Abstract—This work investigates the imager-induced noise components in complex-valued MRI signals. We present background for a noise model, and address the task of validating this model. We identify certain aspects of MRI data which potentially bias the results of goodness-of-fit tests, and suggest a testing formulation that accounts for these issues. This formulation is then applied to sets of echo-planar image sequences, successfully verifying consistency with the proposed model.

Keywords—MRI, Rician, Rayleigh, chi-squared test

I. INTRODUCTION

The noise in MRI images is becoming an increasingly important factor to account for in the analysis of MRI data. Emerging applications which utilize statistical analysis techniques often base their conclusions on assumptions about the underlying noise characteristics.

Functional MRI, for example, has shown enormous potential in both clinical and research settings. However, the primary obstacle in generating reliable activation maps with fMRI is the extremely low signal-to-noise ratio of the acquired data. The detection of a neural response is most commonly carried out in the form of a statistical hypothesis test, which often relies on assumptions of the underlying noise distribution. Therefore, validation of the assumed model is a necessary step if the conclusions are to be considered meaningful.

II. BACKGROUND

Thermal agitation in the receive coil and circuitry induce noise which we will model as Gaussian. This noise is present in both the real and imaginary components of the sampled baseband representation of the free-induction decay (FID) signal. Image reconstruction often consists of a 2D fast Fourier transform which preserves the form of the noise distribution, since the FFT is an orthonormal transformation.

However, when the MRI image is transformed to a magnitude-phase representation, the noise characteristics change significantly. If we may characterize the real and imaginary components as Gaussian with equal variance, $X_R \sim N(A_R, \sigma^2)$ and $X_I \sim N(A_I, \sigma^2)$, then the magnitude $X = \sqrt{X_R^2 + X_I^2}$ is distributed according to the Rician density [1].

$$p(t) = \frac{t}{\sigma^2} e^{-(t^2 + A^2)/2\sigma^2} I_0 \left(\frac{At}{\sigma^2}\right)$$

(1)

Here, $A = \sqrt{A_R^2 + A_I^2}$, and $I_0$ is the modified zero-order Bessel function of the first kind. Notice that when no signal is induced in the receive coil ($A=0$), the magnitude is Rayleigh distributed. The general phase distribution will not be addressed here, but in the no-signal case, the phase is uniformly distributed in $[0, 2\pi]$.

A previous study [2] addressed the model in equation (1), and tested the Rayleigh case ($A=0$) using a set of pixels, or voxels, from a no-signal region of a phantom image. The test was unsuccessful, and the authors suggested this was due to spatial correlation induced by internal processing of the MRI image. However, there are other issues that will potentially affect the results of this test. First, it is possible for the noise distribution to vary with spatial location, as observed in Fig. 1. Second, the use of a phantom in the imager may confound the testing of the no-signal case, since the reconstruction process may alias signal energy to regions outside the phantom. Third, most software in MRI systems internally round data to the nearest integer before writing to the image file, yielding an additional noise source which may strongly affect the test statistic.

III. METHODOLOGY

Model Verification: To verify consistency with the proposed noise model, $\alpha$-level hypothesis tests are performed on each individual voxel time-series. Note that $\alpha$ is the probability of rejecting the null hypothesis (rejecting the notion that the observation has the proposed distribution), given that the null hypothesis is actually true. The tests are repeated for 10 values of $\alpha$ in the range $[0.02, 0.2]$. For each value of $\alpha$, the empirical rejection rate, $\rho$, is computed as the observed fraction of rejections over all the voxels in the image.

Assuming independence between the voxel time-series, under the null hypothesis the number of rejections will be Binomially distributed with parameters $(n, \alpha)$, where $n$ is the total number of tests performed (number of voxels in the image). Note that in this case $\rho$ has mean $\alpha$, variance $\alpha(1-\alpha)/n$, and is approximately Gaussian for large $n$, $\rho \sim N(\alpha, \alpha(1-\alpha)/n)$ by the DeMoivre-Laplace theorem.

Before performing the distribution tests, the parameters $(\sigma^2, A)$ in equation (1) must be estimated. As shown in Fig. 1, these parameters are not necessarily fixed over space, therefore estimates must be determined for each individual time-series. Since estimation of these parameters for the with-signal case ($A > 0$) is extremely non-
The MRI data sets used in the following analyses affect the bin intervals are chosen judiciously, namely from the expected histograms. Therefore, if the boundaries of computed through the variation between the observed and estimated parameters (\(A_R, \sigma^2\)) and (\(A_I, \sigma^2\)), and unbiased estimates may be obtained using the sample mean and sample variance.

**Test Selection:** The \(\chi^2\) goodness-of-fit test [4] is utilized in this study for two reasons. First, this test may be structured so as to eliminate the effect of rounding on the test results. The \(\chi^2\) test statistic is computed through the variation between the observed and expected histograms. Therefore, if the boundaries of the bin intervals are chosen judiciously, namely from the set \{\ldots, -0.5, 0.5, 1.5, \ldots\}, then the observed number of samples falling within a given bin will be the same after rounding as before rounding.

Second, the estimation of the shape parameter, \(\sigma^2\), affects the \(\chi^2\) distribution of the test statistic (under the null hypothesis) by simply subtracting from the degrees of freedom. Therefore the estimation may be performed without biasing the test outcome.

**Data:** The MRI data sets used in the following analyses each consist of a time series of 256 gradient-echo echo-planar (EPI) images (64 \times 64). The images are complex-valued, with rounded magnitude and phase components. The phase component is scaled in thousandths of radians so the rounding error is small relative to the phase noise.

Most MRI applications use either the magnitude or phase component, therefore results will be included of testing the rounded magnitude and phase components for the no-signal case.

**IV. Results and Discussion**

Fig. 2 shows results of the proposed model validation on four sets of image sequences. The \(\alpha\)-level axis represents the significance level of the test, and each curve indicates the observed rejection rate for a data set.

The error bars denote a 95% acceptance region for the Gaussian approximation of the rejection rate, under the null hypothesis. In other words, under the assumption that every voxel in the image is well characterized by the respective test distribution, the observed rejection rate will fall within this acceptance region with probability 0.95.

All four of the magnitude and imaginary data sets fall with the confidence band for the full range of significance levels. The phase and real sets show slightly more deviation from the mean under the null hypothesis, but these results are still strong overall evidence that the Rician model provides an accurate characterization.

**V. Conclusions**

We have developed an appropriate method for carrying out the validation of the Rician noise model. The results obviously contradicts the common assumption that the noise in MRI images is Gaussian distributed. Therefore the issue which always needs to be addressed in such a study is how damaging this assumption may be to the conclusions of the experiment.

**References**


