ABSTRACT

We present a method for estimating the inputs and states in discrete-time switched linear systems with unknown inputs. We first investigate the problem of system invertibility, which reconstructs the unknown inputs based on knowledge of the output of the system, the switching sequence, and the initial system state. We then relax the assumption on the knowledge of the initial system state, and construct observers that asymptotically estimate the state. Our design, which considers a general class of switched linear observers that switch modes based on the known (but arbitrary) switching sequence, shows that system invertibility is necessary in order to construct state observers. Furthermore, some portion of the observer gain must be used to recover the unknown inputs, and the remaining freedom must be used to ensure stability. The state of the observer is then used to asymptotically estimate the unknown inputs (i.e., it forms the dynamic portion of a stable inverter for the given switched system).

1. Overview

Many physical systems can be modeled as having:

- System model: $x_k \in \mathbb{R}^n \ (k = 1, 2, \ldots, N)$ – Specifies the dynamics of the system from $N$ possibilities at time-step $k$
- Known inputs for each mode: $u_k \in \mathbb{R}^m$ – Control signals, etc.
- Unknown inputs for each mode: $d_k \in \mathbb{R}^r$ – Disturbances, faults, modeling uncertainties, etc.
- Internal states: $x_k \in \mathbb{R}^n$ – Evolution: $x_{k+1} = A_k x_k + B_k u_k + E_k d_k$
- Outputs: $y_k \in \mathbb{R}^p$ – At time-step $k$: $y_k = C_k x_k + D_k u_k + F_k d_k$

2. Objectives

Using the output of the system and the switching sequence:

- Estimate the unknown inputs $d_k$ at each time-step
- Estimate the state of the system $x_k$ at each time-step

3. System Invertibility

System is invertible with delay $\alpha$ if, at each time-step $k$, the inputs can be uniquely recovered. Theorem 1 shows that system invertibility is necessary in order to construct state observers. Furthermore, some portion of the observer gain must be used to recover the unknown inputs, and the remaining freedom must be used to ensure stability. The state of the observer is then used to asymptotically estimate the unknown inputs (i.e., it forms the dynamic portion of a stable inverter for the given switched system).

4. State Estimation

- System inverter requires knowledge of initial state $x_0$
- If state is unknown, estimate it by constructing observer of the form $\hat{x}_k = J_{\alpha xk}(x_k, u_k, d_k)$
- Choose $J_{\alpha xk}$ and $K_{\alpha xk}$ so that $\hat{x}_k \rightarrow x_k$ as $k \rightarrow \infty$

5. Stable Inversion

To obtain a stable inverter:

- Replace dynamic portion of system inverter (equation (3)) with state observer (equation (4))
- Since output of observer $\hat{x}_k \rightarrow x_k$ as $k \rightarrow \infty$, we have $d_k \rightarrow d_k$ as $k \rightarrow \infty$
- Resulting inverter asymptotically estimates unknown inputs without knowledge of initial state $x_0$

6. Summary of Contributions

- Studied estimation of unknown inputs and states in switched linear systems
- Presented test for system invertibility, along with a construction for system inverter
- When initial state is unknown, constructed a state observer to asymptotically estimate states
- Showed that system must be invertible in order to construct state observers
- Obtained stable inverter by using state observer as dynamic portion of inverter

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