Transistor – Current Flow

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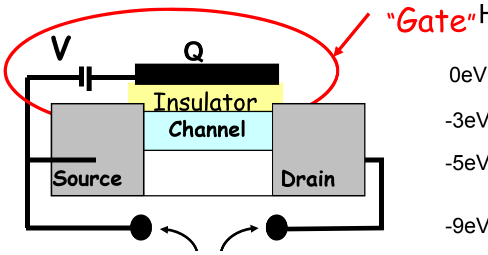
Outline

- Transistors – channel having few energy states
- Energy band diagram
- Current flow & I-V Characteristics
- Subthreshold Leakage
- Generalization to larger transistors

Acknowledgement: Professor Supriyo Datta

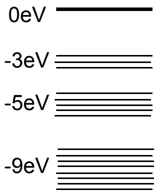
Transistors



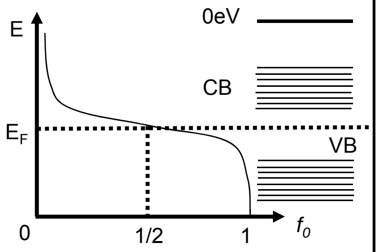
Field Effect Transistor

How to understand I-V characteristics

- Band diagram

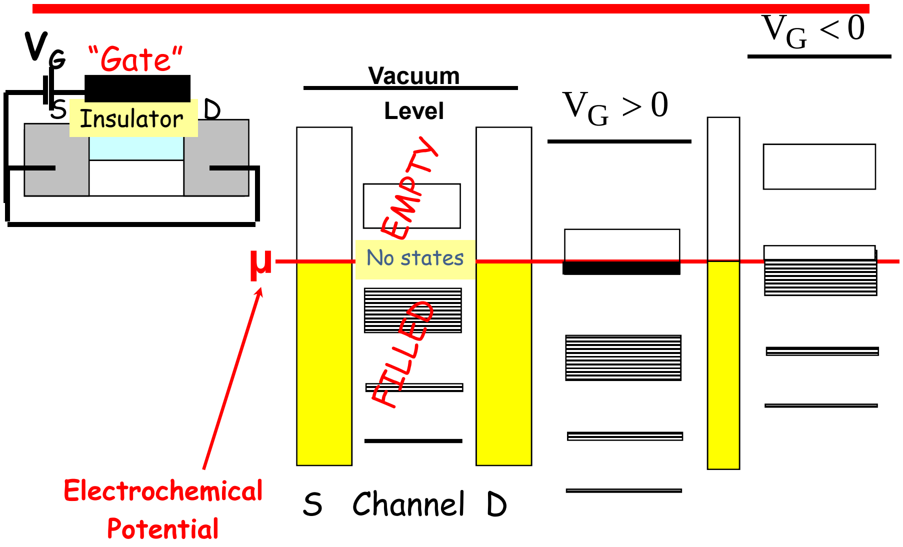


- Attributes of a good switch
  - Good gate control on channel
  - Less control of drain on channel
  - High ON current, less OFF current

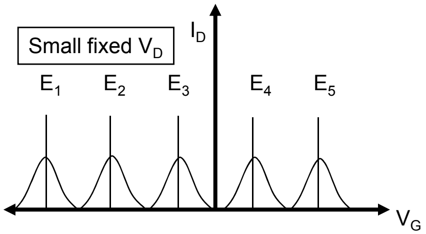


Distribution of electrons over a range of allowed energy levels: $f(E) = 1/(e^{(E-E_F)/kT}+1)$

Gate Bias



Simple Energy Level Scenario



- Peak occurs where E_F crosses an energy level due to an applied V_G bias

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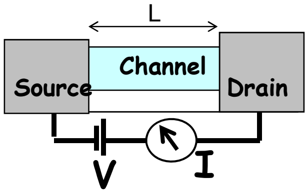
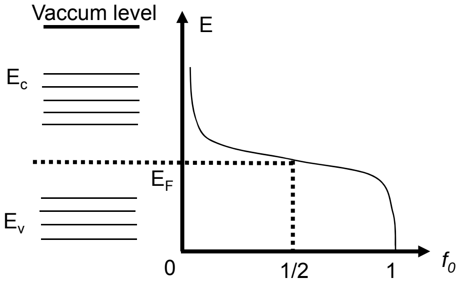
Transistors: Key Concepts





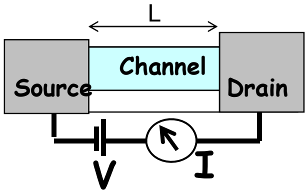
- Key concepts: V_D, V_G, empty and full energy levels, V_T, E_F
- Fermi function is centered at Ø

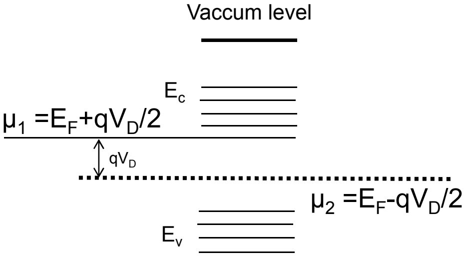
$$f_0(E) = 1/(e^{E/kT}+1)$$

- To get it centered at E_F, shift it

$$f_0(E-E_F) = 1/(e^{(E-EF)/kT}+1)$$

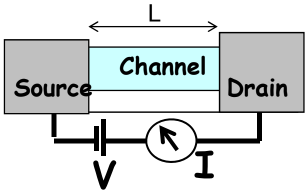
Application of Drain Bias

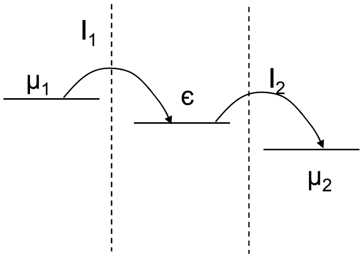




- Total energy difference between μ_1 and μ_2 is $qV_D = 1Vxq = 1eV = 1.6x10^{-19}$ J

Current Flow



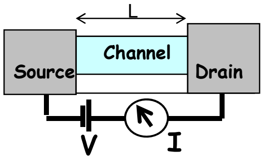


- N_1: Avg. # of electrons that the left contact would like to see $= \boxed{2}f_1(\epsilon) = 2f_0(\epsilon - \mu_1)$
- N_2: $N_2 = \boxed{2}f_2(\epsilon) = 2f_0(\epsilon - \mu_2)$

Accounts for up-spin and down-spin that is possible at a level

Current Flow

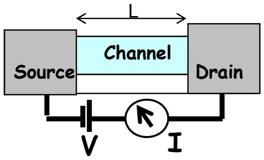
- N: Actual # of electrons at steady state in the channel



- I_1: $q(\gamma_1/\hbar)(N_1-N)$
- I_2: $q(\gamma_2/\hbar)(N-N_2)$
- $\gamma/\hbar$: rate at which electrons cross (escape rate)
- $\hbar = h/2\pi = 1.06\text{x}10^{-34}$ J.sec
- γ_1 and γ_2 are in units of Joule

Ex: γ_1=1meV
$\gamma_1/\hbar = 1.6\text{x}10^{-19}/1.06\text{x}10^{-34} = 10^{-12}$ /sec
= 1psec for electron to escape into the channel

Current Flow



At steady state, $I_1=I_2$
➔ $N = (N_1\gamma_1+N_2\gamma_2)/ (\gamma_1+\gamma_2)$
$I= I_1=I_2=(q/ \hbar)\ (\gamma_1\gamma_2/ \gamma_1+\gamma_2)\ (N_1-N_2)$
$=(2q/\hbar)\ (\gamma_1\gamma_2/ \gamma_1+\gamma_2)\ [f_1(\epsilon)-f_2(\epsilon)]$

N type conduction: go thru level that is empty at equilibrium
P type conduction: go thru level that is full at equilibrium

At small voltages (use Taylor series expansion)
$f_1(\epsilon)=f_0(\epsilon- \mu_1)$, $f_2(\epsilon)=f_0(\epsilon- \mu_2)$
$f_1-f_2= (\delta f_0/\delta E)(\mu_2 - \mu_1)=-(\delta f_0/\delta E)qV_D$
Therefore,
$I= (2q/\hbar)\ (\gamma_1\gamma_2/\gamma_1+\gamma_2)[f_1(\epsilon)-f_2(\epsilon)]$
$= V(2q^2/\hbar)\ (\gamma_1\gamma_2/\gamma_1+\gamma_2)[-\delta f_0/\delta E]$

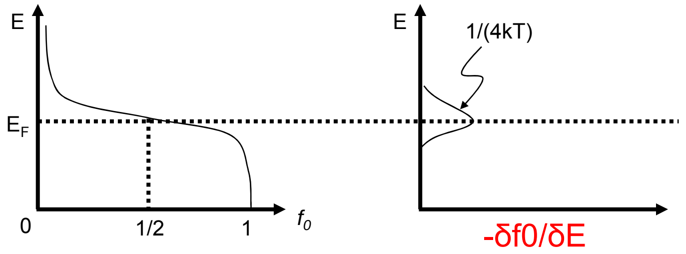
Current Flow

Use $E = \epsilon - E_F$ Since $\mu_1 = E_F + qV_D/2$, $\mu_2 = E_F - qV_D/2$

$2q^2/\hbar$: dimension of conductance

$\gamma_1\gamma_2/\gamma_1+\gamma_2$: dimension of energy

$\delta f_0/\delta E$: dimension of inverse energy



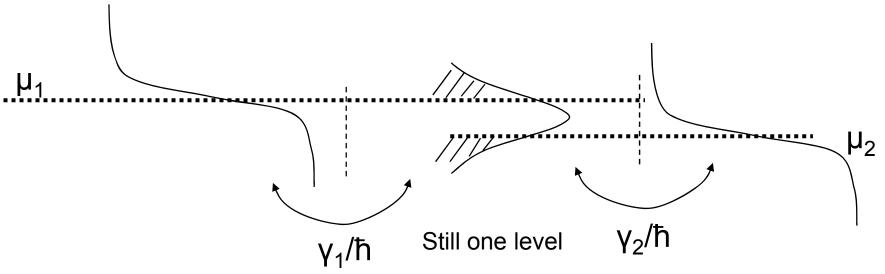
Current Flow

I/V = conductance = $(2q^2/\hbar)\,(\gamma_1\gamma_2/\gamma_1+\gamma_2)[-\delta f_0/\delta E]$

If $f_1 - f_2 = 1$ and $\gamma_1 = \gamma_2$; $I = q\gamma_1/2\hbar$

Seems to indicate that there is no limit to conductance but in reality, we do have a limit

$R_{min} = h/2q^2 = (6.6\text{x}10^{-34}\text{J-s})/2\text{x}(1.6\text{x}10^{-19})^2 = 12.9\text{k}\Omega$



$I = (q/\hbar)\,(\gamma_1/2)(qV_D/2\gamma_1) = q^2V_D/4\hbar$

Broadening

Only a fraction of levels contribute to current

Broadening

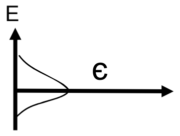
- Each electronic level has a wavefunction Ψ associated with it
  - With no coupling, $\Psi \, \alpha \, e^{-i\epsilon t/\hbar}$ ➔ time domain
  - Energy domain (Fourier transform) we get an impulse response
- Ψ^2 : probability of finding the electron at a point
  - $|\Psi|$ is 1 for the above expression of Ψ^2.
- After coupling the waveform gets modified
  - $\Psi \, \alpha \, e^{-i\epsilon t/\hbar} e^{-t/2\zeta}$ **: lifetime associated with electron**
  - ζ : lifetime --- the probability of finding the electron in the channel
  - Fourier transform of new Ψ gives the density of states

$D(E) = (\gamma/2\pi)/((E- \epsilon)^2+ (\gamma/2)^2)$, $\gamma= \gamma_1 + \gamma_2 = \hbar/2\zeta$

Broadening

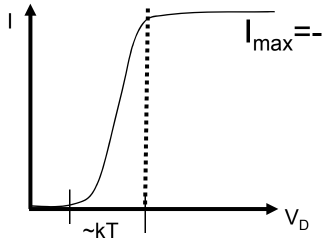
$I= (q/\hbar)\ (\gamma_1\gamma_2/\gamma_1+\gamma_2)[f_1-f_2]$
$= \int dED(E)\ (q/\hbar)\ (\gamma_1\gamma_2/\gamma_1+\gamma_2)[f_1-f_2]$

$N = \int D(E)dE(\gamma_1 f_1+\gamma_2 f_2/\gamma_1+\gamma_2)$



$\int D(E)dE = 1$

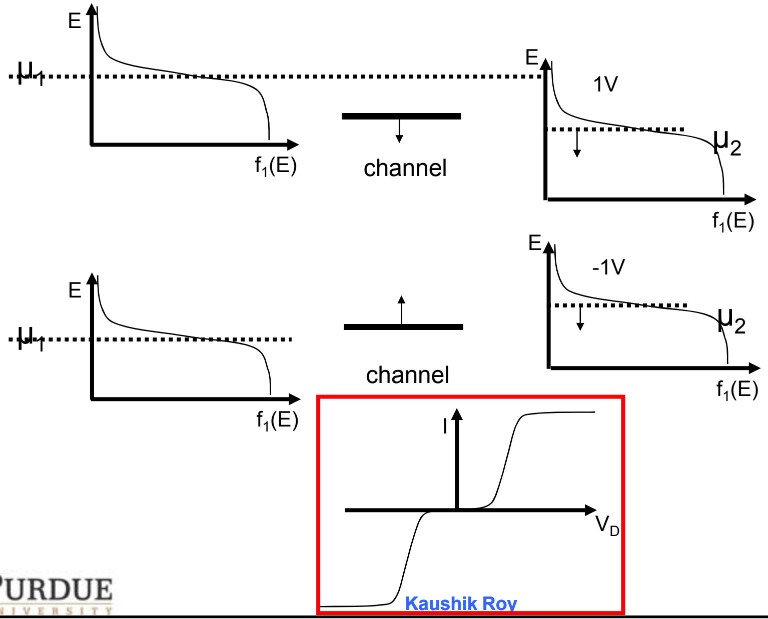
If $f_1-f_2=1$,
$I= -(q/\hbar)\ (\gamma_1\gamma_2/\gamma_1+\gamma_2)\int D(E)\ dE$



$I_{max}=-(q/\hbar)\ (\gamma_1\gamma_2/\gamma_1+\gamma_2)$

But the channel potential gets modulated by the drain voltage!

Current Flow

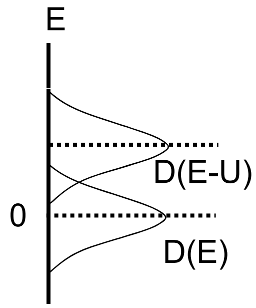


Channel Potential

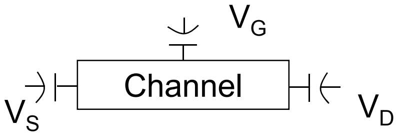
The effect of U (potential in the channel) is to move the density of states up or down depending on the sign of U

$I = -(q/\hbar) \int D(E-U)dE(\gamma_1\gamma_2/\gamma_1+\gamma_2)$
$N = \int D(E-U)dE(\gamma_1 f_1+\gamma_2 f_2/\gamma_1+\gamma_2)$

In order to find U in general, we need to solve Poisson's equation $d^2V/dx^2 = -(q)\Delta n/\epsilon$ (assume U is the same all over the channel)



Channel Potential



U: Potential energy
V: Electric potential in a certain region
U= -qV

Amount of charge in channel = $-q\Delta n= C_SV + C_G(V-V_G)+ C_D(V-V_D)$
With V_S grounded,
$V= (C_GV_G + C_DV_D)/(C_S + C_G+ C_D)+ (-q\Delta n)/(C_S + C_G+ C_D)$
$U = -qV = -q(\ldots\ldots\ldots\ldots\ldots)$
$U = U_L+\boxed{q^2/C_E}\Delta n$; $C_E= C_S + C_G+ C_D$

Single electron charging energy

Small devices, C_E is small and q^2/C_E is large ➔can change a lot of things

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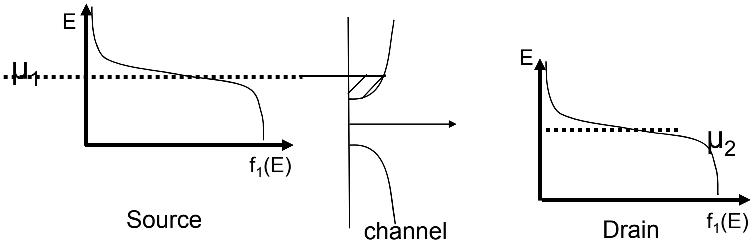
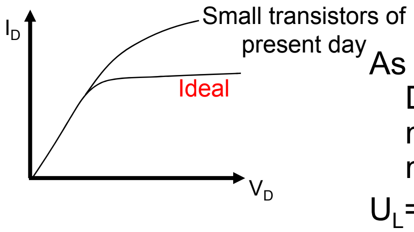
Channel Potential





As drain levels are lowered, the DOS also wants to slide down→ more available states, hence more current

$U_L= -q(C_GV_G + C_DV_D)/(C_S + C_G+ C_D)$

Good transistors: stop DOS sliding in channel. Make U_L as large as possible (C_G) to make effect of V_D negligible!

Good Transistor

- Increase C_G to make the effect of V_D negligible

$$U_L = -q(C_G V_G + C_D V_D)/(C_S + C_G + C_D)$$

- Gate as close as possible to the channel
  - If L=500A, then gate should be as close as 20A to the channel. If L is smaller, gate should be even closer, but gate leakage…

Current

$$I = -(q/\hbar) \int D(E-U)dE(\gamma_1\gamma_2/\gamma_1+\gamma_2)(f_1-f_2)$$

$$N = \int D(E-U)dE(\gamma_1 f_1+\gamma_2 f_2/\gamma_1+\gamma_2)$$

$$U = U_L+U_0\Delta N$$

Need to be solved self-consistently

$$U_0 = q^2/C_E$$

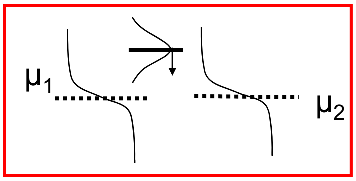
$$C_E = C_S + C_G + C_D$$

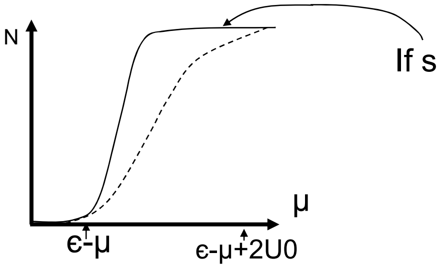
$$U_L = [C_G(-qV_G) + C_D(-qV_D)]/C_E$$

Single Electron Charging Energy

$N = \int D(E-U)dE(\gamma_1 f_1+\gamma_2 f_2/\gamma_1+\gamma_2)$

$U = U_L+U_0\Delta n$

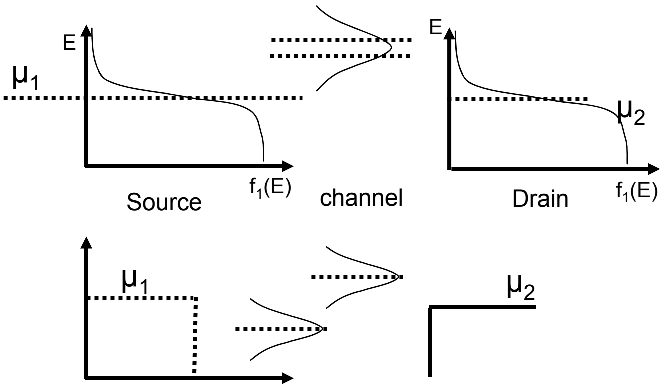




If self consistent solution not considered

Because of term U_0 in self-consistent solution, the level starts floating up as it gets filled with electrons→ making the filling up process slower!

Current Flow



$U_0 >> kT + \gamma$ ➔ Coulomb blockage charging energy U_0 exceeds broadening γ.

Conductance (Revisited)

$I = -(q/\hbar) \int D(E-U)dE(\gamma_1\gamma_2/\gamma_1+\gamma_2)(f_1-f_2)$

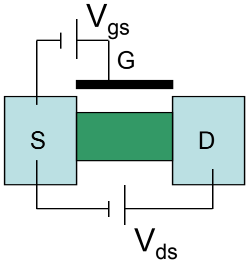
For small applied voltage

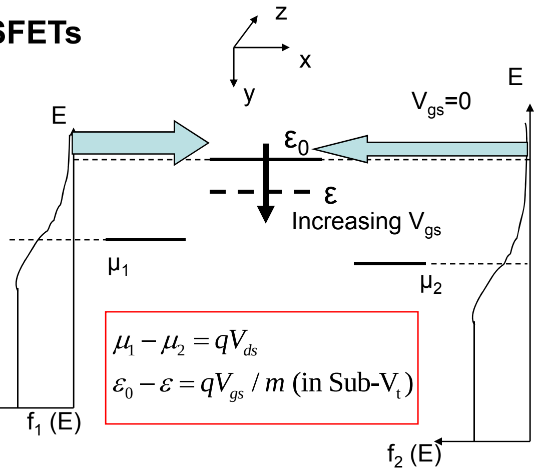
$I = -(q/\hbar)\, D(E)\, (\gamma_1\gamma_2/\gamma_1+\gamma_2)qV_D$

Ohm's law: G α A/L

- More states, more current & larger devices have more states. D α WL➔ contradicts Ohm's law??
  - γ decreases as 1/L; → $\gamma_1\gamma_2/\gamma_1+\gamma_2$ α 1/L
  - L cancels out!
- Conductance independent of L! Ballistic device

Leakage Current in MOSFETs





$$\mu_1 - \mu_2 = qV_{ds}$$

$$\varepsilon_0 - \varepsilon = qV_{gs} / m \text{ (in Sub-V}_t\text{)}$$

$$I = qA(n_x v_x - n_{-x} v_{-x})$$

$$v_x = v_{-x} = v, n_x \to f_1(\varepsilon), n_{-x} \to f_2(\varepsilon)$$

$$\Rightarrow I = qAv\left(f_1(\varepsilon) - f_2(\varepsilon)\right)$$

$$f_j(\varepsilon) = \frac{1}{1+\exp\left(\left(\varepsilon-\mu_j\right)/kT\right)} \approx \exp\left(-\left(\varepsilon-\mu_j\right)/kT\right), j=1,2$$

$$\Rightarrow I = qAv\left(\exp\left(-\left(\varepsilon-\mu_1\right)/kT\right) - \exp\left(-\left(\varepsilon-\mu_2\right)/kT\right)\right)$$

$$\Rightarrow I = \left(qAv\exp\left(\mu_1/kT\right)\right)\exp\left(-\varepsilon/kT\right)\left(1-\exp\left(\left(\mu_2-\mu_1\right)/kT\right)\right)$$

$$\boxed{I = I_0 \exp\left(qV_{gs}/mkT\right)\left(1-\exp\left(-qV_{ds}/kT\right)\right)}$$

$$\boxed{I_{OFF} = I\Big|_{\substack{V_{gs}=0 \\ V_{ds}=V_{dd}}} \approx I_0 = qAv\exp\left(\left(\mu_1-\varepsilon_0\right)/kT\right)}$$

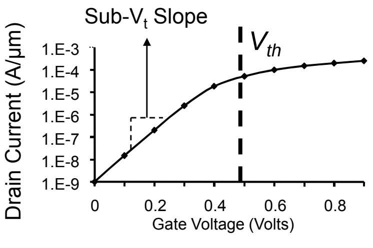
Assumptions in the derivation

- Single energy level in the channel
  - Model can be extended to multiple energy levels by integrating over energy
- Coupling with contacts ignored.
  - No energy broadening
  - ε not a function of x
- Effect of V_{ds} on ε ignored
- Flatband Voltage = 0
  - Equation can be easily corrected by using $V_{gs} – V_{FB}$ instead of V_{gs}
- Fermi function approximated by exponential function
  - A reasonable assumption for sub-threshold region

Sub-threshold Swing

$$I = I_0 \exp\left(qV_{gs} / mkT\right)\left(1 - \exp\left(-qV_{ds} / kT\right)\right)$$

$$S = \frac{\partial V_{gs}}{\partial \log_{10} I}$$



Sub-threshold Swing is inverse of Sub-threshold slope

$$S = \ln 10 m \frac{kT}{q} = 2.303 m \frac{kT}{q} \text{ [mV/decade]}$$

For ideal MOSFETs (m=1), $S = 60\text{mV/decade}$