THE EFFECT OF SENSITIVITY MATRIX FORMULATION ON DAMAGE DETECTION IN CARBON FIBER COMPOSITES WITH SURFACE-MOUNTED ELECTRODES VIA ELECTRICAL IMPEDANCE TOMOGRAPHY

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ABSTRACT

Carbon fiber reinforced polymers (CFRPs) are valued in aerospace and other weight-conscious applications for their high strength-to-weight ratio. However, with the adoption of these lightweight materials emerges challenges not seen in traditional monolithic materials such as complex internal (i.e. outwardly invisible) damages like delamination or fiber failure in the structure. Robust methods of damage detection and health monitoring are therefore important. It is also desirable to utilize an intrinsic property of these materials, such as electrical conductivity, as an indicator of damage to render the material as self-sensing. Electrical impedance tomography (EIT) has been widely explored for damage detection and health monitoring in self-sensing materials. To date, however, studies involving EIT have been largely limited to materials with less electrical anisotropy than is seen in CFRPs and using only edge-placed electrodes (e.g. electrodes placed on the edges of a plate). These limitations are important because the inability to handle highly electrically anisotropic materials precludes EIT from a great number of existing CFRP structures. Furthermore, many real structures lack well-defined edges on which electrodes can be placed. In this paper, we tackle these challenges by presenting a preliminary study into the role of EIT sensitivity matrix formulation and surface-mounted electrodes on damage detection and localization in CFRPs. In our approach, the conductivity is modeled as being anisotropic, and the sensitivity matrix is formed using three methods – with respect to a scalar multiple of the conductivity tensor, the out-of-plane conductivity, and the in-plane conductivity. It was found that through-hole damage can be adeptly identified using the combination of surface-mounted electrodes and a sensitivity matrix formed with respect to either a scalar multiple of the conductivity tensor or the in-plane conductivity. The findings presented in this work take an important step towards translating EIT out of the laboratory and into real applications on CFRPs.

Keywords: Electrical Impedance Tomography (EIT), Carbon Fiber Reinforced Polymer (CFRP), anisotropy, sensitivity matrix

INTRODUCTION

Composites are favored in aerospace, automotive, and other weight-conscious industries for having excellent anisotropic mechanical properties which impart greater strength and stiffness in required directions while simultaneously reducing the weight of the system [1]. However, a drawback of these materials is the various invisible failure modes that exist such as delamination, fiber breakage, and matrix cracking which can go undetected during visual inspections and routine interval-based inspections. As an alternative to traditional interval-based maintenance, structural health monitoring (SHM) can continuously evaluate the integrity of a structure by tracking the extent of damage in materials in real-time and without impeding the functionality of the structure [2, 3].

Much work has been devoted to conductivity-based SHM wherein changes in the electrical properties of materials are used as indicators of condition. Because carbon fiber-reinforced polymer (CFRP) composites are conductive and deleterious effects such as damage change their conductivity, there is potential for real-time monitoring of health and condition in these materials via non-invasive electrical measurements [4 – 8]. Electrical impedance tomography (EIT) is a modality that spatially maps the conductivity distribution of a domain thereby allowing for continuous visualization of deformation and damage in materials. As such, it has been widely researched as a conductivity-based SHM technique. To date, EIT has been most widely employed in materials with relatively low electrical anisotropy such as carbon nanofiller-modified polymers [9], films/paints [10], and cementitious materials [11].

CFRPs, due to having measurable electrical changes in response to damage, seem ideal to integrate with EIT. For example, Baltopoulos et al. [12] presented conductivity maps of CFRP domains under both drilled hole and indentation damage.
modes incorporating post-processing methodologies for optimal solutions. Nonn et al. [13] experimentally found that in case of unidirectional laminates, diagonal current injection patterns along with riveted edge-electrodes led to better reconstructions. It was noted, however, that laminated CFRPs could not be accurately represented by simple orthotropic conductivity distributions. Cagăń et al. [14] utilized a Gaussian anisotropic smoothing filter along with ERT to detect cuts in their specimen which led to significant improvement in the shape reconstruction ability. Furthermore, another study [15] looked at using statistical processing to successfully detect BVID based on amplitude sensitivity and position error.

Despite the success of these studies outlined above, applying EIT to CFRPs remains challenging due to the high electrical anisotropy of these materials. That is, high electrical anisotropy degrades the effectiveness of EIT by increasing the number of unknown parameters to be found (i.e. seeking the spatially varying components of a conductivity tensor as opposed to a spatially varying conductivity scalar) and diminishing the distinguishability of electrical data (i.e. conductivity perturbations result in non-unique boundary voltage data for an anisotropic domain). Another difficulty with practical implementation of EIT for SHM, though not a consequence of anisotropic conductivity, is the lack of well-defined edges on most engineering structures. That is, the traditional EIT formulation is predicated on taking measurements at the edge of a domain using electrodes that span the thickness of the domain (e.g. on the edges of a thin plate). Real structures, however, generally have surface availability rather than edge availability. In light of the potential of EIT for the SHM of CFRPs and the limitations of the state of the art outlined above, the goal of this paper is to explore novel formulations of the EIT inverse problem, by manipulating the formation of the sensitivity matrix, in order to improve damage detection in a CFRP plate with surface-mounted electrodes. This manipulation is done by forming the sensitivity matrix with respect to a scalar multiple of the conductivity tensor (as is traditional for anisotropic EIT), with respect to the in-plane conductivity, and with respect to the out-of-plane conductivity.

As a step towards realizing EIT in CFRPs, we herein present some preliminary proof-of-concept results. The remainder of this paper is organized as follows. First, we present an overview of EIT. Second, we describe our novel sensitivity matrix formulation process. Third, we present our experimental methods. Fourth, the results of this study are discussed. And lastly, we end with a brief summary and conclusions.

**ELECTRICAL IMPEDANCE TOMOGRAPHY**

Here, we use EIT for damage detection and localization in a CFRP laminate by mapping the conductivity distribution changes in the domain. As a nomenclature aside, since this work utilizes direct currents (DC), we are technically using electrical resistance tomography (ERT). EIT refers to imaging with alternating current (AC). However, we will continue to refer to this method as EIT since this is the commonly used verbiage by SHM practitioners. EIT non-invasive technique is composed of a two-part procedure, namely the forward problem and the inverse problem. The forward problem seeks to computationally replicate the experimental process of injecting currents and measuring voltages. The inverse problem involves minimizing the difference between these predicted voltages and those measured experimentally by updating the conductivity distribution of the computational domain.

**EIT Forward Problem**

The EIT forward problem begins with Laplace’s equation in the absence of internal sources as shown below in equation (1) where \( \phi \) is the domain potential and \( \sigma \) is the conductivity distribution. For electrically anisotropic materials, the conductivity is a symmetric, second-order tensor. Laplace’s equation is then subject to the complete electrode model (CEM) boundary conditions. Equation (2) adds an additional degree of freedom for each of \( L \) total electrodes, \( V_i \), while also accounting for the contact impedance between the domain and the \( l \)th electrode, \( z_l \). \( n \) is an outward-pointing normal vector. Equation (3) enforces conservation of charge (i.e. the current entering the domain through the electrodes equals the current leaving the domain through the electrodes). In equation (3), \( E_l \) is the area of the \( l \)th electrode.

\[
\nabla \cdot \sigma \nabla \phi = 0 \quad (1)
\]

\[
\phi + z_l \sigma \nabla \phi \cdot n = V_l \quad (2)
\]

\[
\sum_{l=1}^{L} \int_{E_l} \sigma \nabla \phi \cdot n \, dS_l = 0 \quad (3)
\]

Equations (1)-(3) are readily solved via the finite element method (FEM). Upon discretization, the following system of equations can be formed where \( \Phi \) is a vector of domain potentials, \( V \) is a vector of electrode voltages, and \( I \) is a vector of electrode currents. The matrices \( A_z, A_w, \) and \( A_d \) are formed as shown below. \( A_M \) is the standard steady-state diffusion stiffness matrix. Linear tetrahedral elements were used in this work with in-house FEM code written in Matlab.

\[
\begin{bmatrix}
A_{iw} + A_z & A_w & A_d
\end{bmatrix}
\begin{bmatrix}
\Phi
\end{bmatrix}
= \begin{bmatrix}
0
\end{bmatrix} \quad (4)
\]

\[
A_{Zij} = \sum_{l=1}^{L} \int_{E_l} \frac{1}{z_l} w_i w_j \, dS_l \quad (5)
\]

\[
A_{Wij} = -\int_{E_l} \frac{1}{z_l} w_i \, dS_l \quad (6)
\]

\[
A_{D} = \text{diag} \left( \frac{E_l}{z_l} \right) \quad (7)
\]

**EIT Inverse Problem**

Below, we derive the EIT inverse problem for a case of isotropic conductivity. In the next section, we will specialize this
general formulation to three different sensitivity matrices which account for anisotropy.

To reduce the effect of noise, we shall use difference imaging to minimize an error vector in the least-squares sense as follows: 

\[ \|V_m - W\|_2 \] Here, \( V_m \) represents the difference between experimental voltages measured before and after damage as shown in equation (8). Similarly, \( W \) is a computationally predicted difference vector defined as \( F(\sigma + \Delta \sigma) - F(\sigma) \) where \( F(\cdot) \) is a vector of electrode voltages predicted via the forward problem described previously and \( \Delta \sigma \) is the damage-induced conductivity perturbation we seek (note that \( \Delta \sigma \) is boldfaced as a vector due to the conductivity distribution being discretized by the finite element method). To find the conductivity distribution \( \Delta \sigma \), we linearize \( F(\sigma + \Delta \sigma) \) using a Taylor series expansion as \( F(\sigma_a) + J\Delta \sigma \) as shown in equation (9). Here, \( J = \frac{\partial F(\sigma_a)}{\partial \sigma} \Delta \sigma \) is known as the sensitivity matrix (or the Jacobian matrix) formed using the initial estimate of conductivity, \( \sigma_a \). This leads to the minimization problem shown in equation (10). The first term represents the error minimization while the latter is a regularization term added due to the ill-posed nature of the problem.

\[
V_m = V(t_2) - V(t_1) \tag{8}
\]

\[
F(\sigma + \Delta \sigma) \approx F(\sigma_a) + \frac{\partial F(\sigma_a)}{\partial \sigma} \Delta \sigma \tag{9}
\]

\[
\Delta \sigma = \arg \min_{\Delta \sigma} \|V_m - J\Delta \sigma\|_2 + \alpha \|L\Delta \sigma\|_2 \tag{10}
\]

\[
\Delta \sigma = (J^TJ + \alpha L^TL)^{-1}J^TV_m \tag{11}
\]

The solution to this minimization can also be explicitly written as shown in equation (11). Here, \( L \) represents the discrete Laplace operator, the effect of which is modulated by the scalar value \( \alpha \).

### Sensitivity Matrix Formulations

With the knowledge that the above EIT formulation alone cannot provide a unique solution for anisotropic domains (i.e. cannot find all independent components of an anisotropic conductivity tensor), a common approach is to instead seek a scalar multiple of the conductivity tensor that retains the eigen vectors of the conductivity tensor [16, 17]. In this approach, the conductivity tensor is rewritten as \( \sigma_{ij} = \kappa \sigma_{ij}^\alpha \) where the value of \( \kappa \) is picked such that \( \det(\sigma^\alpha) = 1 \). Note that we are indulging in a slight abuse of index notation – \( \sigma_{ij} \) represents a vector of symmetric second-order conductivity tensors. The subscripts are retained in our description to distinguish this vector of tensors from the previously defined vector of scalar conductivity values, \( \sigma \). We then minimize equation (12) by finding a change in \( \kappa \) across the domain. Hence, we have abandoned trying to find all the independent components of the conductivity tensor and instead seek a scalar field solution. The sensitivity matrix is then formed as \( I_\kappa = \frac{\partial F(\kappa, \sigma_{ij}^\alpha)}{\partial \kappa} \) and the solution to the inverse problem is recast as shown below. An explicit formula for \( I_\kappa \) can be found in [18].

\[
\Delta \kappa^* = \min_{\kappa < \Delta \kappa < 0.01 \kappa} \left\| \frac{I_\kappa}{\alpha L} \Delta \kappa - \left[ \begin{array}{c} V_m \\ 0 \end{array} \right] \right\|_2^2 \tag{12}
\]

Although this scalar coefficient approach is often used for imaging of anisotropic media, it fails to incorporate knowledge of the reduction in conductivity with respect to a particular direction of anisotropy, information that could potentially help distinguish between different damage modes that exist in composite failure. In other words, the solution tends to be skewed in the principal directions of \( \sigma_{ij} \). As an alternative, we are interested in the development of sensitivity matrices formed with respect to in-plane and out-of-plane conductivities. This also allows for some physical insight to be encoded into the sensitivity matrix. For example, it is expected that damages that impede in-plane current flow will be more easily found via an in-plane sensitivity matrix. These sensitivity matrices take the form of \( I_\perp = \frac{\partial F(\sigma_{ij})}{\partial \sigma_{ij}} \) and \( I_\parallel = \frac{\partial F(\sigma_{ij})}{\partial \sigma_{ij}} \) where \( \sigma_{ij} \) is the in-plane conductivity and \( \sigma_\perp \) is the through-thickness conductivity of the laminate. This is shown in equation (13) where the one and two-directions are assumed to be in-plane whereas the three-direction is assumed to be out-of-plane. Solutions to the inverse problem under these conditions are shown below in equations (14) and (15).

\[
\sigma_{ij} = \begin{bmatrix} \sigma_{11} & 0 & 0 \\ 0 & \sigma_{11} & 0 \\ 0 & 0 & \sigma_{11} \end{bmatrix} \tag{13}
\]

\[
\Delta \sigma_{ij}^\perp = \min_{-\sigma_{ij} < \Delta \sigma_{ij} < 0.01 \sigma_{ij}} \left\| \frac{I_\perp}{\alpha L} \Delta \sigma_{ij} - \left[ \begin{array}{c} V_m \\ 0 \end{array} \right] \right\|_2^2 \tag{14}
\]

\[
\Delta \sigma_{ij}^\parallel = \min_{-\sigma_{ij} < \Delta \sigma_{ij} < 0.01 \sigma_{ij}} \left\| \frac{I_\parallel}{\alpha L} \Delta \sigma_{ij} - \left[ \begin{array}{c} V_m \\ 0 \end{array} \right] \right\|_2^2 \tag{15}
\]

Herein, we construct these ‘directional’ sensitivity matrices numerically via the secant method and multiple solutions of the forward problem. The lower limit on the constraints in equations (12), (14) and (15) are enforced based on a realistic scenario of complete (or 100%) loss in conductivity due to damage. The upper limit is set expecting no loss in conductivity at undamaged regions, with a 1% tolerance to the baseline, to account for the noise in experimental data.

### EXPERIMENTAL SETUP

The composite laminate in this study was manufactured using a combination of plain weave carbon fiber fabric and epoxy resin (Fiber Glast 2000). 15 layers of the fabric were stacked using wet-layup technique and cured under vacuum at 60 °C for 5 hours. This gives rise to an orthotropic specimen of...
[0/90]_1; sequence with a net thickness of 0.1” and in-plane dimensions of 4.25” × 4.25”. Additional plates were made in an identical fashion and cut up into small, 0.5” squares such that in-plane and through-thickness conductivity could be measured as shown in Figure 1. To do this, conductive silver paint was applied to these squares on faces perpendicular to the direction in which resistance was to be measured. Cutting the material provided a good electrical connection with the carbon fiber for in-plane measurements. To ensure good electrical contact for through-thickness measurements, the top and bottom surfaces of the small squares were sanded and cleaned acetone before applying silver paint and copper tape electrodes. Resistances were measured through all directions of the 0.5” specimens via a hand-held digital multi-meter (DMM). The conductivity for each of these specimens was the obtained from resistance measurements and their dimensions. A total of 47 specimens were measured in this way. The average conductivities for the in-plane and through-thickness directions are shown in Table 1. These were used as homogeneous background estimates for the EIT forward problem and as linearization points for the inverse problem.

![Figure 1: Small CFRP squares used to estimate pristine conductivity.](image)

**TABLE 1: Conductivity Estimate of Sample**

<table>
<thead>
<tr>
<th>Conductivity Estimate</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_{\parallel} ) (In-plane, ( x_1 ) and ( x_2 ))</td>
<td>5906 ± 1120 S/m</td>
</tr>
<tr>
<td>( \sigma_{\perp} ) (Through-thickness, ( x_3 ))</td>
<td>394 ± 125 S/m</td>
</tr>
</tbody>
</table>

After measuring the conductivity of the small squares, electrodes were attached to the 4.25” × 4.25” CFRP laminate for EIT testing. Recall that using surface-mounted electrodes was another goal of this study. A schematic of the electrode setup, and a comparison to traditional edge-mounted electrodes, is shown in Figure 2. For a 16-electrode model as shown in Figure 2(b), current is injected and grounded at the first pair of adjacent electrodes while voltages are collected at the remaining pairs, excluding the one involved in current injection. This procedure is repeated for all 15 pairs in a ‘snake’ like pattern as indicated by the arrows in the figure. A total of 16 electrodes were painted equidistant to one another as 0.25” squares over the surface of the specimen using highly conductive silver paint. To ensure good electrical contact between the silver paint and the surface of the specimen using highly conductive silver paint. To ensure good electrical contact between the silver paint and the carbon fibers, the same surface preparation procedure was used as was described for the small squares; the laminate was lightly sanded at the electrode locations and cleaned with acetone prior to applying the silver paint. Copper tape was then applied onto these electrode surfaces with extended tabs folded over the acrylic strips (i.e. electrode bars) as shown in Figure 3 for ease of connection to leads with alligator clips. The experimental specimen is shown in Figure 3.

A constant DC of 0.3 A was supplied in each current injection by a BK Precision 9131B power supply and voltages were measured using National Instruments PXIe-6368 data acquisition (DAQ) cards for 10 seconds at a frequency of 100 Hz. Data was collected from each electrode for 10 seconds so that averages could be used in EIT calculations. This helps minimize the effect of noise. After drilling a 3/16” hole in the upper right-hand side of the plate, the same procedure was used to collect post-damage measurements.

**RESULTS AND DISCUSSION**

In this section, we will present the results of this preliminary study and provide discussion. Two sets of results are considered — analyses of the effect of sensitivity matrix formulation on rank and the effect of sensitivity matrix formulation on through-hole detection. Beginning with rank assessment, Figure 4 shows the logarithm of the normalized singular values of \( J_r \), \( J_f \), and \( J_\perp \).
against singular value number. This assessment is important because the quality of EIT reconstructions is largely influenced by the rank of the sensitivity matrix – higher ranks generally provide better imaging capabilities. As seen in Figure 4, the sensitivity values all drop at the same index value in the plot, confirming that different formulations do not affect the rank of the sensitivity matrix. This is important because it gives us confidence that we are not losing data due to new sensitivity matrix formulations. It can also be observed in Figure 4 that the singular values for all the sensitivity matrices do not seem to drop at the same rate (i.e. $J_\perp$ seems to exhibit a delayed drop). The exact reason for this is not known at this time, and an in-depth study of the SVD properties of these matrices exceeds the scope of this manuscript.

Next, we turn our attention to through-hole damage detection for each method. These results are shown in Figure 5. From Figure 5, we notice that the damage is clearly detected in the case of sensitivity matrix formulations with respect to $\kappa$ and with respect to $\sigma_\parallel$ but not with respect to $\sigma_\perp$. These results suggest that both $\kappa$ and $\sigma_\parallel$ are sensitive to this type of damage while $\sigma_\perp$ is not. To better understand this, we recall that in case of CFRPs, it is the carbon fibers that form the conductive pathways. Based on the direction of the fiber layup in the specimen used in this study, the conductivity in the in-plane direction is higher. It is therefore easier for current to flow along these fibers than in the through-thickness direction that owes its conductivity only to inter-laminar fiber contact. This is confirmed by the conductivity estimates in Table 1. Hence, with regards to a through-hole damage, we find that the Jacobian formed is largely influenced by the drastic decrease in in-plane conductivity due to lack of this conductive network of fibers in
SUMMARY AND CONCLUSIONS

This preliminary study was motivated by the potential of EIT for damage detection and SHM in CFRPs and the limitations of existing methods with regard to handling anisotropic conductivity and the inaccessibility of well-defined edges in most engineering structures. To that end, EIT was used to detect a through-hole damage in a CFRP laminate with 16 surface-mounted electrodes. Mathematically, the minimization using these measured voltages was conducted using three different sensitivity matrix formulations. First, a scalar multiplier of the conductivity tensor, \( \kappa \), was introduced in order to obtain a scalar field damage reconstruction in terms of \( \Delta \kappa \). Second, based on the conductivity estimate for this specimen, sensitivity matrices with respect to both in-plane and through thickness directions were introduced to obtain damage reconstructions in these respective directions.

It was noted that sensitivity matrix formulations play an important role in addressing the anisotropy of a composite domain. We see that while forming the sensitivity matrix with respect to a scalar multiplier, the damage is accurately detected. The lack of distortion when using \( \Delta \kappa \) may be a consequence of seeking a circular artifact in a domain having anisotropic conductivity with equal in-plane eigen values. On introducing directionally dependent sensitivity matrices, \( J_1 \) and \( J_\perp \), we notice that through hole damage can be reconstructed based on in-plane conductivity changes but the same cannot be said for the conductivity mapped in the through-thickness direction. We believe that this is due to the influence of larger loss in conductivity due to breakage of fibers in the in-plane direction on the Jacobian that conceals the loss in conductivity due to lack of inter-laminar fiber contact in the out-of-plane direction in the vicinity of the damage. It is hypothesized that damages which cause comparatively greater magnitudes of conductivity loss in the through-thickness direction than the in-plane direction (e.g. a delamination) could be detected with the sensitivity matrix formed in that direction. This, however, will be the subject of future studies.

In light of this preliminary work, it is reasonable to conclude that modifications to EIT’s Jacobian formulation along with surface mounted electrodes can yield satisfactory reconstructions of damages in anisotropic CFRP structures. While this work establishes proof-of-concept for simple through-hole damage reconstructions, numerous avenues of future work can be suggested. Future work should include damages of other forms not considered here, such as the commonly occurring BVID in CFRPs (e.g. delamination, fiber breakage, etc.). Furthermore, particularly with the use of surface-mounted electrodes, the same approach could be extended to complex non-planar structures for practical applications. Future work should consider higher degrees of anisotropy. Herein, a bi-directional weave was used such that in-plane conductivity was more-or-less isotropic. Unidirectional CFRPs with much more pronounced in-plane anisotropy will be considered in the future too. Future works should consider the detection of multiple damages. In this work, proof-of-concept was demonstrated for a simple case of just one through-hole. Exploring the effectiveness of this method in the presence of multiple damages is therefore important. And lastly, future work should more exhaustively explore the SVD properties of these different sensitivity matrix formulation.

REFERENCES


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