PREDICTING RIGHT TURNS ON RED

by

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Abstract

Highway Capacity Manual (1997) does not provide a method of predicting right turns on red signal (RTOR). This paper presents a model for predicting the RTOR volumes at isolated and coordinated signals. The model has been derived using clear assumptions and equations consistent with these assumptions. The method was implemented in the form of a spreadsheet and then evaluated without any previous calibration. CORSIM was used to generate benchmark data. The evaluation results show that the model produces unbiased results if the impedance from queue on the approach with the right turners is the prevailing factor of the RTOR volume. A limited bias appears if the impedance from the flows on the cross street is critical for the RTOR volume. Adequate setting the model parameters (critical gap and follow-up time) should reduce this bias. A way of estimating delays of right turns is shortly addressed in the closure.

Key words: traffic signals, right turns, operational analysis, highway capacity
Introduction

Highway Capacity Manual (1997) recommends adjusting right turn volumes by subtracting the number of vehicles that leave the intersection on a red signal. The manual does not provide any method of predicting such a number. Direct observations, although recommended, are not often exercised because of a common opinion that right turns typically do not experience any capacity shortage. Nevertheless, there may be cases where turning on right significantly reduces right turn and approach delays. It should be noted that average delay is a sole criterion of the quality of traffic. Further, some practicing engineers are not comfortable applying adjustments that are not based on a sound ground. The lack of a practical method of estimating the number of right turns on red at signalized intersection has been being raised from time to time at various occasions.

This paper presents an analytical procedure for predicting the number of right turns on a red signal (RTOR). The method has been derived from clearly stated assumptions, implemented in a spreadsheet, and then evaluated using computer simulation.

Model Derivation

The number of vehicles that turn on red depends on:
1. The number of right-turners scheduled to arrive during the red signal;
2. The level of impedance from the queue on the same approach;
3. The level of impedance from traffic flow into which right turns attempt to merge (vehicles) or to cross (pedestrians).

The first two factors will be considered jointly in the estimation of the volume of RTOR vehicles $v_0$ that would leave the approach if there were no impeding pedestrians and
vehicular flows. The third factor will be represented by the capacity $c$, which is the maximum number of RTOR vehicles that would leave the approach if were continuous supply of turning vehicles at the stop bar. The RTOR volume $v$ is simply

$$v = \min\{v_0, c\}. \quad (1)$$

**Volume $v_0$**

The expected number of right turners arriving on the approach of an isolated intersection during a single red signal is calculated as:

$$a_r = v_r \cdot (C - f_r) / 3600 \quad (2)$$

where $v_r$ is the flow rate of right turners in veh/h and $(C-f_r)$ is the cycle minus phase for right turns. Phase is defined as a green signal for the movement plus a change period following the green signal. The expected lengths should be used if signals are actuated. Progression may reduce or increase the number of right turners arriving during a red phase. A simple adjustment factor $(R_p \cdot (C - f_r) / C)$ should be applied to $a_r$ to account for the progression effect, where $R_p$ is the platoon ratio corresponding to the arrival type AT.

Right-turning vehicles use the rightmost lane when approaching the right-turn bay or to the stop bar (see Fig. 1). Before entering the right-turn bay, they share this lane with other vehicles. If the other vehicles form a queue that is equal or longer than the right-turn bay, then the right turners cannot reach the stop bar and cannot leave the approach during the red signal.

On approaches without right-turn bay, right turning have to share the same lane with other vehicles. Even a single vehicle other than a right turner blocks the right turning movement. This case is equivalent to having a right-turn bay for one vehicle. In the
Let us consider red signals with a fixed total number of vehicles $t$ arriving on red. If $t$ is larger than the length of the right-turn bay $k$ and if the number of right turners arriving during a red signal varies from cycle to cycle, two cases can be distinguished:

1. The total number of non-right turners arriving during a red signal is too low to be able to block the right-turn bay in this red signal. All right turners arriving during a red signal reach the turn bay.

2. The total number of non-right turners arriving during a red signal is large enough to be able to block the right turn bay for some right turners. It must be noted that this condition allow all right turners enter the turn bay if they arrive before the turn bay is blocked.

If the number of non-right-turners arriving during the red signal is too small to block the right-turn bay, all right turns arriving in this red signal can reach the turn bay. The condition on the number of non-right-turners is $t-x < k$ or $x > t-k$. Right turners $x$ can be considered a result of $t$ independent experiments (vehicle arrivals) with the likelihood of success $p$ (proportion of right turners in the rightmost lane). The Binomial distribution is used to calculate the likelihood of $x$:

$$P_B(x) = \binom{t}{x} p^x (1-p)^{t-x},$$

where:

- $x =$ number of unblocked right-turners;
- $t =$ number of vehicles arriving in the rightmost lane;
- $p =$ proportion of right-turns in the rightmost lane.

When the number of non-right turners during a red signal is sufficiently large, then right turners that arrive after the $k^{th}$ non-right-turner are blocked and cannot reach the turn bay.
before a green signal starts. Now, $x$ is the number of right turners arriving before the $k^{th}$ non-right turner and the likelihood of associated with this event can be calculated using the Negative Binomial distribution. Unblocked right turners $x$ can be interpreted as is the number of failures before $k^{th}$ success (arrival of $k^{th}$ non-right turner). The likelihood of success in this case is the proportion of non-right turners in the rightmost lane ($1-p$). Equation 4 applies.

$$ P_{NB}(x) = \binom{x+k-1}{k-1} (1-p)^k p^x. \quad (4) $$

The value of variable $x$ determines which case occurs: $x \leq (t-k)$ is described by the Negative Binomial distribution, while $x > (t-k)$ is described by the Binomial distribution. It can be shown that the likelihood of $(x = 0$ or $1...$ or $t)$ is equal one. Thus, the average value of $x$ can be calculated using the definition of expected value.

$$ b = \sum_{x=0}^{t-k} x \cdot P_{NB}(x) + \sum_{x=t-k+1}^{t} x \cdot P_B(x) \quad (5) $$

Equation 5 has been derived with the assumption that total number of arrivals in the rightmost lane $t$ exceeds length of right-turn bay $k$. If the number of arrivals is smaller, then the right-turn bay cannot be blocks. This is the Binomial case. By setting variable $x$ at 0 if $t < k$, Equation 5 reduces to the second term and yields correct results.

Another assumption made for Equation 5 is that $t$ is single-valued. In fact, $t$ is a random variable and it has its own distribution. From that perspective, Equation 5 gives a conditional mean $b(t)$ for particular $t$. The actual mean can be calculated using the distribution of $t$ and conditional means $b(t)$.

Poisson is the most common and convenient distribution assumed for $t$. Unfortunately, if the traffic is heavy or affected by upstream signals, the Poisson distribution overstates the traffic variability. For the sake of the model simplicity, we will understate the
randomness \( t \) by assuming a fixed value equal to the mean. The performance of the model with this and other simplifying assumptions is tested in the next section using microscopic simulation and Poisson variability of arriving vehicles.

Equation 5 requires integer values of \( t \) while we are using an average number of arrivals that frequently is not integer. A practical method is to calculate \( b \) for the nearest lower and the nearest upper integer values of \( t \) and than to interpolate. The rounding error is very small as indicated by Monte Carlo tests performed by the author.

Equation 5 can be further simplified as follows:

\[
b = \min\{ p \cdot k / (1 - p), p \cdot t \}.
\]

Approximation (6) gives very close estimates of \( b \) if one of the two cases prevails, or in other words, one case occurs much more frequently than the other one. This approximation is not evaluated in this paper.

To be able to use Equations 5 and 6, one has to know \( t \) and \( p \). The expected number of vehicles approaching the intersection in the rightmost lane \( t \) includes all right-turners and the portion of through and left-turning vehicles. This value is easy to estimate after realizing that it applies to a situation with no accumulation of right turners in the rightmost lane. Thus, we may assume that right-turners do not influence the lane choice made by the non-right-turners. We will assume that the non-right-turners split equally between the continuous lanes. The expected number of all vehicles that would use the rightmost straight lane upstream of the right-turn bay is

\[
t = a_n / n + a_r ,
\]

where:

- \( a_r \) = the expected number of right turns arriving during red signal;
- \( a_n \) = the expected number of other vehicles arriving during red signal;
- \( n \) = number of lanes with the through movement.
The proportion \( p \) can be calculated as:

\[
p = \frac{a_r}{a_n / n + a_r}.
\]  \hspace{1cm} (8)

The volume of right turns arriving at the stop bar under the absence of impedance from the perpendicular streams is

\[
v_o = b \cdot m,
\]  \hspace{1cm} (9)

where \( m \) is the expected number of cycles in one hour.

**Capacity \( c \)**

Vehicles turn right on red in two different portions of cycle: (1) when other pedestrians or other vehicles impede motion of right turns, and (2) when right turns are not impeding by any other traffic. The total capacity is \( c = c_1 + c_2 \).

Typically, when red signal is displayed to right turning vehicles, through flows move in the front of right-turners from the left-hand side approach. Right-turners have to yield to the vehicles that use the rightmost lane of the cross street by accepting sufficiently long gaps in the cross-street flow. In addition, right-turners often have to cross a crosswalk when the WALK signal is displayed to the pedestrians. These two impeding factors can be modeled using properly modified HCM equations. The \( c_1 \) capacity is calculated as follows:

\[
c_1 = (1 - \frac{v_p}{2100}) \cdot \frac{v_i \cdot \exp \left( -v_i \cdot t_o \cdot \frac{C}{3600 \cdot f_i} \right)}{1 - \exp \left( -v_i \cdot t_f \cdot \frac{C}{3600 \cdot f_i} \right)}.
\]  \hspace{1cm} (10)

where:
Equation 10 is a modified version of the HCM equation for potential capacity of movements at unsignalized intersections. The modifications are: (1) an adjustment factor for pedestrians \(1 - \frac{v_p}{2100}\), and (2) multiplier \(C/f_i\) to incorporate the flow compression caused by traffic signals.

The impeding flow \(v_i\) is the volume of through vehicles that use the rightmost lane on the approach located to the left of the right turners’ approach. The amount of through traffic using this lane depends on the volume of traffic movements on the approach and on the presence and length of the right-turn bay. For example, the lack of right-turn bay and a strong right-turning volume may discourage through vehicles from using the rightmost continuous lane. In another case, a long right-turn bay for right turners removes them effectively from the rightmost through lane, thus the through traffic is uniformly distributed across the continuous lanes. Figure 2 presents the most difficult case, an approach with a short right-turn bay. The through vehicles are distributed uniformly between lanes from the stop bar until the end of the bay, beyond which some right turners are trapped in the mixed queue. The through traffic departing from the rightmost lane can be approximated with the following equation:

\[
v_i = k \cdot m + \frac{(v_n - k \cdot m \cdot n) + (v_r - b \cdot m)}{n} - (v_r - b \cdot m),
\]

where:

- *first term* = the portion of through flow that forms the queue adjacent to the right-turn bay;
second term = the remainder of the flow in the rightmost continuous lane;  
third term = the portion of the right-turning flow that does not leave the rightmost  
through lane during a red signal.

The components of Equation 11 are further explained in Figure 2. The second term  
reflects the uniform traffic distribution upstream of the right-turn bay. This traffic does  
not include both the through vehicles that are in queue adjacent to the turn bay \((k \cdot m \cdot n)\)  
and the right-turn vehicles that leave the lane on red \((b \cdot m)\).

The assumption of uniform traffic distribution is valid if the through traffic is strong  
enough to use the rightmost lane despite of the presence of right-turning vehicles. The  
condition is

\[
\frac{v_n - kmn}{n - 1} > v_r - bm, \text{ or } v_n > kmn + (n - 1)(v_r - bm). \quad (12)
\]

If the through and left-turn flow is weaker than specified in Equation 12, then the queue  
in the rightmost lane beyond the turning bay consists the right-turning vehicles only. The  
impeding flow is

\[
v_i = k \cdot m \cdot n. \quad (13)
\]

Further, when the queue of through vehicles does not reach the end of the bay (roughly  
when \(v_n < k \cdot m \cdot n\)), then the impeding queue is simply

\[
v_i = v_n / n. \quad (14)
\]

Equations 11, 13, and 14 with the associated conditions for the non-right-turn flow,  
constitute a method of estimating the impeding flow \(v_i\). An impeding flow does not occur  
during the portion of cycle, \((C - f_r - f_l)\), when neither the considered right-turning flow
nor its impeding flow has green signal. The capacity of right-turners during this portion of cycle is estimated with the assumption that a right turner needs $t_f$ seconds to leave the first position in queue. The equation consistent with this assumption is:

$$c_2 = \frac{3600}{t_f} \cdot \frac{C - f_r - f_i}{C}.$$  \hspace{1cm} (15)

Equation 15 concludes the derivation of expressions required to perform calculations of the RTOR volumes.

**Model Evaluation**

The derived model has been implemented in Excel spreadsheet (see Figure 3) to expedite its evaluation. The calculated results were compared to simulated results, which were assumed to replicate a real-world environment. Eighteen isolated intersections were tested, resulting in 72 data points. Intersections varied by traffic volumes, number of lanes, presence and length of right-turn bays, signal timing. No pedestrian traffic has been assumed. CORSIM simulation program was used to model the intersections. Each intersection was simulated for 30 minutes. Because the CORSIM text output does not report right turns on red, the turns were counted by watching computer animation. The hourly RTOR volumes were obtained by doubling the simulated counts.

Figure 3 compares the values obtained from the calculations and from the simulation. In general, the calculated results follow well the simulated values. Table 1 shows that the method overestimates the simulated values only by 2.2 veh/h while the simulated average RTOT volume is 45 veh/h. This systematic bias is probably caused by the overestimation seen in the range between 20 and 60 veh/h. The spread of the points measured with the standard deviation of 11 veh/h is caused by the simulation randomness and by factors that are not incorporated into the model. Please keep in mind that the simulation values are aggregated in short thirty-minutes intervals, thus the random factor can be considerable.
More insight into the results can be gained by separating the results in two distinct cases as indicated by the formula $\min\{v_0, c\}$:

1. Case $v_0$ occurs when all right-turning vehicles that reach the stop bar can depart before the end of red signal. No significant accumulation of right-turners occurs at the stop bar. This case is described by condition $\min\{v_0, c\} = v_0$.

2. Case $c$ occurs when there is a queue of right-turners at the end of red signals. The queue indicates the lack of capacity for right-turners on red and it is represented by condition $\min\{v_0, c\} = c$.

Since each case has its own factors and set of equations, testing the model for each case may help pinpoint specific problems and sources of inaccuracies.

Figure 4 compares the calculated and the simulated RTOR volumes for the first case where the primary factors are traffic volumes and impeding queues on the approach with the right turns. The results are in a good accordance and no bias is obvious in the entire range covered by the results. Indeed, the overall bias is only $-0.3$ veh/h. The standard deviation of the error is around 10 veh/h.

Figure 5 presents the results for the second case, where the effect of the impeding flow on the cross street prevails. A systematic over estimation is clear and confirmed with the numbers in Table 1. The method overestimates the RTOR volumes by 6.3 veh/h while the average true value is believed to be 38.5 veh/h. The standard deviation of the error is again around 10 veh/h, which indicates that the method can be as good as in the first case if the systematic bias is removed.

It has to be emphasized that the model has not been calibrated with the simulated data. All the inputs are either traffic and geometry characteristics, or gap acceptance parameters taken at default values from the HCM. Some tuning is possible to remove the systematic bias present in the second case.
Closure

This paper presents a model for predicting the RTOR volumes at isolated and coordinated signals. The model has been derived using clear assumptions and equations consistent with these assumptions. The method has been implemented in the form of a spreadsheet. The model has been evaluated without any previous calibration. CORSIM was used to generate benchmark data. The evaluation results show that the model produces unbiased results if the impedance on approach on the approach with the right turners determines the RTOR volume. A limited bias appears if the impedance from the flows on the cross street is critical for the RTOR volume. Adequate setting the model parameters (critical gap and follow-up time) should reduce the bias.

The presented model has been derived with the assumption that the through and right-turning flows receive green at the same time, which is the most common case. When these two flows receive green in different times, then the presented method may fail.

The method has been evaluated using micro simulation. The lack of resources did not allow the author to conduct field evaluation. Field data should not be difficult to collect at selected intersections. It would include collecting geometry parameters (number of lanes, length of turn bays), counting arriving volumes, RTORs, and measuring signal lengths.

Estimating the delays of right turning flows seems to be a straightforward task once the RTOR volumes are known. In most cases of negligible queuing, the average delay of RTORs can be approximated with the average time spent at the stop bar, which is $3600/c$. The average delay of the remainder of the right-turn flow can be estimated as other vehicles with one modification though. Subtracting RTORs from the turning flow
reduces the number of arrivals on red considered in this case, thus the arrival type changes towards the one that characterizes better progression. The adjustment that accounts for this effect can be obtained by recalculating progression factor $R_p$ appropriately.

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Literature

Table 1 Summary of prediction errors

<table>
<thead>
<tr>
<th>Case</th>
<th>Number of Runs</th>
<th>Simulated Mean RTOR (veh/h)</th>
<th>Mean Error (veh/h)</th>
<th>Error Standard Deviation (veh/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Both cases</td>
<td>72</td>
<td>45.4</td>
<td>2.2</td>
<td>10.9</td>
</tr>
<tr>
<td>Case $v_0$</td>
<td>45</td>
<td>49.6</td>
<td>-0.3</td>
<td>10.6</td>
</tr>
<tr>
<td>Case $c$</td>
<td>27</td>
<td>38.5</td>
<td>6.3</td>
<td>10.2</td>
</tr>
</tbody>
</table>

Figure 1 Traffic factors of RTOR volumes
Figure 2 An approach with a short right-turn bay

\[ m = \text{# of cycles in one hour} \]
\[ n = \text{# of continuous lanes} \]
\[ k = \text{bay length in veh} \]
\[ b = \text{# of right turners leaving the rightmost lane on red} \]
\[ v_r = \text{right turning volume} \]
\[ v_n = \text{non-right turning volume} \]

\[ \frac{(v_n - k \cdot n \cdot m) - (n - 1) \cdot (v_r - b \cdot m)}{n} \]

Figure 3 Calculated vs. simulated RTOT volumes – both the cases
Figure 4  Calculated vs. simulated RTOT volumes – the $v_0$ case

Figure 5  Calculated vs. simulated RTOT volumes – the $c$ case