EE595S: Class Lecture Notes
Chapter 2* / QD Models for Permanent Magnet Synchronous Machines

S.D. Sudhoff

Fall 2005

***Analysis and Design of Permanent Magnet Synchronous Machines**

S.D. Sudhoff, S.P. Pekarek B. Fahimi

2.7 Extended Bandwidth QD Model

- Objectives

2.7 Extended Bandwidth QD Model

- ABC Variable Model

$$\mathbf{v}_{abc,s} = \mathbf{v}_{abc,z} + \mathbf{v}_{abc,pm} \tag{2.7-1}$$

$$\mathbf{v}_{abc,z} = \mathbf{Z}_{abc,s}(p)\mathbf{i}_{abc,s} \tag{2.7-2}$$

$$\mathbf{Z}_{abc,s}(p) = \begin{bmatrix} Z_{ss}(p) & Z_m(p) & Z_m(p) \\ Z_m(p) & Z_{ss}(p) & Z_m(p) \\ Z_m(p) & Z_m(p) & Z_{ss}(p) \end{bmatrix} \tag{2.7-3}$$

2.7 Extended Bandwidth QD Model

- ABC Variable Model (Cont)

$$\mathbf{v}_{abc,pm} = p\boldsymbol{\lambda}_{abc,pm} \tag{2.7-4}$$

$$\lambda_{abc,pm} = \lambda_m \left[\sin(\theta_r) \quad \sin(\theta_r - 2\pi/3) \quad \sin(\theta_r + 2\pi/3) \right]^T \tag{2.7-5}$$

$$T_e = \frac{P}{2}\lambda_m \left[\cos(\theta_r) \quad \cos(\theta_r - 2\pi/3) \quad \cos(\theta_r + 2\pi/3) \right] \mathbf{i}_{abcs} \tag{2.7-6}$$

2.7 Extended Bandwidth QD Model

- Plan of Attack

2.7 Extended Bandwidth QD Model

- Transformation of Voltage Equation

$$\mathbf{v}_{qd0s}^{s} = \mathbf{v}_{qd0,z}^{s} + \mathbf{v}_{qd0,pm}^{s} \tag{2.7-7}$$

$$\mathbf{v}_{qd0,z}^{s} = \mathbf{Z}_{qd0,s}\mathbf{i}_{qd0,s}^{s} \tag{2.7-8}$$

$$\mathbf{Z}_{qd0,s}(p) = \begin{bmatrix} Z_s(p) & 0 & 0 \\ 0 & Z_s(p) & 0 \\ 0 & 0 & Z_0(p) \end{bmatrix} \tag{2.7-9}$$

$$Z_s(p) = Z_{ss}(p) - Z_m(p) \tag{2.7-10}$$

$$Z_0(p) = Z_{ss}(p) + 2Z_m(p) \tag{2.7-11}$$

Extended Bandwidth QD Model

- Transformation of PM Induced Voltage

$$\mathbf{v}^{s}_{qd0,pm} = p\boldsymbol{\lambda}^{s}_{qd0,pm} \tag{2.7-12}$$

$$\boldsymbol{\lambda}^{s}_{qd0,pm} = \lambda_m \begin{bmatrix}\sin(\theta_r) & \cos(\theta_r) & 0\end{bmatrix}^T \tag{2.7-13}$$

$$\mathbf{v}^{s}_{qd0,pm} = \omega_r \lambda_m \begin{bmatrix}\cos(\theta_r) & -\sin(\theta_r) & 0\end{bmatrix}^T \tag{2.7-14}$$

$$\mathbf{v}^{s}_{qd,s} = Z_s(p)\mathbf{i}^{s}_{qd,s} + \mathbf{v}^{s}_{qd,pm} \tag{2.7-15}$$

2.7 Extended Bandwidth QD Model

- Transformation of Torque Equation

$$T_e = \frac{3}{2}\frac{P}{2}\lambda_m \left(i_{qs}^s \cos\theta_r - i_{ds}^s \sin\theta_r \right) \qquad (2.7\text{-}16)$$

2.7 Extended Bandwidth QD Model

- Simulation Using Extended Bandwidth QD Model

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2.7 Extended Bandwidth QD Model

- Simulation Summary

$$\mathbf{v}_{qd}^{s} = \mathbf{K}_{s}^{s}\Big|_{utr} \mathbf{v}_{abc,r} \qquad (2.7\text{-}17)$$

$$u_{qs}^{s} = v_{qs}^{s} - \omega_r \lambda_m \cos(\theta_r) \qquad (2.7\text{-}20)$$

$$u_{ds}^{s} = v_{ds}^{s} + \omega_r \lambda_m \sin(\theta_r) \qquad (2.7\text{-}21)$$

$$i_{qs}^{s} = Y_s(p) u_{qs}^{s} \qquad (2.7\text{-}18)$$

$$i_{qs}^{s} = Y_s(p) u_{qs}^{s} \qquad (2.7\text{-}19)$$

2.7 Extended Bandwidth QD Model

- Final Note: Representing Transfer Function (Special Case – All Real Poles)

$$Y_s(p) = \sum_{j=1}^{J} \frac{a_j}{\tau_j p + 1} \qquad (2.7\text{-}23)$$

2.7 Extended Bandwidth QD Model

- Continuing…

2.7 Extended Bandwidth QD Model

- Thus, for the all real pole case

$$px_{w,j} = (u_w - x_{w,j}) / \tau_j \qquad (2.7\text{-}24)$$

$$i_{ws}^{s} = \sum_{j=1}^{J} a_j x_{w,j} \qquad (2.7\text{-}25)$$

2.8 Parameter Identification of Extended Bandwidth QD Model

- Procedure for find P and λ_m

- Procedure for finding a_j and τ_j

$$\theta = [a_1\ a_2 \cdots a_J\ \tau_1\ \tau_2 \cdots \tau_J]^T$$

2.8 Parameter Identification of Extended Bandwidth QD Model

- Denote *i*'th frequency f_i (radian frequency ω_i)
- Measure impedance at *i*'th frequency $Z_{m,i}$
- The admittance at the *i*'th frequency is $Y_{s,i}$

$$Y_{s,i} = \frac{3}{2}\frac{1}{Z_{m,i}}$$

2.8 Parameter Identification of Extended Bandwidth QD Model

- Obtaining the parameters. Define the error at a given frequency error as

$$e_{c,i} = \left| \frac{Y_s(\theta_c, j\omega) - Y_{s,i}}{Y_{s,i}} \right|$$

- Define the aggregate error

$$E(\theta_c) = \frac{1}{I} \sum_{i=1}^{I} e_{c,i}$$

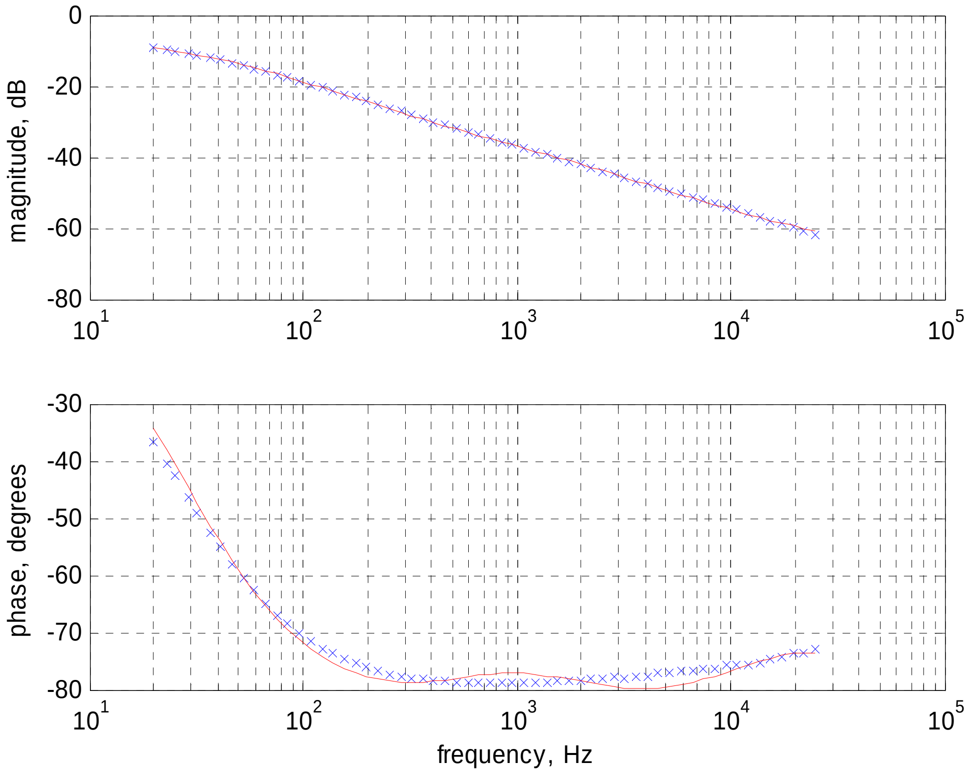
2.8 Parameter Identification of Extended Bandwidth QD Model

- To solve for the parameters, let us maximize

$$F(\theta_c) = \frac{1}{\varepsilon + E(\theta_c)}$$

2.8 Parameter Identification of Extended Bandwidth QD Model

- 3rd Order Fit



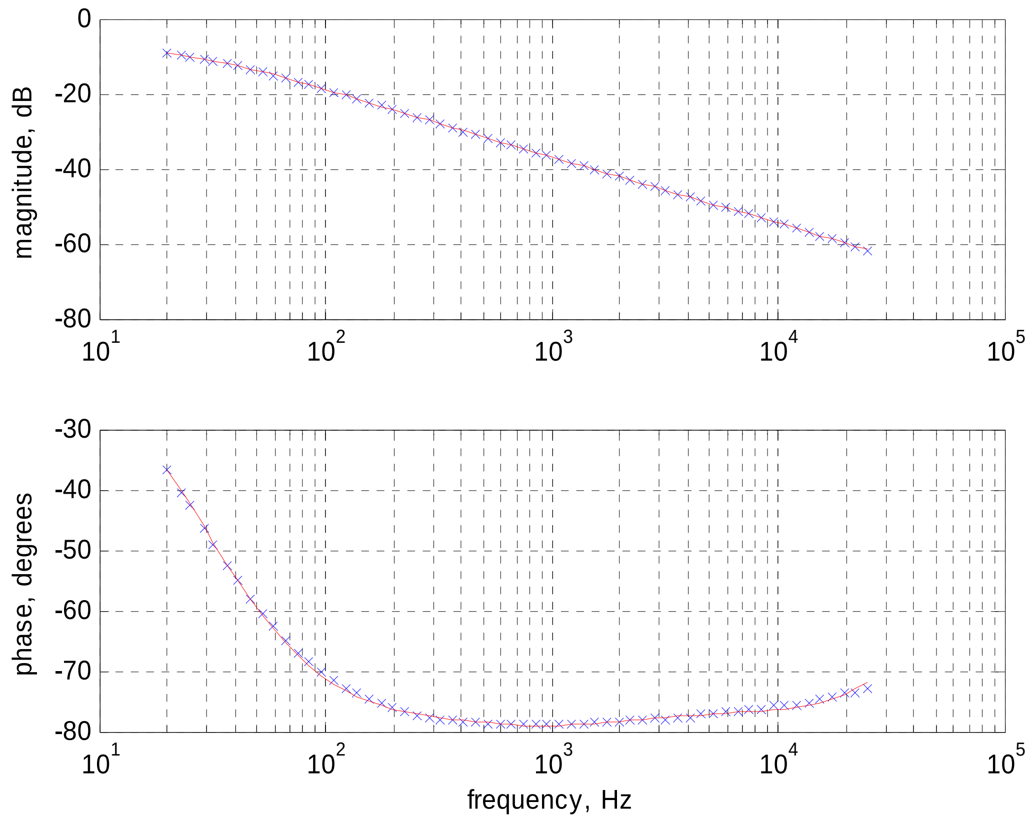
2.8 Parameter Identification of Extended Bandwidth QD Model

Table 2.8-1. Admittance Parameters with 3rd Order Transfer Function

	$m\Omega^{-1}$		ms
a_1	4.14	τ_1	0.134
a_2	411	τ_2	5.54
a_3	0.394	τ_3	0.00508

2.8 Parameter Identification of Extended Bandwidth QD Model

- 6^{th} Order Fit



2.8 Parameter Identification of Extended Bandwidth QD Model

Table 2.8-2. Admittance Parameters with 6th Order Transfer Function

	$m\Omega^{-1}$		ms
a_1	1.10	τ_1	0.0720
a_2	0.203	τ_2	0.0000198
a_3	75.9	τ_3	23.5
a_4	383	τ_4	5.58
a_5	6.11	τ_5	0.410
a_6	0.476	τ_6	0.0202