EE595S: Class Lecture Notes
Chapter 13: Fully Controlled 3-Phase Bridge Converters

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13.2 Fully Controlled 3-Phase Bridge Converter

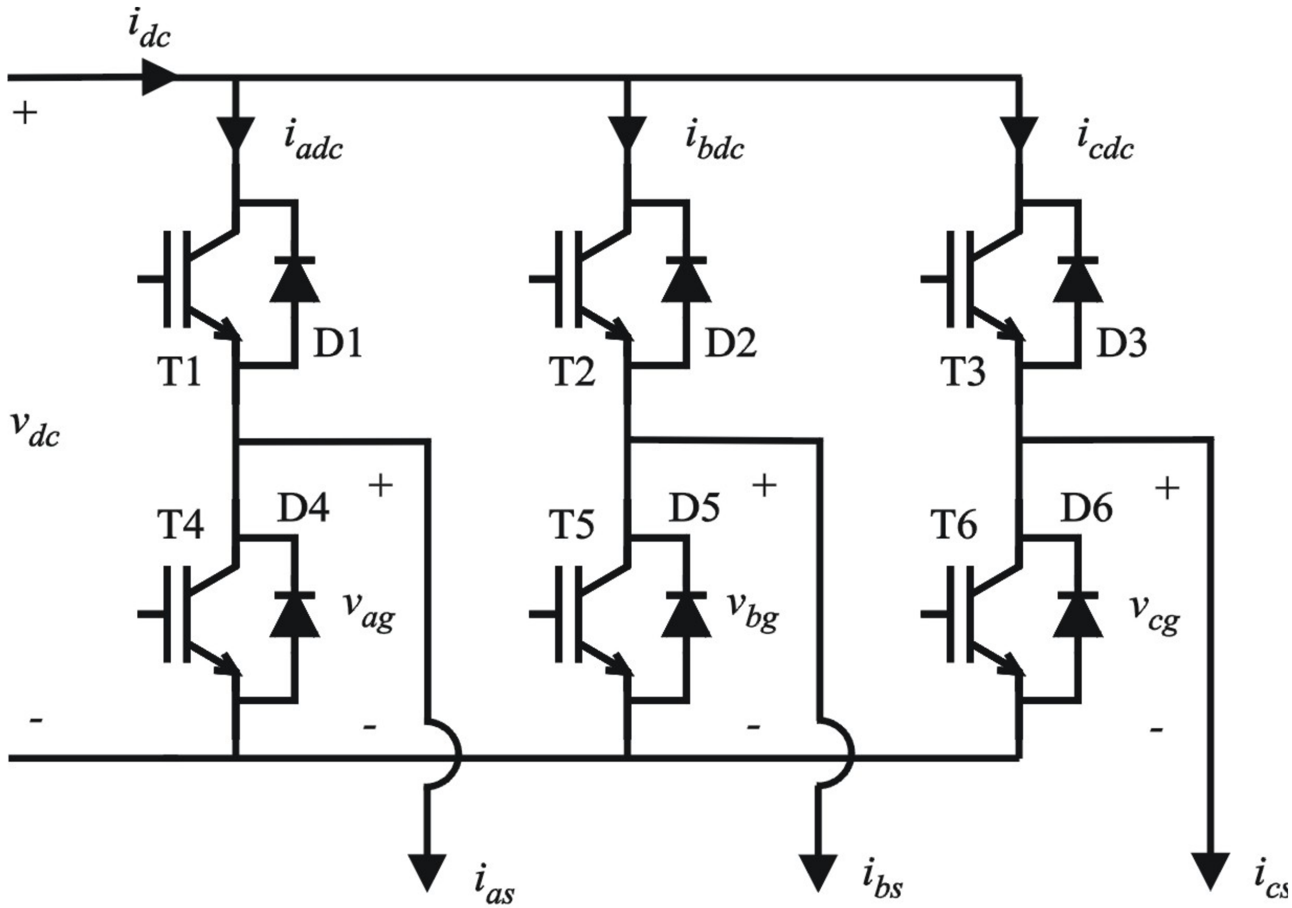
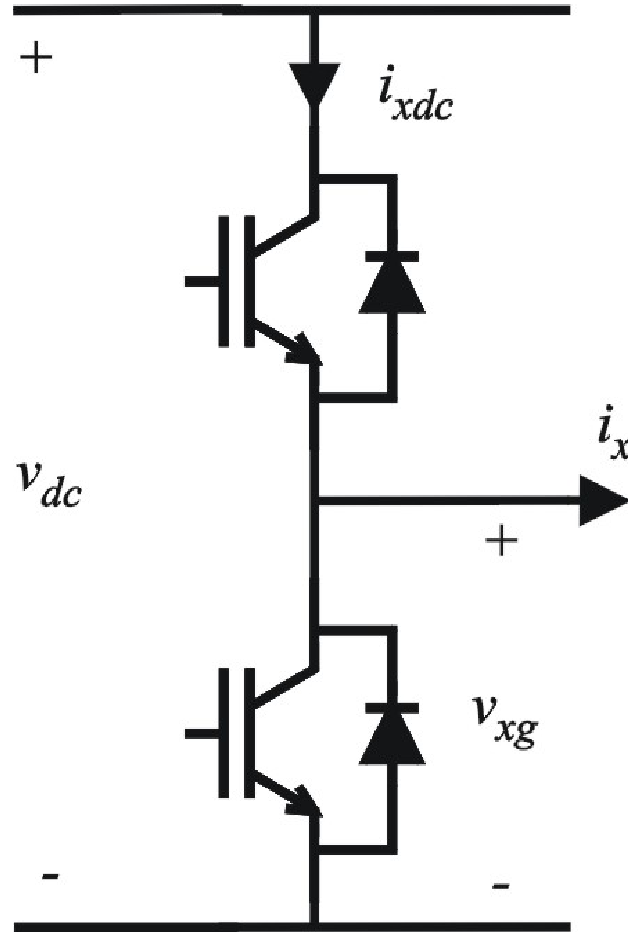
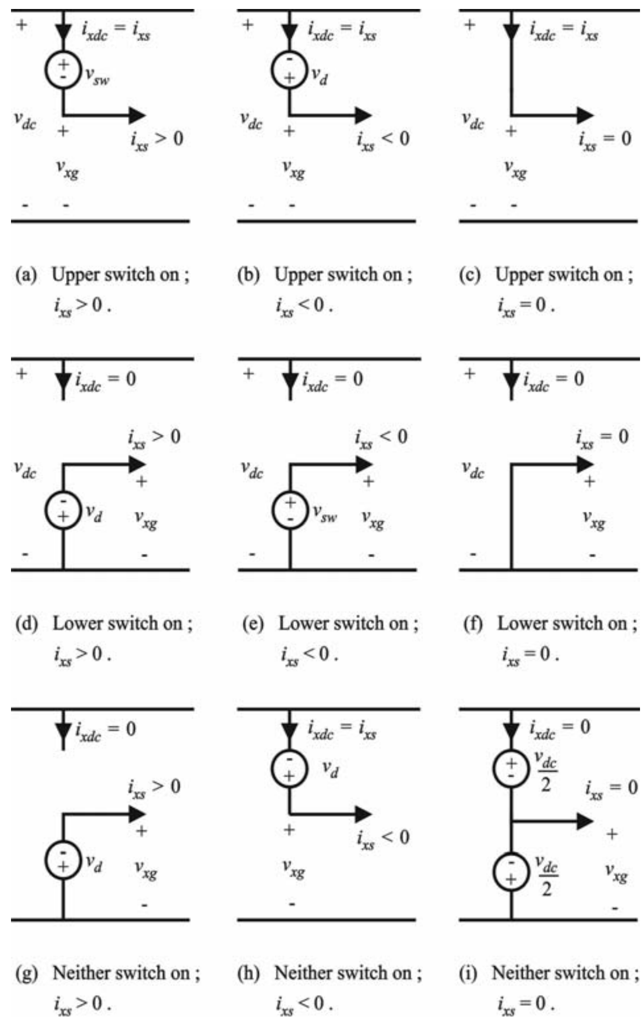


Figure 13.2-1 The three-phase bridge converter toplogy.

One Phase Leg



Phase Leg Equivalent Circuits



Phase Leg Voltages and Currents

Switch On	Current Direction	v_{xg}	i_{xdc}
Upper	Positive	$v_{dc} - v_{sw}$	i_{xs}
	Negative	$v_{dc} + v_d$	i_{xs}
	Zero	v_{dc}	i_{xs}
Lower	Positive	$-v_d$	0
	Negative	v_{sw}	0
	Zero	0	0
Neither	Positive	$-v_d$	0
	Negative	$v_{dc} + v_d$	i_{xs}
	Zero	$v_{dc}/2$	0

Line-to-Line Voltage and DC Current

- Line-to-Line Voltages
  - $v_{abs} = v_{ag} - v_{bg}$ (13.2-1)
  - $v_{bcs} = v_{bg} - v_{cg}$ (13.2-2)
  - $v_{cas} = v_{cg} - v_{ag}$ (13.2-3)

- DC Current
  - $i_{dc} = i_{adc} + i_{bdc} + i_{cdc}$ (13.2-4)

Line-to-Neutral Voltage

- From Fig. 13.2-1
  - $v_{ag} = v_{as} + v_{ng}$ (13.2-5)
  - $v_{bg} = v_{bs} + v_{ng}$ (13.2-6)
  - $v_{cg} = v_{cs} + v_{ng}$ (13.2-7)
- Adding
  - $v_{ng} = \frac{1}{3}(v_{ag} + v_{bg} + v_{cg}) - \frac{1}{3}(v_{as} + v_{bs} + v_{cs})$ (13.2-8)
- Or
  - $v_{ng} = \frac{1}{3}(v_{ag} + v_{bg} + v_{cg}) - v_{0s}$ (13.2-9)

Line-to-Neutral Voltage

- IFF (IFF!)

- Then
  - $$v_{ng} = \frac{1}{3}(v_{ag} + v_{bg} + v_{cg}) \qquad (13.2\text{-}10)$$

Line-to-Neutral Voltage

Backsubstitution of (13.2-10) into (13.2-5)-(13.2-7) yields

- $v_{as} = \frac{2}{3}v_{ag} - \frac{1}{3}v_{bg} - \frac{1}{3}v_{cg}$ (13.2-11)

- $v_{bs} = \frac{2}{3}v_{bg} - \frac{1}{3}v_{ag} - \frac{1}{3}v_{cg}$ (13.2-12)

- $v_{cs} = \frac{2}{3}v_{cg} - \frac{1}{3}v_{ag} - \frac{1}{3}v_{bg}$ (13.2-13)

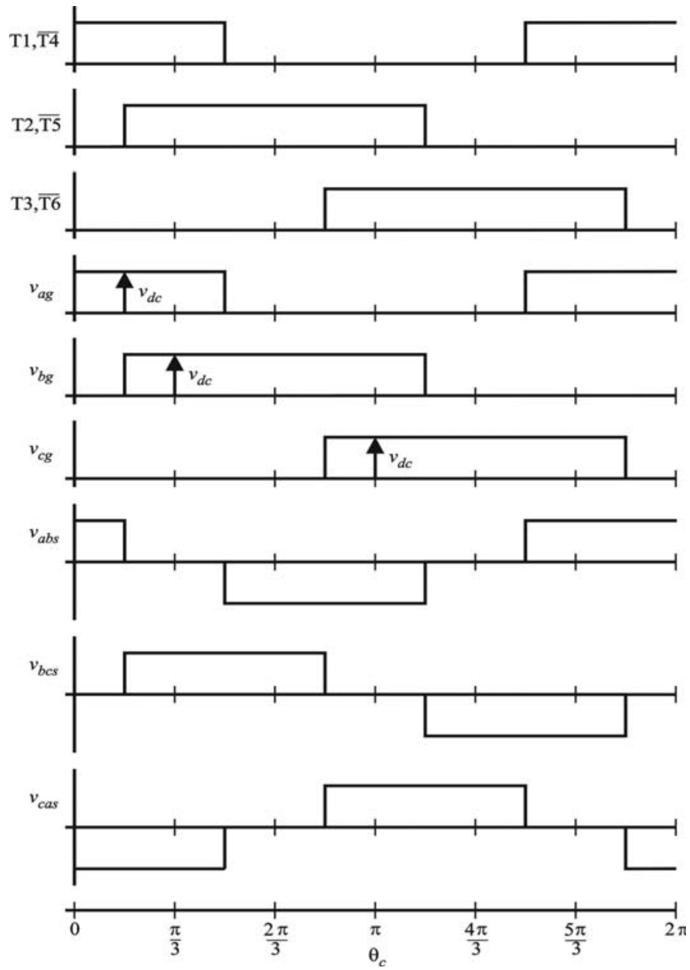
Calculation of QD Voltages

- We do not need line-to-neutral voltages to find QD voltages !

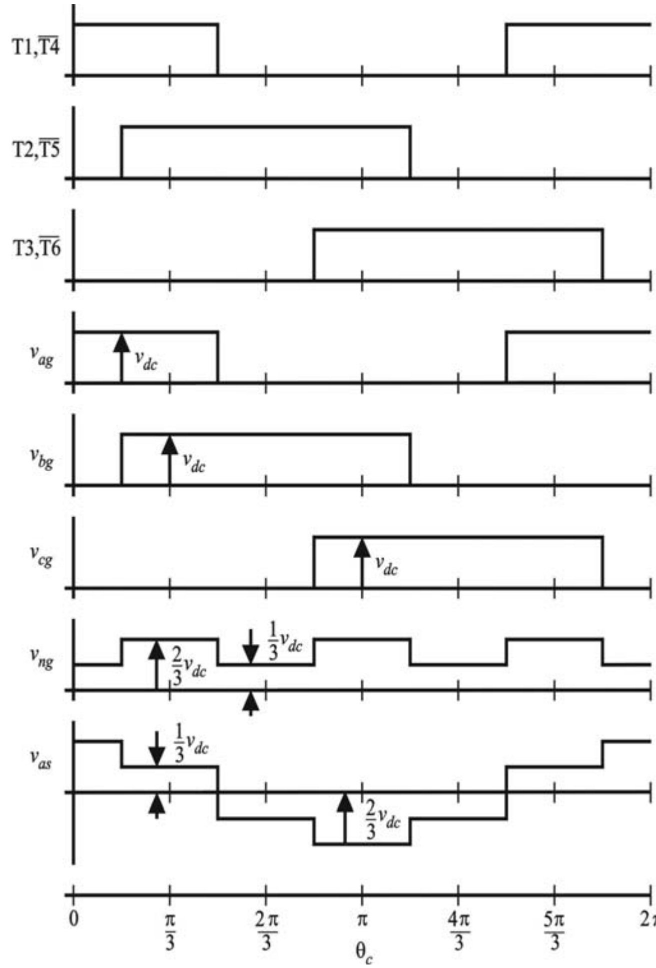
13.3 180^o Voltage Source Inverter (VSI) Operation

- Comments:
  - Six-step operation
  - 120^o VSI used at one time
  -
  -
  -

Line-to-Line Voltages



Line-to-Neutral Voltages (for v_{0s}=0)

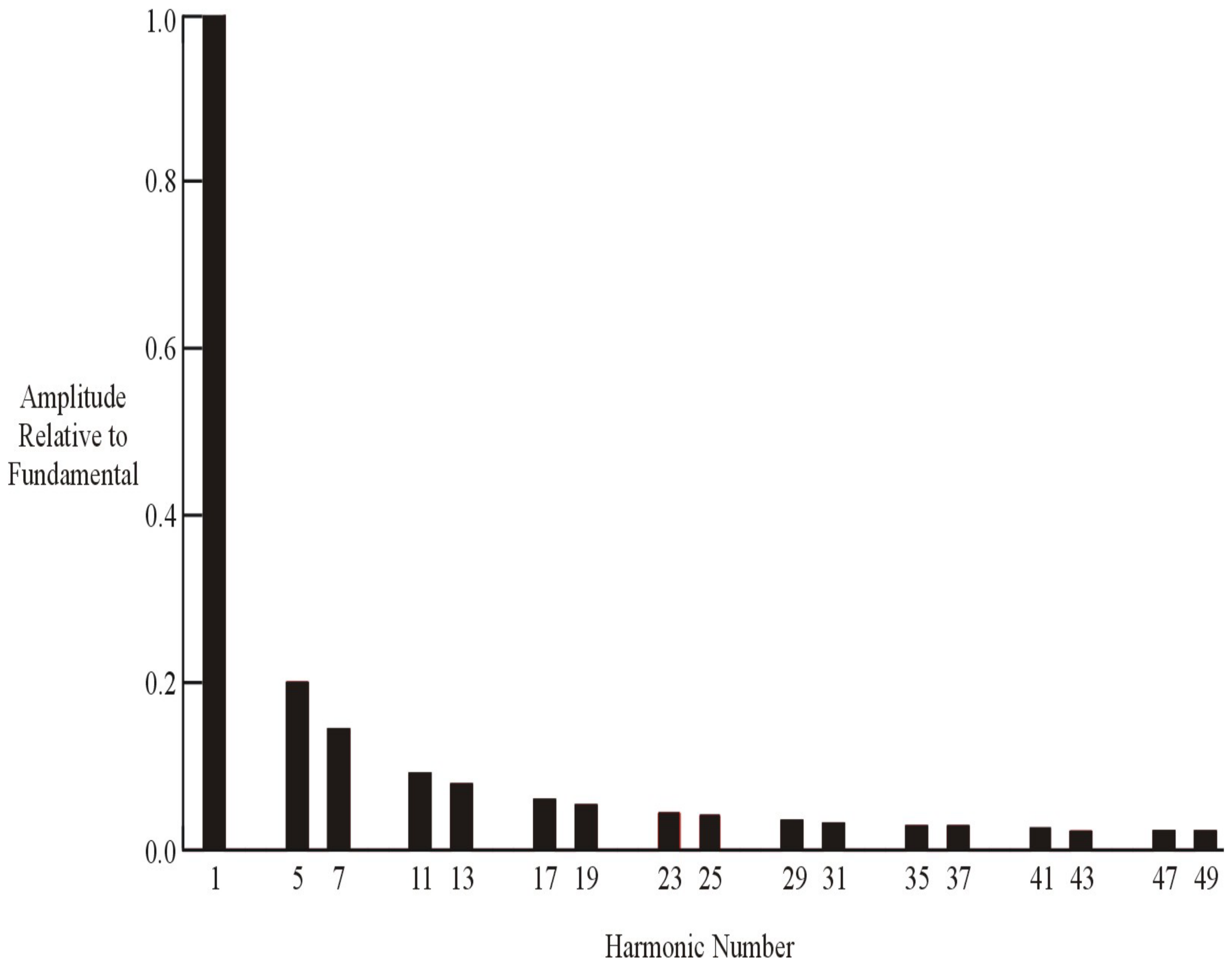


Voltage Spectrum

- Line-to-line voltage
  - $$v_{abs} = \frac{2\sqrt{3}}{\pi} v_{dc} \cos(\theta_c + \frac{\pi}{6}) + \frac{2\sqrt{3}}{\pi} v_{dc} \sum_{j=1}^{\infty} \left(-\frac{1}{6j-1} \cos((6j-1)(\theta_c + \frac{\pi}{6})) + \frac{1}{6j+1} \cos((6j+1)(\theta_c + \frac{\pi}{6})) \right) \quad (13.3\text{-}1)$$
- Line-to-neutral voltage
  - $$v_{as} = \frac{2}{\pi} v_{dc} \cos\theta_c + \frac{2}{\pi} v_{dc} \sum_{j=1}^{\infty} \left(\frac{(-1)^{j+1}}{6j-1} \cos((6j-1)\theta_c) + \frac{(-1)^{j}}{6j+1} \cos((6j+1)\theta_c) \right)$$

(13.3-2)

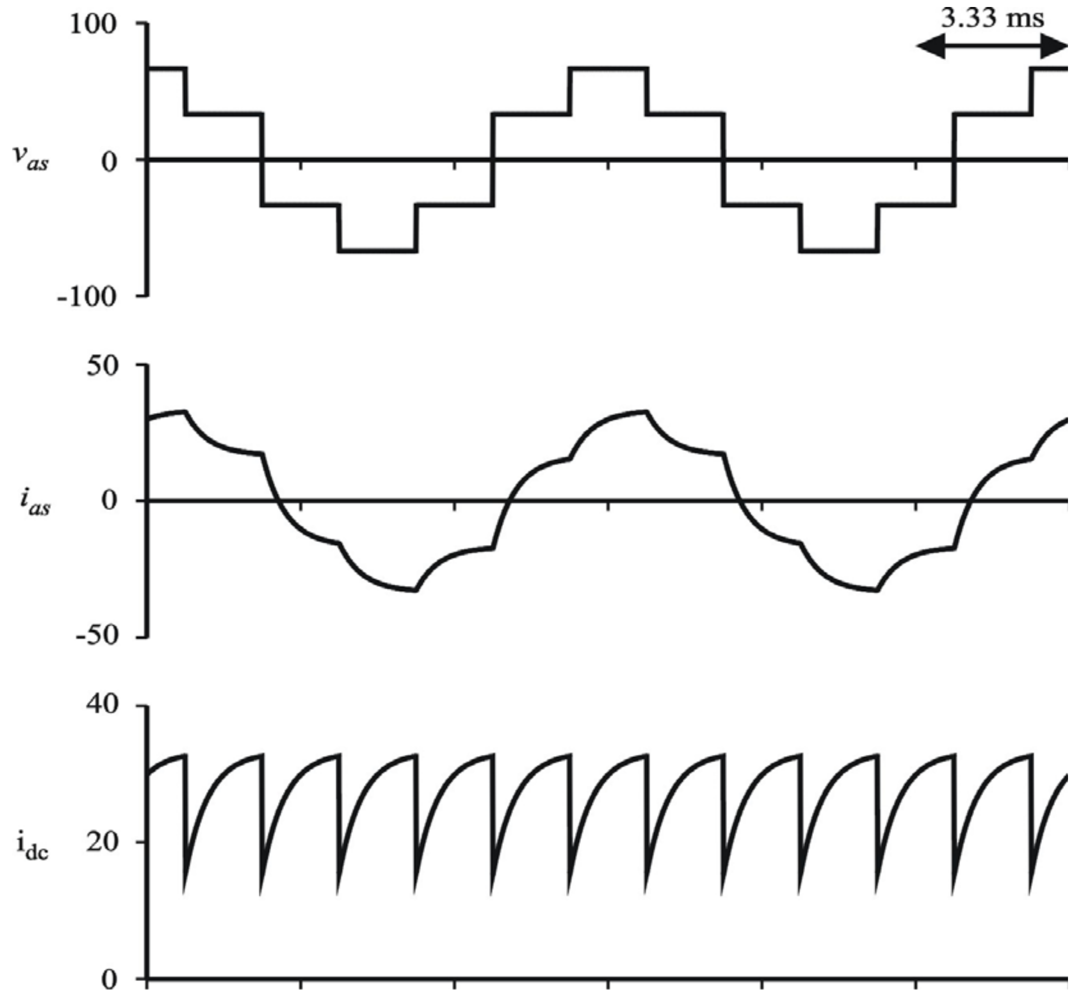
Voltage Spectrum



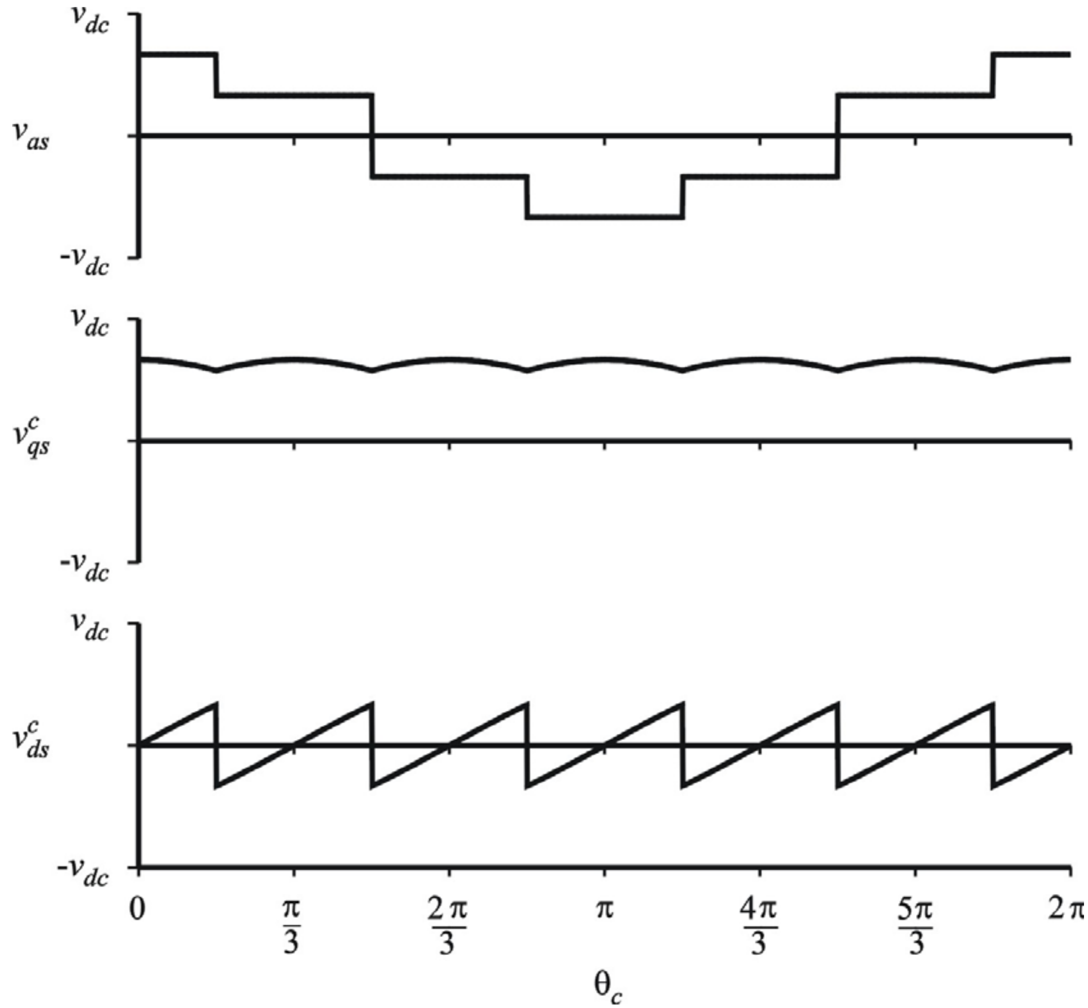
An Example

- Load: Balanced, wye-connected, series RL
  - $R = 2\ \Omega$
  - $L = 1$ mH
- DC Voltage: 100 V
- Fundamental Frequency: 100 Hz

ABC Variables for 180^o VSI



QD Variables



QD Inverter Analysis of 180° VSI

- QD Voltages
  - $$v_{qs}^{c} = \frac{2}{\pi} v_{dc} - \frac{2}{\pi} v_{dc} \sum_{j=1}^{\infty} \frac{2(-1)^{j}}{36 j^{2} - 1} \cos(6 j \theta_{c}) \qquad (13.3\text{-}3)$$
  - $$v_{ds}^{c} = \frac{2}{\pi} v_{dc} \sum_{j=1}^{\infty} \frac{12 j}{36 j^{2} - 1} \sin(6 j \theta_{c}) \qquad (13.3\text{-}4)$$
- Average-Value Analysis
  - $$\bar{v}_{qs}^{c} = \frac{2}{\pi} v_{dc} \qquad (13.3\text{-}5)$$
  - $$\bar{v}_{ds}^{c} = 0 \qquad (13.3\text{-}6)$$

Calculation of DC Current

- We have
  - $P_{in} = i_{dc} v_{dc}$ (13.3-7)
  - $P_{out} = \frac{3}{2}(v_{qs} i_{qs} + v_{ds} i_{ds})$ (13.3-8)
- Thus
  - $i_{dc} = \dfrac{\frac{3}{2}(v_{qs} i_{qs} + v_{ds} i_{ds})}{v_{dc}}$ (13.3-9)
- So
  - $\bar{i}_{dc} = \frac{3}{2}\left(\overline{\dfrac{v_{qs} i_{qs} + v_{ds} i_{ds}}{v_{dc}}}\right)$ (13.3-10)
- Approximately
  - (13.3-11)

Return to Previous Example

- Change resistance to 1 Ohm
- Find DC Current
- Voltages
  - v_{qs}= 63.7 V
  - v_{ds} = 0.0 V
- Currents

Return to Previous Example

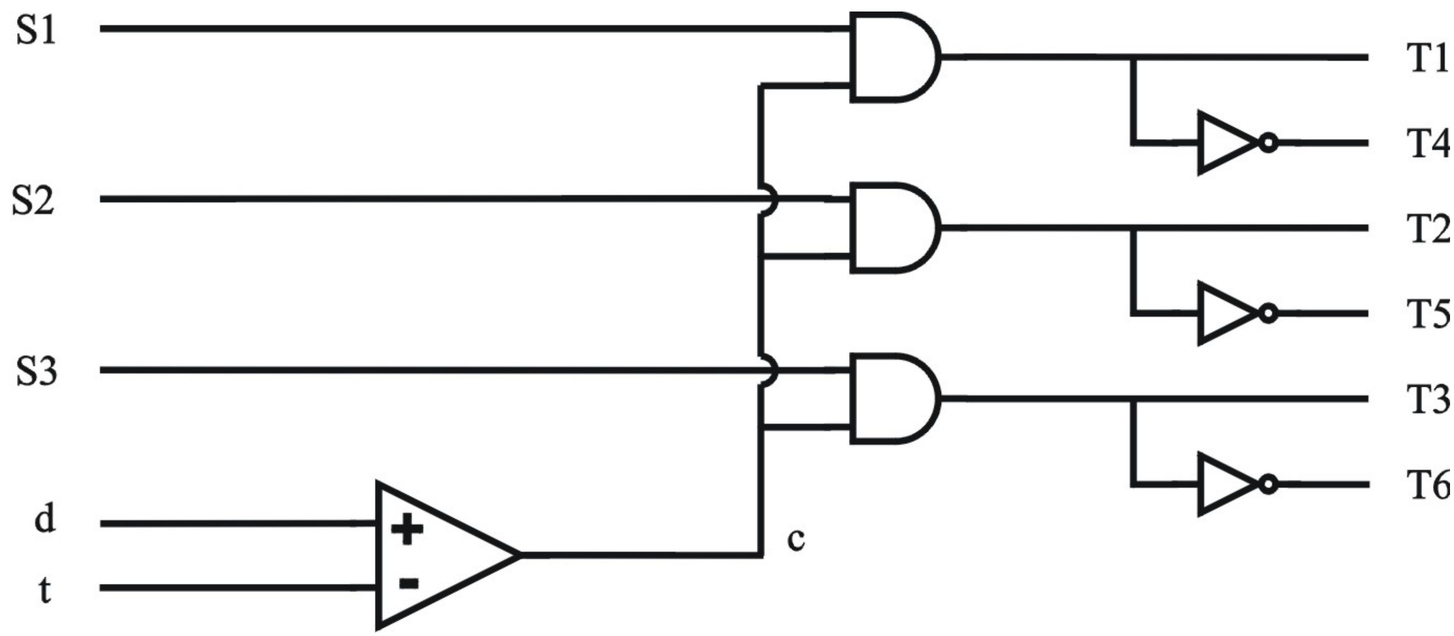
- Results
  - i_{qs}=45.6 A
  - i_{ds}=28.7 A
  - i_{dc}=43.6 A
- Exact Answer
  - i_{dc}=43.9 A

13.4 Pulse-Width Modulation

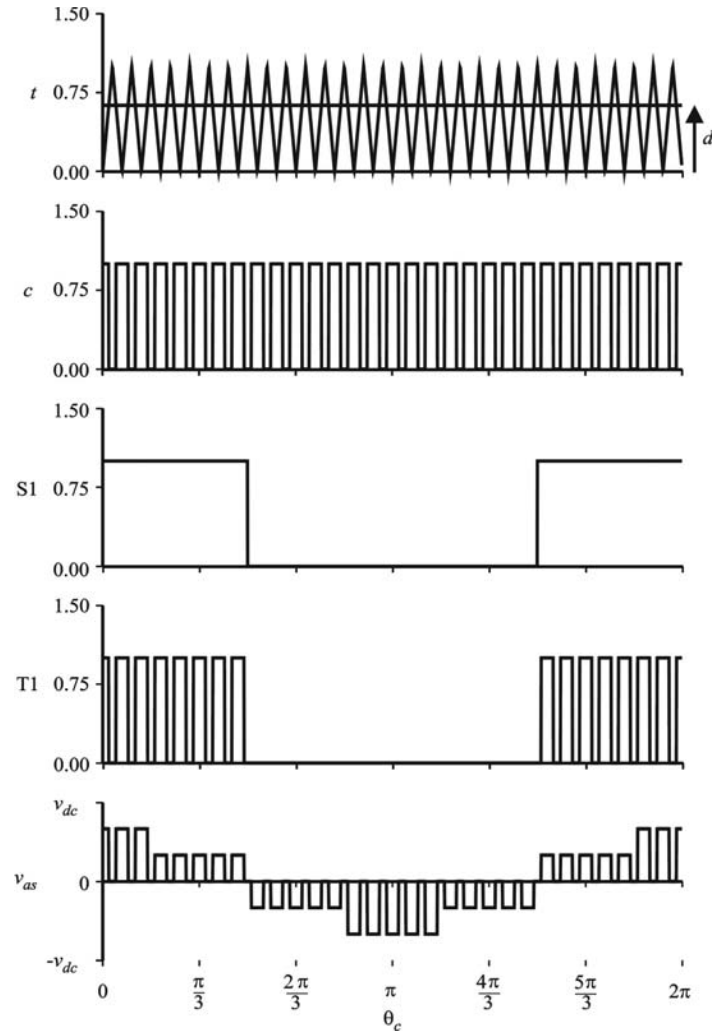
- Current Capability

- Desire

Pulse-Width Modulation



Waveforms for Pulse-Width Modulation



Analysis

- Comparitor output
  - $c = d + 2d \sum_{k=1}^{\infty} \operatorname{sinc}(kd) \cos k\theta_{sw}$ (13.4-1)
- Where
  - $p\theta_{sw} = \omega_{sw}$ (13.4-2)
- Multiplying (13.4-1) by (13.3-2) yields
  - (next page)

Ahhhh….

- $$v_{as} = \frac{2dv_{dc}}{\pi}\left(\cos\theta_c + \sum_{j=1}^{\infty}\left(\frac{(-1)^{j+1}}{6j-1}\cos((6j-1)\theta_c) + \frac{(-1)^j}{6j+1}\cos((6j+1)\theta_c\right)\right) \qquad (13.4\text{-}3)$$

$$\frac{2d}{\pi}V_{dc}\sum_{k=1}^{\infty}\operatorname{sinc}(kd)\cos(k\theta_{sw}-\theta_c)+$$

$$\frac{2dv_{dc}}{\pi}\sum_{k=1}^{\infty}\sin(kd)\sum_{j=1}^{\infty}\left(\frac{(-1)^{j+1}}{6j-1}\cos(k\theta_{sw}-(6j-1)\theta_c)+\frac{(-1)^j}{6j+1}\cos(k\theta_{sw}-(6j+1)\theta_c)\right)$$

$$+\frac{2d}{\pi}V_{dc}\sum_{k=1}^{\infty}\operatorname{sinc}(kd)\cos(k\theta_{sw}+\theta_c)+$$

$$\frac{2dv_{dc}}{\pi}\sum_{k=1}^{\infty}\sin(kd)\sum_{j=1}^{\infty}\left(\frac{(-1)^{j+1}}{6j-1}\cos(k\theta_{sw}+(6j-1)\theta_c)+\frac{(-1)^j}{6j+1}\cos(k\theta_{sw}+(6j+1)\theta_c)\right)$$

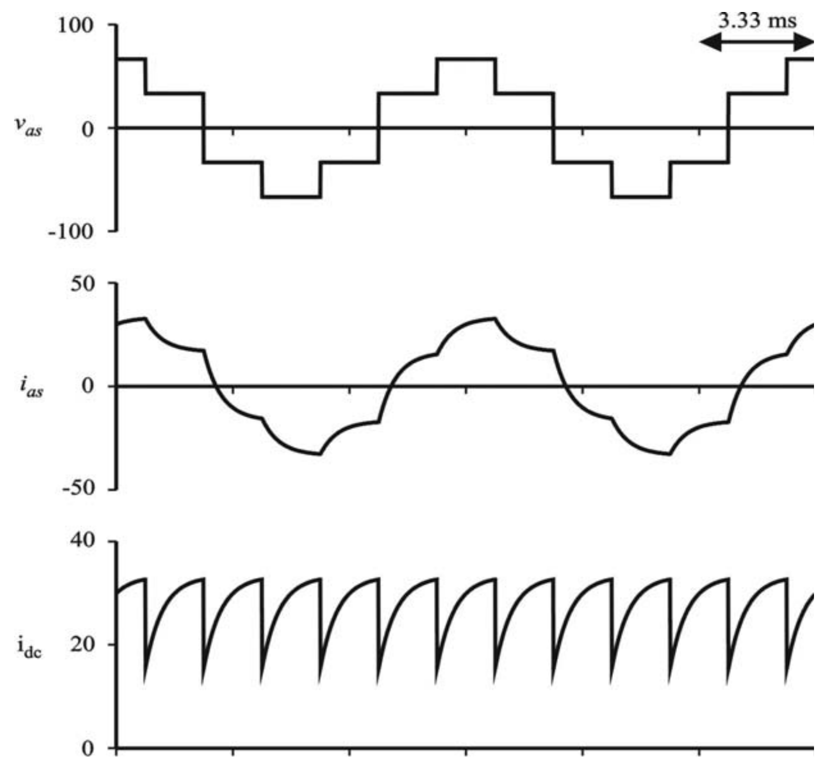
Points of Interest

- Fundamental Component
  - $\left|v_{as}\right|_{fund} = d\frac{2}{\pi}v_{dc}\cos\theta_c$ (13.4-4)
- QD Voltages
  - $\bar{v}_{qs}^{c} = \frac{2}{\pi}dv_{dc}$ (13.4-5)
  - $\bar{v}_{ds}^{c} = 0$ (13.4-6)

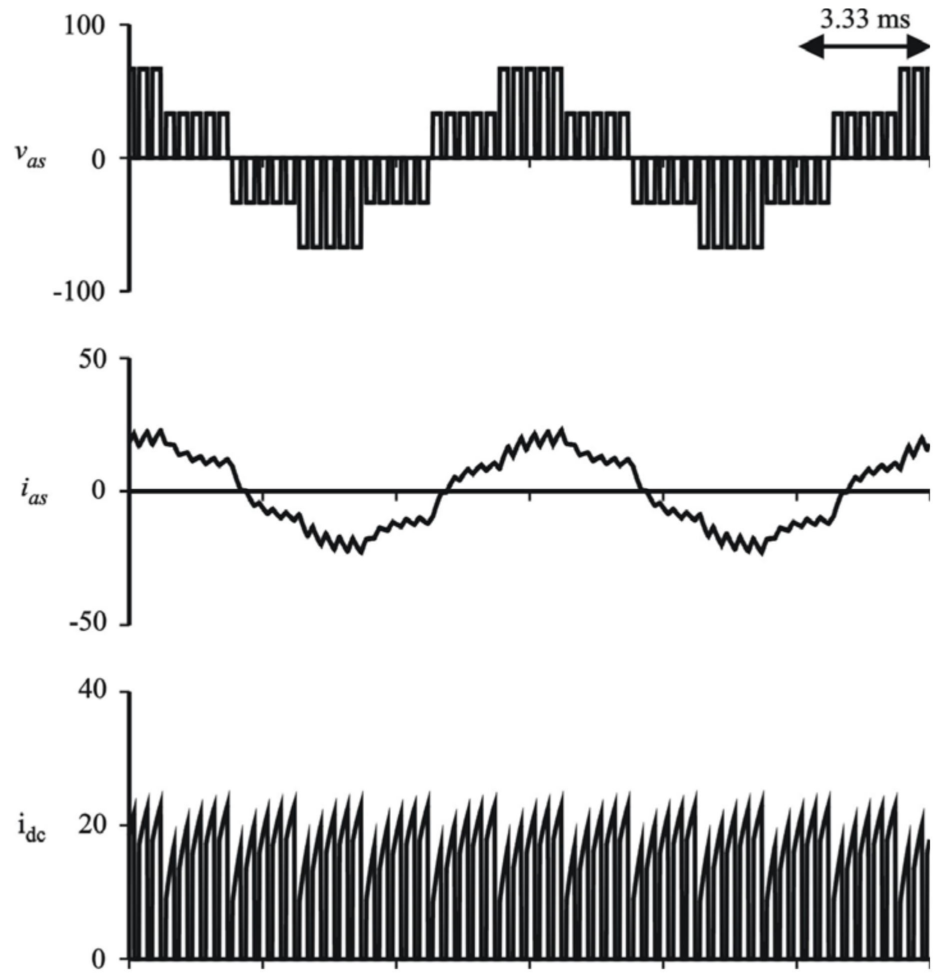
An Example

- Load: Balanced, wye-connected, series RL
  - $R = 2\ \Omega$
  - $L = 1$ mH
- DC Voltage: 100 V
- Fundamental Frequency: 100 Hz
- Duty Cycle: 0.628
- Switching Frequency:3000 Hz

ABC Variables for 180^o VSI



ABC Variables for Duty-Cycle Modulation



Pulse Width Modulation Summary

- Good News

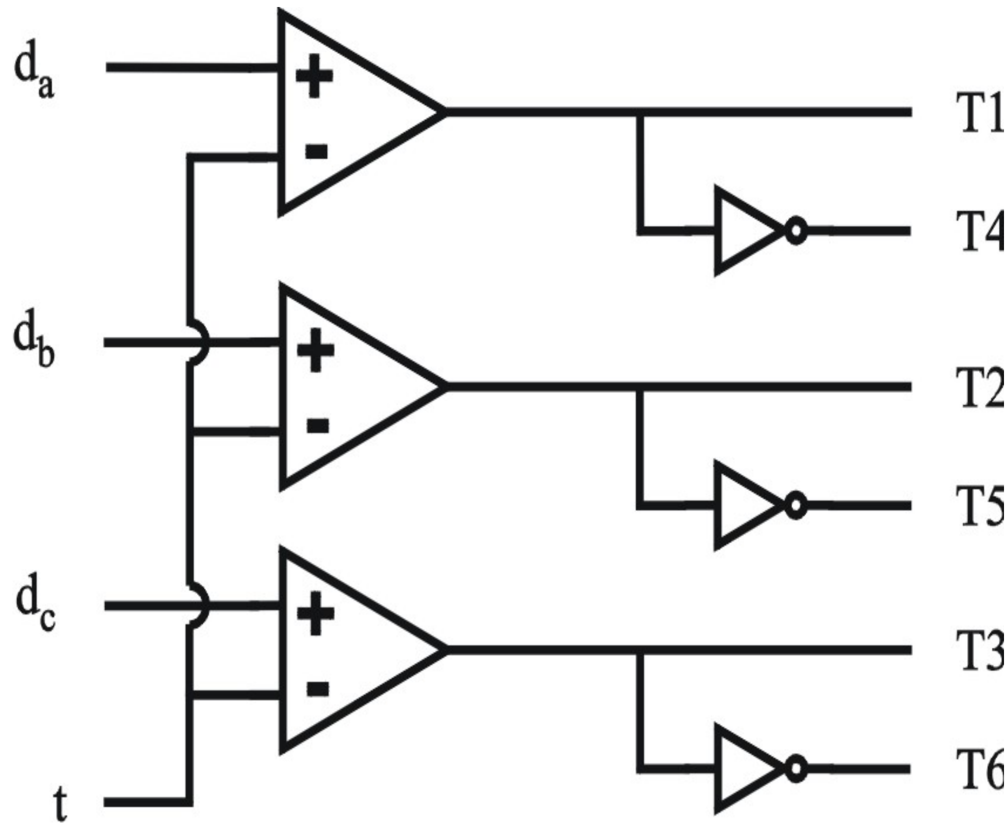
- Bad News

- Applications

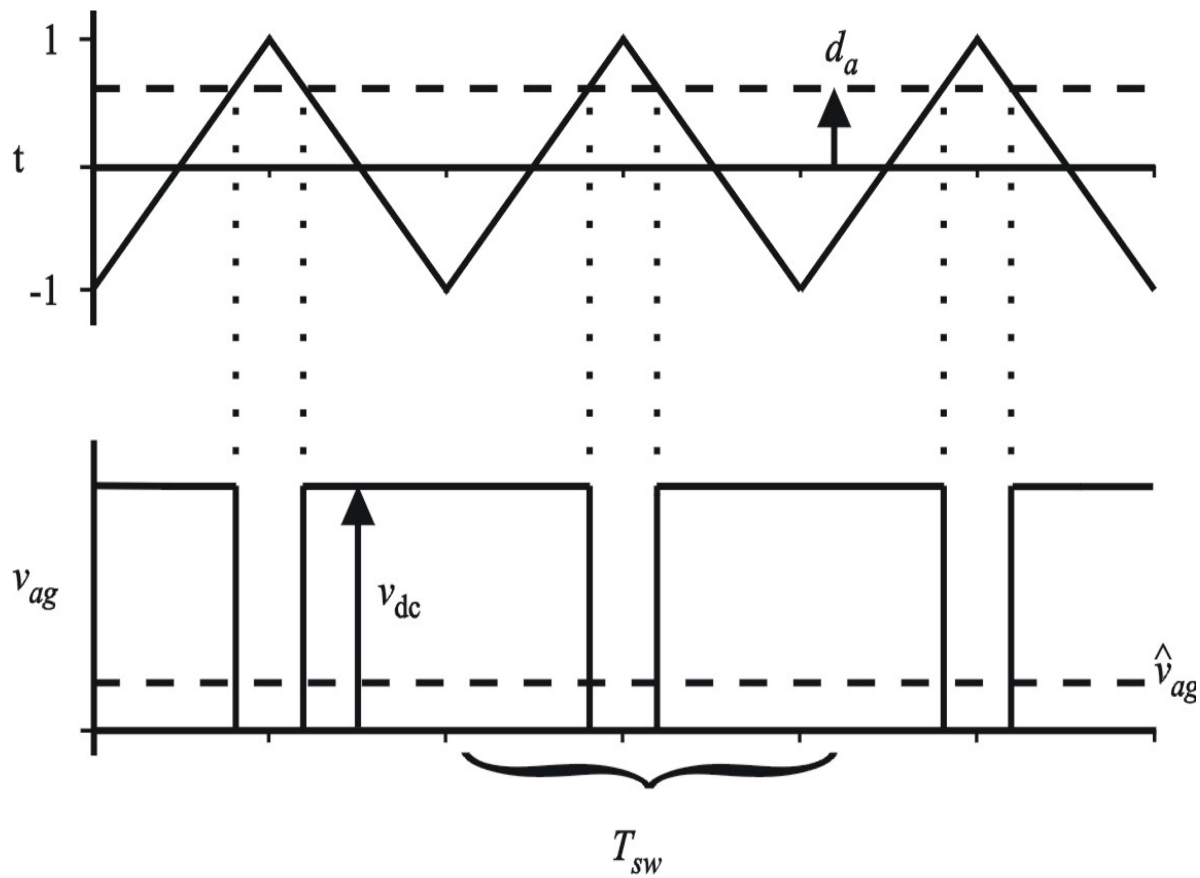
13.5 Sine-Triangle Modulation

- Features
  - Adjustable Voltage Magnitude
  - Adjustable Frequency
  - Low Low-Frequency Harmonic Content
- Comments
  - Originally Meant for Analog Implementation
  - Digital Equivalents Exist

Gate Signal Controls



Operation of A-Phase Leg



Analysis

- Important Concept: The Fast Average
  - $\hat{x}(t) = \frac{1}{T_{sw}} \int_{t-T_{sw}}^{t} x(t)dt$ (13.5-1)
- We can show that
  - $\hat{v}_{ag} = \frac{1}{2}(1 + d_a)v_{dc}$ (13.5-2)
  - $\hat{v}_{bg} = \frac{1}{2}(1 + d_b)v_{dc}$ (13.5-3)
  - $\hat{v}_{cg} = \frac{1}{2}(1 + d_c)v_{dc}$ (13.5-4)

Analysis (2)

- Lets specify duty cycles as
  - $d_a = d\cos\theta_c$ (13.5-5)
  - $d_b = d\cos(\theta_c - \frac{2\pi}{3})$ (13.5-6)
  - $d_c = d\cos(\theta_c + \frac{2\pi}{3})$ (13.5-7)

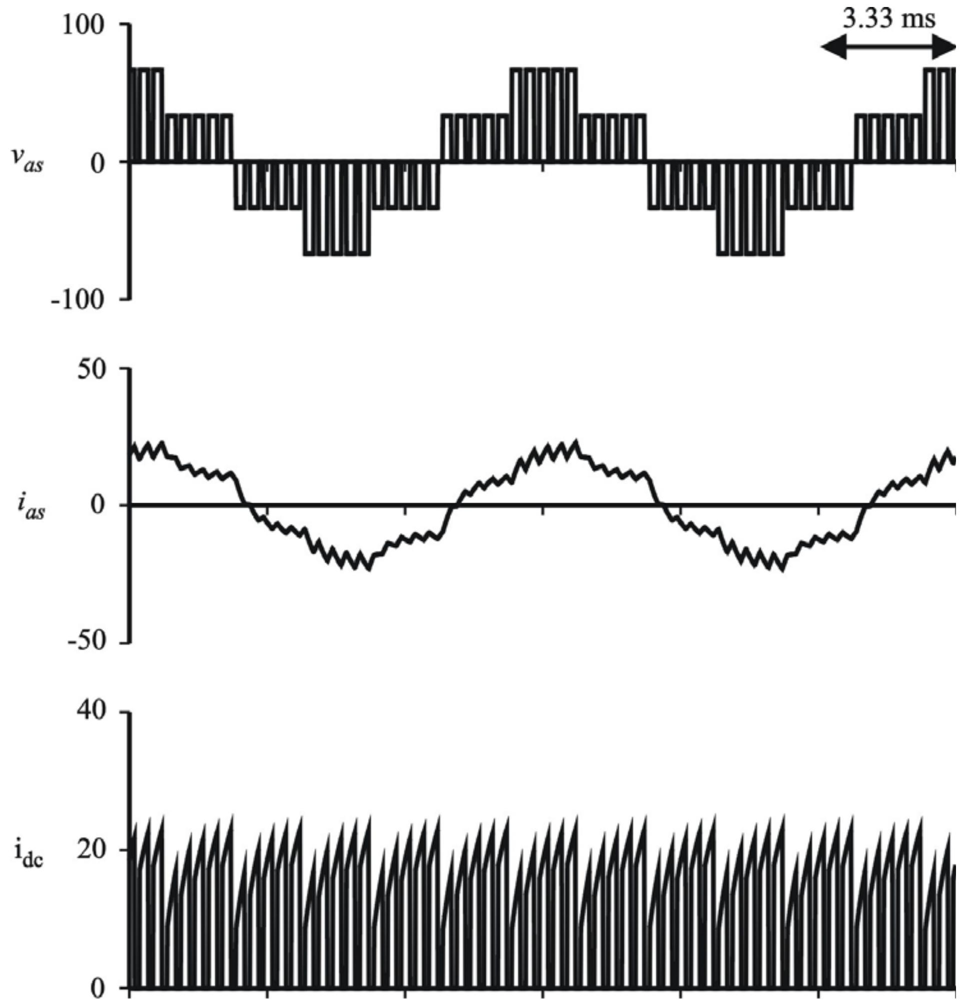
Analysis (3)

- Thus
  - $\hat{v}_{qs}^{c} = \frac{1}{2} d v_{dc}$ (13.5-14)
  - $\hat{v}_{ds}^{c} = 0$ (13.5-15)

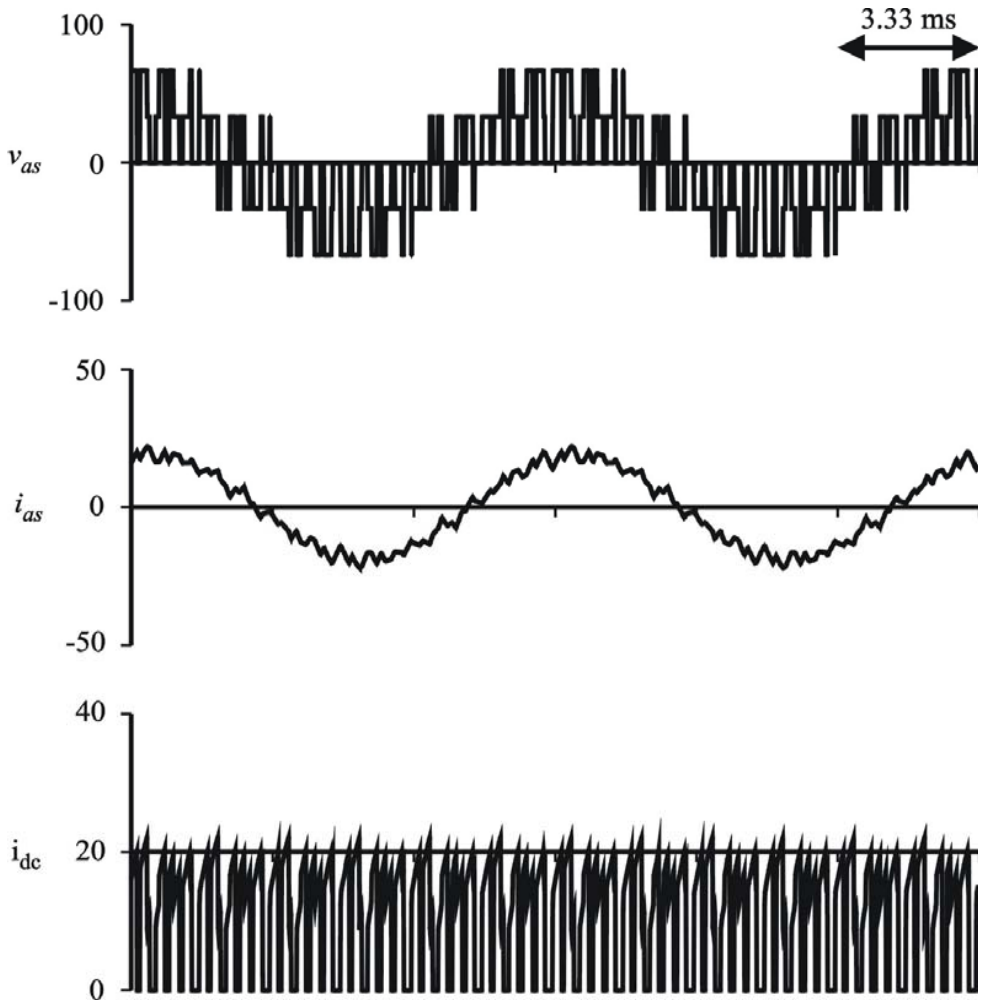
An Example

- Load: Balanced, wye-connected, series RL
  - $R = 2\ \Omega$
  - $L = 1$ mH
- DC Voltage: 100 V
- Fundamental Frequency: 100 Hz
- Duty Cycle: 0.4
- Switching Frequency: 3000 Hz

ABC Variables for Duty-Cycle Modulation (d=0.628)



Sine-Triangle Modulation

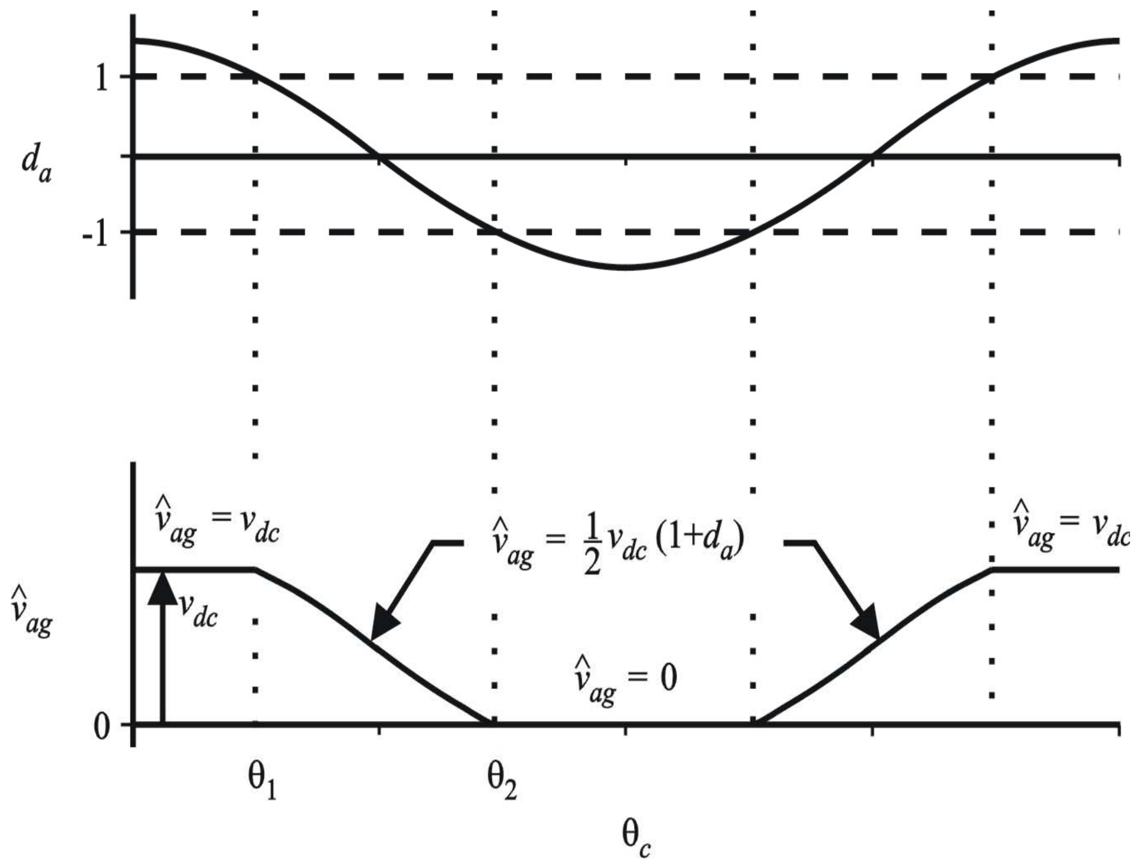


Limitations of Sine-Triangle Modulation

- Sine Triangle Modulation
  - Range for duty cycle:
  - Range for peak voltage (of fund):

- Duty Cycle Modulation:
  - Range for duty cycle:
  - Range for peak voltage (of fund.):

Overmodulation



Analysis of Overmodulation

- Line-to-ground voltage

  - $$\bar{v}_{ag} = \begin{cases} v_{dc} & d_a > 1 \\ \frac{1}{2}(1+d_a)v_{dc} & -1 < d_a < 1 \\ 0 & d_a < 1 \end{cases} \qquad (13.5\text{-}16)$$

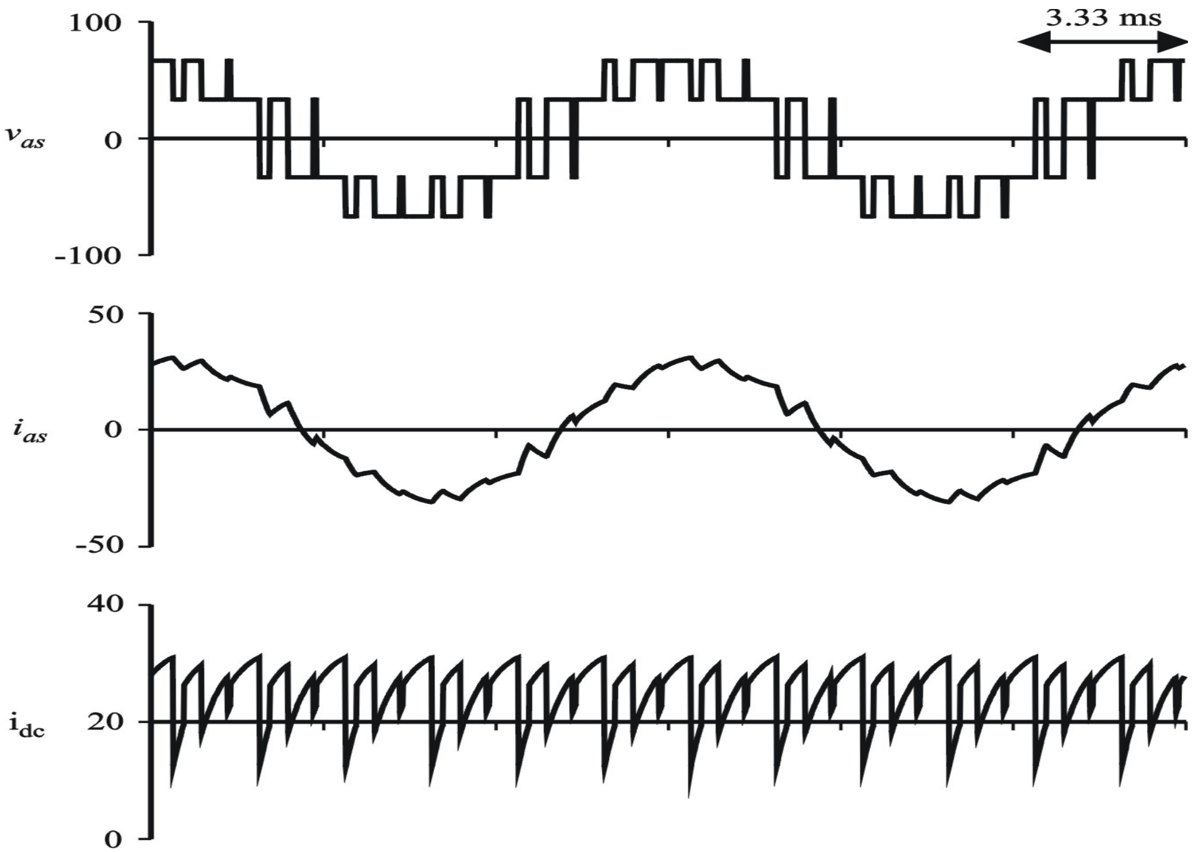
  - $$\left|v_{ag}\right|_{avg+fund} = \frac{v_{dc}}{2} + \frac{2v_{dc}}{\pi} f(d)\cos\theta_c \qquad (13.5\text{-}17)$$

  - $$f(d) = \frac{1}{2}\sqrt{1-\left(\frac{1}{d}\right)^2} + \frac{1}{4}d\left(\pi - 2\arccos\left(\frac{1}{d}\right)\right) \qquad (13.5\text{-}18)$$

Analysis of Overmodulation (2)

- Thus we have
  - $\bar{v}_{qs}^{c} = \frac{2v_{dc}}{\pi} f(d) \quad d \geq 1$ (13.5-20)
  - $\bar{v}_{ds}^{c} = 0 \quad d \geq 0$ (13.5-21)

Overmodulation (d=2)



13.6 Third-Harmonic Injection

- Objective …

The Plan

- Suppose
  - $d_a = d\cos(\theta_c) - d_3\cos(3\theta_c)$ (13.6-1)
  - $d_b = d\cos(\theta_c - \frac{2\pi}{3}) - d_3\cos(3\theta_c)$ (13.6-2)
  - $d_c = d\cos(\theta_c + \frac{2\pi}{3}) - d_3\cos(3\theta_c)$ (13.6-3)

Results

- Working for a little bit …

- Therefore
  - $\hat{v}_{qs}^{c} = \frac{1}{2} d v_{dc}$ (13.5-14)
  - $\hat{v}_{ds}^{c} = 0$ (13.5-15)

So What ?

- Consider this

- Thus
  - $d \le \frac{2}{\sqrt{3}}$ (13.6-12)
- Result
  - 15% Increase in Available Voltage

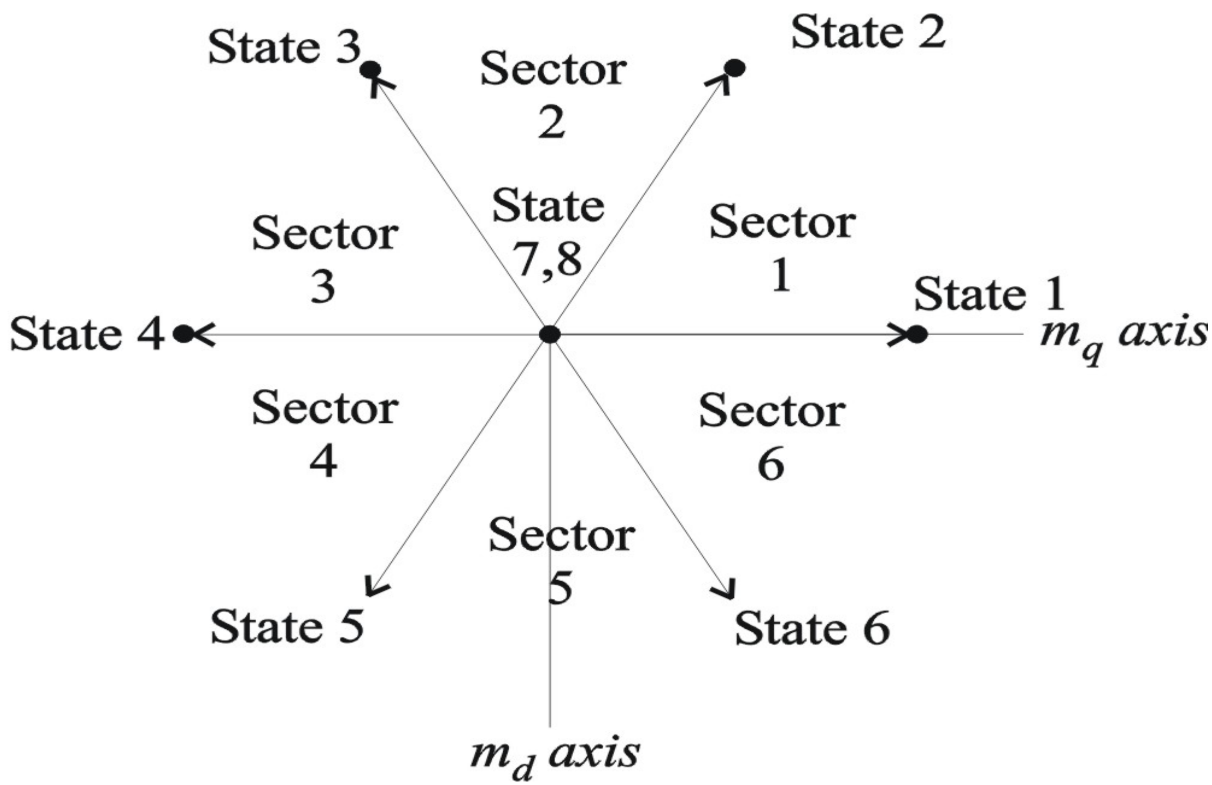
Space Vector Modulation

- General Comments
  - About Same Performance as Sine-Triangle (with 3rd)
  -
  -

Modulation Indices

- Instantaneous
  - $m_q^s = \hat{v}_q^s / v_{dc}$ (13.7-1)
  - $m_d^s = \hat{v}_d^s / v_{dc}$ (13.7-2)

Space-Vector Diagram



Space Vector Diagram

Table 13.7-1 Modulation indexes versus state.

State	$T1/\overline{T4}$	$T2/\overline{T5}$	$T3/\overline{T6}$	$m_{q,x}$	$m_{d,x}$
1	1	0	0	$2/3\cos(0^o)$	$-2/3\sin(0^o)$
2	1	1	0	$2/3\cos(60^o)$	$-2/3\sin(60^o)$
3	0	1	0	$2/3\cos(120^o)$	$-2/3\sin(120^o)$
4	0	1	1	$2/3\cos(180^o)$	$-2/3\sin(180^o)$
5	0	0	1	$2/3\cos(240^o)$	$-2/3\sin(240^o)$
6	1	0	1	$2/3\cos(300^o)$	$-2/3\sin(300^o)$
7	1	1	1	0	0
8	0	0	0	0	0

Space Vector Modulation Algorithm

- Step 1 – Compute Commanded Indices
  - $m_q^{s*} = v_q^{s*} / v_{dc}$ (13.7-3)
  - $m_d^{s*} = v_d^{s*} / v_{dc}$ (13.7-4)
- Step 2 – Compute Commanded Magnitude
  - $m^* = \sqrt{(m_q^{s*})^2 + (m_d^{s*})^2}$ (13.7-5)

Space Vector Modulation Algorithm

- Step 3 – Limit Magnitude of Command

  - $$m_{\max} = \frac{1}{\sqrt{3}} \qquad (13.7\text{-}6)$$

  - $$m_q^{*'} = \begin{cases} m_q^* & m^* \le m_{\max} \\ m_{\max} \dfrac{m_q^*}{|m^*|} & m^* > m_{\max} \end{cases} \qquad (13.7\text{-}7)$$

  - $$m_d^{*'} = \begin{cases} m_d^* & m^* \le m_{\max} \\ m_{\max} \dfrac{m_d^*}{|m^*|} & m^* > m_{\max} \end{cases} \qquad (13.7\text{-}8)$$

Space Vector Modulation Algorithm

- Step 4 – Compute Sector
- $$Sector = \text{ceil}\left(\frac{\text{angle}(m_q^{*'} - j m_d^{*'})3}{\pi}\right) \qquad (13.7\text{-}9)$$
- Step 5 – Compute State Sequence

Space Vector Modulation Algorithm

Table 13.7-2. State Sequence

Sector	Initial State (α)	2^{nd} State (β)	3^{rd} State (γ)	Final State (δ)
1	7	2	1	8
2	7	2	3	8
3	7	4	3	8
4	7	4	5	8
5	7	6	5	8
6	7	6	1	8
1	8	1	2	7
2	8	3	2	7
3	8	3	4	7
4	8	5	4	7
5	8	5	6	7
6	8	1	6	7

State Vector Modulation Algorithm

- Before continuing
  - $$\hat{m}_q = \frac{t_\beta}{T_{sw}} m_{q,\beta} + \frac{t_\gamma}{T_{sw}} m_{q,\gamma} \qquad (13.7\text{-}10)$$
  - $$\hat{m}_d = \frac{t_\beta}{T_{sw}} m_{d,\beta} + \frac{t_\gamma}{T_{sw}} m_{d,\gamma} \qquad (13.7\text{-}11)$$
- Step 5 – Compute Time in Each State
  - $$D = m_{q,\beta} m_{d,\gamma} - m_{q,\gamma} m_{d,\beta} \qquad (13.7\text{-}14)$$
  - $$t_\beta = T_{sw} (m_{d,\gamma} m_q^{*'} - m_{q,\gamma} m_d^{*'}) / D \qquad (13.7\text{-}12)$$
  - $$t_\gamma = T_{sw} (-m_{d,\beta} m_q^{*'} + m_{q,\beta} m_d^{*'}) / D \qquad (13.7\text{-}13)$$

State Vector Modulation Algorithm

- Step 6 – Compute Transition Times
  - $t_A = (T_{sw} - t_\beta - t_\gamma)/2$ (13.7-15)
  - $t_B = t_A + t_\beta$ (13.7-16)
  - $t_C = t_B + t_\gamma$ (13.7-17)

Modeling Space Vector Modulation

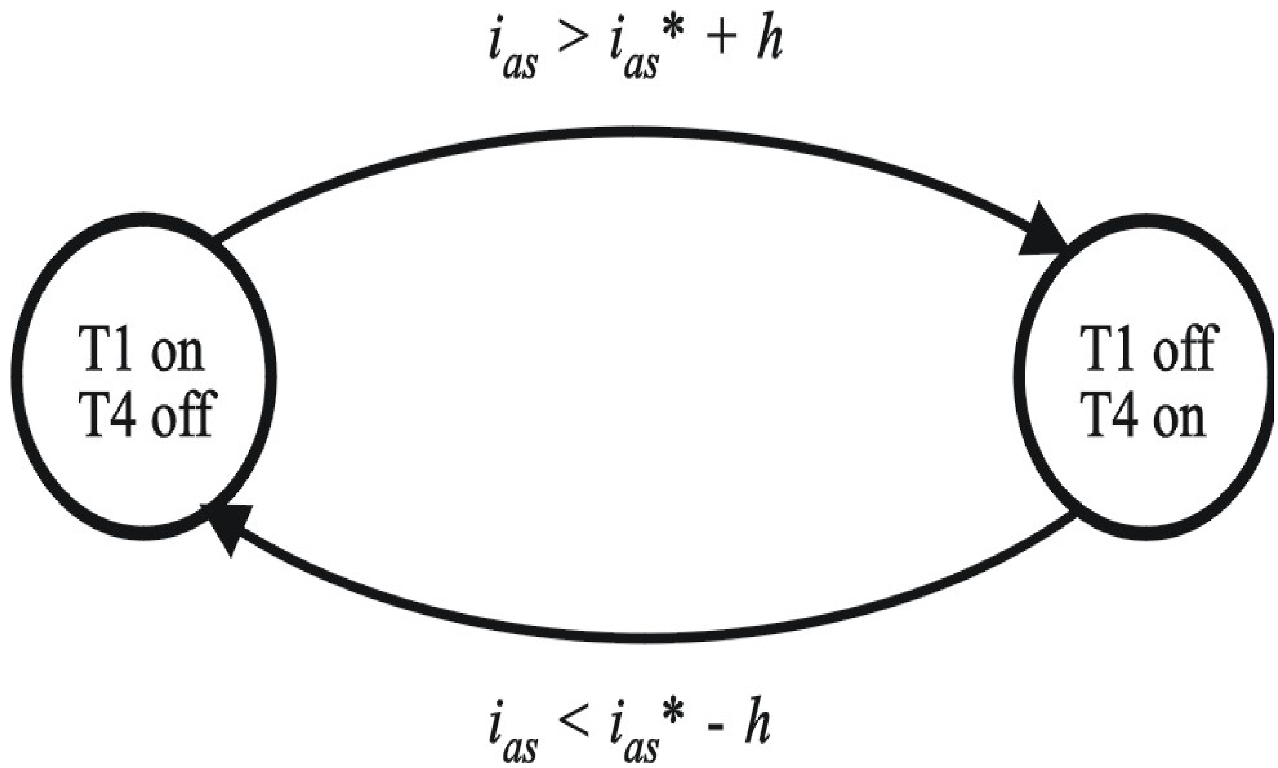
- Comments:

- Thus
  - $\hat{v}_{qs}^{s} = m_q^{**} v_{dc}$ (13.7-18)
  - $\hat{v}_{ds}^{s} = m_d^{**} v_{dc}$ (13.7-19)

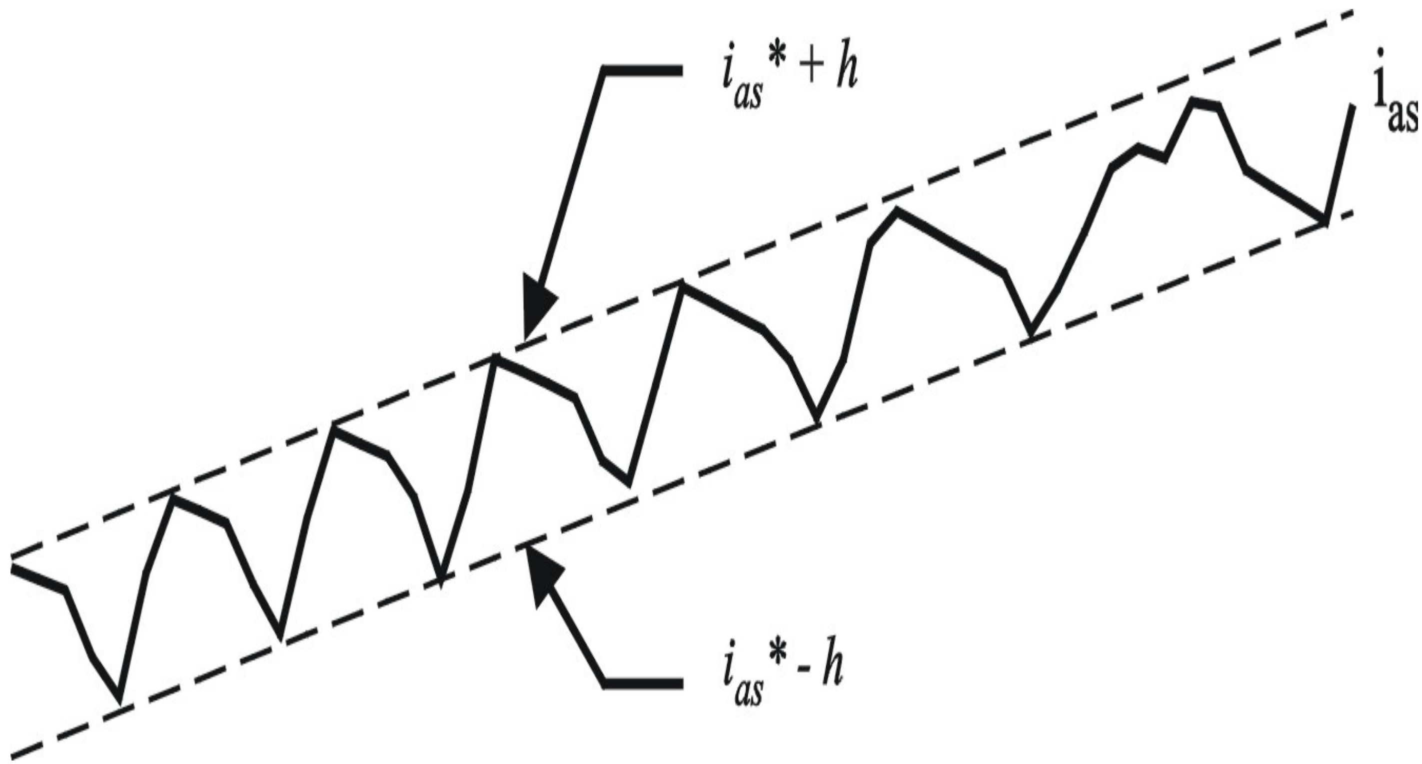
Hysteresis Modulation

- The Good
  - Current Source Based Method
  - High Bandwidth Control
  - Readily Implements Current Limiting
- The Bad
  - No Control Over Switching Frequency

State Transition Diagram



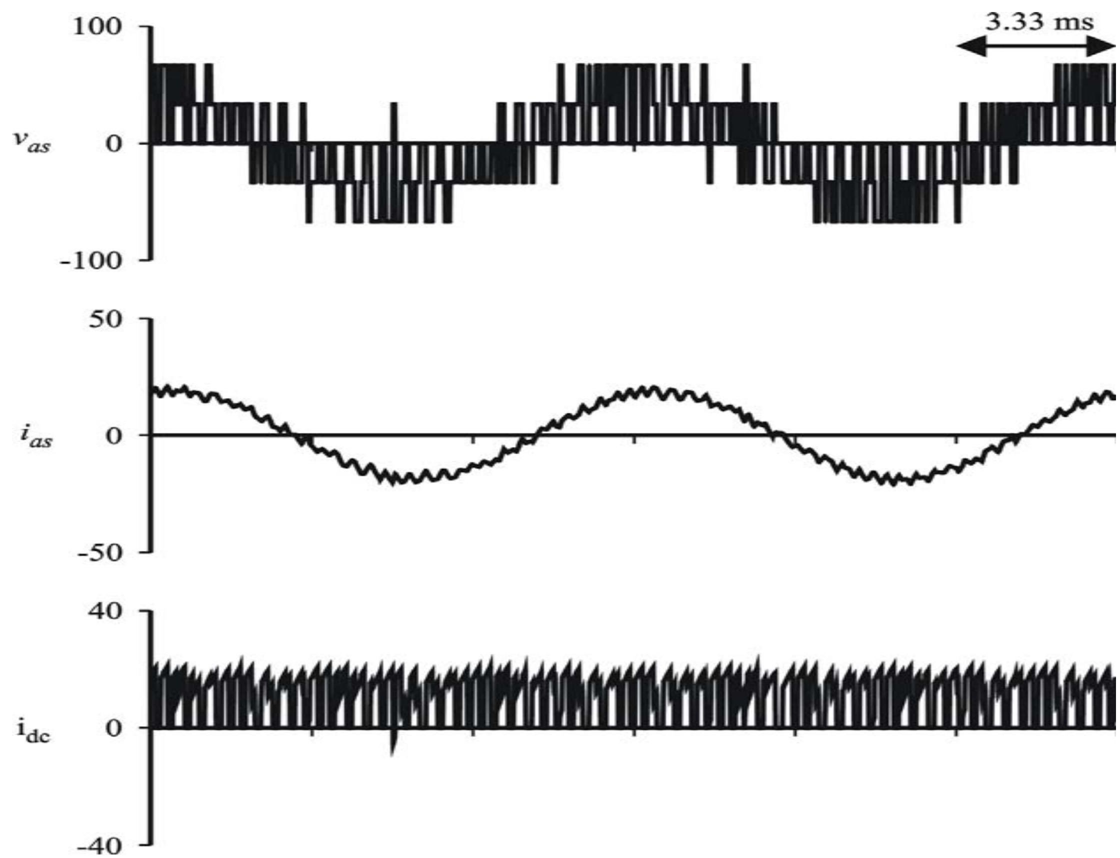
Typical Waveform



An Example

- Load: Balanced, wye-connected, series RL
  - R = 2 Ω
  - L = 1 mH
- DC Voltage: 100 V
- Fundamental Frequency: 100 Hz
- $i_{as}^{*} = 19.1\cos(\theta_c - 17.4^o)$

Typical Waveforms



Limitations of Hysteresis Modulation

- Voltage Constraints
  -
  -
  - Thus
  - $v_{spk} < \dfrac{v_{dc}}{\sqrt{3}}$ (13.8-1)

Delta Modulation

13.10 Open Loop Voltage and Current Control

- Given
  - Commanded q- and d-axis voltages
  - DC Voltage
  - Position of Synchronous Reference Frame
- Find
  - d
  - θ_c

Open Loop Voltage Control for Duty Cycle PWM

- Consider

$$\begin{bmatrix} v_{qs}^{e} \\ v_{ds}^{e} \end{bmatrix} = \begin{bmatrix} \cos\theta_{ce} & \sin\theta_{ce} \\ -\sin\theta_{ce} & \cos\theta_{ce} \end{bmatrix} \begin{bmatrix} v_{qs}^{c} \\ v_{ds}^{c} \end{bmatrix} \qquad (13.10\text{-}1)$$

- Where

$$\theta_{ce} = \theta_c - \theta_e \qquad (13.10\text{-}2)$$

Open Loop Voltage Control for Duty Cycle PWM

- Now

$$\begin{bmatrix} v_{qs}^{e*} \\ v_{ds}^{e*} \end{bmatrix} = \begin{bmatrix} \cos\theta_{ce} & \sin\theta_{ce} \\ -\sin\theta_{ce} & \cos\theta_{ce} \end{bmatrix} \begin{bmatrix} \frac{2}{\pi} d v_{dc} \\ 0 \end{bmatrix} \qquad (13.10\text{-}3)$$

- Manipulating

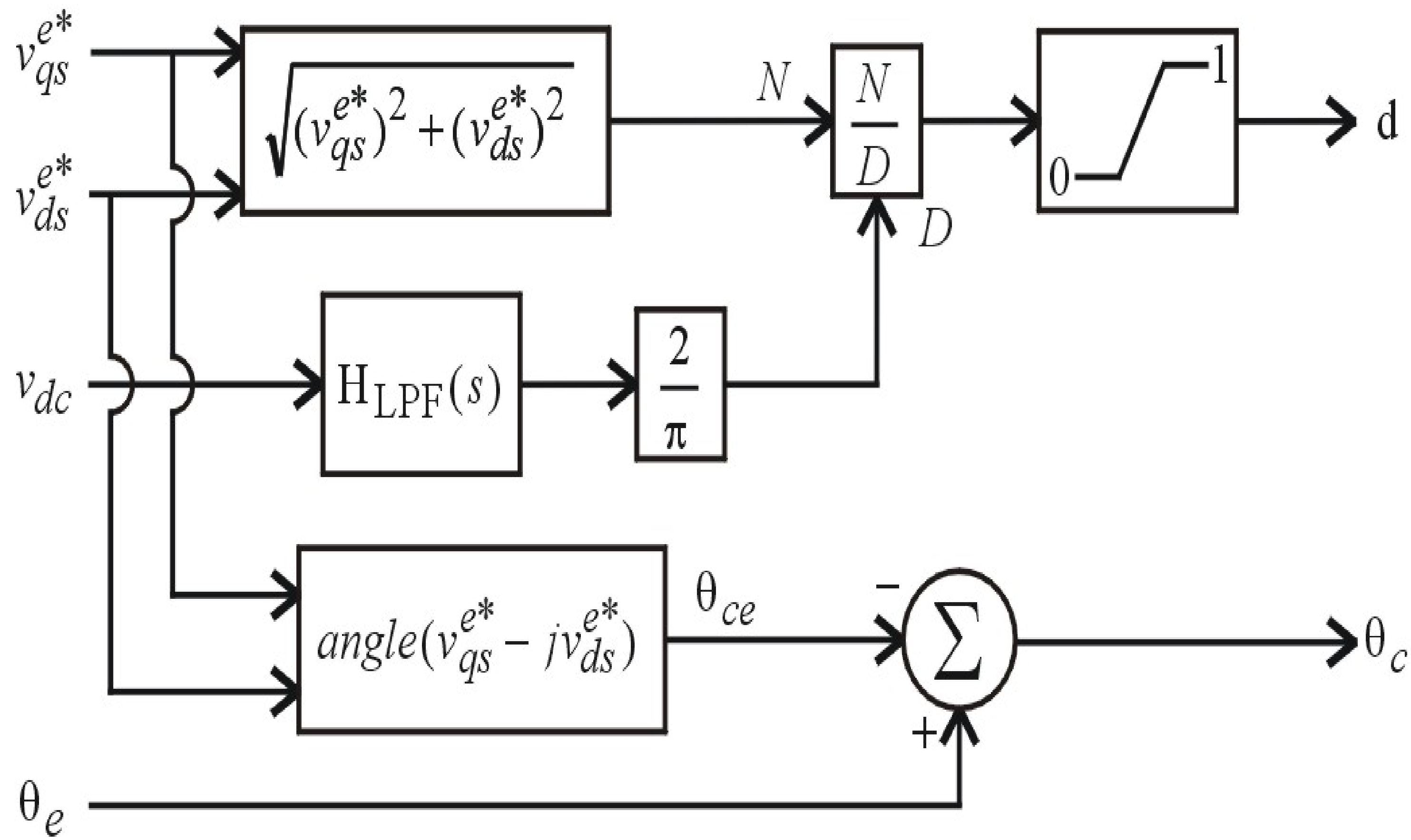
Open Loop Voltage Control for Duty Cycle PWM

- Thus

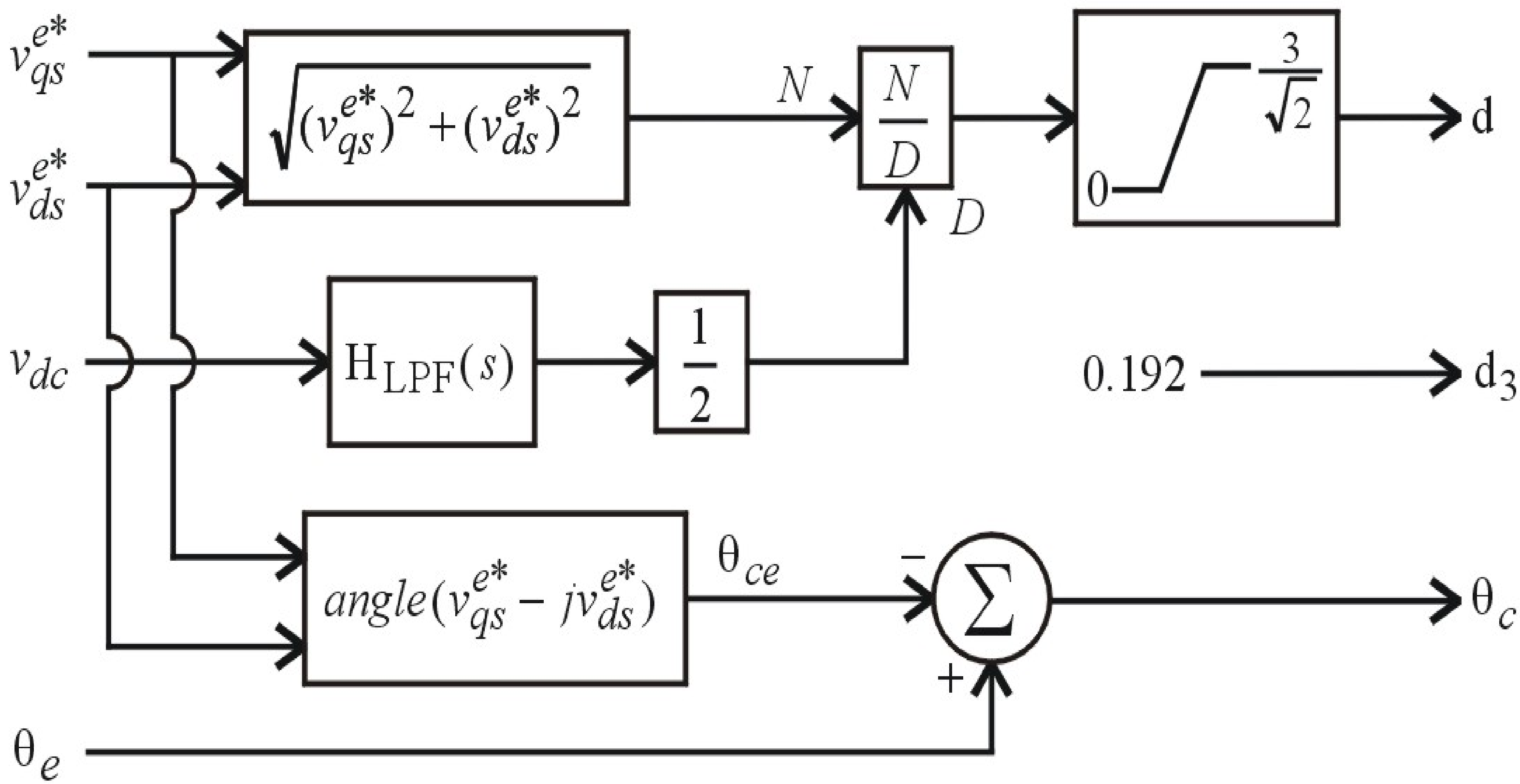
$$d = \frac{\pi}{2v_{dc}}\sqrt{\left(v_{qs}^{e*}\right)^2 + \left(v_{ds}^{e*}\right)^2} \qquad (13.10\text{-}4)$$

$$\theta_{ce} = angle(v_{qs}^{e*} - jv_{ds}^{e*}) \qquad (13.10\text{-}5)$$

Open Loop Voltage Control for Duty Cycle PWM



Open Loop Voltage Control for Sine-Triangle PWM (with 3rd)



Open Loop Voltage Control for Space Vector Modulation

- From the frame-to-frame transformation

$$v_{qs}^{s*} = v_{qs}^{e*} \cos\theta_e + v_{ds}^{e*} \sin\theta_e \tag{13.10-6}$$

$$v_{ds}^{s*} = -v_{qs}^{e*} \sin\theta_e + v_{ds}^{e*} \cos\theta_e \tag{13.10-7}$$

Open Loop Current Control

- From inverse transformation

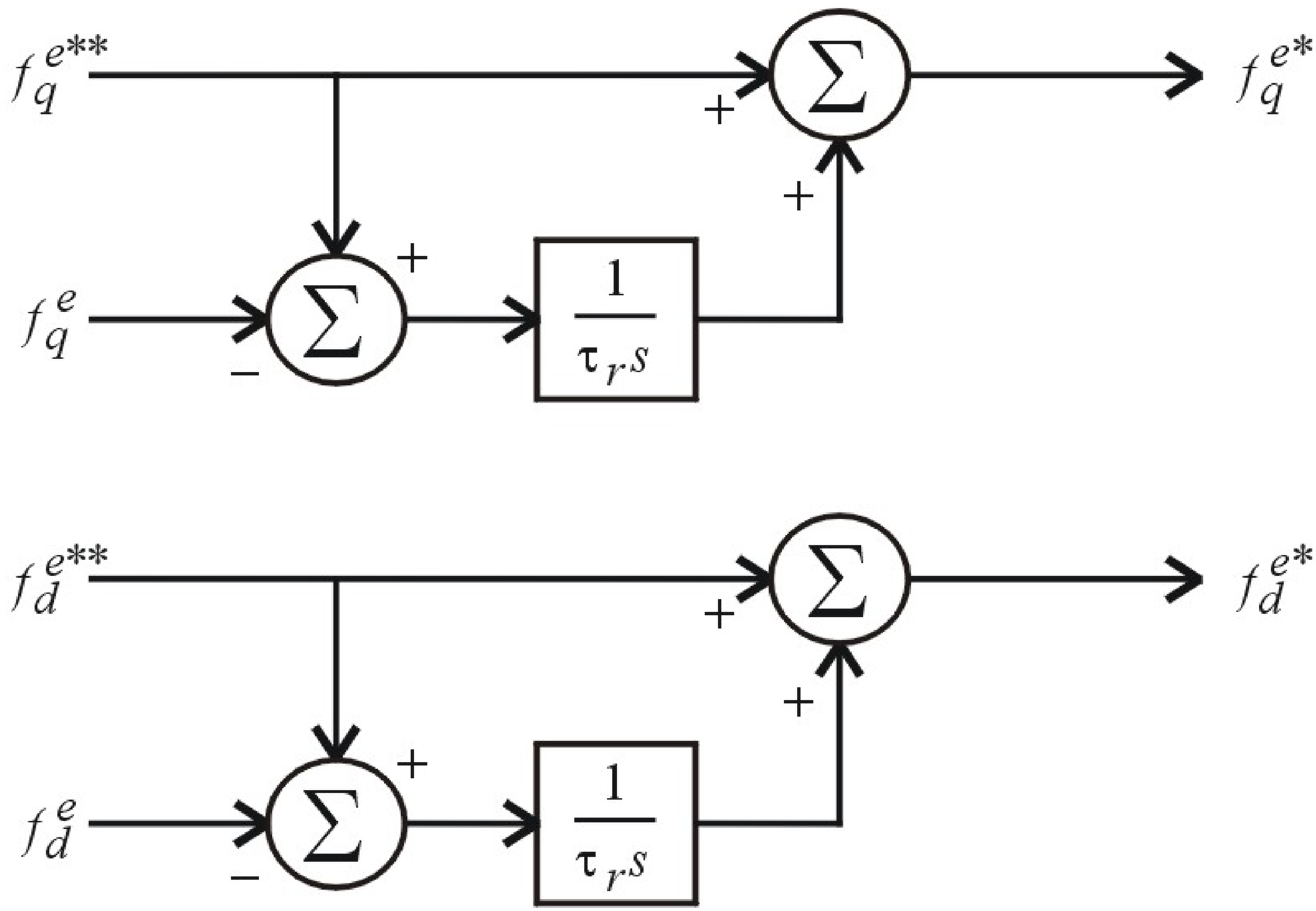
$$i_{abcs}^{*} = K_s^{e^{-1}} i_{qd0s}^{e*} \qquad (13.10\text{-}8)$$

Closed-Loop Current Control

- Current Source Based Current Control

- Voltage Source Based Current Control

Closed Loop Current Source Based Current Control



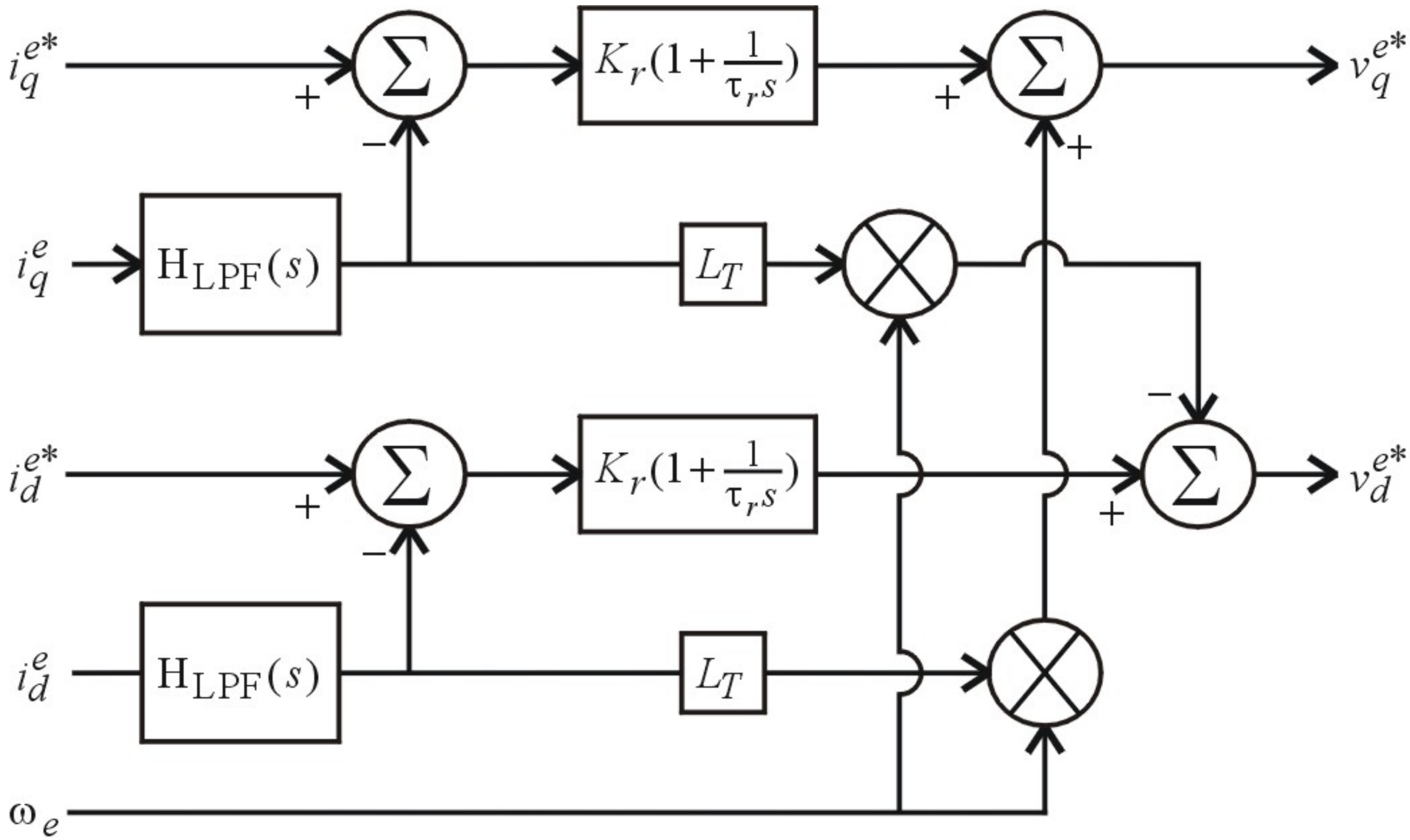
Analysis

- Start with

- Thus

$$f_q^{e*} = f_{qe}^{**} - \frac{1}{\tau_r s + 1} f_{q,err}^{e} \qquad (13.11\text{-}3)$$

Closed Loop Voltage Source Based Current Control



Analysis

- Assumed Load

$$v_{qs}^{e} = \omega_e L_T i_{ds}^{e} + L_T p i_{qs}^{e} + e_{qT} \qquad (13.11\text{-}7)$$

$$v_{ds}^{e} = -\omega_e L_T i_{qs}^{e} + L_T p i_{ds}^{e} + e_{dT} \qquad (13.11\text{-}8)$$

- Thus

$$i_{qs}^{e} = \frac{K_r(\tau_r s + 1) i_{qs}^{e*} - \tau_r s e_{qT}^{e}}{L_T \tau_r \left(s^2 + \frac{K_r}{L_T} s + \frac{K_r}{L_T \tau_r} \right)} \qquad (13.11\text{-}9)$$

Pole Placement

- Note that

$$K_r = L_T(s_1 + s_2) \quad (13.11\text{-}10)$$

$$\tau_r = \frac{1}{s_1} + \frac{1}{s_2} \quad (13.11\text{-}11)$$

- One choice

$$s_1 = s_2 \approx \frac{\pi f_{sw}}{5} \quad (13.12\text{-}12)$$