EE595S: Class Lecture Notes
Chapter 4: Analysis of Induction Machines

S.D. Sudhoff

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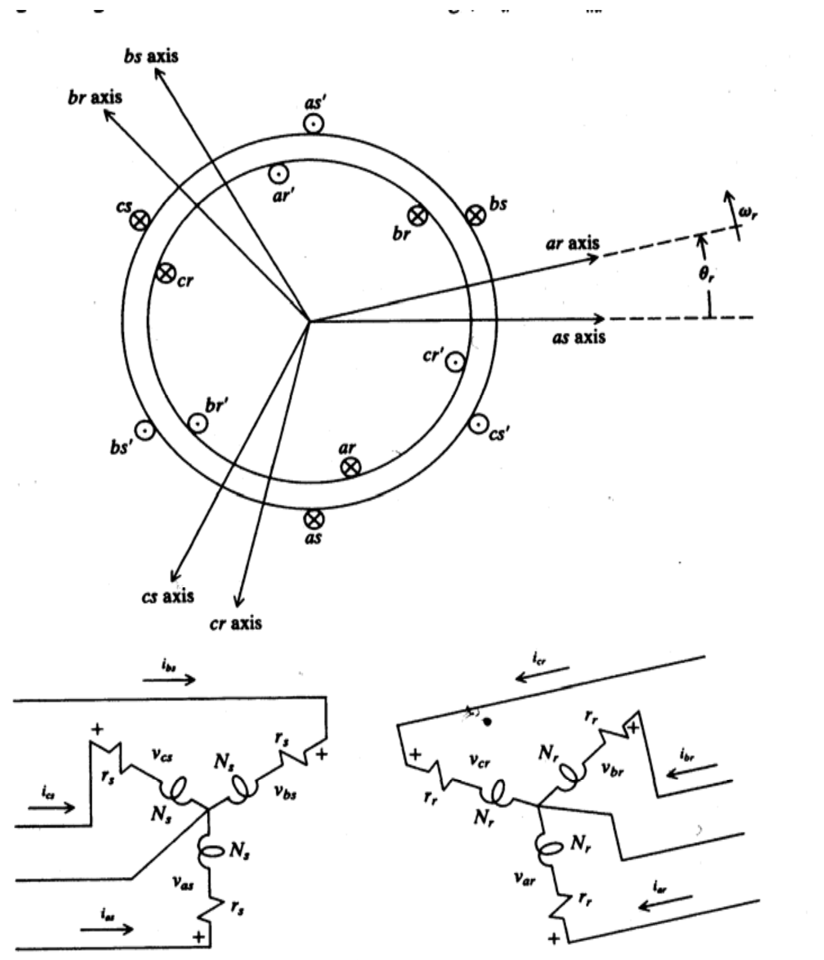
General Comments

- Applications
- Advantages over PMSM
- Disadvantages relative to PMSM
- Comments on Model

4.2/4.3 Machine Variable Model



4.2/4.3 Machine Variable Model

- Types of Rotor Construction
  - Wound Rotor
  - Squirrel Cage Rotor
  - Solid Rotor

4.2/4.3 Machine Variable Model

- General Form
  - $(\mathbf{f}_{abcs})^T = [f_{as} \quad f_{bs} \quad f_{cs}]$ (4.2-3)
  - $(\mathbf{f}_{abcr})^T = [f_{ar} \quad f_{br} \quad f_{cr}]$ (4.2-4)
- Voltage Equations
  - $\mathbf{v}_{abcs} = \mathbf{r}_s \mathbf{i}_{abcs} + p\boldsymbol{\lambda}_{abcs}$ (4.2-1)
  - $\mathbf{v}_{abcr} = \mathbf{r}_r \mathbf{i}_{abcr} + p\boldsymbol{\lambda}_{abcr}$ (4.2-2)

4.2/4.3 Machine Variable Model

- Flux Linkage Equations
  - $$\begin{bmatrix} \boldsymbol{\lambda}_{abcs} \\ \boldsymbol{\lambda}_{abcr} \end{bmatrix} = \begin{bmatrix} \mathbf{L}_s & \mathbf{L}_{sr} \\ (\mathbf{L}_{sr})^T & \mathbf{L}_r \end{bmatrix} \begin{bmatrix} \mathbf{i}_{abcs} \\ \mathbf{i}_{abcr} \end{bmatrix} \qquad (4.2\text{-}5)$$
  - $$\mathbf{L}_s = \begin{bmatrix} L_{ls} + L_{ms} & -\frac{1}{2}L_{ms} & -\frac{1}{2}L_{ms} \\ -\frac{1}{2}L_{ms} & L_{ls} + L_{ms} & -\frac{1}{2}L_{ms} \\ -\frac{1}{2}L_{ms} & -\frac{1}{2}L_{ms} & L_{ls} + L_{ms} \end{bmatrix} \qquad (4.2\text{-}6)$$
  - $$\mathbf{L}_r = \begin{bmatrix} L_{lr} + L_{ms} & -\frac{1}{2}L_{mr} & -\frac{1}{2}L_{mr} \\ -\frac{1}{2}L_{mr} & L_{lr} + L_{mr} & -\frac{1}{2}L_{mr} \\ -\frac{1}{2}L_{mr} & -\frac{1}{2}L_{mr} & L_{lr} + L_{mr} \end{bmatrix} \qquad (4.2\text{-}7)$$

4.2/4.3 Machine Variable Model

- Flux Linkage Equations

  - $$\mathbf{L}_{sr} = L_{sr}\begin{bmatrix} \cos\theta_r & \cos\left(\theta_r + \frac{2\pi}{3}\right) & \cos\left(\theta_r - \frac{2\pi}{3}\right) \\ \cos\left(\theta_r - \frac{2\pi}{3}\right) & \cos\theta_r & \cos\left(\theta_r + \frac{2\pi}{3}\right) \\ \cos\left(\theta_r + \frac{2\pi}{3}\right) & \cos\left(\theta_r - \frac{2\pi}{3}\right) & \cos\theta_r \end{bmatrix} \quad (4.2\text{-}8)$$

4.2/4.3 Machine Variable Model

- Relationship Between Magnetizing Inductances

- Referral of Rotor Quantities
  - $$\mathbf{i}'_{abcr} = \frac{N_r}{N_s}\mathbf{i}_{abcr} \quad (4.2\text{-}9)$$
  - $$\mathbf{v}'_{abcr} = \frac{N_s}{N_r}\mathbf{v}_{abcr} \quad (4.2\text{-}10)$$
  - $$\boldsymbol{\lambda}'_{abcr} = \frac{N_s}{N_r}\boldsymbol{\lambda}_{abcr} \quad (4.2\text{-}11)$$

4.2/4.3 Machine Variable Model

- Referred Flux Linkage Equations
  - $$\begin{bmatrix} \boldsymbol{\lambda}_{abcs} \\ \boldsymbol{\lambda}'_{abcr} \end{bmatrix} = \begin{bmatrix} \mathbf{L}_{s} & \mathbf{L}'_{sr} \\ (\mathbf{L}'_{sr})^{T} & \mathbf{L}'_{r} \end{bmatrix} \begin{bmatrix} \mathbf{i}_{abcs} \\ \mathbf{i}'_{abcr} \end{bmatrix} \qquad (4.2\text{-}18)$$

4.2/4.3 Machine Variable Model

- Where
  - $$\mathbf{L}'_{sr} = \frac{N_s}{N_r}\mathbf{L}_{sr} = L_{ms}\begin{bmatrix} \cos\theta_r & \cos\left(\theta_r + \frac{2\pi}{3}\right) & \cos\left(\theta_r - \frac{2\pi}{3}\right) \\ \cos\left(\theta_r - \frac{2\pi}{3}\right) & \cos\theta_r & \cos\left(\theta_r + \frac{2\pi}{3}\right) \\ \cos\left(\theta_r + \frac{2\pi}{3}\right) & \cos\left(\theta_r - \frac{2\pi}{3}\right) & \cos\theta_r \end{bmatrix} \quad (4.2\text{-}13)$$
  - $$\mathbf{L}'_r = \begin{bmatrix} L'_{lr} + L_{ms} & -\frac{1}{2}L_{ms} & -\frac{1}{2}L_{ms} \\ -\frac{1}{2}L_{ms} & L'_{lr} + L_{ms} & -\frac{1}{2}L_{ms} \\ -\frac{1}{2}L_{ms} & -\frac{1}{2}L_{ms} & L'_{lr} + L_{ms} \end{bmatrix} \quad (4.2\text{-}16)$$
  - $$L'_{lr} = \left(\frac{N_s}{N_r}\right)^2 L_{lr} \quad (4.2\text{-}17)$$

4.2/4.3 Machine Variable Model

- Referred Voltage Equations

- Where
  - $$\mathbf{r}_r' = \left(\frac{N_s}{N_r}\right)^2 \mathbf{r}_r \qquad (4.2\text{-}20)$$

4.4 Equations of Transformation

- We Have Worked With a Transformation for Stator Variables
- We'll Also Need a Rotor Transformation for Rotor Variables
- Lets Define
  - $(\mathbf{f}'_{abcr})^T = [f'_{ar} \quad f'_{br} \quad f'_{cr}]$ (4.4-3)
  - $(\mathbf{f}'_{qd0r})^T = [f'_{qr} \quad f'_{dr} \quad f'_{0r}]$ (4.4-2)

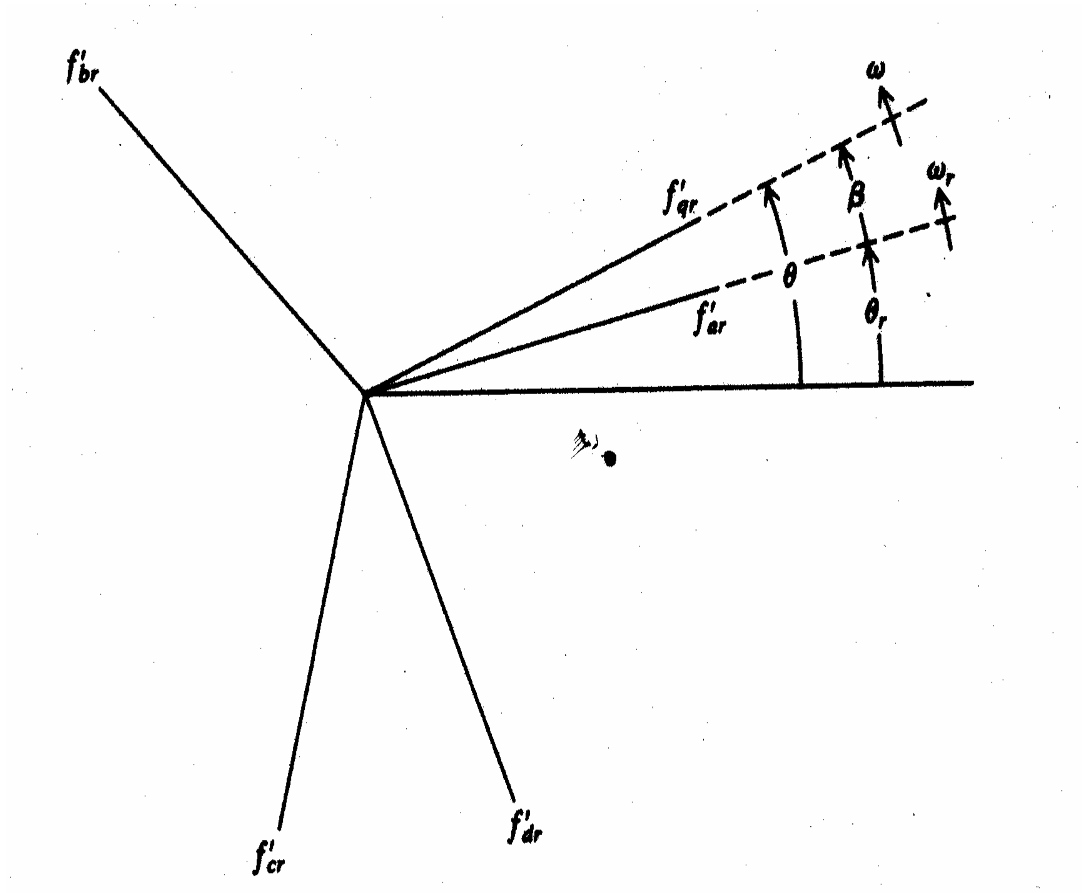
4.4 Equations of Transformation

- The Transformation is Given By
  - $\mathbf{f}'_{qd0r} = \mathbf{K}_r \mathbf{f}'_{abcr}$ (4.4-1)
  - Where
  - $$\mathbf{K}_r = \frac{2}{3}\begin{bmatrix} \cos\beta & \cos\left(\beta - \frac{2\pi}{3}\right) & \cos\left(\beta + \frac{2\pi}{2}\right) \\ \sin\beta & \sin\left(\beta - \frac{2\pi}{3}\right) & \sin\left(\beta + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \quad (4.4\text{-}4)$$

4.4 Equations of Transformation

- $$(\mathbf{K}_r)^{-1} = \begin{bmatrix} \cos\beta & \sin\beta & 1 \\ \cos\left(\beta - \frac{2\pi}{3}\right) & \sin\left(\beta - \frac{2\pi}{3}\right) & 1 \\ \cos\left(\beta + \frac{2\pi}{3}\right) & \sin\left(\beta + \frac{2\pi}{3}\right) & 1 \end{bmatrix} \quad (4.4\text{-}7)$$
- And Where
- $\beta = \theta - \theta_r$ (4.4-5)

4.4 Equations of Transformation



4.5 QD Machine Model

- Voltage Equations
  - $\mathbf{v}_{qd0s} = \mathbf{r}_s \mathbf{i}_{qd0s} + \omega \boldsymbol{\lambda}_{dqs} + p\boldsymbol{\lambda}_{qd0s}$ (4.5-1)
  - $\mathbf{v}'_{qd0r} = \mathbf{r}'_r \mathbf{i}'_{qd0r} + (\omega - \omega_r)\boldsymbol{\lambda}'_{dqr} + p\boldsymbol{\lambda}'_{qd0r}$ (4.5-2)
  - Where
  - $(\boldsymbol{\lambda}_{dqs})^T = \begin{bmatrix} \lambda_{ds} & -\lambda_{qs} & 0 \end{bmatrix}$ (4.5-3)
  - $(\boldsymbol{\lambda}'_{dqr})^T = \begin{bmatrix} \lambda'_{dr} & -\lambda'_{qr} & 0 \end{bmatrix}$ (4.5-4)

4.5 QD Machine Model

- Flux Linkage Equations
  - $$\begin{bmatrix} \boldsymbol{\lambda}_{qd0s} \\ \boldsymbol{\lambda}'_{qd0r} \end{bmatrix} = \begin{bmatrix} \mathbf{K}_s \mathbf{L}_s (\mathbf{K}_s)^{-1} & \mathbf{K}_s \mathbf{L}'_{sr} (\mathbf{K}_r)^{-1} \\ \mathbf{K}_r (\mathbf{L}'_{sr})^T (\mathbf{K}_s)^{-1} & \mathbf{K}_r \mathbf{L}'_r (\mathbf{K}_r)^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{i}_{qd0s} \\ \mathbf{i}'_{qd0r} \end{bmatrix} \qquad (4.5\text{-}5)$$
- Now
  - $$\mathbf{K}_s \mathbf{L}_s (\mathbf{K}_s)^{-1} = \begin{bmatrix} L_{ls} + L_M & 0 & 0 \\ 0 & L_{ls} + L_M & 0 \\ 0 & 0 & L_{ls} \end{bmatrix} \qquad (4.5\text{-}6)$$
  - Where
  - $$L_M = \frac{3}{2} L_{ms} \qquad (4.5\text{-}7)$$

4.5 QD Machine Model

- Next
  - $$\mathbf{K}_r \mathbf{L}'_r (\mathbf{K}_r)^{-1} = \begin{bmatrix} L'_{lr} + L_M & 0 & 0 \\ 0 & L'_{lr} + L_M & 0 \\ 0 & 0 & L'_{lr} \end{bmatrix} \quad (4.5\text{-}8)$$
- Finally
  - $$\mathbf{K}_s \mathbf{L}'_{sr} (\mathbf{K}_r)^{-1} = \mathbf{K}_r (\mathbf{L}'_{sr})^T (\mathbf{K}_s)^{-1} = \begin{bmatrix} L_M & 0 & 0 \\ 0 & L_M & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (4.5\text{-}9)$$

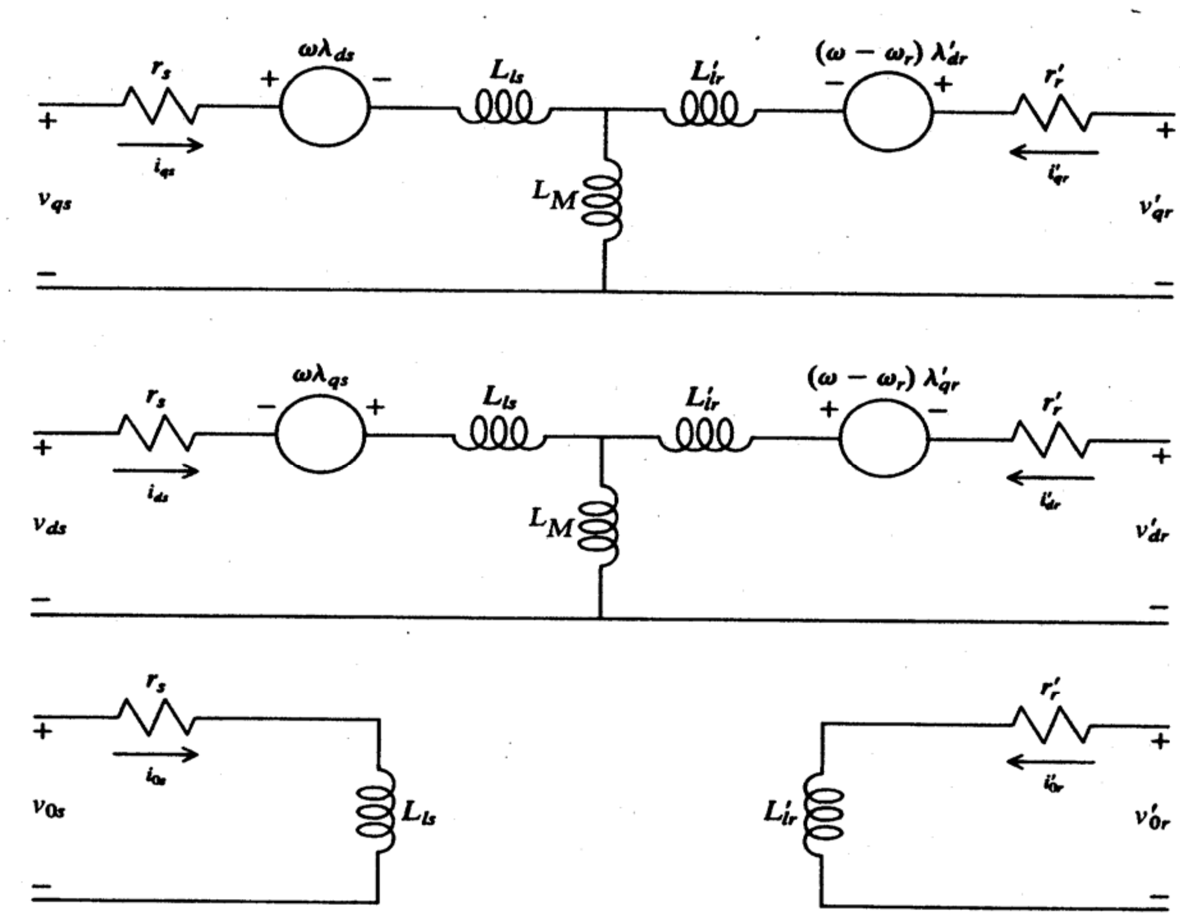
4.5 QD Machine Model

- Voltage Equations in Scalar From
  - $v_{qs} = r_s i_{qs} + \omega\lambda_{ds} + p\lambda_{qs}$ (4.5-10)
  - $v_{ds} = r_s i_{ds} - \omega\lambda_{qs} + p\lambda_{ds}$ (4.5-11)
  - $v_{0s} = r_s i_{0s} + p\lambda_{0s}$ (4.5-12)
  - $v'_{qr} = r'_r i'_{qr} + (\omega - \omega_r)\lambda'_{dr} + p\lambda'_{qr}$ (4.5-13)
  - $v'_{dr} = r'_r i'_{dr} - (\omega - \omega_r)\lambda'_{qr} + p\lambda'_{dr}$ (4.5-14)
  - $v'_{0r} = r'_r i'_{0r} + p\lambda'_{0r}$ (4.5-15)

4.5 QD Machine Model

- Flux Linkage Equations in Scalar Form
  - $\lambda_{qs} = L_{ls} i_{qs} + L_M (i_{qs} + i'_{qr})$ (4.5-16)
  - $\lambda_{ds} = L_{ls} i_{ds} + L_M (i_{ds} + i'_{dr})$ (4.5-17)
  - $\lambda_{0s} = L_{ls} i_{0s}$ (4.5-18)
  - $\lambda'_{qr} = L'_{lr} i'_{qr} + L_M (i_{qs} + i'_{qr})$ (4.5-19)
  - $\lambda'_{dr} = L'_{lr} i'_{dr} + L_M (i_{ds} + i'_{dr})$ (4.5-20)
  - $\lambda'_{0r} = L'_{lr} i'_{0r}$ (4.5-21)

4.5 QD Machine Model



4.5 QD Machine Model

- Electromagnetic Torque
  - $T_e = \left(\frac{3}{2}\right)\left(\frac{P}{2}\right)(\lambda_{ds} i_{qs} - \lambda_{qs} i_{ds})$ (4.6-4)
- Comments on Electromagnetic Torque

4.9 Analysis of Steady-State Operation

- Recall
  - $\tilde{F}_{qs} = \tilde{F}_{as}$ (4.9-1)
  - $\tilde{F}_{ds} = j\tilde{F}_{qs}$ (4.9-2)
- Similarly
  - $\tilde{F}'_{dr} = j\tilde{F}'_{qr}$ (4.9-3)

4.9 Analysis of Steady State Operation

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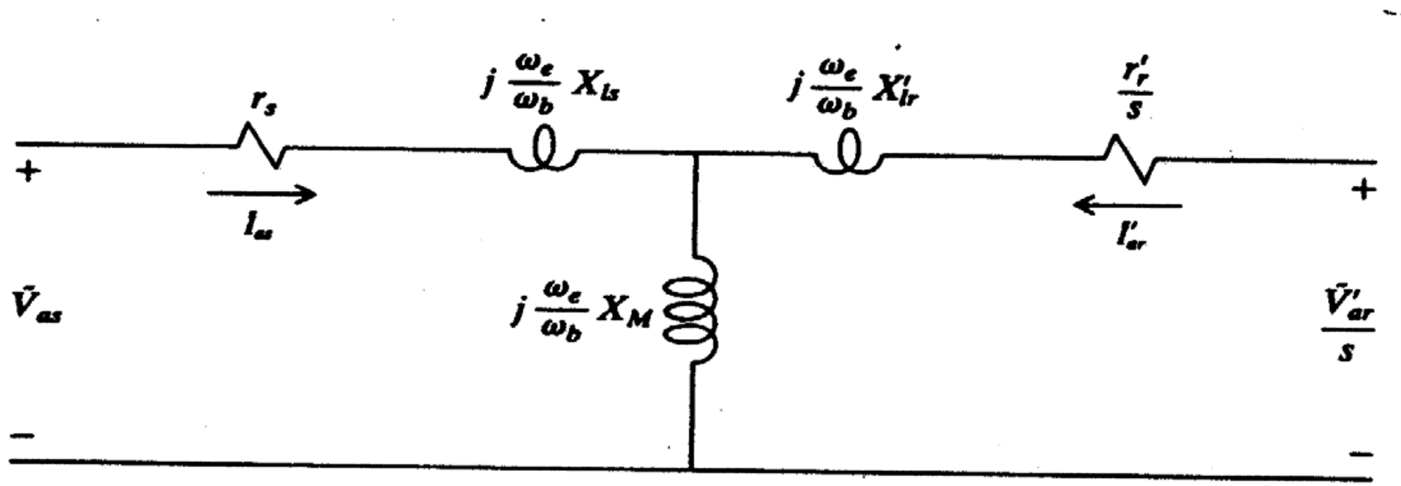


4.9 Analysis of Steady-State Operation

- After Length Bit Of Algebra
  - $$T_e = 3\left(\frac{P}{2}\right)\left(\frac{X_M}{\omega_b}\right)\mathrm{Re}[\, j\tilde{I}_{as}^{*}\tilde{I}_{ar}'\,] \qquad (4.9\text{-}14)$$

4.9 Analysis of Steady State Operation

- Derivation of Torque in Terms of Current
  - By Current Division
    - $$\tilde{I}'_{ar} = -\frac{j(\omega_e/\omega_b)X_M}{r'_r/s + j(\omega_e/\omega_b)X'_{rr}}\tilde{I}_{as} \qquad (4.9\text{-}15)$$
    - So
    - $$T_e = \frac{3\left(\frac{P}{2}\right)\left(\frac{\omega_e}{\omega_b}\right)\left(\frac{X_M^2}{\omega_b}\right)\left(\frac{r'_r}{s}\right)\left|\tilde{I}_{as}\right|^2}{\left(\frac{r'_r}{s}\right)^2 + \left(\frac{\omega_e}{\omega_b}\right)^2 X'^2_{rr}} \qquad (4.9\text{-}16)$$

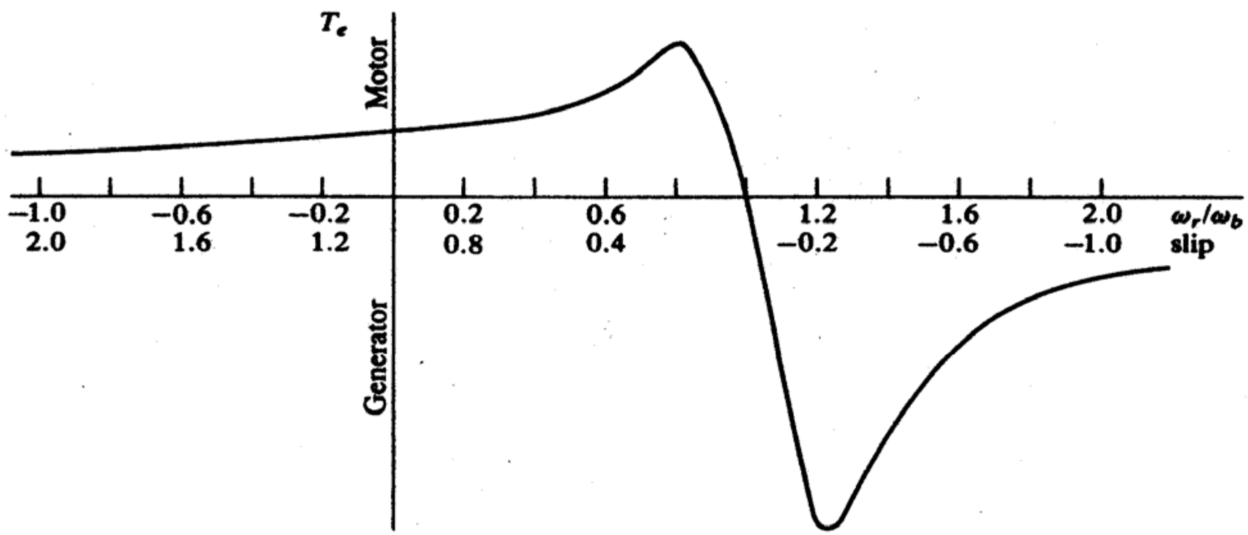
4.9 Analysis of Steady-State Operation

- Derivation of Torque in Terms of Voltage
  - ➢Impedance Looking Into Drive
    - $Z = \dfrac{\dfrac{r_s r_r'}{s} + \left(\dfrac{\omega_e}{\omega_b}\right)^2 \left(X_M^2 - X_{ss} X_{rr}'\right) + j\dfrac{\omega_e}{\omega_b}\left(\dfrac{r_r'}{s} X_{ss} + r_s X_{rr}'\right)}{\dfrac{r_r'}{s} + j\dfrac{\omega}{\omega_b} X_{rr}'}$ (4.9-17)

4.9 Analysis of Steady-State Operation

- Now We Have
  - $$\left|\tilde{I}_{as}\right| = \frac{\left|\tilde{V}_{as}\right|}{|Z|} \qquad (4.9\text{-}18)$$
- Substitution of (4.9-17) into (4.9-18) and the Result Into (4.9-16) Yields (Ahhh…)
  - $$T_e = \frac{\frac{\omega_e}{\omega_b} X_M^2 r_r' s \left|\tilde{V}_{as}\right|^2}{\left[r_s r_r' + s\left(\frac{\omega_e}{\omega_b}\right)^2 (X_M^2 - X_{ss} X_{rr}')\right]^2 + \left(\frac{\omega_e}{\omega_b}\right)^2 (r_r' X_{ss} + s r_s X_{rr}')^2} \qquad (4.9\text{-}20)$$

4.9 Analysis of Steady-State Operation



4.9 Analysis of Steady-State Operation

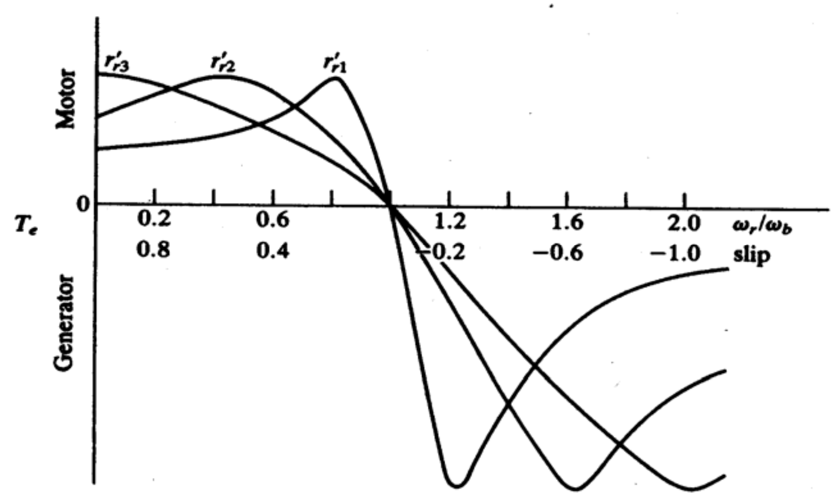
- Taking the Partial Derivative of Torque wrt Slip, The Slip At Which Maximum Torque Occurs Is
  - $s_m = r_r' G$ (4.9-21)

4.9 Analysis of Steady-State Operation

- Where
  - $$G = \pm \left[\frac{(\omega_e/\omega_b)^{-2} r_s^2 + X_{ss}^2}{(X_M^2 - X_{ss}X'_{rr})^2 \left(\frac{\omega_e}{\omega_b}\right)^2 + r_s^2 X'^2_{rr}} \right] \qquad (4.9\text{-}22)$$
- Maximum Torque Point May Be Expressed
  - $$T_{e(max)} = \frac{3\left(\frac{P}{2}\right)\left(\frac{\omega_e}{\omega_b}\right)\left(\frac{X_M^2}{\omega_b}\right) G \left|\tilde{V}_{as}\right|^2}{\left[r_s + G\left(\frac{\omega_e}{\omega_b}\right)^2 (X_M^2 - X_{ss}X'_{rr})\right]^2 + \left(\frac{\omega_e}{\omega_b}\right)^2 (X_{ss} + G r_s X'_{rr})^2} \qquad (4.9\text{-}23)$$

4.9 Analysis of Steady-State Operation

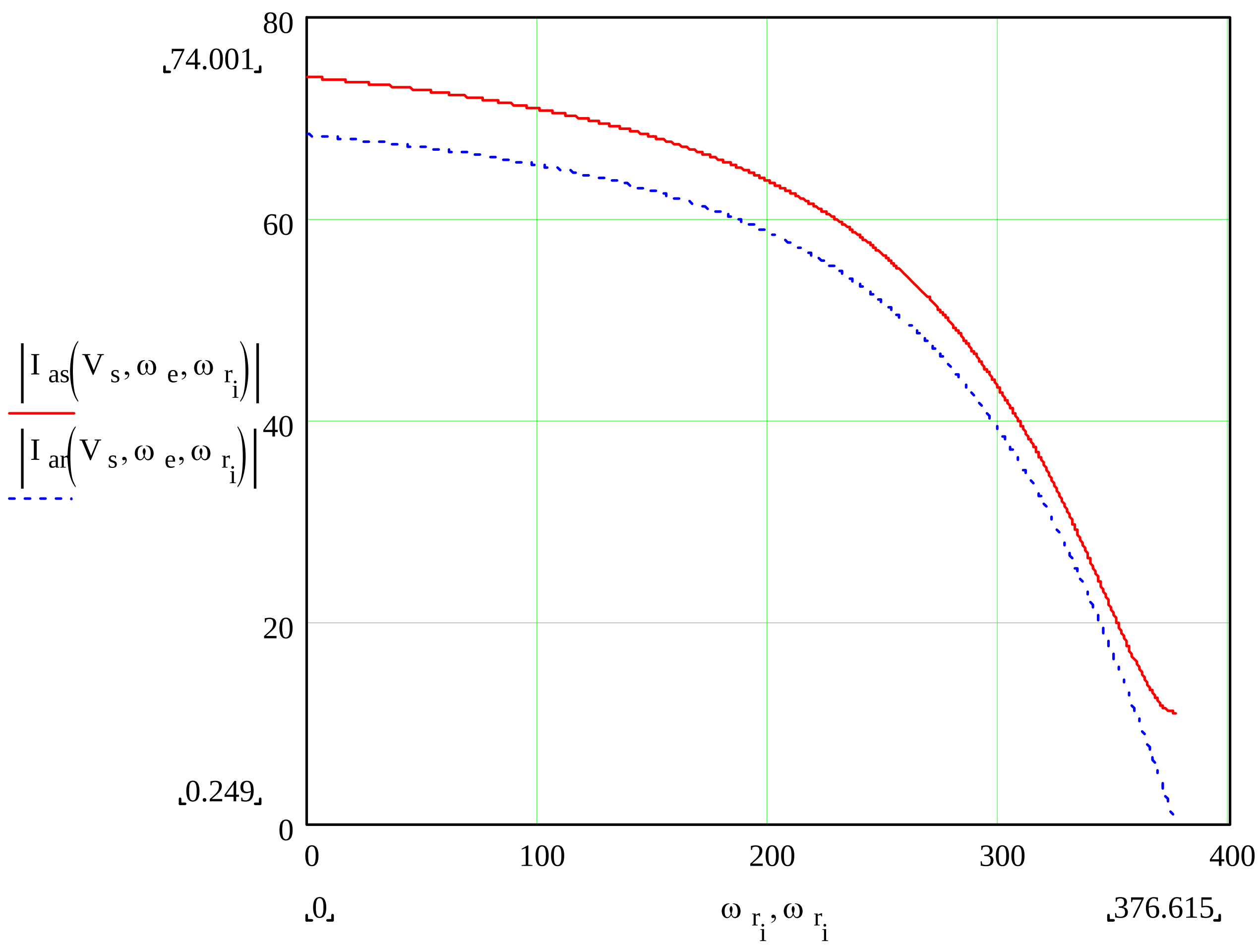
- Observation on Maximum Torque For Constant Voltage



4.9 Analysis of Steady-State Operation

- Let's consider a numerical example
  - Stator resistance: 0.3 Ohms
  - Stator leakage inductance: 2 mH
  - Magnetizing inductance: 30 mH (L_M)
  - Rotor leakage inductance: 2.5 mH (referred)
  - Rotor resistance: 0.5 Ohms (referred)
  - Poles: 4
  - Phases: 3
  - Voltage: 230 V, l-l, rms
  - Frequency: 60 Hz

4.9 Analysis of Steady-State Operation



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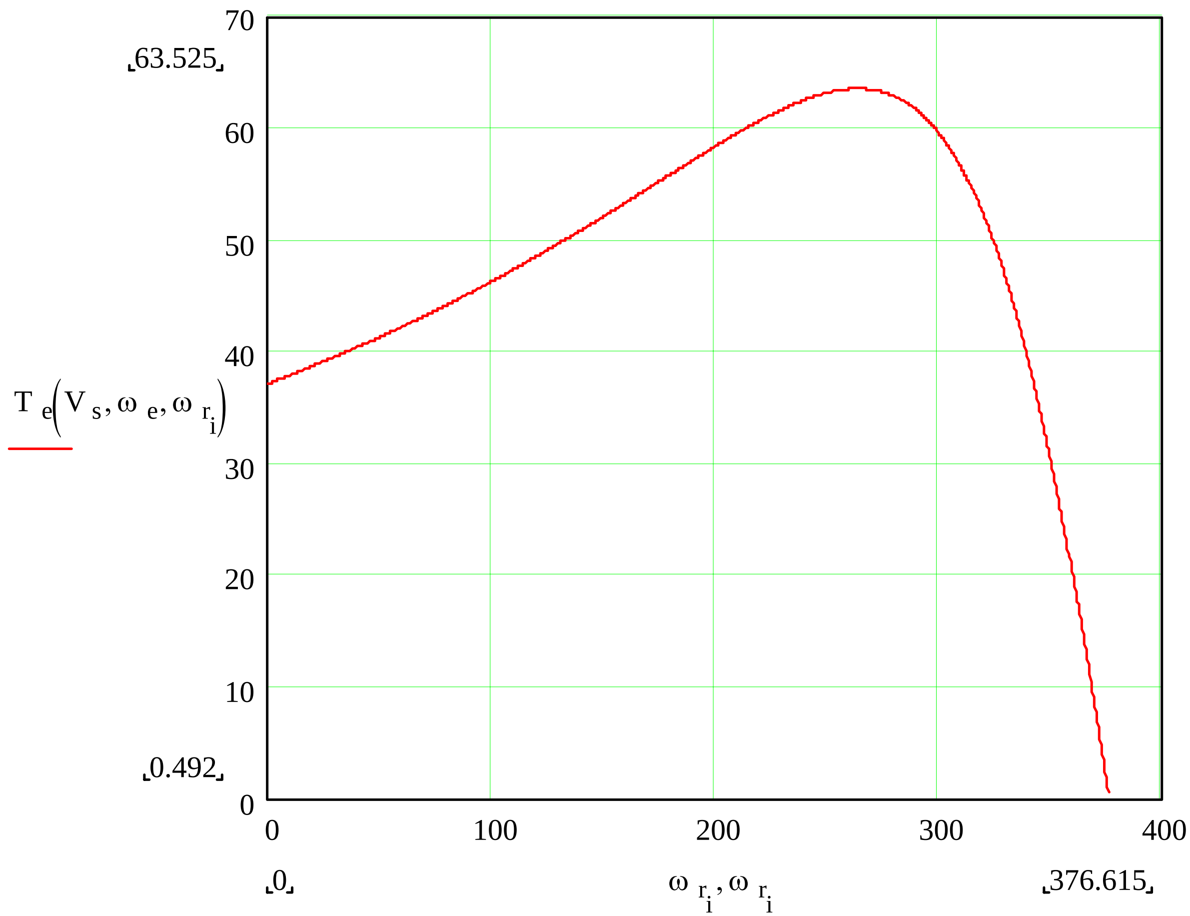

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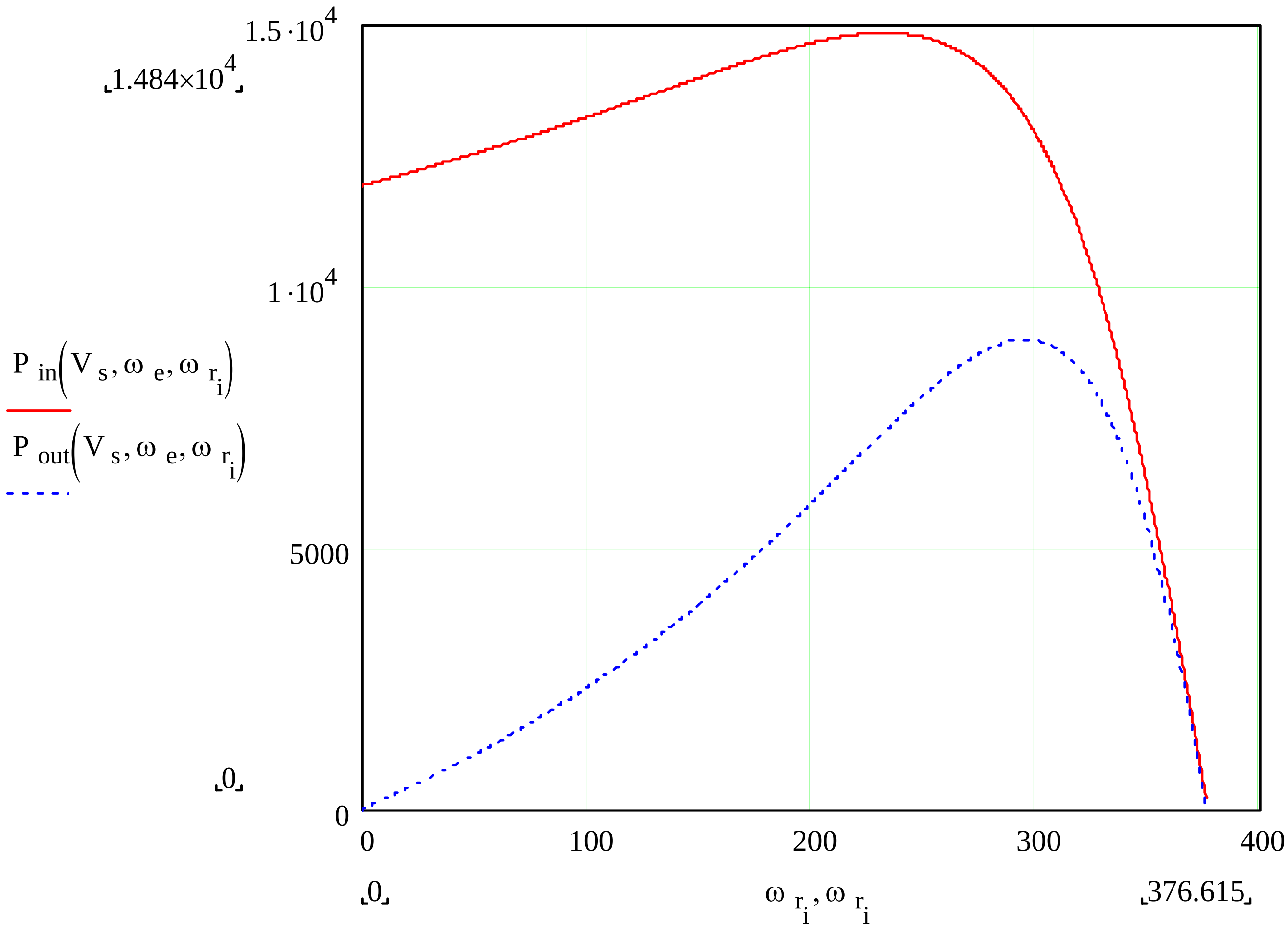

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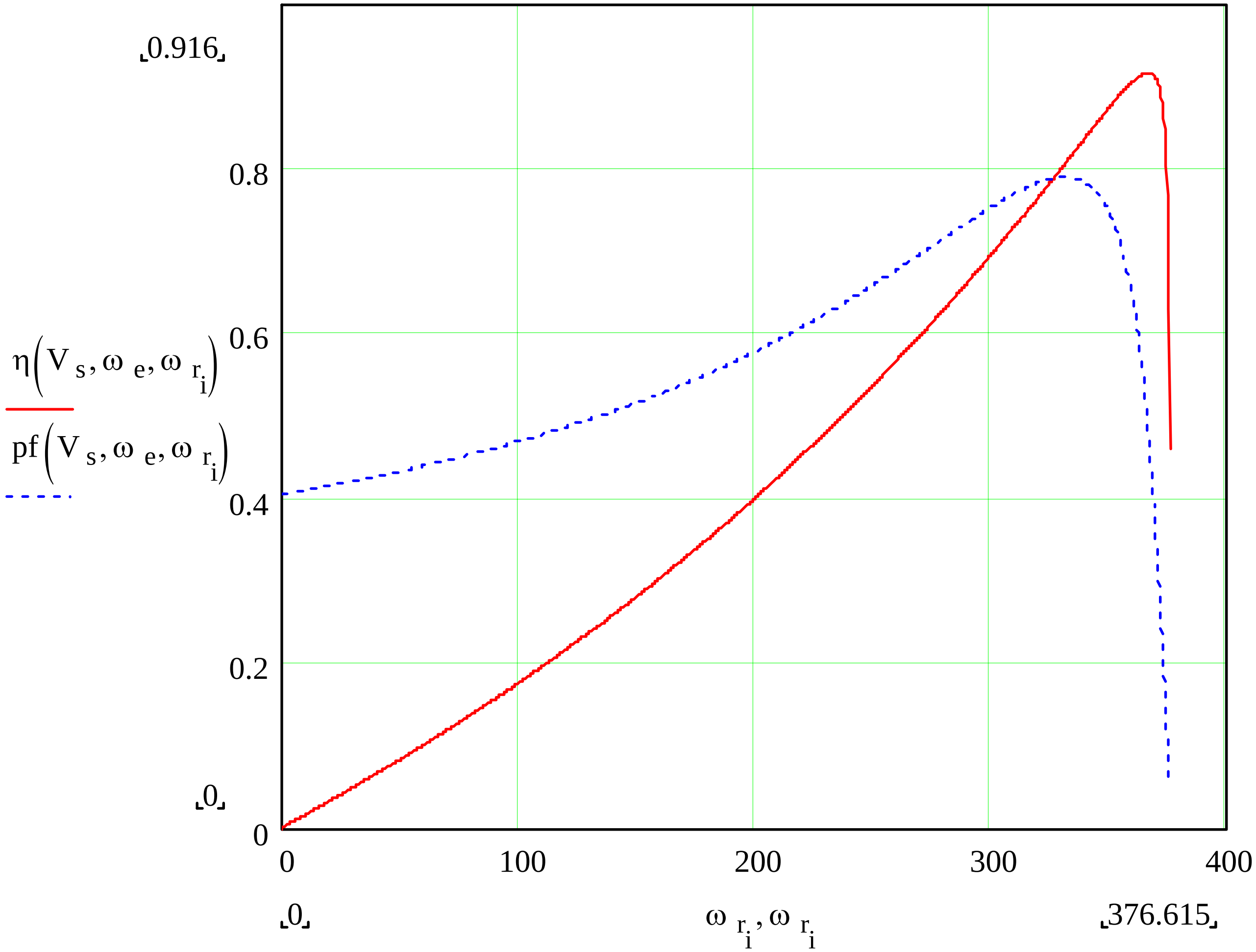

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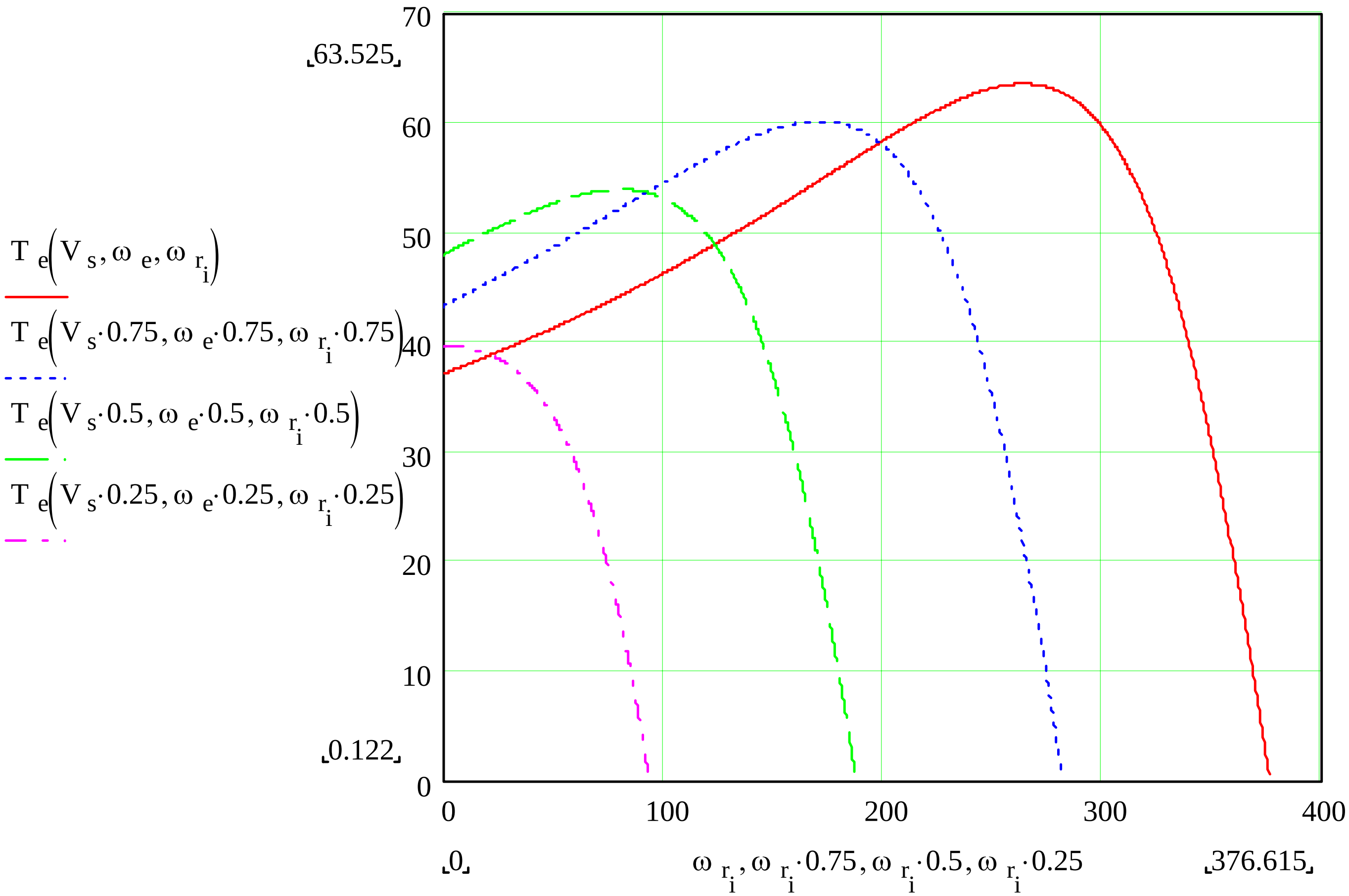
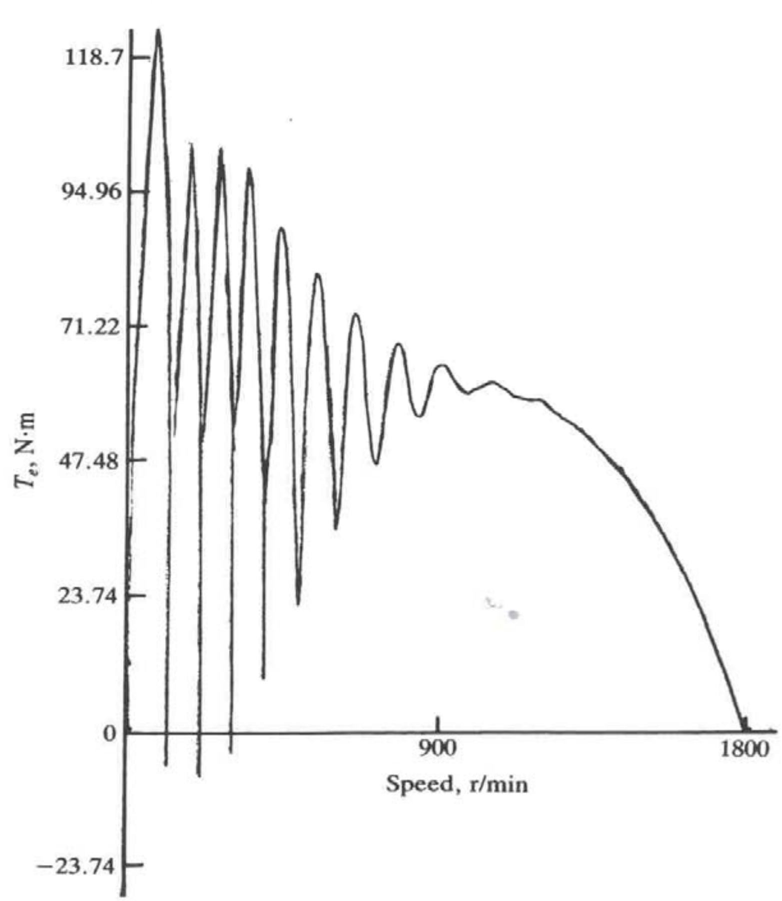


4.10 Free Acceleration Characteristics

Machine rating			T_B	$I_{B(abc)}$	r_s	X_{ls}	X_M	X'_{lr}	r'_r	J
hp	volts	rpm	$\mathrm{N \cdot m}$	amps	ohms	ohms	ohms	ohms	ohms	$\mathrm{kg \cdot m^2}$
3	220	1710	11.9	5.8	0.435	0.754	26.13	0.754	0.816	0.089
50	460	1705	198	46.8	0.087	0.302	13.08	0.302	0.228	1.662
500	2300	1773	1.98×10^3	93.6	0.262	1.206	54.02	1.206	0.187	11.06
2250	2300	1786	8.9×10^3	421.2	0.029	0.226	13.04	0.226	0.022	63.87

4.10 Free Acceleration Characteristics



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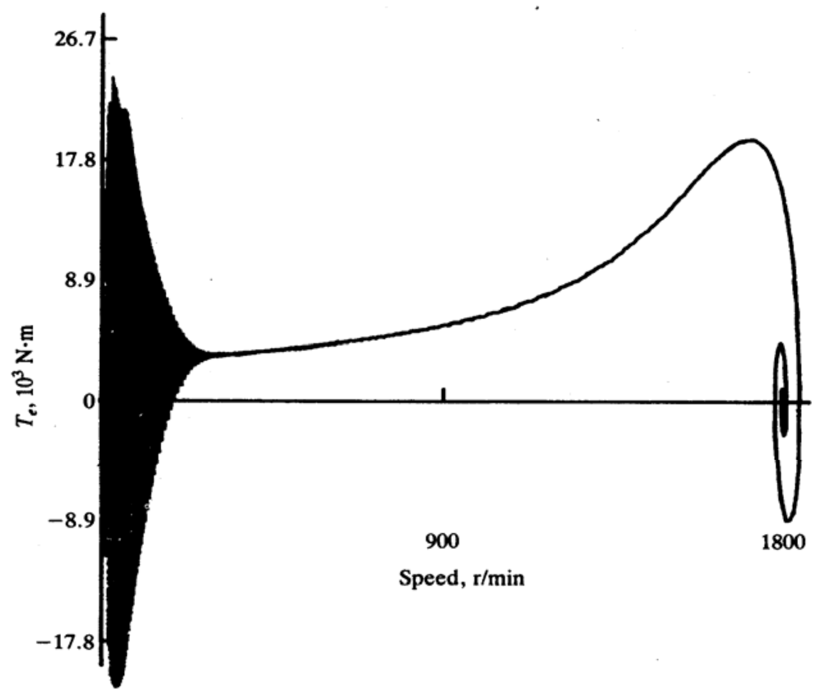

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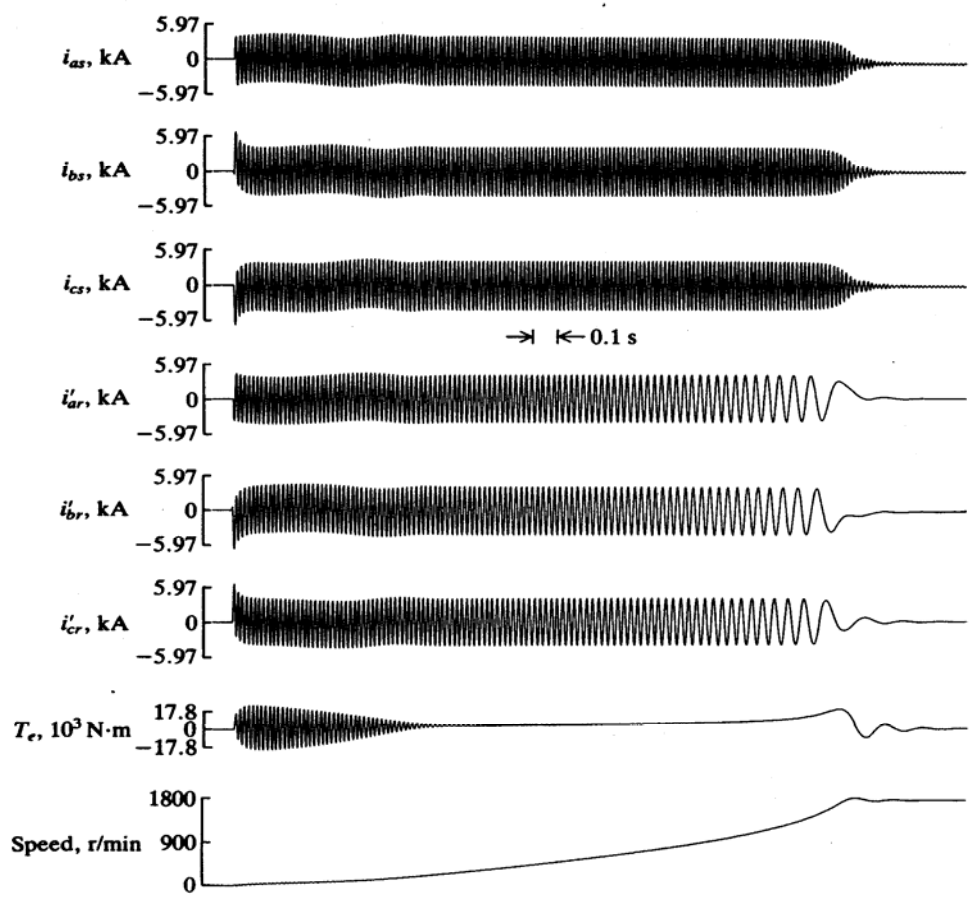
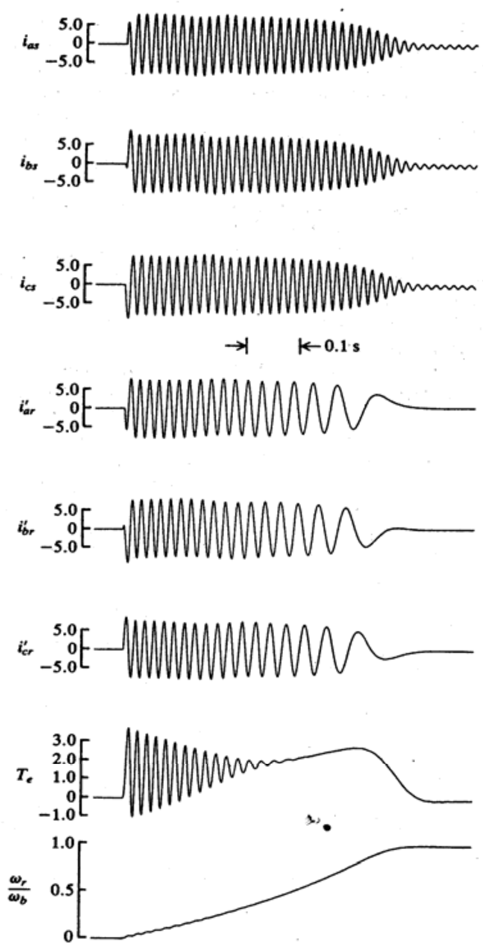


4.11 Free Acceleration Viewed From Several Reference Frames



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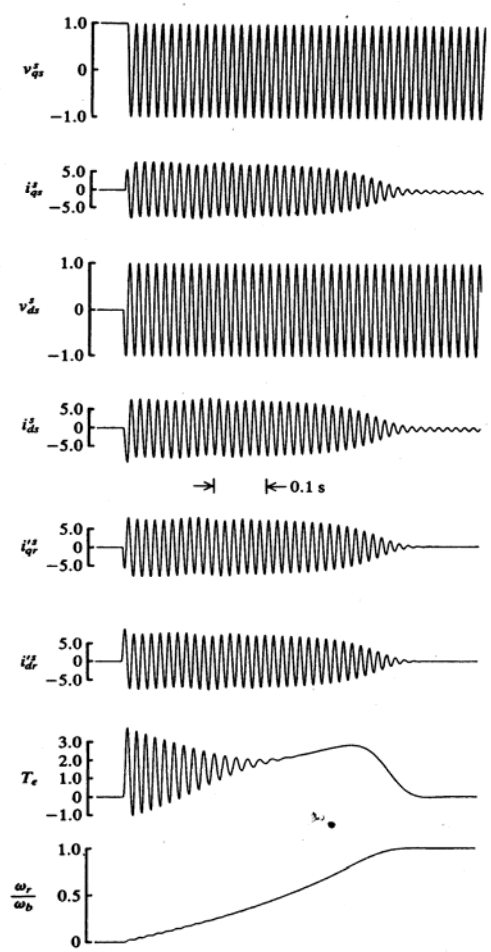

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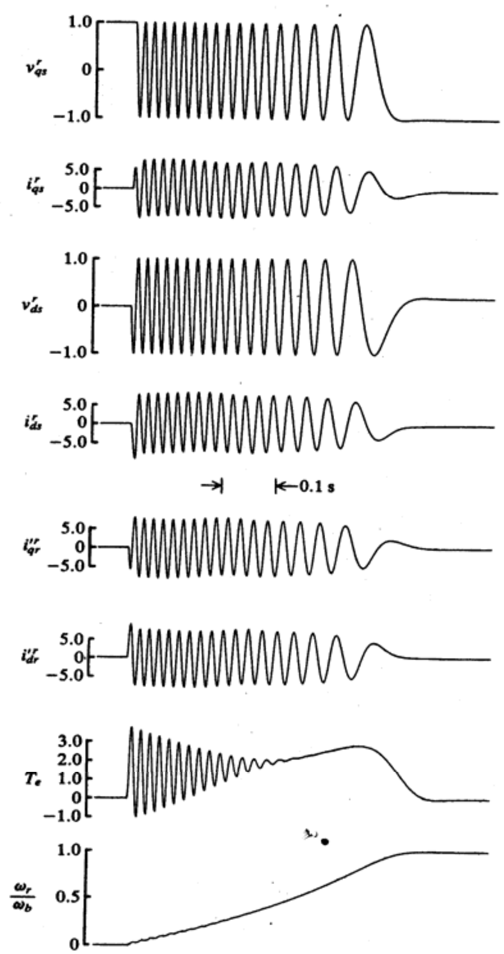

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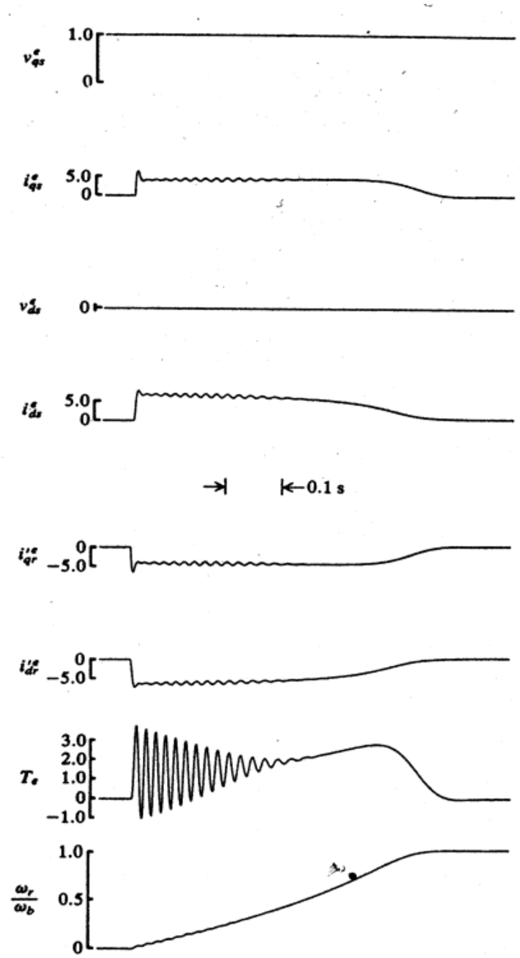

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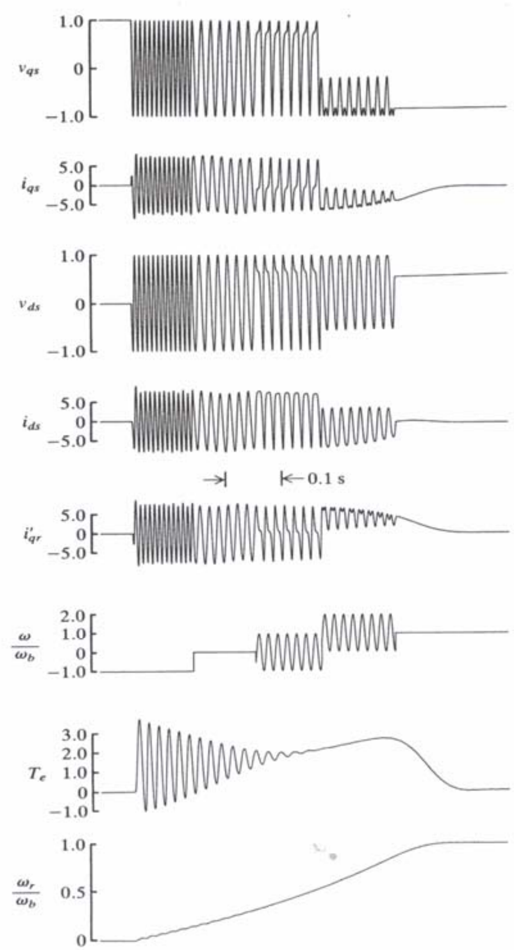
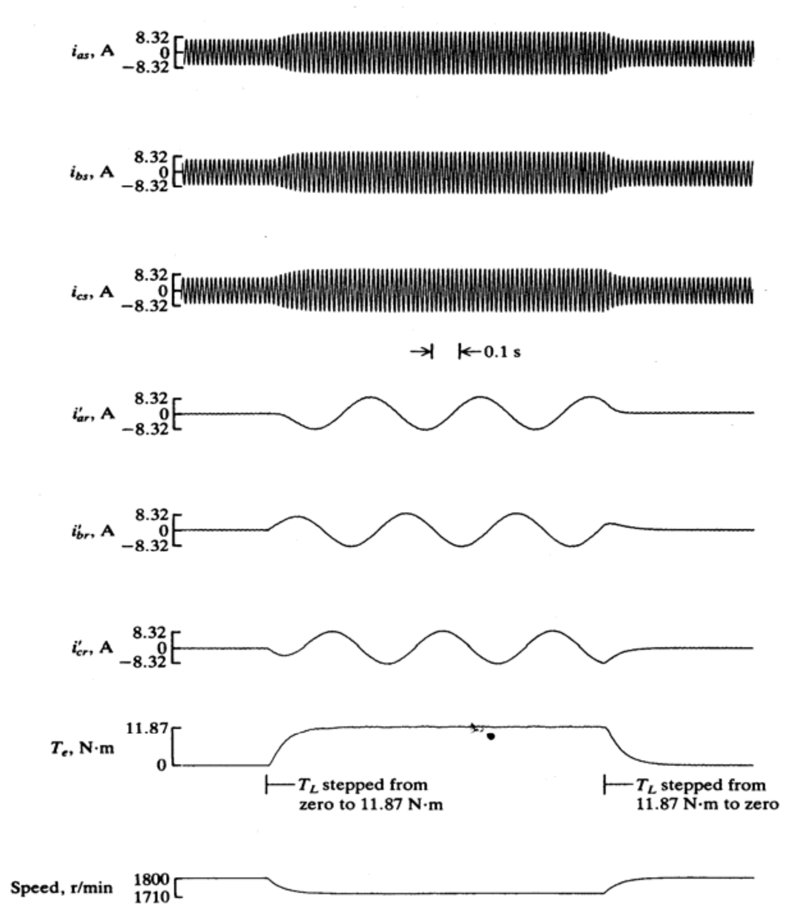


4.12 Dynamic Performance During Step Changes in Load



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