EE595S: Class Lecture Notes
Chapter 2* / QD Models for Permanent Magnet Synchronous Machines

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Fall 2005

***Analysis and Design of Permanent Magnet Synchronous Machines**

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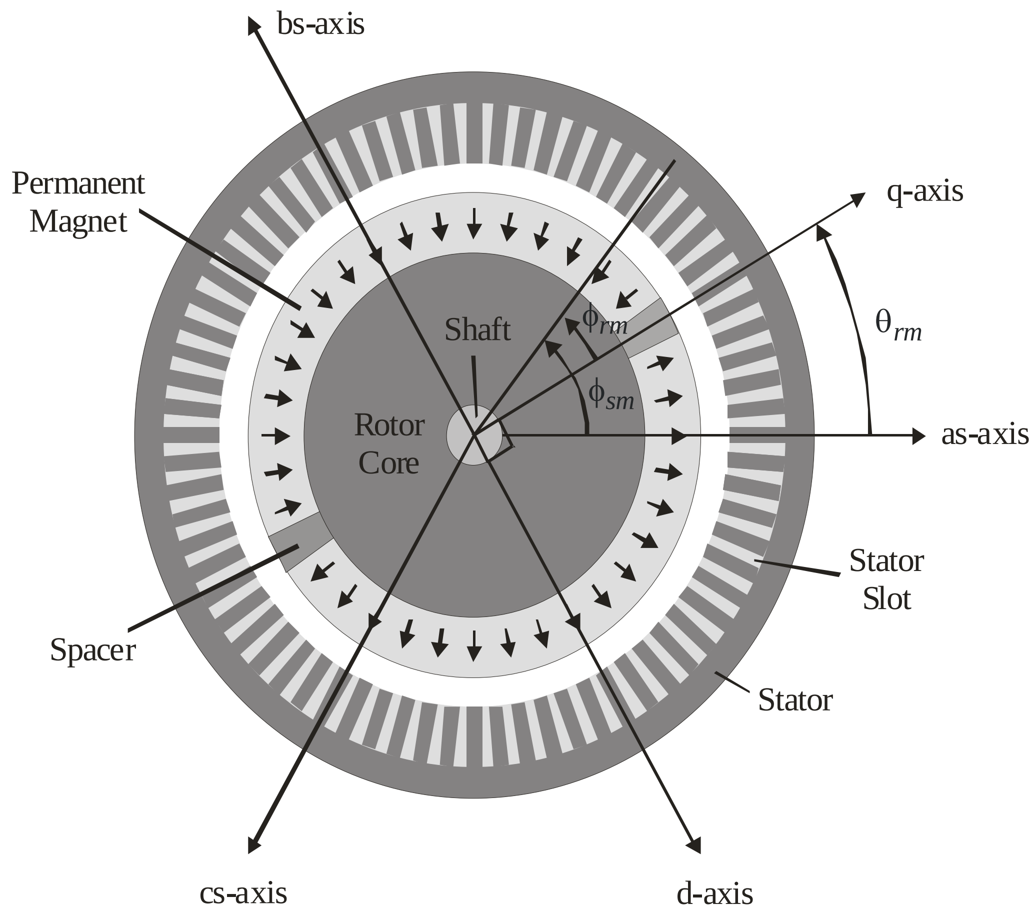
2.1 SMPM: Configuration



2.1 Position Relationships

$$\phi_{sm} = \theta_{rm} + \phi_{rm}$$

$$\theta_r = \frac{P}{2}\theta_{rm} \qquad \omega_r = \frac{P}{2}\omega_{rm}$$

$$\phi_r = \frac{P}{2}\phi_{rm} \qquad \phi_s = \frac{P}{2}\phi_{sm}$$

$$\phi_s = \theta_r + \phi_r$$

2.1 Turns Distribution

$$n_{as}(\phi_{sm}) = n_p \sin(\phi_s) \qquad (2.1\text{-}7)$$

$$n_{bs}(\phi_{sm}) = n_p \sin(\phi_s - 2\pi/3) \qquad (2.1\text{-}8)$$

$$n_{cs}(\phi_{sm}) = n_p \sin(\phi_s + 2\pi/3) \qquad (2.1\text{-}9)$$

2.1 Winding Function

$$w_{as}(\phi_s) = \frac{2n_p}{P}\cos(\phi_s) \tag{2.1-10}$$

$$w_{bs}(\phi_s) = \frac{2n_p}{P}\cos(\phi_s - 2\pi/3) \tag{2.1-11}$$

$$w_{cs}(\phi_s) = \frac{2n_p}{P}\cos(\phi_s + 2\pi/3) \tag{2.1-12}$$

2.1 Comments on Winding Function

- Properties

2.1 Comments on Winding Function

- Derivation

2.2 ABC Model

- Voltage Equation

$$\mathbf{v}_{abcs} = r_s \mathbf{i}_{abcs} + p\boldsymbol{\lambda}_{abcs} \qquad (2.2\text{-}1)$$

- Flux Linkage Equation

$$\boldsymbol{\lambda}_{abcs} = \boldsymbol{\lambda}_{abcs,l} + \boldsymbol{\lambda}_{abcs,m} \qquad (2.2\text{-}2)$$

2.2 ABC Model

- Leakage Flux

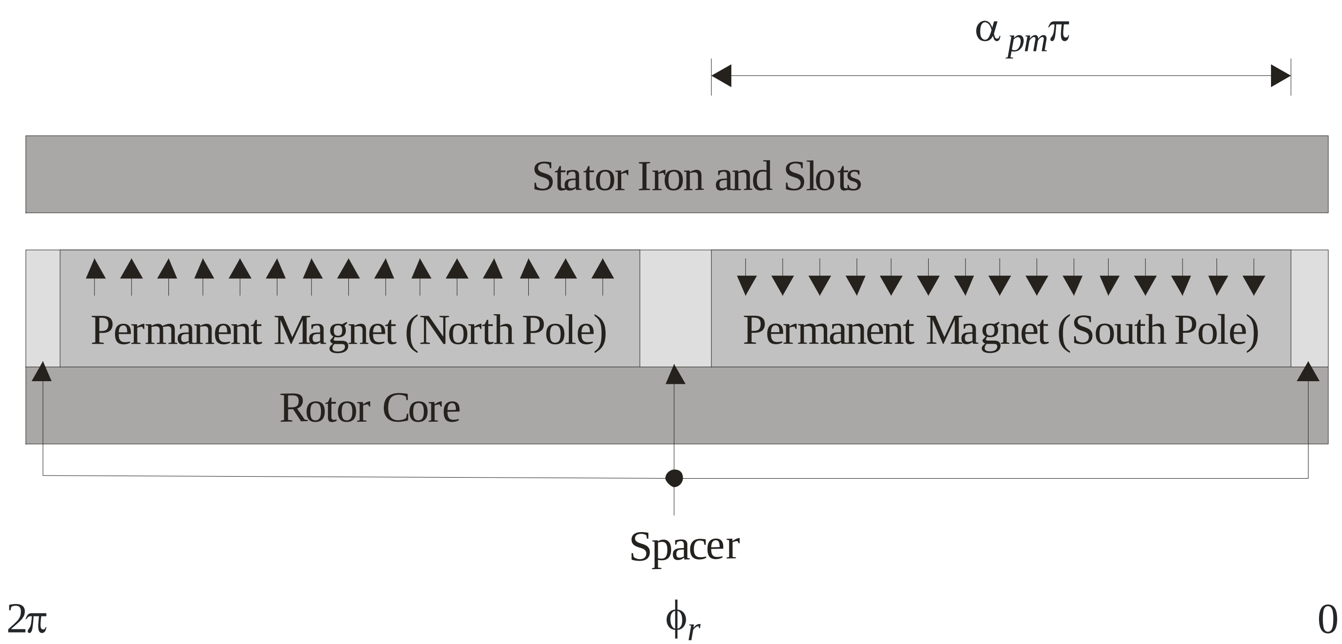
$$\boldsymbol{\lambda}_{abcs,l} = \begin{bmatrix} L_{lp} & L_{lm} & L_{lm} \\ L_{lm} & L_{lp} & L_{lm} \\ L_{lm} & L_{lm} & L_{lp} \end{bmatrix} \mathbf{i}_{abcs} \qquad (2.2\text{-}3)$$

- Magnetizing Flux

$$\lambda_{as,m} = \int_0^{2\pi} w_{as}(\phi_s) B(\phi_s) dr_s \, d\phi_{sm} \qquad (2.2\text{-}4)$$

2.2 ABC Model

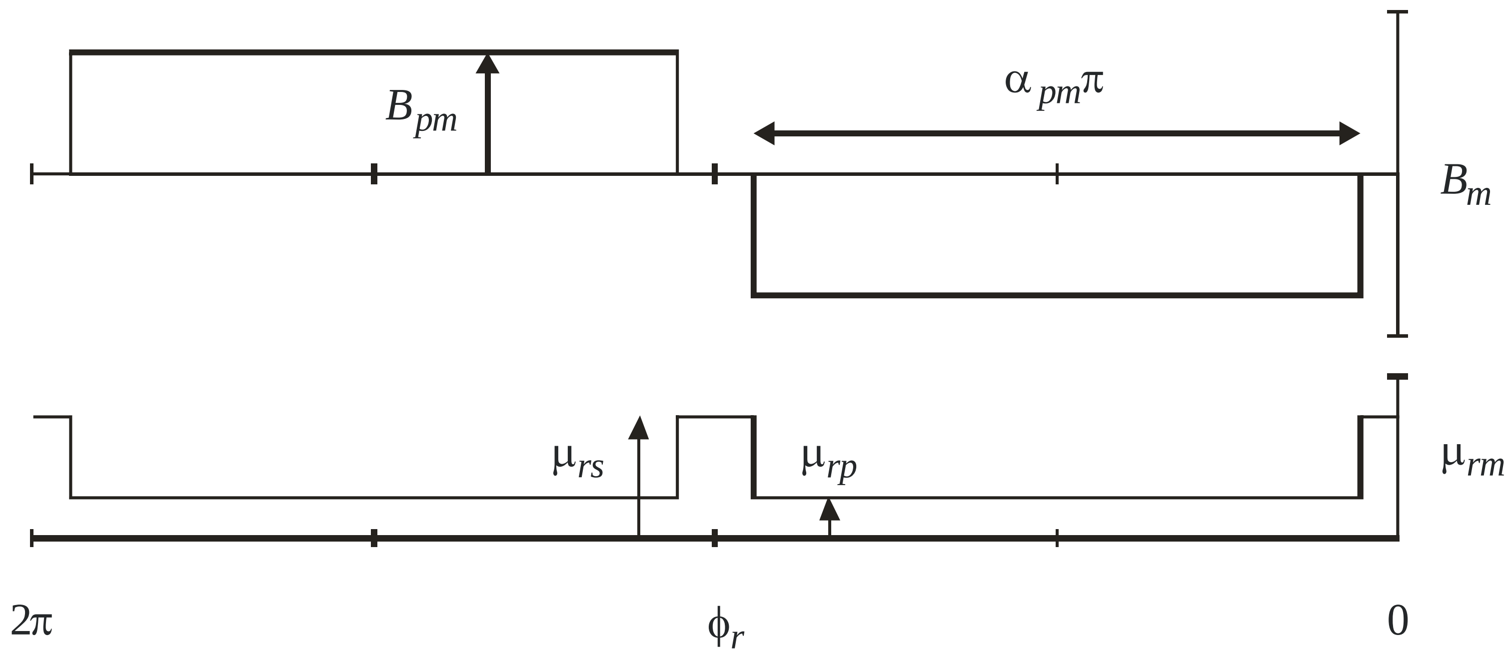
- Magnetic Material



2.2 ABC Model

- In Magnetic Material $B = \mu_{rm}\mu_0 H_m + B_m$

(2.2-5)



2.2 ABC Model

- In the magnetic material: $B = \mu_{rm}\mu_0 H_m + B_m$ (2.2-5)
- In the gap: $B = \mu_0 H_g$ (2.2-6)

- Thus $H_m = \dfrac{1}{\mu_{rm}}\left(H_g - \dfrac{1}{\mu_o}B_m\right)$ (2.2-7)

2.2 ABC Model

- Recall that MMF drop is defined as

$$F = \int_{rotor}^{stator} H dr \qquad (2.2\text{-}8)$$

- Thus

$$F = H_g g + H_m d_m \qquad (2.2\text{-}9)$$

2.2 ABC Model

- After Manipulation,

$$B = c_{bf} F + B_{m,eff} \qquad (2.2\text{-}10)$$

- where

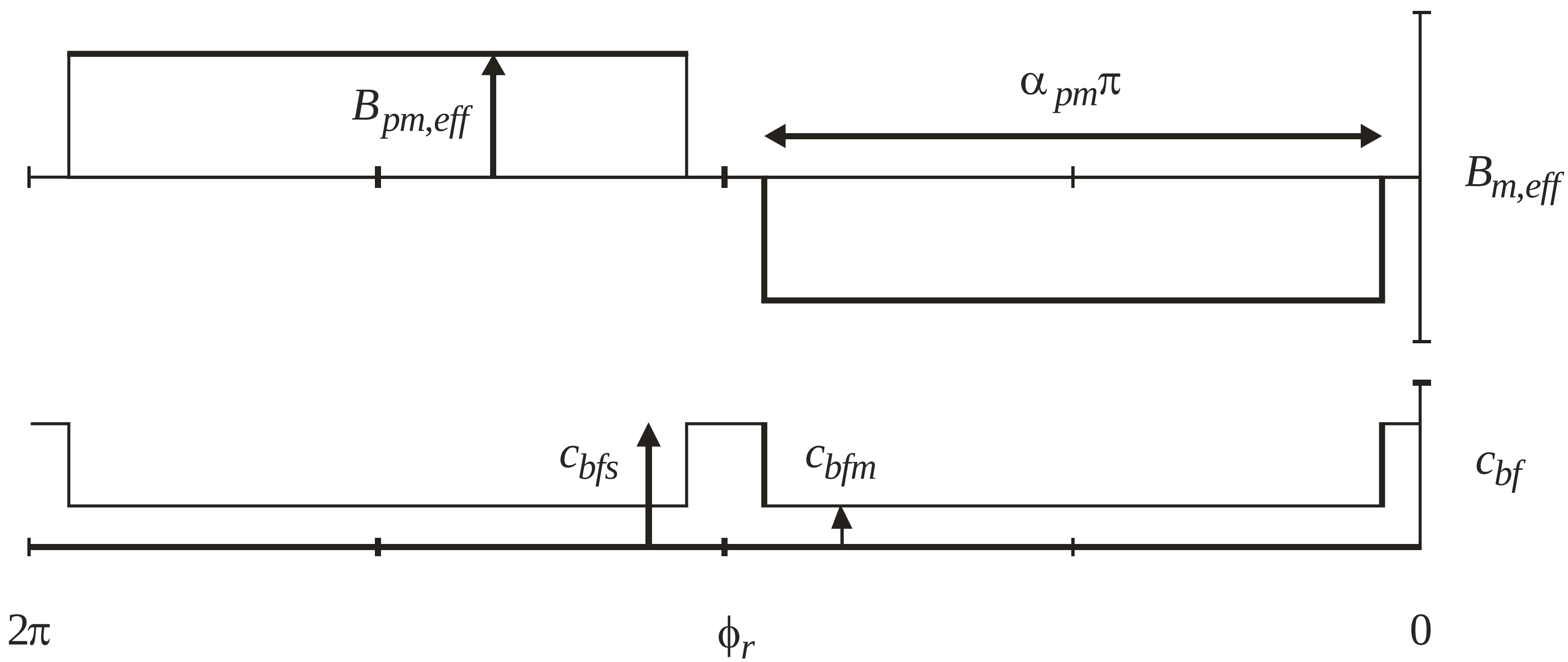
$$c_{bf} = \frac{\mu_0 \mu_{rm}}{g\mu_{rm} + d_m} \qquad (2.2\text{-}11)$$

$$B_{m,eff} = \frac{d_m}{g\mu_{rm} + d_m} B_m \qquad (2.2\text{-}12)$$

2.2 ABC Model

- Derivation:

2.2 ABC Model



2.2 ABC Model

- Note: we have

$$c_{bfm} = \frac{\mu_0 \mu_{rp}}{g\mu_{rp} + d_m} \tag{2.2-13}$$

$$c_{bfs} = \frac{\mu_0 \mu_{rs}}{g\mu_{rs} + d_m} \tag{2.2-14}$$

$$B_{pm,eff} = \frac{d_m}{g\mu_{rp} + d_m} B_{pm} \tag{2.2-15}$$

2.2 ABC Model

- Recapping

$$\lambda_{as,m} = \int_0^{2\pi} w_{as}(\phi_s) B(\phi_s) dr_s \, d\phi_{sm} \qquad (2.2\text{-}4)$$

$$B = c_{bf} F + B_{m,eff} \qquad (2.2\text{-}10)$$

- Also, we have

$$F = w_{as} i_{as} + w_{bs} i_{bs} + w_{cs} i_{cs} \qquad (2.2\text{-}16)$$

2.2 ABC Model

- Substituting (2.2-16) into (2.2-10) and result into (2.2-4)

$$\lambda_{as,m} = dr_s \int_0^{2\pi} w_{as}\left(c_{bf}\left(w_{as}i_{as} + w_{bs}i_{bs} + w_{cs}i_{cs}\right) + B_{m,eff}\right) d\phi_{sm} \qquad (2.2\text{-}17)$$

$$\lambda_{as,m} = dr_s \int_0^{2\pi} w_{as}\left(c_{bf}\left(w_{as}i_{as} + w_{bs}i_{bs} + w_{cs}i_{cs}\right) + B_{m,eff}\right) d\phi_{rm} \qquad (2.2\text{-}18)$$

$$\lambda_{as,m} = dr_s P \int_0^{2\pi / P} w_{as}\left(c_{bf}\left(w_{as}i_{as} + w_{bs}i_{bs} + w_{cs}i_{cs}\right) + B_{m,eff}\right) d\phi_{rm} \qquad (2.2\text{-}19)$$

2.2 ABC Model

- Evaluating (2.2-19) yields

$$\lambda_{as,m} = L_{asas} i_{as} + L_{asbs} i_{bs} + L_{ascs} i_{cs} + \lambda_{as,pm} \qquad (2.2\text{-}20)$$

$$L_{asy} = 2 d r_s \int_0^{\pi} w_{as} w_y c_{bf} \, d\phi_r \qquad (2.2\text{-}21)$$

$$\lambda_{as,pm} = 2 d r_s \int_0^{\pi} w_{as} B_{m,eff} \, d\phi_r \qquad (2.2\text{-}22)$$

2.2 ABC Model

- Inductance term

$$L_{asy} = 2dr_s \int_0^{\pi} w_{as} w_y c_{bf}\, d\phi_r \tag{2.2-21}$$

$$L_{asy} = 2dr_s \left[c_{bfs} \int_0^{\pi} w_{as} w_y\, d\phi_r + (c_{bfm} - c_{bfs}) \int_{\frac{1}{2}(1-\alpha_{pm})\pi}^{\frac{1}{2}(1+\alpha_{pm})\pi} w_{as} w_y\, d\phi_r \right] \tag{2.2-23}$$

2.2 ABC Model

- Self Inductance using (2.2-23)

$$L_{asas} = \frac{4dr_s n_p^2}{P^2}\left[c_{bfs}\pi + (c_{bfm} - c_{bfs})\left(\pi\alpha_{pm} - \sin(\pi\alpha_{pm})\cos(2\theta_r)\right)\right] \quad (2.2\text{-}24)$$

$$L_{asas} = L_A + L_B\cos(2\theta_r) \quad (2.2\text{-}25)$$

$$L_A = \frac{4\pi n_p^2 dr_s}{P^2}\left(c_{bfs} + \alpha_{pm}(c_{bfm} - c_{bfs})\right) \quad (2.2\text{-}26)$$

$$L_B = \frac{4dr_s}{P^2}(c_{bfs} - c_{bfm})\sin(\pi\alpha_{pm}) \quad (2.2\text{-}27)$$

2.2 ABC Model

- Mutual inductance using(2.2-23)

$$L_{asbs} = -\frac{2dr_s n_p^2}{P^2}\left[c_{bfs}\pi + (c_{bfm} - c_{bfs})\left(\pi\alpha_{pm} - 2\sin(\pi\alpha_{pm})\cos\left(2\theta_r + \frac{\pi}{3}\right)\right)\right] \quad (2.2\text{-}28)$$

$$L_{asbs} = -\frac{1}{2}L_A + L_B\cos\left(2\theta_r + \frac{\pi}{3}\right) \quad (2.2\text{-}29)$$

$$L_{ascs} = -\frac{1}{2}L_A + L_B\cos\left(2\theta_r - \frac{\pi}{3}\right) \quad (2.2\text{-}30)$$

2.2 ABC Model

- PM term

$$\lambda_{as,pm} = 2dr_s \int_0^{\pi} w_{as} B_{m,eff} \, d\phi_r \qquad (2.2\text{-}22)$$

$$\lambda_{as,pm} = \lambda_m \sin(\theta_r) \qquad (2.2\text{-}31)$$

$$\lambda_{as,pm} = \frac{8 n_p d r_s B_{pm}}{P} \qquad (2.2\text{-}32)$$

2.2 Final ABC Model

- Finally, we arrive at

$$\boldsymbol{\lambda}_{abcs,m} = \frac{L_A}{2}\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}\mathbf{i}_{abcs} +$$

$$L_B\begin{bmatrix} \cos(2\theta_r) & \cos\left(2\theta_r + \pi/3\right) & \cos(2\theta_r - \pi/3) \\ \cos(2\theta_r + \pi/3) & \cos(2\theta_r + 2\pi/3) & \cos(2\theta_r) \\ \cos(2\theta_r - \pi/3) & \cos(2\theta_r) & \cos(2\theta_r - 2\pi/3) \end{bmatrix}\mathbf{i}_{abcs} +$$

$$\lambda_m\begin{bmatrix} \sin(\theta_r) \\ \sin(\theta_r - 2\pi/3) \\ \sin(\theta_r + 2\pi/3) \end{bmatrix} \qquad (2.2\text{-}33)$$

2.2 ABC Model

- Comments on ABC Model

2.3 Standard QD0 Model

- As usual

$$\mathbf{f}_{qd0s}^{r} = \mathbf{K}_s^r \mathbf{f}_{abcs} \tag{2.3-1}$$

$$\mathbf{f}_{abcs} = \begin{bmatrix} f_{as} & f_{bs} & f_{cs} \end{bmatrix}^T \tag{2.3-2}$$

$$\mathbf{f}_{qd0s}^{r} = \begin{bmatrix} f_{qs}^r & f_{ds}^r & f_{0s} \end{bmatrix}^T \tag{2.3-3}$$

$$\mathbf{K}_s^r = \frac{2}{3}\begin{bmatrix} \cos(\theta_r) & \cos(\theta_r - 2\pi/3) & \cos(\theta_r + 2\pi/3) \\ \sin(\theta_r) & \sin(\theta_r - 2\pi/3) & \cos(\theta_r + 2\pi/3)` \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \tag{2.3-4}$$

2.3 Standard QD0 Model

- Transformation of voltage equation

$$\mathbf{v}_{qd0s}^{r} = r_s \mathbf{i}_{qd0s}^{r} + \omega_r \mathbf{S} \boldsymbol{\lambda}_{qd0s}^{r} + p \boldsymbol{\lambda}_{qd0s}^{r} \qquad (2.3\text{-}5)$$

$$\mathbf{S} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad (2.3\text{-}6)$$

2.3 Standard QD0 Model

- Transformation of flux-linkage equation

$$\boldsymbol{\lambda}_{abcs} = \boldsymbol{\lambda}_{abcs,l} + \boldsymbol{\lambda}_{abcs,m} \qquad (2.2\text{-}2)$$

$$\boldsymbol{\lambda}_{qd0s}^{r} = \boldsymbol{\lambda}_{qd0s,l}^{r} + \boldsymbol{\lambda}_{qd0s,m}^{r} \qquad (2.3\text{-}7)$$

2.3 Standard QD0 Model

- Transformation of leakage flux linkage

$$\boldsymbol{\lambda}_{abcs,l} = \begin{bmatrix} L_{lp} & L_{lm} & L_{lm} \\ L_{lm} & L_{lp} & L_{lm} \\ L_{lm} & L_{lm} & L_{lp} \end{bmatrix} \mathbf{i}_{abcs} \qquad (2.2\text{-}3)$$

- In qd0 variables

$$\boldsymbol{\lambda}^{r}_{qd0s,l} = \begin{bmatrix} L_{ls} & 0 & 0 \\ 0 & L_{ls} & 0 \\ 0 & 0 & L_{0} \end{bmatrix} \mathbf{i}^{r}_{qd0s} \qquad (2.3\text{-}8)$$

$$L_{ls} = L_{lp} - L_{lm} \qquad (2.3\text{-}9) \qquad\qquad L_{0} = L_{lp} + 2L_{lm} \qquad (2.3\text{-}10)$$

2.3 Standard QD0 Model

- Transformation of magnetizing flux linkage

$$\boldsymbol{\lambda}_{abcs,m} = \frac{L_A}{2}\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}\mathbf{i}_{abcs} +$$

$$L_B\begin{bmatrix} \cos(2\theta_r) & \cos(2\theta_r + \pi/3) & \cos(2\theta_r - \pi/3) \\ \cos(2\theta_r + \pi/3) & \cos(2\theta_r + 2\pi/3) & \cos(2\theta_r) \\ \cos(2\theta_r - \pi/3) & \cos(2\theta_r) & \cos(2\theta_r - 2\pi/3) \end{bmatrix}\mathbf{i}_{abcs} + \qquad (2.2\text{-}33)$$

$$\lambda_m\begin{bmatrix} \sin(\theta_r) \\ \sin(\theta_r - 2\pi/3) \\ \sin(\theta_r + 2\pi/3) \end{bmatrix}$$

- In qd0 variables

$$\boldsymbol{\lambda}^r_{qd0s,m} = \frac{3}{2}L_A\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\mathbf{i}^r_{qd0s} + \frac{3}{2}L_B\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\mathbf{i}^r_{qd0s} + \lambda_m\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad (2.3\text{-}11)$$

2.3 Standard QD0 Model

- In qd0 variables, it is convenient to write

$$\boldsymbol{\lambda}_{qd0s}^{r} = \begin{bmatrix} L_q & 0 & 0 \\ 0 & L_d & 0 \\ 0 & 0 & L_0 \end{bmatrix} \mathbf{i}_{qd0s}^{r} + \lambda_m \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad (2.3\text{-}12)$$

$$L_q = L_{ls} + \frac{3}{2}(L_A + L_B) \qquad (2.3\text{-}13)$$

$$L_d = L_{ls} + \frac{3}{2}\left(L_A - L_B\right) \qquad (2.3\text{-}14)$$

2.3 Standard QD0 Model

- Torque

$$T_e = \frac{3}{2}\frac{P}{2}\left(\lambda_{ds}^r i_{qs}^r - \lambda_{qs}^r i_{ds}^r\right) \qquad (2.3\text{-}15)$$

- See Chapter 5 of *Techniques for Analysis and Design of Electromechanical Systems*, S.D. Sudhoff, S. P. Pekarek, monograph in preparation for publication.

2.4 Steady State Operation

- Steady state voltage equations

2.4 Steady State Operation

- Steady state torque equation

2.4 Steady State Operation

- Steady state model summary:

$$v_{qs}^{r} = r_s i_{qs}^{r} + \omega_r L_d i_{ds}^{r} + \omega_r \lambda_m \qquad (2.4\text{-}1)$$

$$v_{ds}^{r} = r_s i_{ds}^{r} - \omega_r L_q i_{qs}^{r} \qquad (2.4\text{-}2)$$

$$T_e = \frac{3}{2}\frac{P}{2}\left(\lambda_m i_{qs}^{r} + (L_d - L_q) i_{qs}^{r} i_{ds}^{r}\right) \qquad (2.4\text{-}3)$$

2.4 Steady State Operation

- Operation from a VSI

$$v_{as} = \sqrt{2} v_s \cos(\theta_r + \phi_v) \qquad (2.4\text{-}4)$$

$$v_{bs} = \sqrt{2} v_s \cos(\theta_r - 2\pi / 3 + \phi_v) \qquad (2.4\text{-}5)$$

$$v_{cs} = \sqrt{2} v_s \cos(\theta_r + 2\pi / 3 + \phi_v) \qquad (2.4\text{-}6)$$

- In qd0 variables

$$v_{qs}^r = \sqrt{2} v_s \cos(\phi_v) \qquad (2.4\text{-}7)$$

$$v_{ds}^r = -\sqrt{2} v_s \sin(\phi_v) \qquad (2.4\text{-}8)$$

2.4 Steady State Operation

- Solving for the qd0 currents

2.4 Steady State Operation

- Thus

$$i_{qs}^r = \frac{r_s(v_{qs}^r - \omega_r \lambda_m) - \omega_r L_d}{r_s^2 + \omega_r^2 L_d L_q} \qquad (2.4\text{-}9)$$

$$i_{ds}^r = \frac{\omega_r L_q(v_{ds}^r - \omega_r \lambda_m) + r_s v_{ds}^r}{r_s^2 + \omega_r^2 L_d L_q} \qquad (2.4\text{-}10)$$

2.4 Steady State Operation

- Other handy relationships

$$i_s = \frac{1}{\sqrt{2}}\sqrt{\left(i_{qs}^r\right)^2 + \left(i_{ds}^r\right)^2} \qquad v_s = \frac{1}{\sqrt{2}}\sqrt{\left(v_{qs}^r\right)^2 + \left(v_{ds}^r\right)^2}$$

$$P_{out} = \frac{2}{P}\omega_r T_e$$

$$P_{in} = \frac{3}{2}\left(v_{qs}^r i_{qs}^r + v_{ds}^r i_{ds}^r\right)$$

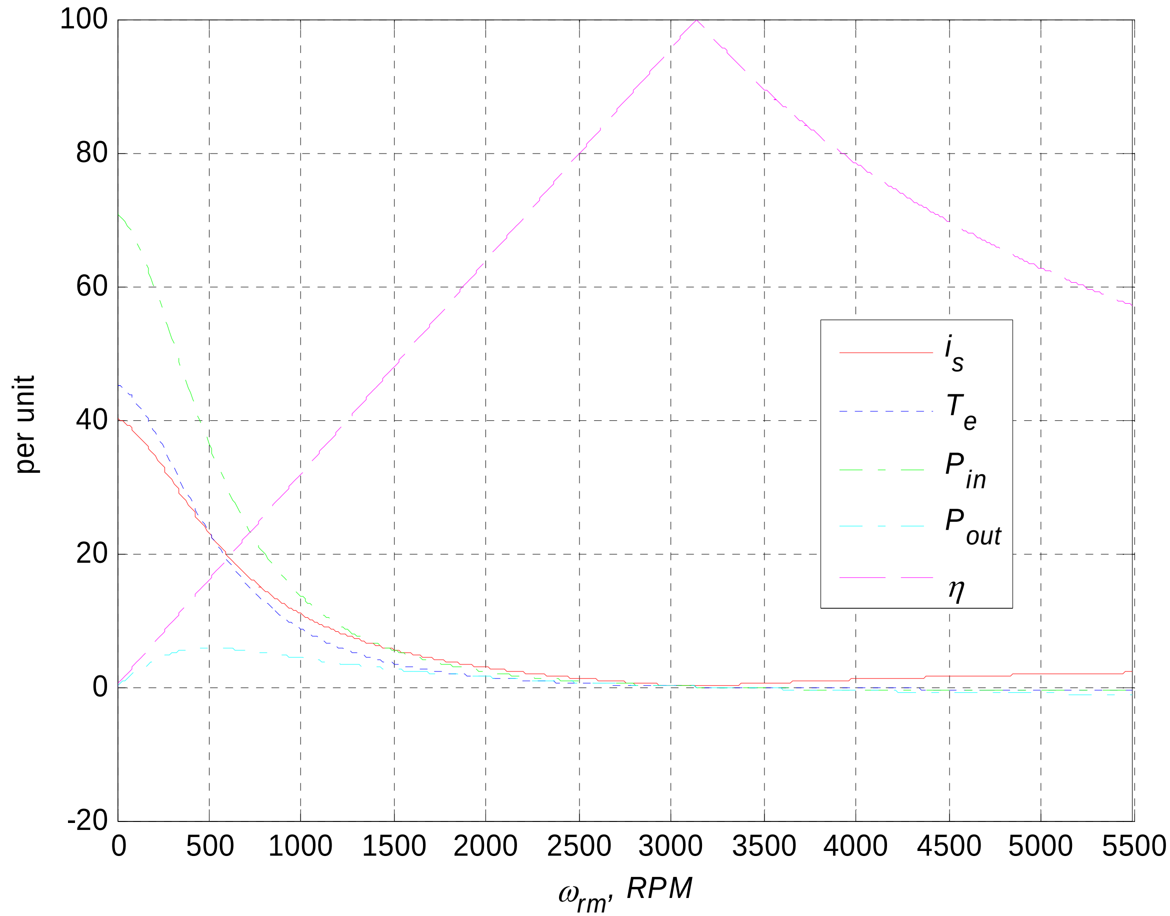
$$\eta = \begin{cases} \dfrac{P_{out}}{P_{in}} & P_{out} \geq 0, P_{in} > 0 \\ \dfrac{P_{in}}{P_{out}} & P_{out} < 0, P_{in} \leq 0 \\ 0 & P_{in} > 0, P_{out} < 0 \end{cases}$$

2.4 Steady State Operation

- Consider a machine
  - r_s=2.6 W
  - L_q=L_d=12.4 mH
  - λ_m=286 Vs
  - P=4
- Rated values
  - Current 3.3 A rms
  - Voltage 230 V l-l rms
  - Power: 746 W at 2000 rpm
  - Speed: 5500 rpm (max)

2.4 Steady State Operation

- Operation at rated voltage (no phase shift)



2.4 Steady State Operation

- Operation from a CSI

$$i_{as} = \sqrt{2} i_s \cos(\theta_r + \phi_i) \qquad (2.4\text{-}11)$$

$$i_{bs} = \sqrt{2} i_s \cos(\theta_r - 2\pi / 3 + \phi_i) \qquad (2.4\text{-}12)$$

$$i_{cs} = \sqrt{2} i_s \cos(\theta_r + 2\pi / 3 + \phi_i) \qquad (2.4\text{-}13)$$

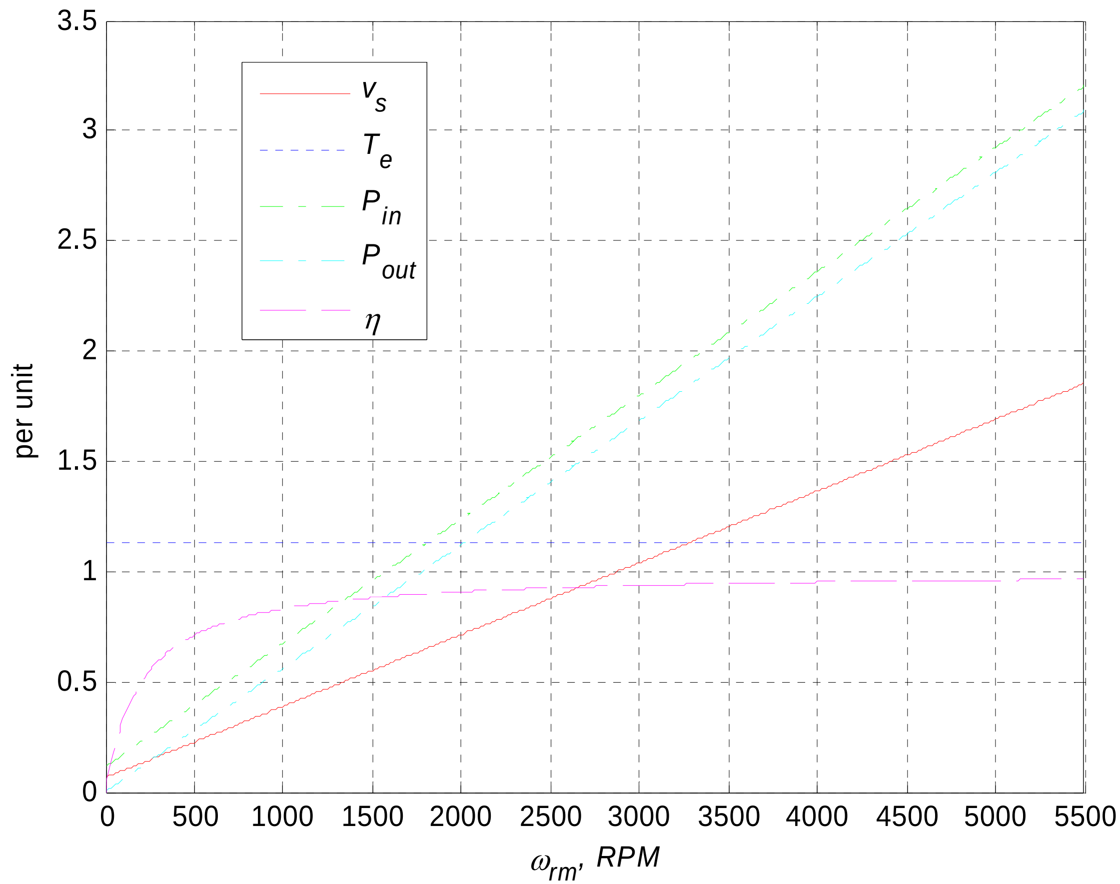
- In qd0 variables

$$i_{ds}^r = -\sqrt{2} i_s \sin(\phi_i) \qquad (2.4\text{-}14)$$

$$i_{qs}^r = \sqrt{2} i_s \cos(\phi_i) \qquad (2.4\text{-}15)$$

2.4 Steady State Operation

- Operation at rated current (no phase shift)



2.5 Parameter Identification

- For no load conditions

$$\mathbf{v}_{qd0s}^{r} = \omega_r \lambda_m \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad (2.5\text{-}1)$$

- Thus

$$\lambda_m = \frac{v_{abcs}\big|_{pk\ \ fnd}}{\sqrt{3}\omega_r} \qquad (2.5\text{-}6)$$

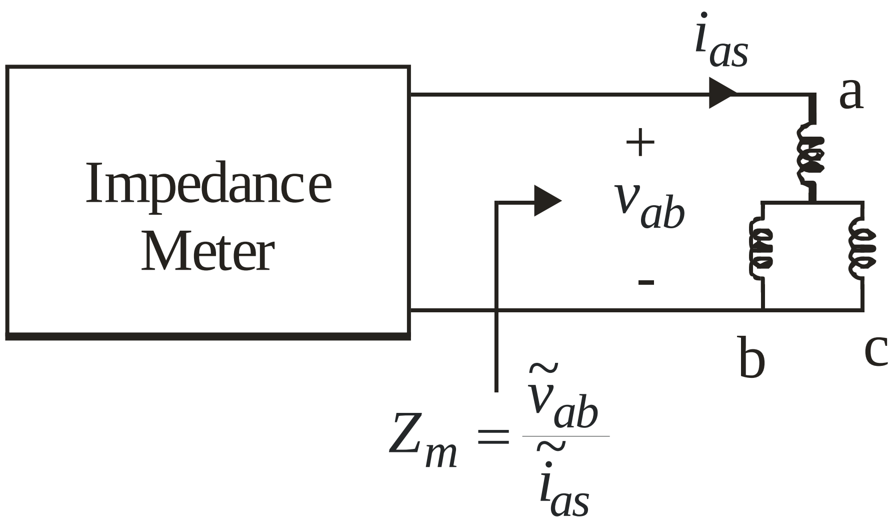
$$P = 2round(\omega_r / \omega_{rm}) \qquad (2.5\text{-}7)$$

2.5 Parameter Identification

- Derivation

2.5 Parameter Identification

- Measurement of d-axis parameters



$$\theta_r = \pi / 2$$

2.5 Parameter Identification

- Pertinent relationships

$$Z_d = \frac{2}{3}\frac{\tilde{v}_{ab}}{\tilde{i}_{as}} = \frac{2}{3} Z_m \qquad (2.5\text{-}17)$$

$$r_s = \mathrm{Re}(Z_d) \qquad (2.5\text{-}20)$$

$$L_d = \frac{1}{\omega}\mathrm{Im}(Z_d) \qquad (2.5\text{-}21)$$

2.5 Parameter Identification

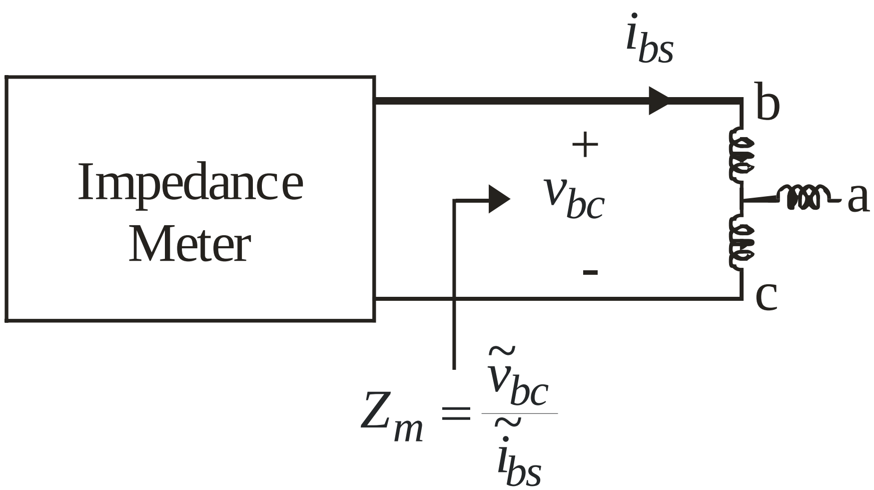
- Comment on positioning the rotor

2.5 Parameter Identification

- Derivation of d-axis results

2.5 Parameter Identification

- Measurement of q-axis parameters



$$\theta_r = \pi / 2$$

2.5 Parameter Identification

- Pertinent results

$$Z_q = \frac{1}{2}\frac{\tilde{v}_{ab}}{\tilde{i}_{as}} = \frac{1}{2} Z_m \qquad (2.5\text{-}29)$$

$$L_q = \frac{1}{\omega}\operatorname{Re}(Z_q) \qquad (2.5\text{-}32)$$

2.6 Simulation

- Derivation of state model

2.6 Simulation

2.6 Simulation

- Model summary:

$$\mathbf{v}_{qd}^r = \mathbf{K}_s^r\Big|_{utr} \mathbf{v}_{abc,x}$$

$$T_e = \frac{3}{2}\frac{P}{2}\left(\lambda_m i_{qs}^r + (L_d - L_q) i_{qs}^r i_{ds}^r\right)$$

$$\omega_r = \frac{P}{2}\omega_{rm}$$

$$p\omega_{rm} = \frac{T_e - T_l}{J}$$

$$p i_{qs}^r = \frac{(v_{qs}^r - r_s i_{qs}^r) - \omega_r \lambda_m}{L_q}$$

$$p\theta_r = \frac{P}{2}\omega_r$$

$$p i_{ds}^r = \frac{v_{ds}^r - r_s i_{ds}^r}{L_d}$$