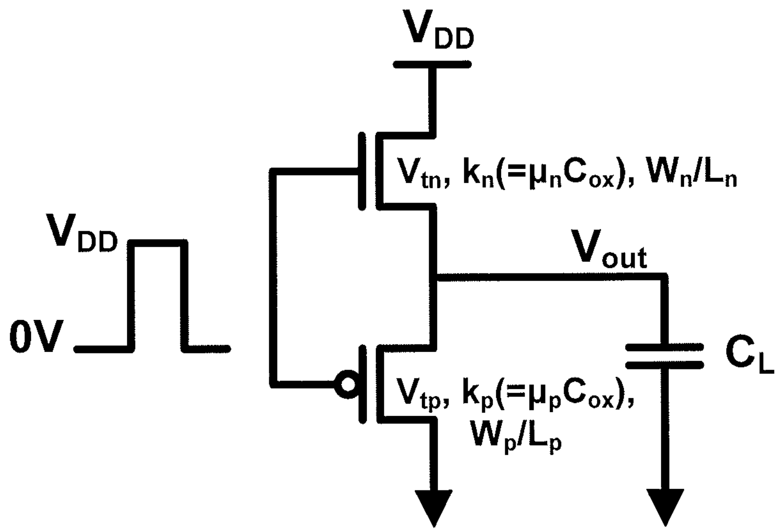
1. **For the circuit shown below, derive the energy consumption when (a) input switches from 0 to V_{DD} (b) followed by input switching from V_{DD} to 0V. Assume that V_{out} = 0V initially. (20 pt.)**



(a)

$$E_{supply} = \int_0^{V_{DD}-V_{tn}} i(t)\, V_{DD}\, dt = \int_0^{V_{DD}-V_{tn}} V_{DD}\,(C_L \times dV_{out})$$

$$= C_L V_{DD} (V_{DD} - V_{tn})$$

$E_{\cancel{save}}$ (in cap) $= \frac{1}{2} C_L (V_{DD} - V_{tn})^2$
stored

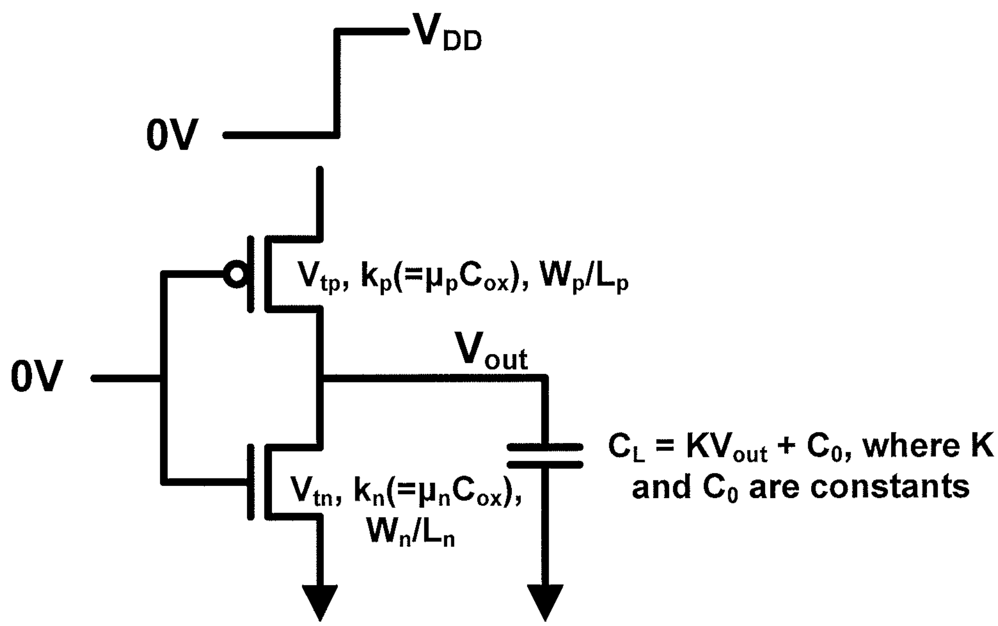
$$E_{diss} = E_{supply} - E_{save} = C_L V_{DD} (V_{DD} - V_{tn}) - \frac{1}{2} C_L (V_{DD} - V_{tn})^2$$

$$= \frac{1}{2} C_L (V_{DD}^2 - V_{tn}^2)$$

(b) V_{out} : $V_{DD} - V_{tn} \longrightarrow |V_{tp}|$

$$E_{diss} = \underbrace{\frac{1}{2} C_L (V_{DD} - V_{tn})^2}_{\text{Stored \cancel{Saved} energy before switching}} - \underbrace{\frac{1}{2} C_L V_{tp}^2}_{\text{Stored \cancel{Saved} energy after switching}}$$

2. **Derive the energy dissipation with the rising transition of the signal shown in the following figure. (20 pt.)**



$Q = C_L V_{out}$ $\qquad i(t) = \frac{dQ}{dt} = \frac{d}{dt}(C_L V_{out})$

$= \frac{d}{dt}(K V_{out}^2 + C_0 V_{out}) = (2K V_{out} + C_0)\frac{dV_{out}}{dt}$

(※ C_L is not constant here!)

$E_{supply} = \int i(t) V_{DD}\, dt = \int_0^{V_{DD}} V_{DD}(2K V_{out} + C_0)\, dV_{out}$

$= K V_{DD}^3 + C_0 V_{DD}^2 \quad \cdots ①$

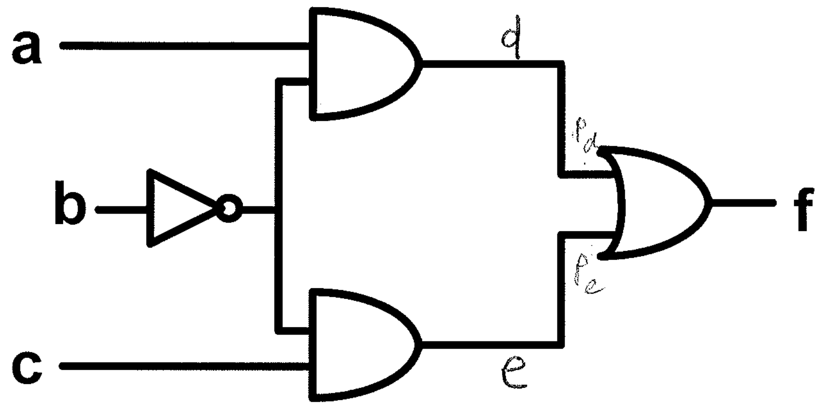
E_{save} (in cap) $= \int i(t) V_{out}\, dt = \int_0^{V_{DD}} V_{out}(2K V_{out} + C_0)\, dV_{out}$

$= \frac{2}{3} K V_{DD}^3 + \frac{1}{2} C_0 V_{DD}^2 \quad \cdots ②$

$\therefore E_{diss} = ① - ②$

$= \frac{1}{3} K V_{DD}^3 + \frac{1}{2} C_0 V_{DD}^2$

3. **Consider the following logic.**



The signal probabilities of the inputs are P_a, P_b and P_c while the signal activity factors are α_a, α_b and α_c, respectively. Assume that the inputs are spatially uncorrelated; Determine activity factor α_f of the output (f) in terms of P_a, P_b, P_c, α_a, α_b and α_c. (10 pt.)

Sol 1)

All inputs are spatially uncorrelated

$$\alpha_f = \sum_{i=1}^{n} P\left(\frac{\partial f}{\partial a_i}\right)\alpha_i \quad \left(\frac{\partial f}{\partial a_i} = f(a_i=1) \oplus f(a_i=0) \right)$$

$$f = a\bar{b} + \bar{b}c$$

$$\therefore \alpha_f = P\{f(a=1) \oplus f(a=0)\}\alpha_a + P\{f(b=1) \oplus f(b=0)\}\alpha_b$$
$$+ P\{f(c=1) \oplus f(c=0)\}\alpha_c \quad \cdots \text{①}$$

$$f(a=1) \oplus f(a=0) = (\bar{b} + \bar{b}c) \oplus (\bar{b}c)$$
$$= (\overline{\bar{b} + \bar{b}c})\bar{b}c + (\bar{b} + \bar{b}c)(\overline{\bar{b}c})$$
$$= \bar{b}\bar{c}$$

$$\therefore P[f(a=1) \oplus f(a=0)] = (1 - P_b)(1 - P_c)$$

(Question 3 cont.)

$$f(b=1) \oplus f(b=0) = a + c$$

$$\therefore p[f(b=1) \oplus f(b=0)] = P_a + P_c - P_a P_c \quad \cdots ②$$

$$f(c=1) \oplus f(c=0) = \bar{a}.\bar{b}$$

$$P[f(c=1) \oplus f(c=0)] = (1-P_a)(1-P_b)$$

$$\therefore \alpha_f = (1-P_b)(1-P_c)\alpha_a + (P_a + P_c - P_a P_c)\alpha_b$$

$$+ (1-P_a)(1-P_b)\alpha_c$$