Fall 2005 EE595S
Homework Assignment Number 4

Note: Since this will not be collected, there is no due date. It is recommended that this be completed before we start the next set of lecture notes.

Problem 1

Our state vector will be

$$x = \begin{bmatrix} v_{dc} & i_{qs}^r & i_{ds}^r & \omega_r \end{bmatrix}^T$$

we have

$$pv_{dc} = \frac{1}{C_{dc}}(i_r - i_{dc})$$

Since

$$i_r = \frac{v_{batt} - v_{dc}}{r_{batt}}$$

and

$$i_{dc} = \frac{3}{2} m (i_{qs}^r \cos\phi_v - i_{ds}^r \sin\phi_v)$$

it follows that

$$pv_{dc} = \frac{1}{C_{dc}}\left(\frac{v_{batt} - v_{dc}}{r_{batt}} - \frac{3}{2} m \left(i_{qs}^r \cos\phi_v - i_{ds}^r \sin\phi_v\right)\right)$$

The state equations for i_{qs}^r, i_{ds}^r, and ω_{rm} are unchanged. Thus

$$p\begin{bmatrix} \bar{v}_{dc} \\ \bar{i}_{qs}^r \\ \bar{i}_{ds}^r \\ \bar{\omega}_r \end{bmatrix} = \begin{bmatrix} -\frac{1}{C_{dc} r_{batt}} & 0 & 0 & 0 \\ 0 & -\frac{r_s}{L_q} & 0 & -\frac{\lambda_m}{L_q} \\ 0 & 0 & -\frac{r_s}{L_d} & 0 \\ 0 & \frac{3}{2}\left(\frac{P}{2}\right)^2 \frac{1}{J}\lambda_m & 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{v}_{dc} \\ \bar{i}_{qs}^r \\ \bar{i}_{ds}^r \\ \bar{\omega}_r \end{bmatrix} + \begin{bmatrix} \frac{v_{batt}}{C_{dc} r_{batt}} - \frac{3m}{2C_{dc}}\left(\hat{i}_{qs}^r \cos\phi_v - \bar{i}_{ds}^r \sin\phi_v\right) \\ \frac{1}{L_q}\bar{v}_{dc} m \cos\phi_v - \frac{L_d}{L_q}\bar{\omega}_r \bar{i}_{ds}^r \\ -\frac{1}{L_q}\bar{v}_{dc} m \sin\phi_v + \frac{L_q}{L_d}\bar{\omega}_r \bar{i}_{qs}^r \\ \frac{P}{2}\frac{1}{J}\left(\frac{3}{2}\frac{P}{2}\left(L_d - L_q\right)\bar{i}_{qs}^r \bar{i}_{ds}^r - \bar{T}_L\right) \end{bmatrix}$$

Problem 2

Derive a linearized model similar to (15.6-1) of the text [1].

Our input vector will be

$$u = \begin{bmatrix} m & \phi_v & T_L \end{bmatrix}^T$$

Linearizing our result from problem 2, we have

$$p\begin{bmatrix} \Delta\bar{v}_{dc} \\ \Delta\bar{i}_{qs}^{r} \\ \Delta\bar{i}_{ds}^{r} \\ \Delta\bar{\omega}_r \end{bmatrix} = \begin{bmatrix} -\dfrac{1}{C_{dc} r_{batt}} & -\dfrac{3m_0}{2C_{dc}}\cos\phi_{v0} & \dfrac{3m_0}{2C_{dc}}\sin\phi_{v0} & 0 \\ \dfrac{1}{L_q} m_0 \cos\phi_{v0} & -\dfrac{r_s}{L_q} & -\dfrac{L_d}{L_q}\bar{\omega}_{r0} & -\dfrac{\lambda_m}{L_q} - \dfrac{L_d}{L_q}\bar{i}_{ds0}^{r} \\ -\dfrac{1}{L_q} m_0 \sin\phi_{v0} & \dfrac{L_q}{L_d}\bar{\omega}_{r0} & -\dfrac{r_s}{L_d} & \dfrac{L_q}{L_d}\bar{i}_{qs0}^{r} \\ 0 & \dfrac{1}{J}\dfrac{3}{2}\left(\dfrac{P}{2}\right)^2\left(\left(L_d - L_q\right)\bar{i}_{ds0}^{r} + \lambda_m\right) & \dfrac{1}{J}\dfrac{3}{2}\left(\dfrac{P}{2}\right)^2\left(L_d - L_q\right)\bar{i}_{qs0}^{r} & 0 \end{bmatrix} \begin{bmatrix} \Delta\bar{v}_{dc} \\ \Delta\bar{i}_{qs}^{r} \\ \Delta\bar{i}_{ds}^{r} \\ \Delta\bar{\omega}_r \end{bmatrix} +$$

$$\begin{bmatrix} -\dfrac{3}{2C_{dc}}\left(\hat{i}_{qs0}^{r}\cos\phi_{v0} - \bar{i}_{ds0}^{r}\sin\phi_{v0}\right) & \dfrac{3m}{2C_{dc}}\left(\hat{i}_{qs0}^{r}\sin\phi_{v0} + \bar{i}_{ds0}^{r}\cos\phi_0\right) & 0 \\ \dfrac{1}{L_q}\bar{v}_{dc0}\cos\phi_{v0} & -\dfrac{1}{L_q}\bar{v}_{dc0} m_0 \sin\phi_{v0} & 0 \\ -\dfrac{1}{L_d}\bar{v}_{dc0}\sin\phi_{v0} & -\dfrac{1}{L_d}\bar{v}_{dc0} m_0 \cos\phi_{v0} & 0 \\ 0 & 0 & -\dfrac{1}{J}\dfrac{P}{2} \end{bmatrix} \begin{bmatrix} \Delta m \\ \Delta\phi_v \\ \Delta T_L \end{bmatrix}$$

Problem 3

We start with

$$v_{dc} = v_{batt} - r_{batt} i_r$$

Since the capacitor current is zero in the steady-state,

$$i_r = i_{dc}$$

Thus

$$v_{dc} = v_{batt} - r_{batt} i_{dc}$$

Note that (15.4-14) still holds, thus

$$v_{dc} = v_{batt} - r_{batt} \frac{3}{2} m (i_{qs}^r \cos\phi_v - i_{ds}^r \sin\phi_v)$$

Expressions (15.5-7), and (15.5-8) still hold. Substitution of these into the above and manipulating yields

$$v_{dc} = \left(r_s^2 + \omega_r^2 L_q L_d\right) \frac{v_{batt} + \frac{3}{2} r_b m \lambda_m \omega_r (r_s \cos\phi_v + \omega_r L_q \sin\phi_v)}{r_s^2 + \omega_r^2 L_q L_d + \frac{3}{2} m^2 r_{batt} \left(r_s + \frac{1}{2} \omega_r (L_d - L_q) \sin 2\phi_v \right)}$$

Problem 4

Calculation of the q- and d-axis voltages is straightforward. Numerical values are attached. To find the dc voltage

$$v_{dc} = v_{batt} - \frac{3}{2} m\left(i_{qs}^r \cos\phi_v - i_{ds}^r \sin\phi_v\right)$$

We also have that

$$v_{qs}^r = m v_{dc} \cos\phi_v$$

Thus

$$\frac{v_{qs}^r}{m\cos\phi_v} = v_{batt} - \frac{3}{2} m\left(i_{qs}^r \cos\phi_v - i_{ds}^r \sin\phi_v\right)$$

Solving the above yields

$$m = \frac{v_{batt}\cos\phi_v \pm \sqrt{v_{batt}^2 \cos^2\phi_v - 6\cos\phi_v v_{qs}^r (i_{qs}^r \cos\phi_v - i_{ds}^r \sin\phi_v)}}{3\cos\phi_v (i_{qs}^r \cos\phi_v - i_{ds}^r \sin\phi_v)}$$

The negative root gives the most reasonable answer. See mathcad sheet for numerics.

Problem 5

See mathcad sheet (attached).