
  - High ON current, less OFF current



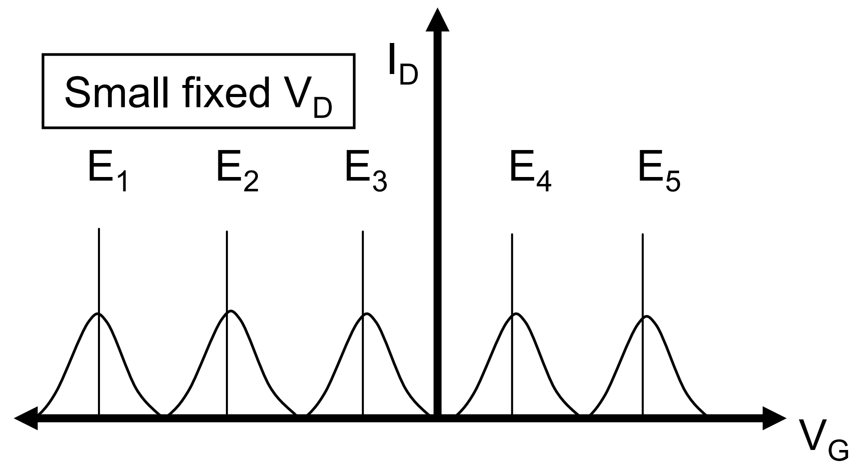
Distribution of electrons over a range of allowed energy levels: $f(E) = 1/(e^{(E-EF)/kT}+1)$

Gate Bias



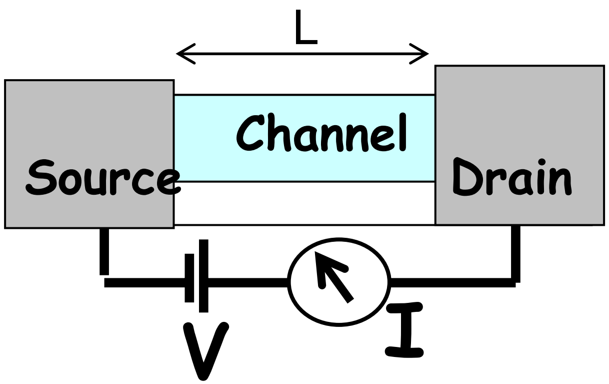
$V_G < 0$
V_G
"Gate"
S
Insulator
D
Vacuum
Level
$V_G > 0$
EMPTY
μ
No states
FILLED
Electrochemical
Potential
S Channel D

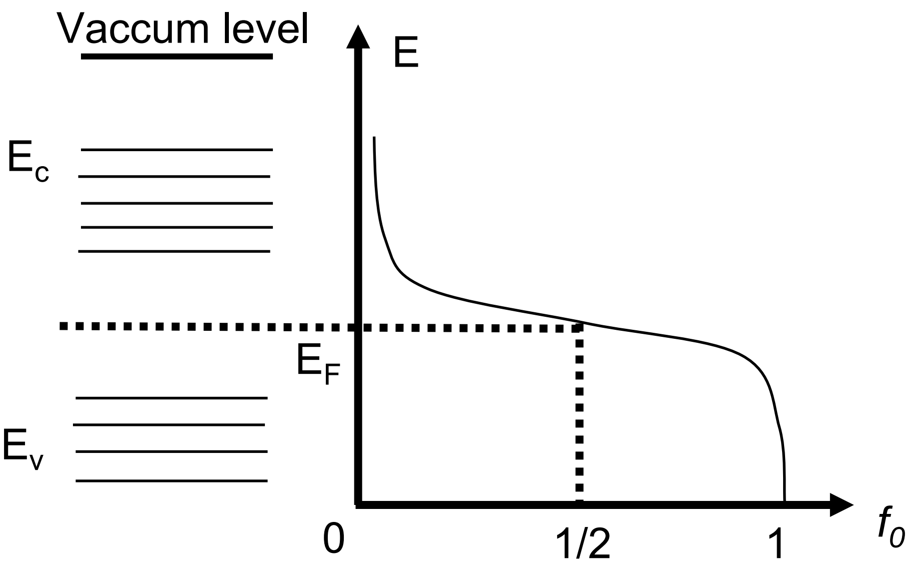
Simple Energy Level Scenario



- Peak occurs where E_F crosses an energy level due to an applied V_G bias

Transistors: Key Concepts





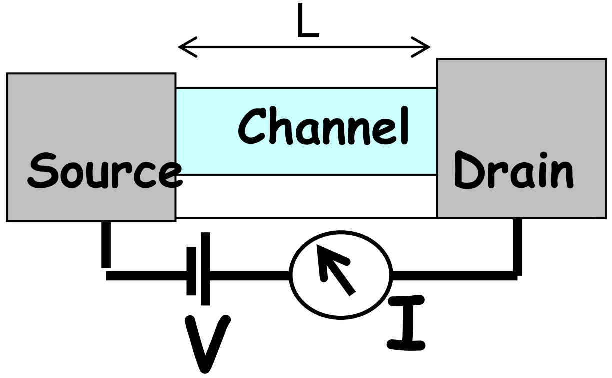
- Key concepts: V_D, V_G, empty and full energy levels, V_T, E_F
- Fermi function is centered at Ø

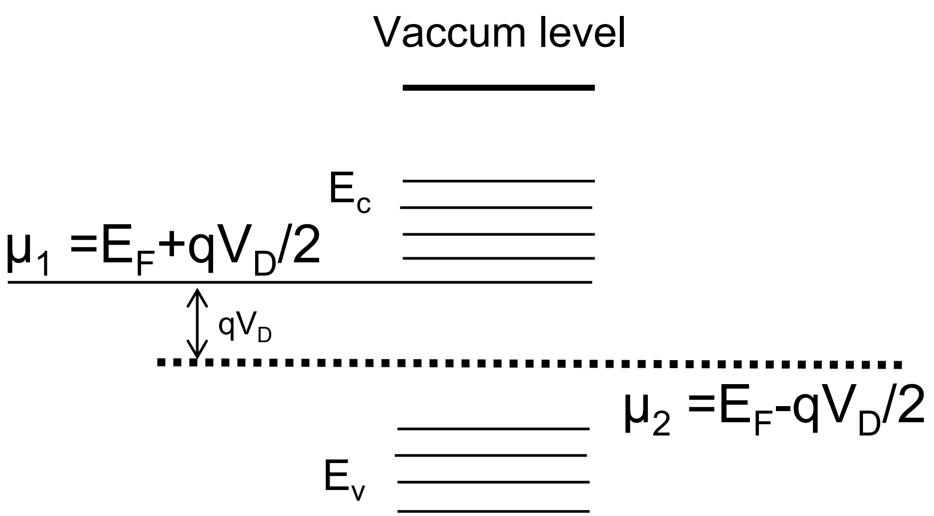
$$f_0(E) = 1/(e^{E/kT}+1)$$

- To get it centered at E_F, shift it

$$f_0(E-E_F) = 1/(e^{(E-EF)/kT}+1)$$

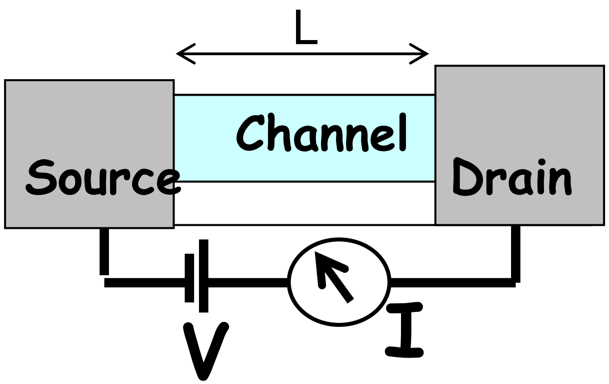
Application of Drain Bias

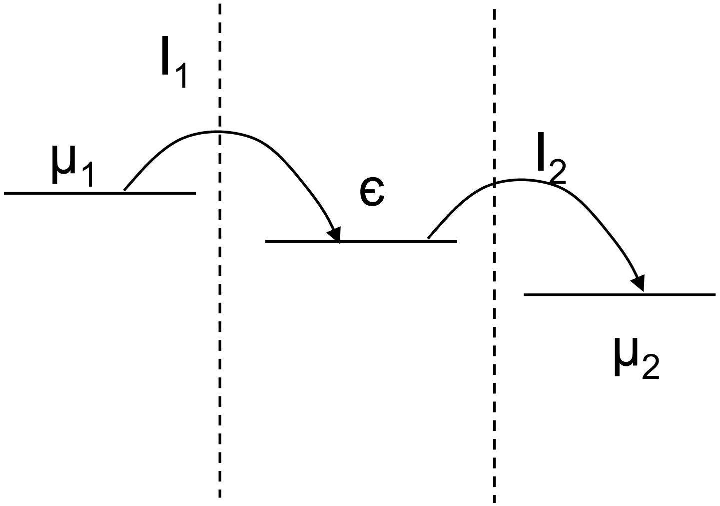




- Total energy difference between μ_1 and μ_2 is $qV_D = 1\text{V} \times q = 1\text{eV} = 1.6 \times 10^{-19}$ J

Current Flow



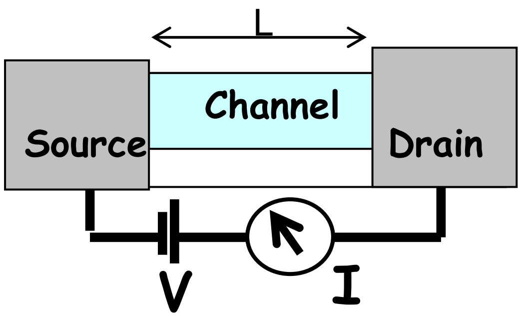


- N_1: Avg. # of electrons that the left contact would like to see = $2f_1(\epsilon) = 2f_0(\epsilon - \mu_1)$
- N_2: $N_2 = 2f_2(\epsilon) = 2f_0(\epsilon - \mu_2)$

Accounts for up-spin and down-spin that is possible at a level

Current Flow

- N: Actual # of electrons at steady state in the channel



- I_1: $q(\gamma_1/\hbar)(N_1\text{-}N)$
- I_2: $q(\gamma_2/\hbar)(N\text{-}N_2)$
- $\gamma/\hbar$: rate at which electrons cross (escape rate)
- $\hbar = h/2\pi = 1.06\times10^{-34}$ J.sec
- γ_1 and γ_2 are in units of Joule

Ex: γ_1=1meV

$\gamma_1/\hbar = 1.6\times10^{-19}/1.06\times10^{-34} = 10^{-12}$ /sec

= 1psec for electron to escape into the channel

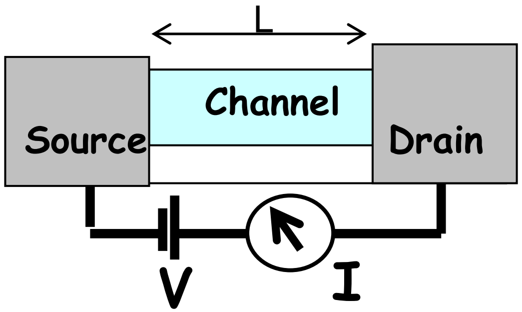
Current Flow

At steady state, $I_1 = I_2$

➔ $N = (N_1\gamma_1 + N_2\gamma_2) / (\gamma_1 + \gamma_2)$

$I = I_1 = I_2 = (q/\hbar)\,(\gamma_1\gamma_2 /\, \gamma_1 + \gamma_2)\,(N_1 - N_2)$

$= (2q/\hbar)\,(\gamma_1\gamma_2 /\, \gamma_1 + \gamma_2)\,[f_1(\epsilon) - f_2(\epsilon)]$



N type conduction: go thru level that is empty at equilibrium

P type conduction: go thru level that is full at equilibrium

At small voltages (use Taylor series expansion)

$f_1(\epsilon) = f_0(\epsilon - \mu_1)$, $f_2(\epsilon) = f_0(\epsilon - \mu_2)$

$f_1 - f_2 = (\delta f_0/\delta E)(\mu_2 - \mu_1) = -(\delta f_0/\delta E)qV_D$

Therefore,

$I = (2q/\hbar)\,(\gamma_1\gamma_2/\gamma_1 + \gamma_2)[f_1(\epsilon) - f_2(\epsilon)]$

$= V(2q^2/\hbar)\,(\gamma_1\gamma_2/\gamma_1 + \gamma_2)[-\delta f_0/\delta E]$

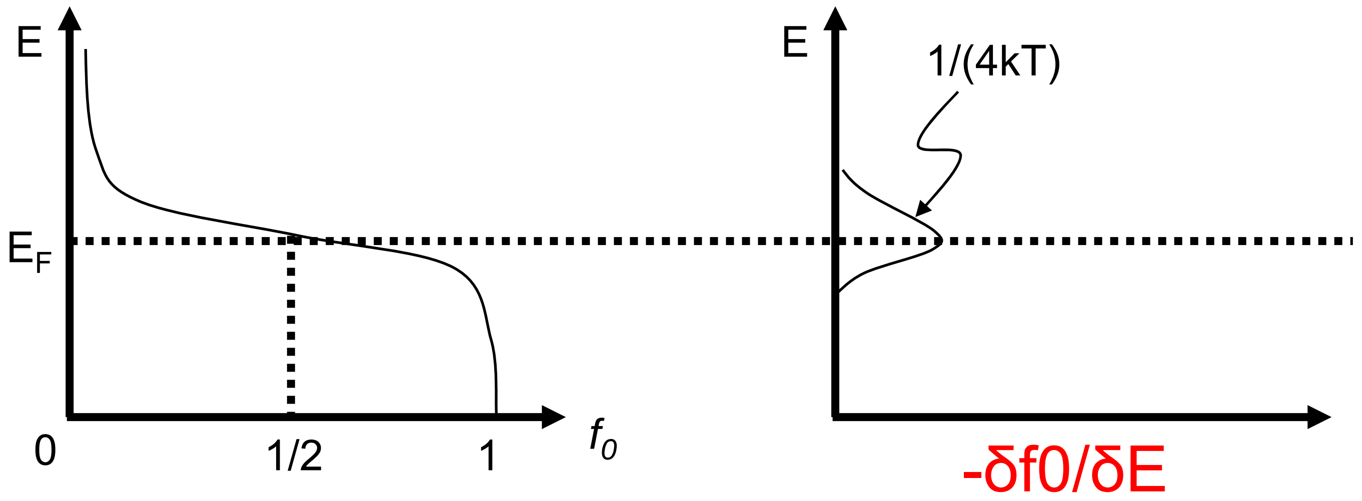
Current Flow

Use $E = \epsilon - E_F$ Since $\mu_1 = E_F + qV_D/2$, $\mu_2 = E_F - qV_D/2$

$2q^2/\hbar$: dimension of conductance

$\gamma_1\gamma_2/\gamma_1+\gamma_2$: dimension of energy

$\delta f_0/\delta E$: dimension of inverse energy



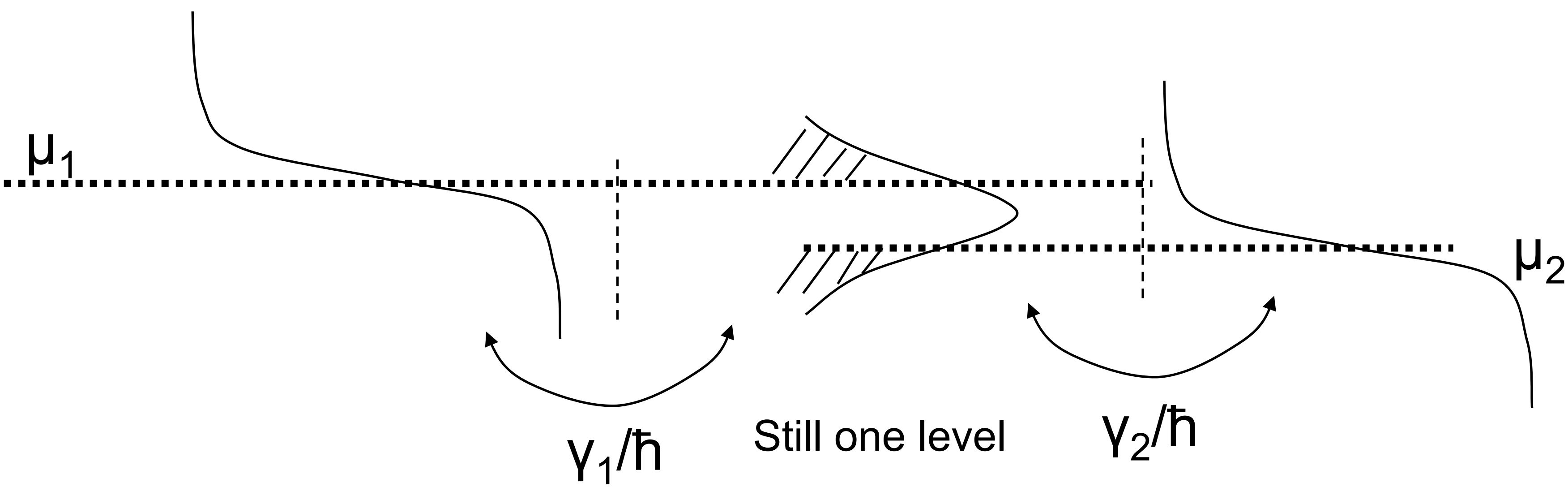
Current Flow

I/V = conductance = $(2q^2/\hbar)\,(\gamma_1\gamma_2/\gamma_1+\gamma_2)[-\delta f_0/\delta E]$

If $f_1 - f_2 = 1$ and $\gamma_1 = \gamma_2$; $I = q\gamma_1/2\hbar$

Seems to indicate that there is no limit to conductance but in reality, we do have a limit

$R_{min} = h/2q^2 = (6.6\text{x}10^{-34}\text{J-s})/2\text{x}(1.6\text{x}10^{-19})^2 = 12.9\text{k}\Omega$



$I=(q/\hbar)\,(\gamma_1/2)(qV_D/2\gamma_1) = q^2V_D/4\hbar$

Broadening

Only a fraction of levels contribute to current

Broadening

- Each electronic level has a wavefunction Ψ associated with it
  - With no coupling, $\Psi \propto e^{-i\epsilon t/\hbar}$ ➔ time domain
  - Energy domain (Fourier transform) we get an impulse response
- Ψ^2 : probability of finding the electron at a point
  - $|\Psi|$ is 1 for the above expression of Ψ^2.
- After coupling the waveform gets modified
  - $\Psi \propto e^{-i\epsilon t/\hbar} e^{-t/2\zeta}$ **: lifetime associated with electron**
  - ζ : lifetime --- the probability of finding the electron in the channel
  - Fourier transform of new Ψ gives the density of states

$$D(E) = (\gamma/2\pi)/((E-\epsilon)^2 + (\gamma/2)^2),\ \gamma = \gamma_1 + \gamma_2 = \hbar/2\zeta$$

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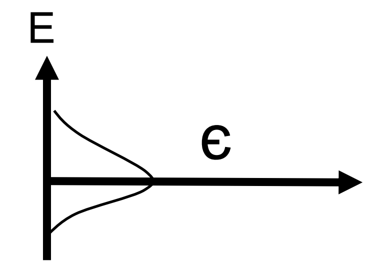
Broadening

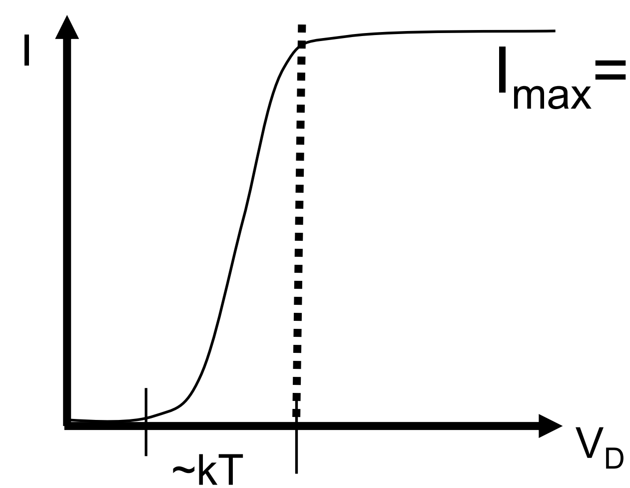
$I = (q/\hbar)\,(\gamma_1\gamma_2/\gamma_1+\gamma_2)[f_1-f_2]$
$\;\; = \int dE D(E)\,(q/\hbar)\,(\gamma_1\gamma_2/\gamma_1+\gamma_2)[f_1-f_2]$



$N = \int D(E)dE(\gamma_1 f_1+\gamma_2 f_2/\gamma_1+\gamma_2)$

$\int D(E)dE = 1$

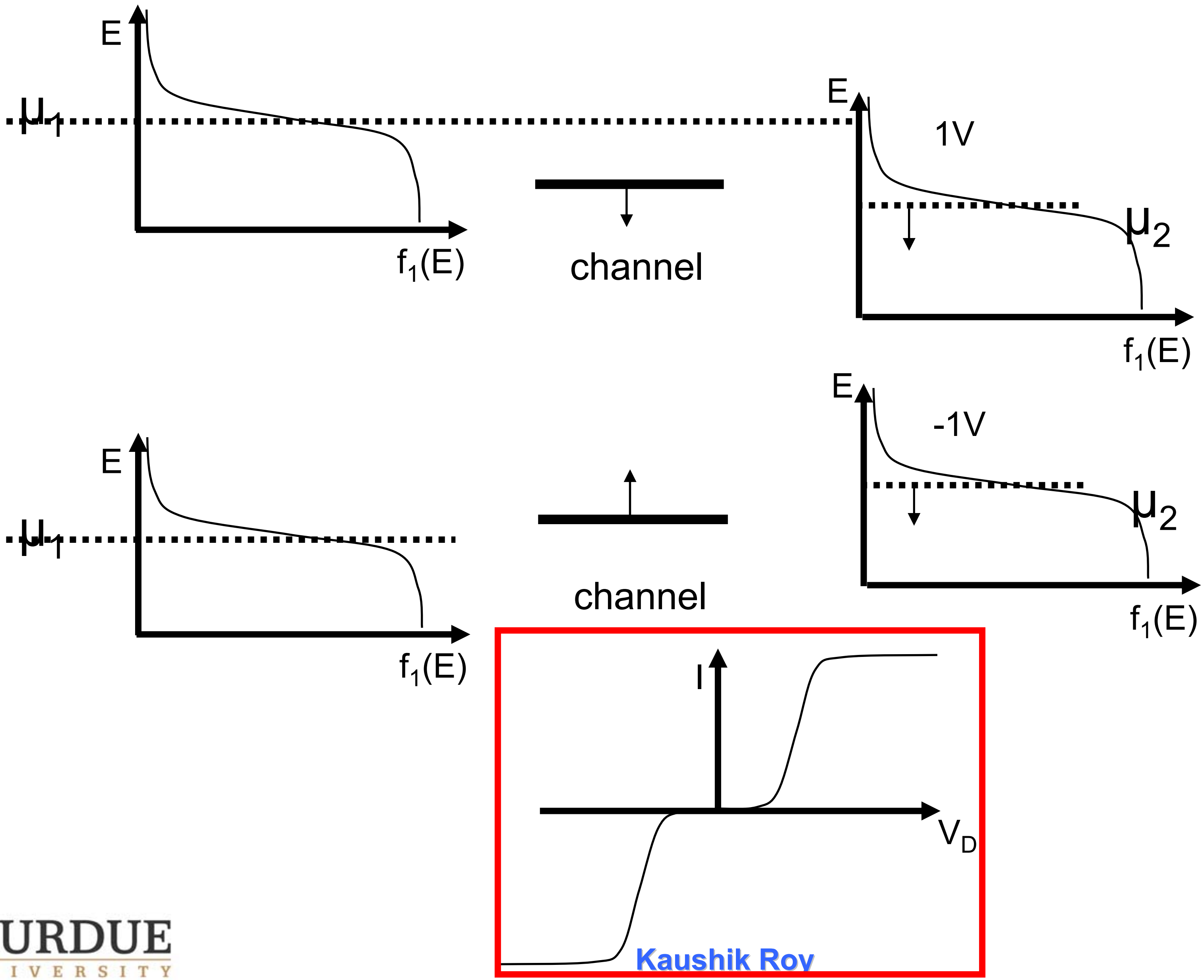
If $f_1-f_2=1$,
$I = -(q/\hbar)\,(\gamma_1\gamma_2/\gamma_1+\gamma_2)\int D(E)\,dE$



$I_{max} = -(q/\hbar)\,(\gamma_1\gamma_2/\gamma_1+\gamma_2)$

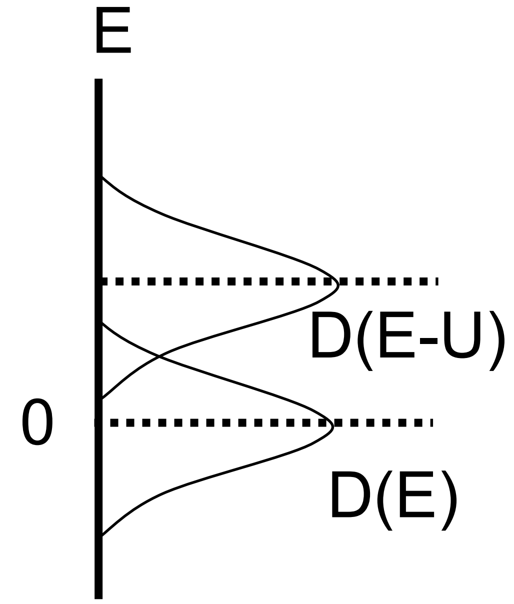
But the channel potential gets modulated by the drain voltage!

Current Flow



Channel Potential

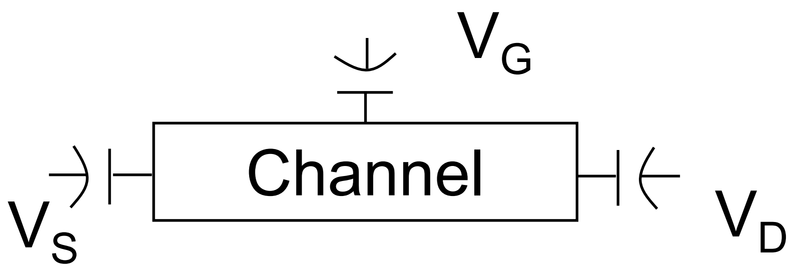
The effect of U (potential in the channel) is to move the density of states up or down depending on the sign of U



$I = -(q/\hbar) \int D(E-U)dE(\gamma_1\gamma_2/\gamma_1+\gamma_2)$
$N = \int D(E-U)dE(\gamma_1 f_1+\gamma_2 f_2/\gamma_1+\gamma_2)$

In order to find U in general, we need to solve Poisson's equation $d^2V/dx^2 = -(q)\Delta n/\epsilon$ (assume U is the same all over the channel)

Channel Potential



U: Potential energy

V: Electric potential in a certain region

$U = -qV$

Amount of charge in channel = $-q\Delta n = C_S V + C_G(V-V_G) + C_D(V-V_D)$

With V_S grounded,

$V = (C_G V_G + C_D V_D)/(C_S + C_G + C_D) + (-q\Delta n)/(C_S + C_G + C_D)$

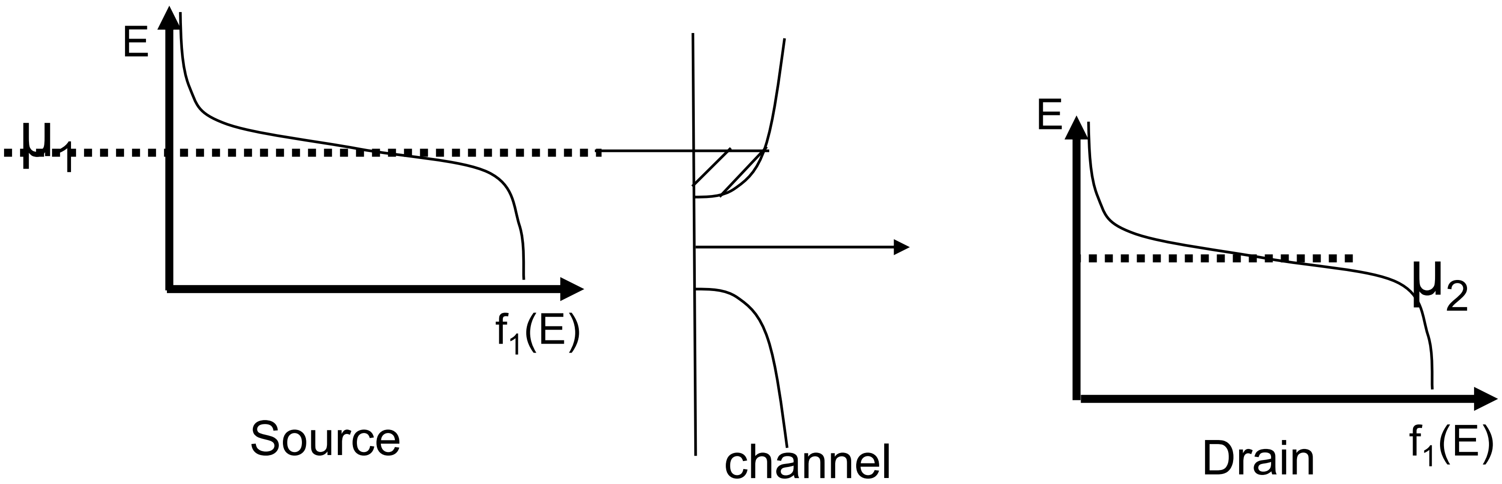
$U = -qV = -q(\ldots\ldots\ldots\ldots\ldots\ldots)$

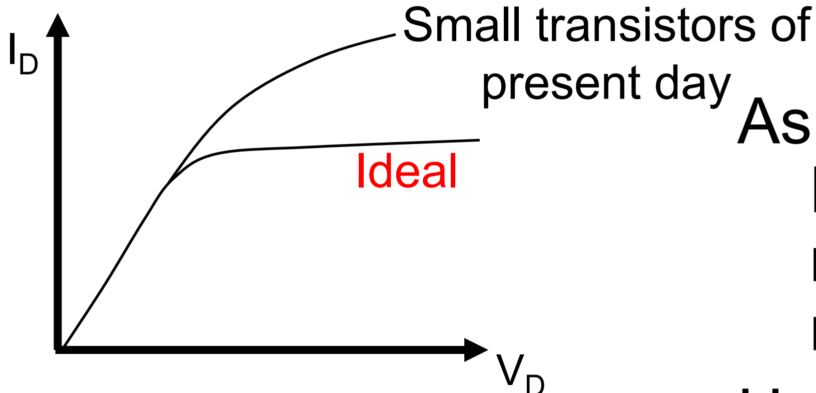
$U = U_L + q^2/C_E \Delta n$; $\quad C_E = C_S + C_G + C_D$

Single electron charging energy

Small devices, C_E is small and q^2/C_E is large → can change a lot of things

Channel Potential





As drain levels are lowered, the DOS also wants to slide down→ more available states, hence more current

$$U_L = -q(C_G V_G + C_D V_D)/(C_S + C_G + C_D)$$

Good transistors: stop DOS sliding in channel. Make U_L as large as possible (C_G) to make effect of V_D negligible!

Good Transistor

- Increase C_G to make the effect of V_D negligible

$$U_L = -q(C_G V_G + C_D V_D)/(C_S + C_G + C_D)$$

- Gate as close as possible to the channel
  - If L=500A, then gate should be as close as 20A to the channel. If L is smaller, gate should be even closer, but gate leakage…

Current

$$I = -(q/\hbar) \int D(E-U)dE(\gamma_1\gamma_2/\gamma_1+\gamma_2)(f_1-f_2)$$

$$N = \int D(E-U)dE(\gamma_1 f_1+\gamma_2 f_2/\gamma_1+\gamma_2)$$

$$U = U_L+U_0\Delta N$$

Need to be solved self-consistently

$$U_0 = q^2/C_E$$

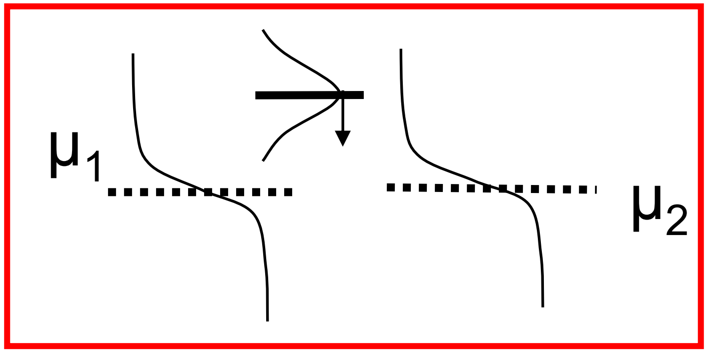
$$C_E = C_S + C_G + C_D$$

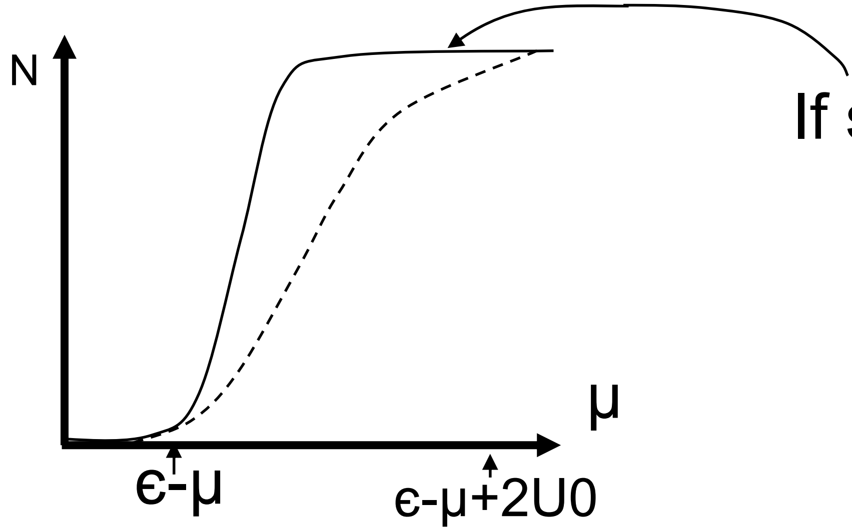
$$U_L = [C_G(-qV_G) + C_D(-qV_D)]/C_E$$

Single Electron Charging Energy

$N = \int D(E-U)dE(\gamma_1 f_1+\gamma_2 f_2/\gamma_1+\gamma_2)$

$U = U_L+U_0\Delta n$



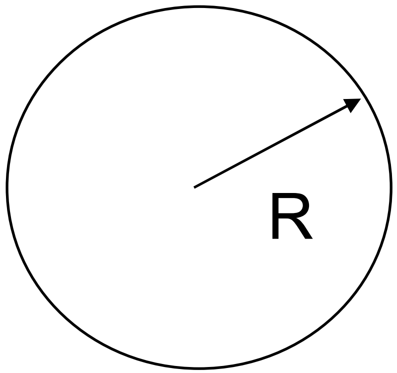


If self consistent solution not considered

Because of term U_0 in self-consistent solution, the level starts floating up as it gets filled with electrons→ making the filling up process slower!

Example

Sphere of charge



$U=q^2/4\pi\epsilon R$

$= 1.6x10^{-19}Coul/4x3.14x8.85x10^{12}\ F/mx10^{-7}m$

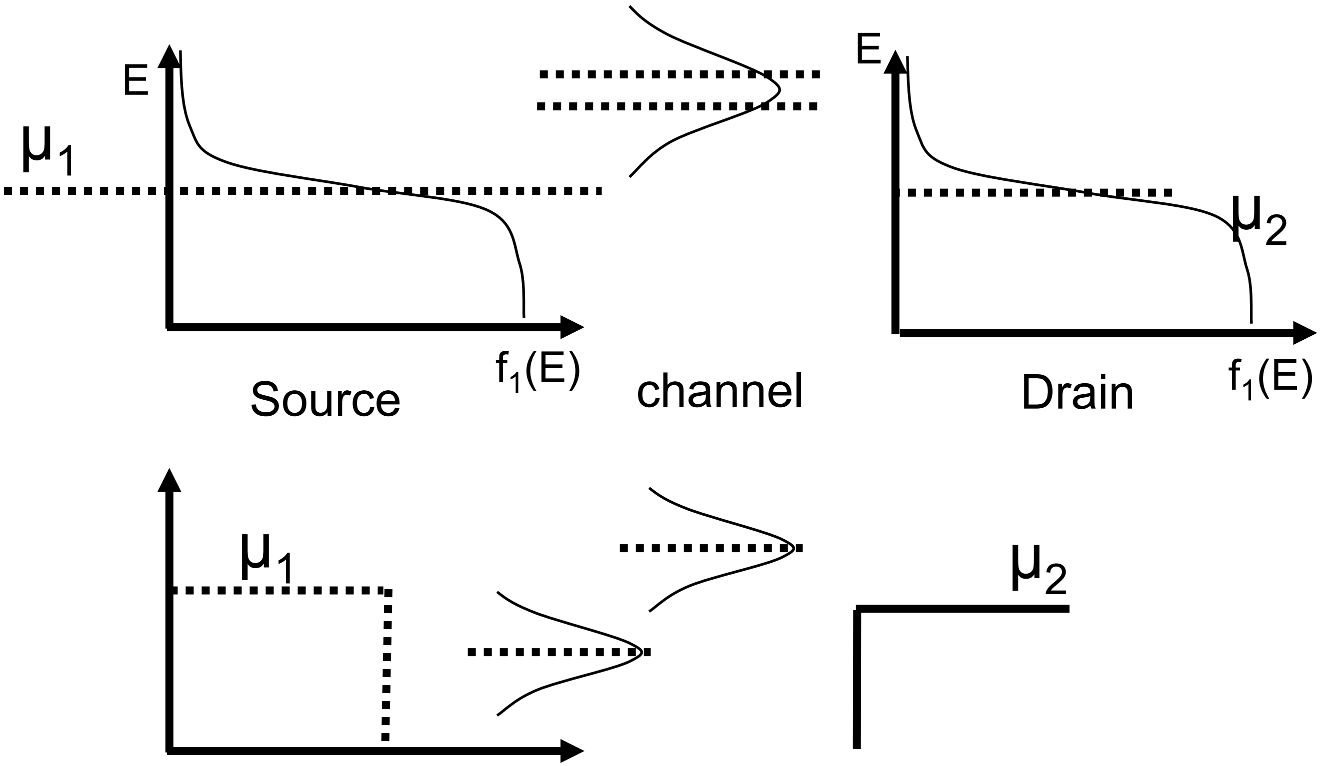
$\approx$14meV ~ order of kT

For small devices U_0 will be larger

$U = U_L+U_0(N-N_0)$

Big devices, U_0 is smaller, but $(N-N_0)$ is large

Current Flow



$U_0 >> kT + \gamma$ ➔ Coulomb blockage charging energy U_0 exceeds broadening γ.

Conductance (Revisited)

$I = -(q/\hbar) \int D(E-U)dE(\gamma_1\gamma_2/\gamma_1+\gamma_2)(f_1-f_2)$

For small applied voltage

$I = -(q/\hbar)\, D(E)\, (\gamma_1\gamma_2/\gamma_1+\gamma_2)qV_D$

Ohm's law: G α A/L

- More states, more current & larger devices have more states. D α WL➔ contradicts Ohm's law??
  - γ decreases as 1/L; → $\gamma_1\gamma_2/\gamma_1+\gamma_2$ α 1/L
  - L cancels out!
- Conductance independent of L! Ballistic device

xxx

Bottom-up view leads to ($\gamma_1=\gamma_2$ and U=0)

I= $(q\ \gamma_1/\hbar) \int D(E)dE\ (f_1(E)-f_2(E))$

For small applied voltage

Usual top down view yields an expression

I= $-(S/L)n\bar{\mu}(\mu_1 - \mu_2)$

S: cross sectional area; (1/L): inverse of length; n: electron density; $\bar{\mu}$: mobility

- Relate broadening to diffusion instant D, $\gamma_1=2\hbar D/L^2$
- Fermi fn approximated by Boltzmann fn ("non-degenerate" assumption) $f_1(E)-f_2(E))=e^{(E-\mu 1)/kT}(\mu_1 - \mu_2)/kT$
- D/ $\bar{\mu}$ = kT/q (Einstein's relation) and

$nSL = \int D(E)dEe^{(E-\mu 1)/kT}$

Subthreshold

$n_s = \int D(E)dE\, f(E-\mu)$
$= \int D(E)dE\, e^{(E-\mu)/kT}$
$= e^{\mu/kT} \int D(E)dE\, e^{-\mu/kT}$

With gate bias:

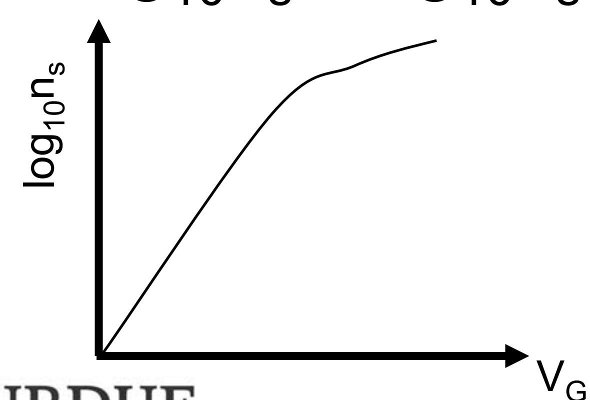
$n_s = e^{qVG/kT}[\ldots]$

(equivalent to lowering D(E) or raising μ)

$n_s = n_s(0)e^{qVG/kT}$

$\log_{10}n_s = \log_{10}n_s(0) + qV_G/kT\log_{10}e;\ \log_{10}e = 0.43$

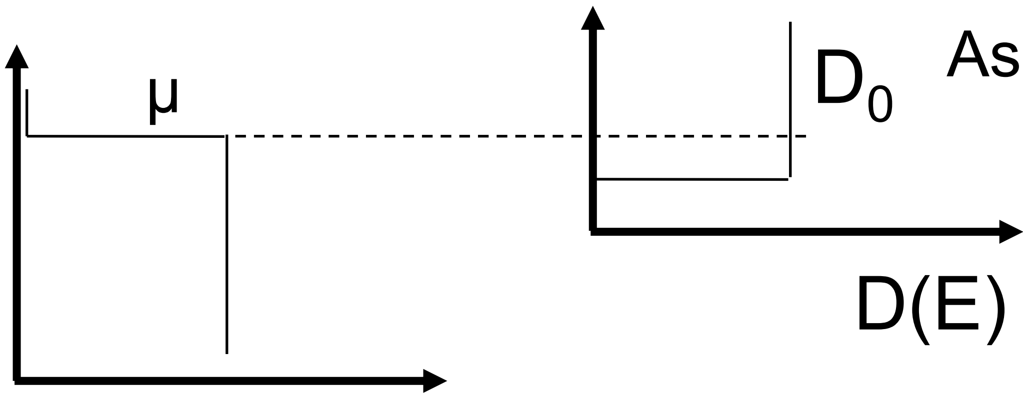
$f(E)=1/(1+e^{(E-\mu)/kT})$
$\approx e^{-(E-\mu)/kT}$
When everything are way above μ, $1 \ll e^{(E-\mu)/kT}$



$V_G/((kT/q)\,(1/0.43))$

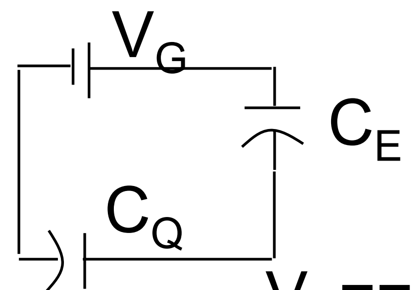
~60mV at room temperature

Superthreshold



Assume constant D_0

$D_0 = mS/\pi\hbar^2$



V_C== not V_G

$n_s = \int D(E)dE\, f(E-\mu)$

$= D_0(\mu - E_c)$

$= D_0(\mu - E_c) + D_0 qV_G$

$qn_s = q(D_0(\mu - E_c)) + D_0 q^2 V_G$

$d(qn_s)/dV_G = D_0 q^2$ =Quantum capacitance C_Q

$d(Q)/dV_G = \epsilon S/d$ =Electrostatic capacitance C_E

D_0: small C_Q is small and cannot be ignored!

D_0: virtually Φ C_Q is sub-vt region. $C_Q C_E / C_Q + C_E$