Your Name: Solution

EE595S - Fall 2005
Exam 2

Open Book (Clean Copy)

Open Chapter 3, "Analysis and Design of Permanent Magnet Synchronous Machines"
(Clean Copy – Available from Course Web Site)

No Calculator
No PDA
No Computer

Exam Duration: 50 minutes

Notes

(1) On some problems, there is considerably more paper than you may need
(2) Handy trig formulas are on the back of the exam
(3) There are 4 problems in this exam

Don't Forget that November 12 is National Induction Motor Appreciation Day

Show Figure below

1.) 25 pts. Consider the drive shown ~~in Fig 15.2-1.~~ Consider the following changes. First, remove the stabilizing filter. Second, remove the ac source, rectifier, and inductor L_{dc}. Replace the rectifier source with a battery source, which is governed by the overly simplistic model that the current out of the battery is given by

$$i_r = \frac{v_{batt} - v_{dc}}{r_{batt}}$$

where v_{batt} and r_{batt} are Thevenin equivalent battery source voltage and resistance, respectively. Derive a nonlinear model similar to (15.4-21) of the text *Analysis of Electric Machinery*.

See HW #4 Solution.

2.) 25 pts. Consider a PMSM machine in which $\lambda_m = 0$ and in which $L_d > L_q$ (i.e. a reluctance machine). The torque may be expressed

$$T_e = \frac{3}{2}\frac{P}{2}\left(L_d - L_q\right)i_{qs}^r i_{ds}^r$$

Derive an expression for the q- and d-axis current command such as to achieve a desired torque T_e^* ~~with the minimum rms current~~. Show work (i.e. don't make an educated guess). Assume $T_e^* > 0$.

$$\sqrt{2}\, I_s = \sqrt{I_q^2 + I_d^2}$$

$$2 I_s^2 = I_q^2 + I_d^2$$

$$I_d = \cancel{\frac{\frac{3}{2}\frac{P}{2} I_q}{L_d - L_q}} \quad \frac{T_e}{\frac{3}{2}\frac{P}{2}(L_d - L_q) I_q}$$

$$2 I_s^2 = I_q^2 + \left[\frac{T_e}{\frac{3}{2}\frac{P}{2}(L_d - L_q)}\right]^2 \frac{1}{I_q^2}$$

$$\frac{\partial (2 I_s^2)}{\partial I_q} = 2 I_q + \left[\frac{T_e}{\frac{3}{2}\frac{P}{2}(L_d - L_q)}\right]^2 \left(-2 I_q^{-3}\right) = 0$$

$$\therefore \quad I_q = \pm \sqrt{\frac{T_e^*}{\frac{3}{2}\frac{P}{2}(L_d - L_q)}}$$

Let's choose positive root

$$I_d = \sqrt{\frac{T_e^*}{\frac{3}{2}\frac{P}{2}(L_d - L_q)}}$$

As in problem 2

3.) 25 pts. ~~Again~~ consider a reluctance machine. Suppose that the d-axis current command is given by $i_{ds}^{r*} = ki_{qs}^{r*}$, and that the desired toque is T_e^*. Derive an expression for the maximum electrical rotor speed at which obtainable in this mode, given that the rms l-n voltage is limited to v_s^*. Your answer should be in terms of L_q, L_d, r_s, P, and T_e^*.

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$$T_e = \frac{3}{2}\frac{P}{2}(L_d - L_q) K I_q^2$$

$$I_q^* = \sqrt{\frac{T_e}{\frac{3}{2}\frac{P}{2}(L_d - L_q) K}}$$

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$$V_q = r_s I_q + \omega_r L_d I_d$$

$$V_d = r_s I_d - \omega_r L_q I_q$$

$$2V_s^2 = V_q^2 + V_d^2$$

$$= r_s^2 I_q^2 + 2 r_s I_q \omega_r L_d I_d + \omega_r^2 L_d^2 I_d^2 + r_s^2 I_d^2 - 2 r_s I_d \omega_r L_q I_q + \omega_r^2 L_q^2 I_q^2$$

$$2V_s^2 = I_q^2 r_s^2 (1 + K^2) + 2 r_s (L_d - L_q) \omega_r I_q^2 K + \omega_r^2 (L_d^2 K^2 + L_q^2) I_q^2$$

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$$0 = -\frac{2V_s^2}{I_q^2} + r_s^2(1+K^2) + 2 r_s (L_d - L_q) K \omega_r + (L_d^2 K^2 + L_q^2)\omega_r^2$$

$$\omega_{r,mx} = \frac{-\cancel{2} r_s (L_d - L_q) K + \sqrt{\cancel{4} r_s^2 (L_d - L_q)^2 K^2 - \cancel{4}\left(r_s^2(1+K^2) - \frac{2V_s^2}{I_q^2}\right)(L_d^2 K^2 + L_q^2)}}{\cancel{2}(L_d^2 K^2 + L_q^2)}$$

$$\omega_{r,mx} = \frac{-r_s (L_d - L_q) K^2 + \sqrt{r_s^2 (L_d - L_q)^2 K^4 + \left(\frac{\partial V_s^2}{\frac{T_e}{\frac{3}{2}\frac{P}{2}(L_d - L_q) K}} - r^2 (1 + K^2) \right) (L_d^2 K^2 + L_q^2)}}{(L_d^2 K^2 + L_q^2)}$$

4.) 25 pts. Consider an induction machine operating in the steady state. Suppose that the a-phase current is i_{dc}, the b-phase current is $-i_{dc}/2$, and the c-phase current is $-i_{dc}/2$. Using the q-d model in the stationary reference frame, derive an expression for torque in terms of P, L_M, L_{lr}', r_r', i_{dc}, and ω_r.

L_{rr}

with $\theta = 0$

$$i_{qs} = \frac{2}{3}\left[1 \quad -\tfrac{1}{2}\left(-\tfrac{1}{2}\right) \quad -\tfrac{1}{2}\left(-\tfrac{1}{2}\right)\right] i_{dc} = i_{dc}$$

$$i_{ds} = \frac{2}{3}\left[0(1) \quad -\tfrac{\sqrt{3}}{2}\left(-\tfrac{1}{2}\right) \quad +\tfrac{\sqrt{3}}{2}\left(-\tfrac{1}{2}\right)\right] i_{dc} = 0$$

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$$0 = r_r i_{qr} + (\cancel{\omega}^{0} - \omega_r)\lambda_{dr} + \cancel{p\lambda_{qr}}^{0}$$

$$0 = r_r i_{dr} - (\cancel{\omega}^{0} - \omega_r)\lambda_{qr} + \cancel{p\lambda_{dr}}^{0}$$

$$\lambda_{dr} = L_{rr} i_{dr} + L_M i_{ds}$$

$$\lambda_{qr} = L_{rr} i_{qr} + L_M i_{qs}$$

$$0 = r_r i_{qr} - \omega_r L_{rr} i_{dr} - L_M i_{ds}$$

$$0 = r_r i_{dr} + \omega_r L_{rr} i_{qr} + L_M i_{qs}$$

$$\begin{bmatrix} r_r & -\omega_r L_{rr} \\ \omega_r L_{rr} & r_r \end{bmatrix} \begin{bmatrix} i_{qr} \\ i_{dr} \end{bmatrix} = \begin{bmatrix} \omega_r L_M i_{ds} \\ -\omega_r L_M i_{qs} \end{bmatrix}$$

$$\begin{bmatrix} i_{qr} \\ i_{dr} \end{bmatrix} = \frac{\begin{bmatrix} r_r & \omega_r L_{rr} \\ -\omega_r L_{rr} & r_r \end{bmatrix}}{r_r^2 + \omega_r^2 L_{rr}^2} \begin{bmatrix} \cancel{L_M i_{ds}}^{0} \\ -\omega_r L_M i_{qs} \end{bmatrix}$$

$$\therefore \quad i_{qr} = -\frac{\omega_r^2 L_{rr} L_M i_{qs}}{r_r^2 + \omega_r^2 L_{rr}^2}$$

$$i_{dr} = -\frac{r_r \omega_r L_M i_{qs}}{r_r^2 + \omega_r^2 L_{rr}^2}$$

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$$T_e = \frac{3}{2}\frac{P}{2}\left(\lambda_{ds} i_{qs} - \cancel{\lambda_{qs} i_{ds}}^{\,0}\right)$$

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$$\lambda_{ds} = \cancel{L_{ss} i_{ds}}^{\,0} + L_M i_{dr}$$

$$\therefore \quad T_e = -\frac{3}{2}\frac{P}{2} L_M \frac{r_r \omega_r L_M}{r_r^2 + \omega_r^2 L_{rr}^2} i_{qs}^2$$

$$\boxed{T_e = -\frac{3}{2}\frac{P}{2} \frac{r_r \omega_r L_M^2}{r_r^2 + \omega_r^2 L_{rr}^2} i_{ac}^2}$$

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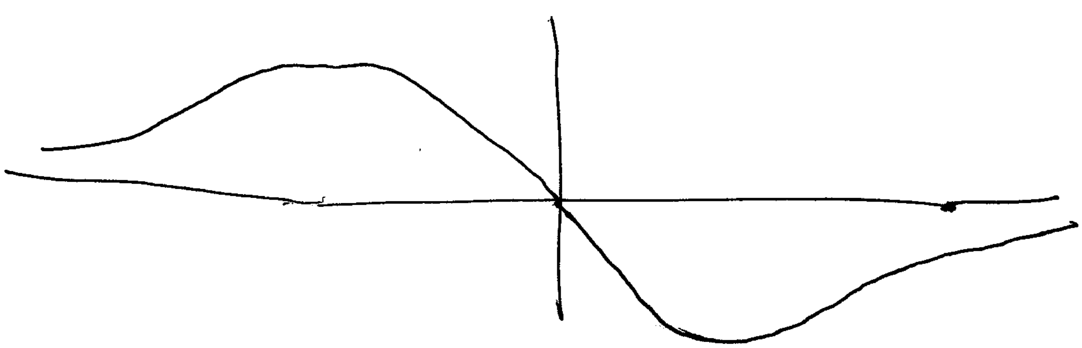

Appendix A

TRIGONOMETRIC RELATIONS, CONSTANTS AND CONVERSION FACTORS, AND ABBREVIATIONS

TRIGONOMETRIC RELATIONS

$$\cos^2 x + \cos^2\left(x - \frac{2\pi}{3}\right) + \cos^2\left(x + \frac{2\pi}{3}\right) = \frac{3}{2}$$

$$\sin^2 x + \sin^2\left(x - \frac{2\pi}{3}\right) + \sin^2\left(x + \frac{2\pi}{3}\right) = \frac{3}{2}$$

$$\sin x \cos x + \sin\left(x - \frac{2\pi}{3}\right)\cos\left(x - \frac{2\pi}{3}\right) + \sin\left(x + \frac{2\pi}{3}\right)\cos\left(x + \frac{2\pi}{3}\right) = 0$$

$$\cos x + \cos\left(x - \frac{2\pi}{3}\right) + \cos\left(x + \frac{2\pi}{3}\right) = 0$$

$$\sin x + \sin\left(x - \frac{2\pi}{3}\right) + \sin\left(x + \frac{2\pi}{3}\right) = 0$$

$$\sin x \cos y + \sin\left(x - \frac{2\pi}{3}\right)\cos\left(y - \frac{2\pi}{3}\right) + \sin\left(x + \frac{2\pi}{3}\right)\cos\left(x + \frac{2\pi}{3}\right) = \frac{3}{2}\sin(x - y)$$

$$\sin x \sin y + \sin\left(x - \frac{2\pi}{3}\right)\sin\left(y - \frac{2\pi}{3}\right) + \sin\left(x + \frac{2\pi}{3}\right)\sin\left(y + \frac{2\pi}{3}\right) = \frac{3}{2}\cos(x - y)$$

$$\cos x \sin y + \cos\left(x - \frac{2\pi}{3}\right)\sin\left(y - \frac{2\pi}{3}\right) + \cos\left(x + \frac{2\pi}{3}\right)\sin\left(y + \frac{2\pi}{3}\right) = -\frac{3}{2}\sin(x - y)$$

$$\cos x \cos y + \cos\left(x - \frac{2\pi}{3}\right)\cos\left(y - \frac{2\pi}{3}\right) + \cos\left(x + \frac{2\pi}{3}\right)\cos\left(y + \frac{2\pi}{3}\right) = \frac{3}{2}\cos(x - y)$$

$$\sin x \cos y + \sin\left(x + \frac{2\pi}{3}\right)\cos\left(y - \frac{2\pi}{3}\right) + \sin\left(x - \frac{2\pi}{3}\right)\cos\left(y + \frac{2\pi}{3}\right) = \frac{3}{2}\sin(x + y)$$

$$\sin x \sin y + \sin\left(x + \frac{2\pi}{3}\right)\sin\left(y - \frac{2\pi}{3}\right) + \sin\left(x - \frac{2\pi}{3}\right)\sin\left(y + \frac{2\pi}{3}\right) = -\frac{3}{2}\cos(x + y)$$

$$\cos x \sin y + \cos\left(x + \frac{2\pi}{3}\right)\sin\left(y - \frac{2\pi}{3}\right) + \cos\left(x - \frac{2\pi}{3}\right)\sin\left(y + \frac{2\pi}{3}\right) = \frac{3}{2}\sin(x + y)$$

$$\cos x \cos y + \cos\left(x + \frac{2\pi}{3}\right)\cos\left(y - \frac{2\pi}{3}\right) + \cos\left(x - \frac{2\pi}{3}\right)\cos\left(y + \frac{2\pi}{3}\right) = \frac{3}{2}\cos(x + y)$$

A

Mathematical Tables

Table A-1 Trigonometric Identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\cos A \cos B = \frac{1}{2}[\cos(A + B) + \cos(A - B)]$

$\sin A \sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$

$\sin A \cos B = \frac{1}{2}[\sin(A + B) + \sin(A - B)]$

$\sin A + \sin B = 2 \sin \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B)$

$\sin A - \sin B = 2 \sin \frac{1}{2}(A - B) \cos \frac{1}{2}(A + B)$

$\cos A + \cos B = 2 \cos \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B)$

$\cos A - \cos B = -2 \sin \frac{1}{2}(A + B) \sin \frac{1}{2}(A - B)$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A = \cos^2 A - \sin^2 A$

$\sin \frac{1}{2}A = \sqrt{\frac{1}{2}(1 - \cos A)}$ $\quad$ $\cos \frac{1}{2}A = \sqrt{\frac{1}{2}(1 + \cos A)}$

$\sin^2 A = \frac{1}{2}(1 - \cos 2A)$ $\quad$ $\cos^2 A = \frac{1}{2}(1 + \cos 2A)$

$$\sin x = \frac{e^{jx} - e^{-jx}}{2j} \qquad \cos x = \frac{e^{jx} + e^{-jx}}{2} \qquad e^{jx} = \cos x + j \sin x$$

$A \cos(\omega t + \phi_1) + B \cos(\omega t + \phi_2) = C \cos(\omega t + \phi_3)$

where

$$C = \sqrt{A^2 + B^2 - 2AB \cos(\phi_2 - \phi_1)}$$

$$\phi_3 = \tan^{-1}\left[\frac{A \sin \phi_1 + B \sin \phi_2}{A \cos \phi_1 + B \cos \phi_2}\right]$$

$$\sin(\omega t + \phi) = \cos\left(\omega t + \phi - \frac{\pi}{2}\right)$$

Table A-2 Derivatives

$$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$$

$$\frac{d}{dx}u^n=nu^{n-1}\frac{du}{dx}$$

$$\frac{d}{dx}\log u=\frac{\log e}{u}\frac{du}{dx}$$

$$\frac{d}{dx}\ln u=\frac{1}{u}\frac{du}{dx}$$

$$\frac{d}{dx}a^u=a^u\ln a\frac{du}{dx}$$

$$\frac{d}{dx}e^u=e^u\frac{du}{dx}$$

$$\frac{d}{dx}u^v=vu^{v-1}\frac{du}{dx}+u^v\ln u\frac{dv}{dx}$$

$$\frac{d}{dx}\sin u=\cos u\frac{du}{dx}$$

$$\frac{d}{dx}\cos u=-\sin u\frac{du}{dx}$$

$$\frac{d}{dx}\tan u=\frac{1}{\cos^2 u}\frac{du}{dx}$$

Table A-3 Approximations

Taylor's series:

$$f(x)=f(a)+\dot{f}(a)\frac{(x-a)}{1!}+\ddot{f}(a)\frac{(x-a)^2}{2!}+\cdots$$

Maclaurin's series:

$$f(x)=f(0)+\dot{f}(0)\frac{x}{1!}+\ddot{f}(0)\frac{x^2}{2!}+\cdots$$

For small values of x:

$$\frac{1}{1+x}\doteq 1-x$$

$$(1+x)^n\doteq 1+nx$$

$$e^x\doteq 1+x$$

$$\ln(1+x)\doteq x$$

$$\sin x\doteq x$$

$$\cos x\doteq 1-\frac{x^2}{2}$$

$$\tan x\doteq x$$