UST/DME: A Clock Tree Router for General Skew Constraints

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4/10/01 1

Outline of Talk

- Introduction
- Problem Formulation
- Incremental Skew Scheduler
- The UST/DME Algorithm
- Experimental Results
- Future Works

4/10/01 2

Clock Skew Constraints

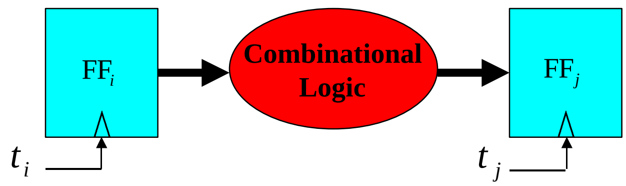
- To avoid zero-clocking (Upper-bound)

$$t_i + t_{logic,\max} + t_{setup} \le t_j + P$$

$$\Rightarrow t_i - t_j \le P - t_{logic,\max} - t_{setup} - d_u$$

- To avoid double-clocking (Lower-bound)

$$t_i - t_j \ge -t_{logic,\min} + t_{hold} + d_l$$



Review of Previous Works

- Clock skew scheduling optimization
  - Linear programming [Fishburn 90]
  - Graph-based algorithms [Deokar/Sapatnekar 94, Neves/Friedman 96]
- Clock tree routing optimization
  - Zero-skew tree (ZST) routing [Jackson/Kuh 90, Tsay 93]
  - ZST routing by Deferred-Merge Embedding (DME) Paradigm [Chao/Hsu/Ho 92, Boese/Kahng 92, Edahiro 92]
  - Bounded-skew tree (BST) routing by DME [Cong/Kahng/Koh/Tsao 98]
- Useful skew scheduling and tree routing (UST-BP) [Xi /Dai, 96]

Problem Formulation

Given

- Set of n sinks $S = \{s_1, s_2, \ldots, s_n\}$
- Optional clock source s_0
- Set of local skew constraints:

  $C = \{l_{ij} \le t_i - t_j \le u_{ij}\} = \{t_i - t_j \in [l_{ij}, u_{ij}]\}$

Find tree T over $S \Rightarrow$

- Total wire length is minimized
- Schedule $\{t_i\}$ satisfies skew constraints C

4/10/01 5

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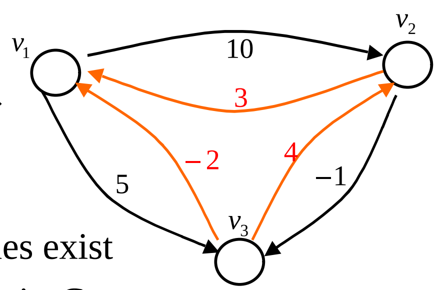
4/10/01 6

Graph-based Skew Scheduling

- Skew Constraints $C \Rightarrow$ Graph $G_c(V,E)$

$$\begin{cases} t_1 - t_2 \in [-10, \ 3\ ] \\ t_1 - t_3 \in [-5, \ -2\ ] \\ t_2 - t_3 \in [1, \quad 4\ ] \end{cases} \Rightarrow$$



- Feasible skew schedules exist

  $\Leftrightarrow$ No negative cycles in G_c

  [Cormen/Leiserson/Rivest]

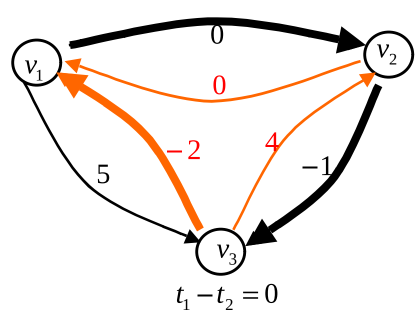
4/10/01 7

Feasible Skew Range (FSR)

- Maximum skew range for committing skew between sinks such that final skew schedule is feasible

$$\text{Given} \begin{cases} t_1 - t_2 \in [-10,3] \\ t_1 - t_3 \in [-5,-2] \\ t_2 - t_3 \in [1,4] \end{cases}$$

$$\Rightarrow \quad \begin{aligned} t_1 - t_2 &= 0 \\ t_1 - t_3 &= ? \\ t_2 - t_3 &= ? \end{aligned}$$



4/10/01 8

Feasible Skew Range (FSR)

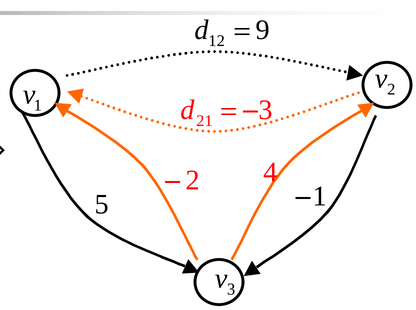
$$\text{Given}\begin{cases} t_1 - t_2 \in [-10,3] \\ t_1 - t_3 \in [-5,-2] \\ t_2 - t_3 \in [1,4] \end{cases} \Rightarrow FSR_{12} = [-9,-3]$$

$$\Rightarrow \quad \begin{matrix} t_1 - t_2 = -9 & t_1 - t_2 = -6 & t_1 - t_2 = -3 \\ t_1 - t_3 = -5 & t_1 - t_3 = -2 & t_1 - t_3 = -2 \\ t_2 - t_3 = 4 \checkmark & t_2 - t_3 = 4 \checkmark & t_2 - t_3 = 1 \checkmark \end{matrix}$$

4/10/01 9

Compute an *FSR*

$$\text{Given}\begin{cases} t_1 - t_2 \in [-10,3] \\ t_1 - t_3 \in [-5,-2] \\ t_2 - t_3 \in [1,4] \end{cases} \Rightarrow$$



$$\Downarrow$$

$$-10 \le t_1 - t_2 \le 3$$
$$-5 \le t_1 - t_3 \le -2$$
$$-4 \le t_3 - t_2 \le -1$$
$$\Rightarrow -9 \le t_1 - t_2 \le -3$$

d_{ij}: Shortest distance $v_i \rightarrow v_j$

$$\Downarrow$$

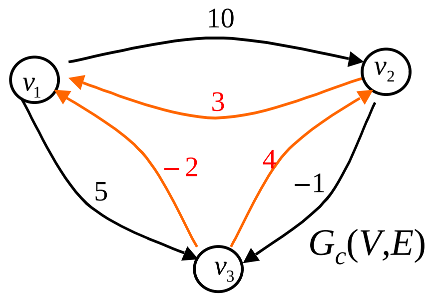
$$\Rightarrow \text{FSR}_{12} = [-d_{12}, d_{21}] = [-9,-3]$$

4/10/01 10

Compute all *FSR's*

	v_1	v_2	v_3
v_1	0	9	5
v_2	-3	0	-1
v_3	-2	4	0

$\Leftarrow$



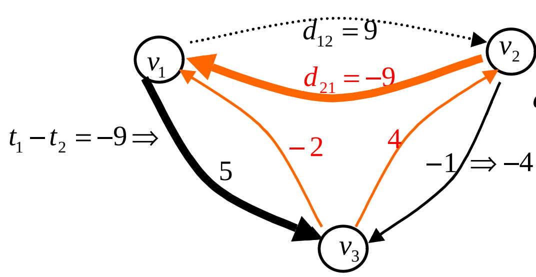
All-pair shortest distance matrix $D = \{ d_{ij} \}$ $\Rightarrow$ $FSR_{ij} = \lfloor -d_{ij}, d_{ji} \rfloor$

4/10/01 11

Update Distance Matrix

Skew Commitment $t_i - t_j = x$

- $\Rightarrow$ New constraint $x \le t_i - t_j \le x$
- $\Rightarrow$ Edges in G_c updated with $d_{ij} = -x$, $d_{ji} = x$
- $\Rightarrow$ Matrix D updated in $O(n^2)$



$$d_{kl} = \min \begin{Bmatrix} d_{kl} \\ d_{ki} + d_{ij} + d_{jl} \\ d_{kj} + d_{ji} + d_{il} \end{Bmatrix}$$

$$d_{23} = d_{21} + d_{13} = -4$$

4/10/01 12

Incremental Skew Scheduler

- Build G_c from skew constraints C : $O(|E|)$
- Build distance matrix D from G_c : $O(n^3)$
- For any non-trivial FSR_{ij} : $n - 1$ steps
  - Commit skew at $x \in FSR_{ij}$ (best for routing)
  - Update distance matrix D : $O(n^2)$ time
- Complexity: $O(n^3)$

4/10/01 13

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4/10/01 14

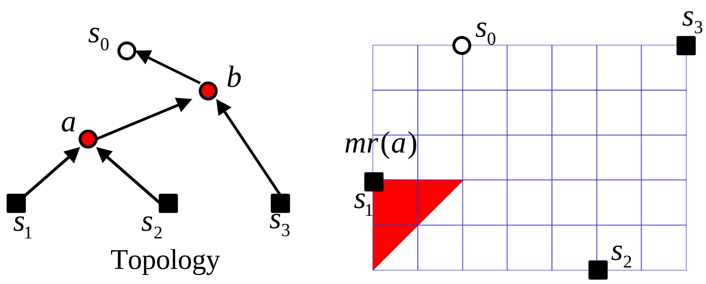
The UST/DME Algorithm

- Incremental skew scheduler + DME
  - Extension from a global skew bound to general skew constraints
- Two-phase tree construction
  - Bottom-up: Construct a binary tree of merging regions for internal nodes
  - Top-down: Embed nodes in merging regions

4/10/01 15

UST/DME: Bottom-Up Phase

- Merging regions
  - Loci of min-length locations of internal nodes
  - Constructed between child regions with feasible skew

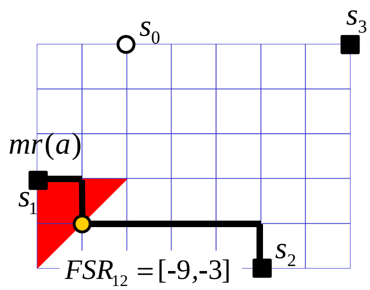


4/10/01 16

UST/DME: Bottom-Up Phase

- Merging regions
  - Loci of min-length locations of internal nodes
  - Constructed between child regions with feasible skew

	v_1	v_2	v_3
v_1	0	9	5
v_2	−3	0	−1
v_3	−2	4	0



4/10/01 17

Merge a, $s_3 \Rightarrow b$

- Choose segment $L_a \in mr(a)$ closest to s_3
- Compute $skew_{12}$ = –3 on L_a for sinks s_1, s_2
- Update matrix D for committing $skew_{12}$ = –3

	v_1	v_2	v_3
v_1	0	9	5
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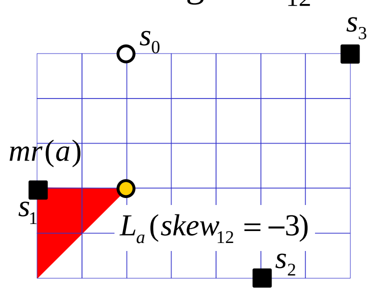
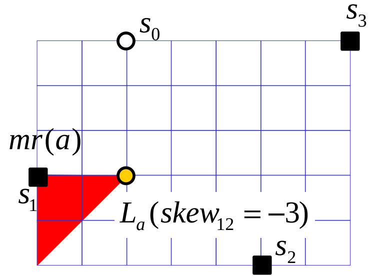


4/10/01 18

Merge a, $s_3 \Rightarrow b$

- Choose segment $L_a \in mr(a)$ closest to s_3
- Compute $skew_{12}$ = –3 on L_a for sinks s_1, s_2
- Update matrix D for committing $skew_{12}$ = –3

	v_1	v_2	v_3
v_1	0	3	2
v_2	−3	0	−4
v_3	−2	4	0

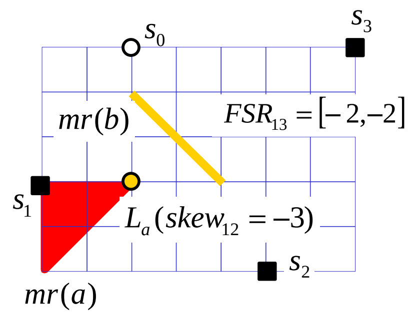


4/10/01 19

Merge a, $s_3 \Rightarrow b$ (cont.)

- Get FSR_{13} from matrix D for sinks s_1 and s_3
- Build $mr(b)$ between L_a, s_3 where $skew_{13} \in FSR_{13}$

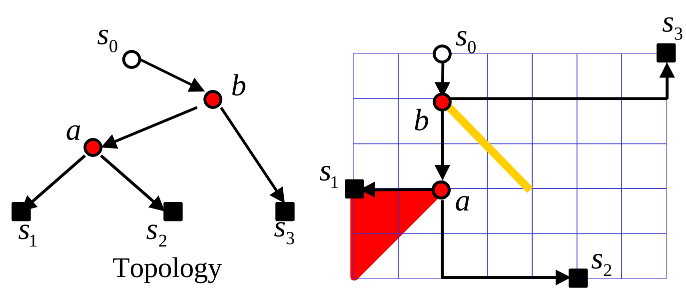
	v_1	v_2	v_3
v_1	0	3	2
v_2	−3	0	−4
v_3	−2	4	0



4/10/01 20

Top-Down Embedding

- Embed internal nodes in merging regions where closest to parent nodes



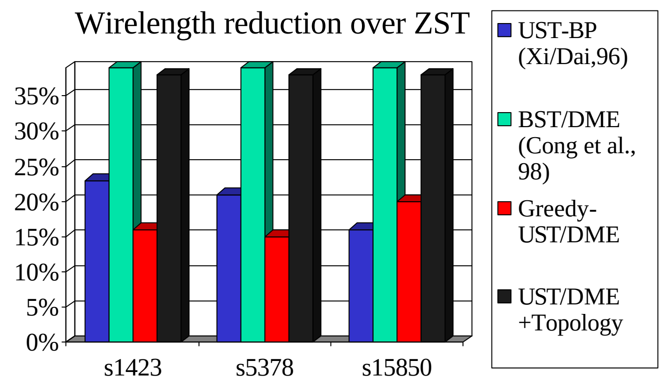
4/10/01 21

Experiments

ISCAS89 circuits	#sinks	#skew constraints	Max. global skew bound (ns)
s1423	74	78	1.4
s5378	179	175	0.3
s15850	597	318	0.2

4/10/01 22

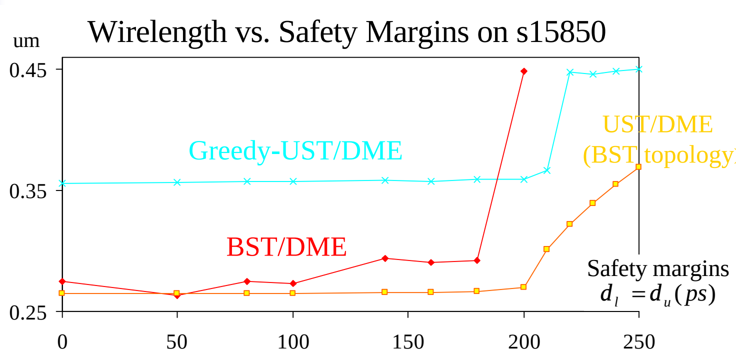
Experimental Results



4/10/01

23

Experimental Results (cont.)



$$t_{hold} - t_{logic,\min} + d_l \le t_i - t_j \le P - t_{logic,\max} - t_{setup} - d_u$$

4/10/01

24

Conclusions

- UST/DME:
  - Simultaneous clock skew scheduling and tree routing
  - Extension of DME paradigm for general skew constraints
- Future Work
  - Reduce $O(n^3)$ complexity of UST/DME
  - Improve topology generated by Greedy-UST/DME
  - Integration with buffer insertions