3 / Generalized Immittance Based Stability Analysis

In our work to this point, we have been concentrating on design techniques that are meant to be applied to known systems at an operating point. In other words, using the vocabulary of Chapter 1, we have been focused on the problem of local dynamic stability. However, power electronics based systems are normally operated through a host of operating points, and it is desirable, in a single analysis, to show that all operating points are stable. In other words, we wish to address the problem of regional dynamic stability. As it turns out, we can approach the problem of regional dynamic stability in much the same way as the problem of local dynamic stability, using the concept of generalized immittance.

3.1 GENERALIZED IMMITTANCE

Power electronics based systems exhibit nonlinear dynamics and therefore the linearized component model changes as loading conditions vary. Furthermore, parameter variations result in further variations of the linearized plant model. As an example, consider the laboratory dc power system depicted in Fig. 3.1-1. This system is typical of a turbine electric propulsion system. In this system, a turbine prime mover (implemented in the laboratory as a dynamometer emulating a turbine) drives a synchronous machine rectifier system whose output voltage is regulated at 400 V. The rectifier output is connected via a short transmission line to an induction motor drive turning a mechanical load (emulated by a second dynamometer in the laboratory). In this case, the induction motor is controlled using an indirect field oriented control so that torque control is nearly instantaneous. The parameters of this system and a detailed description are set forth in [1]. That work described two control systems, a nominal control and a dc link stabilizing control (later referred to as a Nonlinear Stabilizing Control in [2]). The nominal control is used herein.

Fig. 3.1-2 depicts the linearized source impedance of the synchronous machine rectifier system as power varies from 0 to 1 p.u. in 0.2 p.u. steps. Not unexpectedly, the impedance is a function of output power due to dynamic non-linearities and will also vary as other aspects of the operating point (speed, voltage...
reference) and parameters vary. The techniques of the previous section may be extended to this practical system by representing the source impedance and load admittance as generalized sets (denoted $Z^G_s(s)$ and $Y^G_l(s)$, respectively), which encompass the full range of possible plants. This is done by characterizing the source/load over the full range of operating conditions and parameter values and then bounding the result. The generalized impedance/admittance is defined as this bounded region. As an example, Fig. 3.1-3 depicts the generalized source impedance of the generator rectifier. The generalized impedance for this example was constructed by defining a set of nominal speeds from 228-279 rad/s in 5 increments, defining a set of output voltage references from 380-420 V in 5 increments, and defining a set of output powers from 0 to 4.07 kW in 5 increments. The plant was linearized about every combination of speed, voltage reference, and output power obtainable using these sets (therefore yielding 125 plants). The individual source impedances of the resulting 125 plants were then bounded and the generalized source impedance was taken to be the interior region of this bound, which provides a good representation of all possible values of source impedance.

Figure 3.1-2. Linearized source impedance of the synchronous machine rectifier system.
Figure 3.1-3. Generalized source impedance of the generator rectifier.

Fig. 3.1-4 depicts the generalized load admittance of the tie line and the induction motor drive. In this case, the generalized load admittance was based on 216 plants obtained by every combination of linearized plants which could be arrived from the set of speed values (ranging from 0-200 rad/s in 6 steps), torque commands (ranging from 0-20.9 Nm in 6 steps), and input voltage (ranging from 380-420 V in 6 steps).

Figure 3.1-4. Generalized load admittance of the tie line and induction motor drive.
The use of generalized impedance sets and generalized admittance sets can be used to account for nonlinearities. By taking into account every possible plant model, small signal stability of the plant can be guaranteed for all operating points considered. Likewise, the use of generalized impedances and admittances can be used to take into account uncertainties, and variations in parameters and operating conditions. For example, in the case the synchronous machine/rectifier, the source impedance set was formulated by linearizing the plant over the entire range of operating conditions and therefore represents a good approximation to the full range of impedance values that could be encountered. In the example shown, this involved varying the set points and loading conditions, but could have just as easily included parameter variation as well.

There are many methods to create such a boundary from the set of linearized plants; herein the boundary is taken to be an irregular N-sided polygon. This polygon as a function of frequency forms the generalized source impedance or generalized load admittance, as appropriate.

From a computation point of view, the automatic calculation of the impedances/admittances is readily achievable by linearizing time-domain Nonlinear Average Value Models (NLAMs) [3-5] of the components. In particular, NLAMs are automatically linearizable in a variety of state variable based simulation languages such as ACSL [6] and Simulink [7]. This facilitates rapid and easy calculation of the generalized source impedance and the generalized load admittance.

Although NLAM models are probably the easiest method to use, the impedance/admittance sets required can also be obtained experimentally (by measuring impedance/admittance over a large range of operating points) or by using a detailed computer simulation to ‘measure’ the input impedance.

3.2 GENERALIZED NYQUIST EVALUATION

The Nyquist Impittance Criterion can be used not only to test for local dynamic stability but for regional dynamic stability as well, by using generalized impedance and admittance concepts. Whereas the impedance \( Z(s) \) (or admittance \( Y(s) \)) is a unique complex number at a given frequency; the generalized impedance \( Z^G(s) \) (or admittance \( Y^G(s) \)) is a bounded set of numbers. In other words, whereas a standard impedance (or admittance) is a single point in the s-plane, the generalized impedance (or admittance) is a bounded area in the s-plane.

The product of two generalized quantities \( X^G(s) \) and \( Y^G(s) \) (which could either be impedances or admittances for the purposes at hand) is defined as the bounded set of values that can be achieved by multiplication of any value \( x(s) \in X^G(s) \) and \( y(s) \in Y^G(s) \). As a result, the Nyquist contour \( Z^G(s)Y^G(s) \) is not a line in the s-plane, but rather a bounded set (representing, in essence, a ‘thick line’). If the Nyquist contour of \( Z^G(s)Y^G(s) \) does not encircle the –1 point then
regional dynamic stability in ensured because it can readily be shown that
\( Z(s)Y_f(s) \) could not have encircled the –1 point for all \( Z(s) \in Z^G(s) \) and all
\( Y_f(s) \in Y^G(s) \). Hence regional dynamic stability can be guaranteed.

Let us consider a brief example. Consider the circuit shown in Fig. 3.2-1. Figure 3.2-2 illustrates the standard Nyquist evaluation of \( Z(s)Y_f(s) \) with
\( R_{load} = 97.3 \, \Omega \) and also with \( R_{load} = 24.3 \, \Omega \). As can be seen, in the first case there are no encirclements of the –1 point and so the system is stable; in the second case two clockwise encirclement are present so the system is unstable.

Figure 3.2-1. Example system for Nyquist evaluation.

Figure 3.2-2. Nyquist evaluation for example system.

Now let us consider the generalized case. The generalized Nyquist evaluation of \( Z^G(s)Y^G_f(s) \) is shown for the circuit in Fig. 3.2-1 with
\( R_{load} = 24.3 \, \Omega \), with the exception that all parameter have been allowed to vary by
\( \pm 1\% \). The contour itself is made up a collection of regions; where each region bounds \( Z^G(s)Y^G_f(s) \) for a particular value of \( s \). For example, the set of values comprising \( Z^G(s)Y^G_f(s) |_{s=\omega_1} \) where \( \omega_1 = 2\pi 104.7 \, \text{rad/s} \) are highlighted. This amalgamation of all such regions as \( s \) is swept over a range of values which encircles the right half plane makes up the generalized Nyquist evaluation. It is
interesting (and important) to observe that even though the range of parameter variation was small, the Nyquist contour ‘thickened’ considerably.

Figure 3.2-3. Generalized Nyquist evaluation for example system.

3.3 GENERALIZED MIDDLEBROOK

The Middlebrook design constraint is based on confining the Nyquist contour of \( Z_s(s)Y_I(s) \) to the interior of a circle or radius \( 1/GM \) in the s-plane, where the gain margin \( (GM) \) is greater than one. Recall that in Section 2, wherein we discussed local dynamics stability, the Middlebrook Criteria could be stated

\[
|Z_s(s)Y_I(s)| < \frac{1}{GM} \quad (3.3-1)
\]

When examining regional dynamic stability using generalized admittances, it is convenient to define a generalized impedance (admittance) norm on \( X^G \) as

\[
\|X\|_G = \max_{x \in X^G} \|x\|_G
\]

whereupon the Middlebrook design constraint to ensure regional dynamic stability becomes

\[
\left\|Z_s^G(s)\right\|_G \left\|Y_I^G(s)\right\|_G < \frac{1}{GM} \quad (3.3-3)
\]

Provided that (3.3-3) is satisfied then it follows that (3.3-1) will be satisfied for all \( Z_s(s) \in Z_s^G(s) \) and all \( Y_I(s) \in Y_I^G(s) \), which in turn means that the Nyquist
criteria will be satisfied for all \( Z_s(s) \in Z_s^G(s) \) and all \( Y_f(s) \in Y_f^G(s) \), thereby insuring regional dynamic stability.

From (3.3-3), we can readily form design specifications. In particular, suppose the generalized source impedance is known. From (3.3-3) we require that the generalized load admittance satisfy

\[
\left\| Y_f^G(s) \right\|_G < \frac{1}{GM} \left\| Z_s^G(s) \right\|_G \tag{3.3-4}
\]

Alternately, if the load admittance is known, we can formulate the following specification on the source impedance

\[
\left\| Z_l^G(s) \right\|_G < \frac{1}{GM} \left\| Y_s^G(s) \right\|_G \tag{3.3-5}
\]

### 3.4 GENERALIZED GAIN AND PHASE MARGIN CRITERIA

The Gain Margin, Phase Margin (GMPM) design method is based on confining the Nyquist contour of \( Z_s(s)Y_f(s) \) to the allowable region indicated in Figure 2.5-1. From Section 2.5, the GMPM design constraint is satisfied in a local sense provided that (3.4-1) \textit{or} (3.4-2) is satisfied

\[
\left\| Y_f(s) \right\| Z_s(s) \leq \frac{1}{GM} \tag{3.4-1}
\]

\[
\angle Y_f(s) + \angle Z_s(s) \leq \left[180^\circ - PM\right] \quad \text{and} \quad \angle Y_f(s) + \angle Z_s(s) \geq \left[-180^\circ + PM\right] \tag{3.4-2}
\]

Similarly, the GMPM criterion can be used to ensure regional dynamic stability provided that either (3.4-3) \textit{or} (3.4-4) is satisfied.

\[
\left\| Y_f^G(s) \right\|_G \left\| Z_s^G(s) \right\|_G < \frac{1}{GM} \tag{3.4-3}
\]

\[
\angle_{\text{max}} Y_f^G(s) + \angle_{\text{max}} Z_s^G(s) \leq \left[180^\circ - PM\right] \quad \text{and} \quad \angle_{\text{min}} Y_f^G(s) + \angle_{\text{min}} Z_s^G(s) \geq \left[-180^\circ + PM\right] \tag{3.4-4}
\]
where the definition of $\angle_{G}\text{max}^X$ and $\angle_{G}\text{min}^X$ is illustrated in Fig. 3.4-1. In particular, therein shows the boundary of a generalized set $X^G$ in the Nyquist plane. The angles $-\phi_{\text{max}}$ and $\phi_{\text{min}}$ are measured in the clockwise and counterclockwise directions, respectively, and are both restricted to the interval $[0 \ 2\pi)$. They are selected so that the indicated lines are tangent to the generalized set, whereupon $\angle_{G}\text{max}^X$ is defined as $\phi_{\text{max}}$ as mapped to the interval $[0 \ \pi)$ and $\angle_{G}\text{min}^X$ is defined as $\phi_{\text{min}}$ as mapped to the interval $(-\pi \ \pi)$.

![Figure 3.4-1. Definition of $\angle_{G}\text{max}^X$ and $\angle_{G}\text{min}^X$.](image)

From (3.4-3)-(3.4-4), if the source impedance is known, the design specifications on the load are such that either (3.5-5) or (3.4-6) is satisfied.

\[
\left\| Y_l^G(s) \right\|_G \leq \frac{1}{\left\| Z_s^G(s) \right\|_G}\ GM
\]  
(3.4-5)

\[
\angle_{G}\text{max} Y_l^G(s) \leq \left(180^\circ - \text{PM}\right)-\angle_{G}\text{max} Z_s^G(s)
\]

\[
\angle_{G}\text{min} Y_l^G(s) \geq \left(-180^\circ - \text{PM}\right)-\angle_{G}\text{min} Z_s^G(s)
\]  
(3.4-6)

Conversely, if the load is characterized then the design constraints on the load are that either (3.4-7) or (3.4-8) are satisfied.

\[
\left\| Z_s^G(s) \right\|_G \leq \frac{1}{\left\| Y_l^G(s) \right\|_G}\ GM
\]  
(3.4-7)
\[ \angle_{\text{max}} Z_s^G(s) \leq (180^\circ - PM) \angle_{\text{max}} Y_l^G(s) \]
\[ \angle_{\text{min}} Z_s^G(s) \geq -(180^\circ - PM) \angle_{\text{min}} Y_l^G(s) \]

and

\[ (3.4-8) \]

3.5 GENERALIZED ESAC CRITERIA

From our work in Section 2.7, the reader will recall that the formulation of immittance specifications using the ESAC Criterion is more involved than for the Middlebrook or GMPM Criteria. Not surprisingly, this is also true in the generalized case.

The process for constructing the load admittance constraint from a generalized source impedance is illustrated in Figure 3.5-1. Therein the ESAC Criteria stability constraint is shown, a partial Nyquist evaluation of the (nongeneralized) source impedance of a nominal plant (the same as shown as is shown in Figure 2.6-1(a)) along with generalized source impedance \( Z_s^G \) at a frequency \( f_a \) (this is based on the nominal source plant with a parameter variation of ±10%).

For the purposes at hand it is convenient to define \( Z_{s,a}^G = Z_s^G (j2\pi f_a) \); it is also convenient to define \( Z_{s,a}^P \) as the path in the s-plane that bounds \( Z_{s,a}^G \). Point \( S_b \) in Fig. 3.5-1 represents a point on the stability criteria. The immediate problem at hand is to find the load admittance \( Y_{l,ab} \) which will cause \( Y_{l,ab} Z_s \) to be tangent to the stability criteria curve at point \( S_b \) for one value \( Z_s \in Z_{s,a}^G \), and be in the allowable region for all other value \( Z_s \in Z_{s,a}^G \). As in turns out, this may be calculated as

\[ Y_{l,ab} = \frac{S_b}{Z_{s,ab}} \]

(3.5-1)

where \( Z_{s,ab} \) is the value of source impedance which will define be used to calculate the load admittance constraint at point \( S_b \) and at frequency \( f_a \). In order to determine \( Z_{s,ab} \), it is convenient to define a point \( t_b \) which is an incremental distance \( \varepsilon \) from \( S_b \), in the forbidden region, and located in a direction normal to the stability criteria curve. Geometrical considerations reveal that \( Z_{s,ab} \) is the value of \( Z_s \in Z_{s,a}^P \) which maximizes

\[ \int_{Z_{s,a}^P} \left| t_b - \frac{S_b}{Z_{s,ab}^P} Z_{s,ab}^P(s) \right| ds \]

(3.5-2)
Figure 3.5-1. Construction of load admittance constraint from a generalized source impedance.

By sweeping point $S_b$ over the entire stability constraint curve (which is truncated at points $S_c$ to $S_d$ for boundedness) a constraint is placed on the load admittance at frequency $f_a$. Repeating this process over all frequencies of interest results in the three dimensional load admittance constraint curve. As can be seen, except for the calculation $Z_{s,ab}$ this part of the procedure is identical to the procedure for local dynamic stability.

The converse problem of determining a design specification on the source impedance from the load impedance utilizes an identical procedure with the exception that the roles of source impedance and load admittance are interchanged and the result is plotted in impedance rather than in admittance space.

3.6 CASE STUDY

In order to illustrate some of the concepts presented, let us return to the sample system first discussed in Section 3.1. Figure 3.6-1 depicts the resulting load admittance constraint and generalized load admittance. Therein, the ESAC Criterion was used with a 3 dB gain margin and a 20° phase margin. In this case it can be seen that the generalized load admittance intersects the load admittance constraint, which could result in instability.
Fig. 3.6-1. Load admittance constraint and generalized load admittance for system shown in Fig 3.1-1.

Fig. 3.6-2 depicts the measured system performance during a ramp increase in torque command. Initially the system is operating in the steady state with a commanded bus voltage of 400 V, the generator speed is at 253.4 rad/s, and the induction motor speed is at 188.5 rad/s (the mechanical speed was fixed by a dynamometer). Approximately 100 ms into the study, a ramped torque command is issued. The traces are the commanded current, the actual current, and the dc bus voltage at the input to the converter. As can be seen, an instability results, which is consistent with Figure 3.6-1.

Fig. 3.6-2. Measured system performance during a ramp increase in torque command.

It should be kept in mind that satisfying the stability criterion is a sufficient but not necessary condition for stability, and so it would have been possible for the system to be stable even though a violation of the stability criterion was observed. If a necessary and sufficient condition criterion is desired, this can be approximated by using the ESAC Criterion (because it introduces less artificial conservatism than the other methods) with very low gain and phase margins.
It is interesting to observe the changes that can occur in the stability of the example system as the control is modified. In particular, incorporating the dc link stabilizing control first introduced in [1] and later referred to as a nonlinear stabilizing control (NSC) in [2] yields a dramatic change in the generalized load admittance, as seen in Fig. 3.6-3. In this case, there is no intersection of the generalized load admittance and so stability is guaranteed (although there is an apparent intersection in Fig. 3.6-3, rotation of the figure will reveal that the intersection is merely a line-of-sight effect). The experiment shown in Fig. 3.6-2 is repeated in Fig. 3.6-4 with the Nonlinear Stabilizing Control demonstrating that for this scenario system operation is stable.

Fig. 3.6-3. Load admittance constraint and generalized load admittance for system shown in Fig 3.1-1 with addition of Nonlinear Stabilizing Control.

Fig. 3.6-4. Measured system performance during a ramp increase in torque command while utilizing Nonlinear Stabilizing Control.
3.7 CLOSING REMARKS

In this Section, we have modified our original immittance base approach so that we can treat nonlinearities, and uncertainties, and thus consider the problem of regional dynamic stability. However, at this point, we are still only considering simple source load systems. In the next section, we will extend this technique to the consideration of full systems.

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3.9 REFERENCES