Your Name: Solution

EE595S - Fall 2005
Exam 1

Open Book (Clean Copy)

Open Chapter 2, "Techniques for Analysis and Design of Electromechanical Systems" (Clean Copy – Available from Course Web Site)

Calculator Allowed

No PDA
No Computer

Exam Duration: 50 minutes

Notes

(1) On some problems, there is considerably more paper than you may need
(2) Handy trig formulas are on the back of the exam

May the Flux Be With You !

1.) 25 pts. Consider a 2 phase salient pole reluctance machine whose flux linkage equations may be expressed.

$$\lambda_{abs} = \begin{bmatrix} L_A - L_B\cos(2\theta_r) & -L_B\sin(2\theta_r) \\ -L_B\sin(2\theta_r) & L_A + L_B\cos(2\theta_r) \end{bmatrix} i_{abcs} \tag{1}$$

Also consider the transformation

$$f_{qds}^{r} = \begin{bmatrix} \cos\theta_r & \sin\theta_r \\ \sin\theta_r & -\cos\theta_r \end{bmatrix} f_{abs} \tag{2}$$

Express the flux linkage equation (1) in terms of qd variables.

15

$$\lambda_{abs} = \left[L_A I_2 - L_B \underbrace{\begin{bmatrix} \cos 2\theta_r & \sin 2\theta_r \\ \sin 2\theta_r & -\cos 2\theta_r \end{bmatrix}}_{R} \right] i_{abs}$$

$$\lambda_{qds}^{r} = \underbrace{K_s^r \left[L_A I_2 - L_B R \right] K_s^{r^{-1}}}_{T} i_{qds}^{r}$$

$$K_s^r = K_s^{r^{-1}}$$

$$T = K_s^r L_A I_2 K_s^{r^{-1}} - K_s^r L_B R K_s^{r^{-1}}$$

$$= L_A I_2 - L_B K_s^r \begin{bmatrix} \cos 2\theta_r & \sin 2\theta_r \\ \sin 2\theta_r & -\cos 2\theta_r \end{bmatrix} \begin{bmatrix} \cos\theta_r & \sin\theta_r \\ \sin\theta_r & -\cos\theta_r \end{bmatrix}$$

$$= L_A I_2 - L_B \begin{bmatrix} \cos\theta_r & +\sin\theta_r \\ \sin\theta_r & -\cos\theta_r \end{bmatrix} \begin{bmatrix} \cos\theta_r & -\sin\theta_r \\ \sin\theta_r & \cos\theta_r \end{bmatrix}$$

10

$$= L_A I_2 - L_B \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\therefore \lambda_{qds}^{r} = \begin{bmatrix} L_A - L_B & 0 \\ 0 & L_A + L_B \end{bmatrix} i_{qds}^{r}$$

5

−10

2.) 25 pts. An inverter is operating from a 100 V dc source. It is desired to obtain, in the rotor reference frame, a q-axis voltage of 50 V and a d-axis voltage of ~~10~~ V. A sine triangle modulator (with 3rd harmonic injection) is used. The a-phase duty cycle waveform for this mode is given by the usual expression, i.e.

$$d_a = d\cos(\theta_c) - d_3\cos(3\theta_c)$$

where in this case $\theta_c = \theta_r + \phi$. Calculate (1) a numerical value for d, (2) a numerical value of ϕ, and (3) a possible value for d_3.

$$\begin{bmatrix} V_{qs}^r \\ V_{ds}^r \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\theta_{rc} & -\sin\theta_{rc} \\ \sin\theta_{rc} & \cos\theta_{rc} \end{bmatrix}}_{{}^cK^r} \begin{bmatrix} \frac{1}{2}dV_{dc} \\ 0 \end{bmatrix} \quad (1)$$

V_{qs}^c V_{ds}^c

where $\theta_{rc} = \theta_r - \theta_c \quad (2)$

From (1)

$$V_{qs}^r = \tfrac{1}{2} d V_{dc} \cos\theta_{rc}$$

$$V_{ds}^r = \tfrac{1}{2} d V_{dc} \sin\theta_{rc}$$

$$V_{qs}^r + jV_{ds}^r = \tfrac{1}{2} d V_{dc} e^{j\theta_{rc}}$$

$$\therefore \quad \tfrac{1}{2} d V_{dc} = \sqrt{(V_{qs}^r)^2 + (V_{ds}^r)^2}$$

$$d = \frac{2}{V_{dc}}\sqrt{(V_{qs}^r)^2 + (V_{ds}^r)^2}$$

$$\boxed{d = 1.063}$$

$$\boxed{\begin{aligned} d_3 &= \tfrac{1}{6} d \\ d_3 &= 0.177 \end{aligned}}$$

$$\theta_{rc} = \text{angle}(V_{qs}^r + jV_{ds}^r)$$

$$\theta_r - \theta_c = -41.19^\circ$$

$$\theta_c = \theta_r + 41.19^\circ$$

$$\boxed{\phi = 41.19 \ (0.719 \text{ rad})}$$

8

∂ $h = 0.1\,A$

3.) 25 pts. A brushless dc machine is ~~connected~~ to an inverter. The parameters are $r_s = 2\Omega$, $L_q = 10\,\text{mH}$, $L_d = 5\,\text{mH}$, $\lambda_m = 0.2\,\text{Vs}$, P=4. The inverter is operating with a 200 V dc voltage. The transistor drop is 2 V, and the diode drop is 1 V. The inverter is controlled with a hysteresis modulator. At a given instant in time, the current command is $i^*_{abcs} = [10\ \ -5\ \ -5]^T$. At that same instant, the actual currents are $i_{abcs} = [9\ \ -6\ \ -3]^T$. At that instant, the electrical rotor speed and position are 200 rad/s and ~~1~~ rad, respectively. What are the time derivatives of the q- and d-axis current at that instant ?

From currents and commands

9 $\left\{ \begin{array}{ll} S_a = 1 & V_{ag} = V_{dc} - V_{sw} = 198\,V \\ S_b = 1 & V_{bg} = V_{dc} + V_d = \cancel{192}\ 201\,V \\ S_c = 0 & V_{cg} = \cancel{V_d} = \cancel{2V} \\ & \qquad\ \ V_{sw} \qquad 2V \end{array} \right.$

Next

8 $\left\{ \begin{array}{l} V^r_{qs} = \frac{2}{3}\left[V_{ag}\cos\theta_r + V_{bs}\cos(\theta_r - \frac{2\pi}{3}) + V_{cg}\cos(\theta_r + \frac{2\pi}{3})\right] = 65.3\,V \\ V^r_{ds} = \frac{2}{3}\left[V_{as}\sin\theta_r + V_{bs}\sin(\theta_r - \frac{2\pi}{3}) + V_{cg}\sin(\theta_r + \frac{2\pi}{3})\right] = -116.6\,V \end{array} \right.$

Similarly

8 $\left\{ \begin{array}{l} i^r_{qs} = \frac{2}{3}\left[i_{as}\cos\theta_r + i_{bs}\cos(\theta_r - \frac{2\pi}{3}) + i_{cs}\cos(\theta_r + \frac{2\pi}{3})\right] = 9\,A \\ i^r_{ds} = \frac{2}{3}\left[i_{as}\sin\theta_r + i_{bs}\sin(\theta_r - \frac{2\pi}{3}) + i_{cs}\sin(\theta_r + \frac{2\pi}{3})\right] = 1.73\,A \end{array} \right.$

Finally

8 $\left\{ \begin{array}{l} p i^r_{qs} = \dfrac{V^r_{qs} - r_s i^r_{qs} - \omega_r L_d i^r_{ds} - \omega_r \lambda_m}{L_q} = 0.560\,\text{KA/s} \\ p i^r_{ds} = \dfrac{V^r_{ds} - r_s i^r_{ds} - \omega_r L_q i^r_{qs}}{L_d} = -20.4\,\text{KA/s} \end{array} \right.$

 10 min

4.) 20 pts. A 2 pole PMSM has a stator radius r_s, and an active length l. Suppose that for open circuit conditions, the radial flux density is given by $B = B_s \cos(\phi_r)$. Suppose the a-phase conductor density out of the machine is $n_{as} = -N\sin(\phi_s)$. Derive an expression for the a-phase flux linking the machine in terms of r_s, l, B_s, N, and θ_r.

$$n_{as} = -N \sin\phi_s$$

$$\frac{dW_{as}}{d\phi_s} = -n_{as} = N\sin\phi_s$$

$c' = -s$

$s' = c$

5 $\longrightarrow$ $W_{as} = -N\cos\phi_s$

5 $\longrightarrow$ $\lambda_{as} = r_s l \int_0^{2\pi} W_{as} B \, d\phi_s$

$\phi_s = \theta_r + \phi_r$

$\downarrow$

10

$$= r_s l \int_0^{2\pi} (-N\cos\phi_s) B_s \cos(\phi_r) d\phi_s$$

$$= -r_s l N B_s \int_0^{2\pi} \cos\phi_s \cos(\phi_s - \theta_r)\, d\phi_s$$

$$= -\frac{r_s l N B_s}{2} \int_0^{2\pi} \cos(2\phi_s - \theta_r) + \cos(\theta_r)\, d\phi_s$$

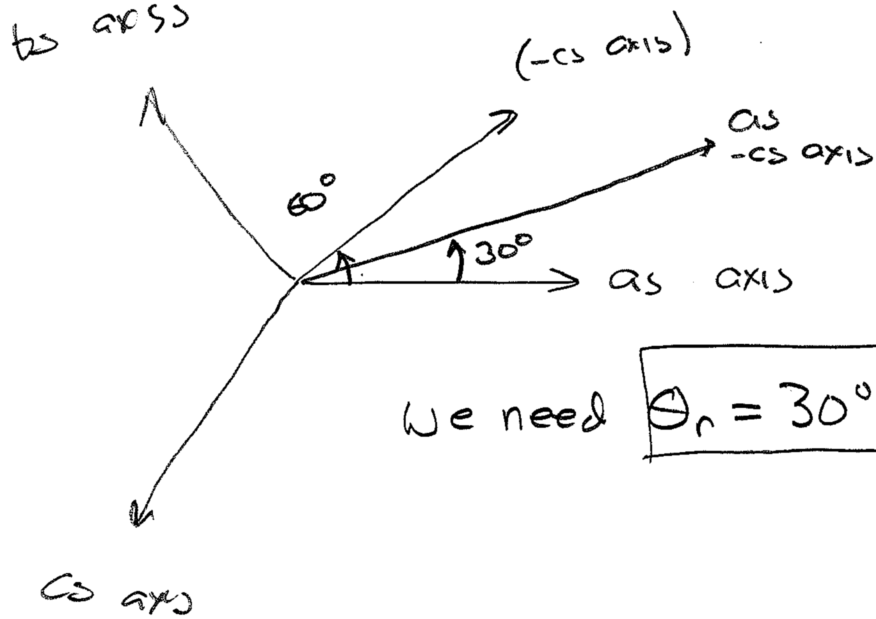
$$= -\frac{r_s l N B_s}{\cancel{2}} \cos(\theta_r) (\cancel{2}\pi)$$

$$\boxed{\lambda_{as} = -r_s l N B_s \pi \cos(\theta_r)}$$

5

PMSM

5.) 5 pts. Consider the parameter identification of an ~~induction machine~~. Suppose it is desired to measure the q-axis impedance using a a- to c- phase measurement (with the b-phase open circuited). What should the rotor position be ?



We need $\boxed{\theta_r = 30^\circ}$

1 min

Appendix A

TRIGONOMETRIC RELATIONS, CONSTANTS AND CONVERSION FACTORS, AND ABBREVIATIONS

TRIGONOMETRIC RELATIONS

$$\cos^2 x + \cos^2\left(x - \frac{2\pi}{3}\right) + \cos^2\left(x + \frac{2\pi}{3}\right) = \frac{3}{2}$$

$$\sin^2 x + \sin^2\left(x - \frac{2\pi}{3}\right) + \sin^2\left(x + \frac{2\pi}{3}\right) = \frac{3}{2}$$

$$\sin x \cos x + \sin\left(x - \frac{2\pi}{3}\right)\cos\left(x - \frac{2\pi}{3}\right) + \sin\left(x + \frac{2\pi}{3}\right)\cos\left(x + \frac{2\pi}{3}\right) = 0$$

$$\cos x + \cos\left(x - \frac{2\pi}{3}\right) + \cos\left(x + \frac{2\pi}{3}\right) = 0$$

$$\sin x + \sin\left(x - \frac{2\pi}{3}\right) + \sin\left(x + \frac{2\pi}{3}\right) = 0$$

$$\sin x \cos y + \sin\left(x - \frac{2\pi}{3}\right)\cos\left(y - \frac{2\pi}{3}\right) + \sin\left(x + \frac{2\pi}{3}\right)\cos\left(x + \frac{2\pi}{3}\right) = \frac{3}{2}\sin(x - y)$$

$$\sin x \sin y + \sin\left(x - \frac{2\pi}{3}\right)\sin\left(y - \frac{2\pi}{3}\right) + \sin\left(x + \frac{2\pi}{3}\right)\sin\left(y + \frac{2\pi}{3}\right) = \frac{3}{2}\cos(x - y)$$

$$\cos x \sin y + \cos\left(x - \frac{2\pi}{3}\right)\sin\left(y - \frac{2\pi}{3}\right) + \cos\left(x + \frac{2\pi}{3}\right)\sin\left(y + \frac{2\pi}{3}\right) = -\frac{3}{2}\sin(x - y)$$

$$\cos x \cos y + \cos\left(x - \frac{2\pi}{3}\right)\cos\left(y - \frac{2\pi}{3}\right) + \cos\left(x + \frac{2\pi}{3}\right)\cos\left(y + \frac{2\pi}{3}\right) = \frac{3}{2}\cos(x - y)$$

$$\sin x \cos y + \sin\left(x + \frac{2\pi}{3}\right)\cos\left(y - \frac{2\pi}{3}\right) + \sin\left(x - \frac{2\pi}{3}\right)\cos\left(y + \frac{2\pi}{3}\right) = \frac{3}{2}\sin(x + y)$$

$$\sin x \sin y + \sin\left(x + \frac{2\pi}{3}\right)\sin\left(y - \frac{2\pi}{3}\right) + \sin\left(x - \frac{2\pi}{3}\right)\sin\left(y + \frac{2\pi}{3}\right) = -\frac{3}{2}\cos(x + y)$$

$$\cos x \sin y + \cos\left(x + \frac{2\pi}{3}\right)\sin\left(y - \frac{2\pi}{3}\right) + \cos\left(x - \frac{2\pi}{3}\right)\sin\left(y + \frac{2\pi}{3}\right) = \frac{3}{2}\sin(x + y)$$

$$\cos x \cos y + \cos\left(x + \frac{2\pi}{3}\right)\cos\left(y - \frac{2\pi}{3}\right) + \cos\left(x - \frac{2\pi}{3}\right)\cos\left(y + \frac{2\pi}{3}\right) = \frac{3}{2}\cos(x + y)$$

A

Mathematical Tables

Table A-1 Trigonometric Identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\cos A \cos B = \tfrac{1}{2}[\cos(A+B) + \cos(A-B)]$$
$$\sin A \sin B = \tfrac{1}{2}[\cos(A-B) - \cos(A+B)]$$
$$\sin A \cos B = \tfrac{1}{2}[\sin(A+B) + \sin(A-B)]$$
$$\sin A + \sin B = 2 \sin \tfrac{1}{2}(A+B) \cos \tfrac{1}{2}(A-B)$$
$$\sin A - \sin B = 2 \sin \tfrac{1}{2}(A-B) \cos \tfrac{1}{2}(A+B)$$
$$\cos A + \cos B = 2 \cos \tfrac{1}{2}(A+B) \cos \tfrac{1}{2}(A-B)$$
$$\cos A - \cos B = -2 \sin \tfrac{1}{2}(A+B) \sin \tfrac{1}{2}(A-B)$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A = \cos^2 A - \sin^2 A$$
$$\sin \tfrac{1}{2}A = \sqrt{\tfrac{1}{2}(1 - \cos A)} \qquad \cos \tfrac{1}{2}A = \sqrt{\tfrac{1}{2}(1 + \cos A)}$$
$$\sin^2 A = \tfrac{1}{2}(1 - \cos 2A) \qquad \cos^2 A = \tfrac{1}{2}(1 + \cos 2A)$$
$$\sin x = \frac{e^{jx} - e^{-jx}}{2j} \qquad \cos x = \frac{e^{jx} + e^{-jx}}{2} \qquad e^{jx} = \cos x + j \sin x$$
$$A \cos(\omega t + \phi_1) + B \cos(\omega t + \phi_2) = C \cos(\omega t + \phi_3)$$

where

$$C = \sqrt{A^2 + B^2 - 2AB\cos(\phi_2 - \phi_1)}$$
$$\phi_3 = \tan^{-1}\left[\frac{A \sin \phi_1 + B \sin \phi_2}{A \cos \phi_1 + B \cos \phi_2}\right]$$
$$\sin(\omega t + \phi) = \cos\left(\omega t + \phi - \frac{\pi}{2}\right)$$

Table A-2 Derivatives

$$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\dfrac{du}{dx}-u\dfrac{dv}{dx}}{v^2}$$

$$\frac{d}{dx}u^n=nu^{n-1}\frac{du}{dx}$$

$$\frac{d}{dx}\log u=\frac{\log e}{u}\frac{du}{dx}$$

$$\frac{d}{dx}\ln u=\frac{1}{u}\frac{du}{dx}$$

$$\frac{d}{dx}a^u=a^u\ln a\frac{du}{dx}$$

$$\frac{d}{dx}e^u=e^u\frac{du}{dx}$$

$$\frac{d}{dx}u^v=vu^{v-1}\frac{du}{dx}+u^v\ln u\frac{dv}{dx}$$

$$\frac{d}{dx}\sin u=\cos u\frac{du}{dx}$$

$$\frac{d}{dx}\cos u=-\sin u\frac{du}{dx}$$

$$\frac{d}{dx}\tan u=\frac{1}{\cos^2 u}\frac{du}{dx}$$

Table A-3 Approximations

Taylor's series:

$$f(x)=f(a)+\dot{f}(a)\frac{(x-a)}{1!}+\ddot{f}(a)\frac{(x-a)^2}{2!}+\cdots$$

Maclaurin's series:

$$f(x)=f(0)+\dot{f}(0)\frac{x}{1!}+\ddot{f}(0)\frac{x^2}{2!}+\cdots$$

For small values of x:

$$\frac{1}{1+x}\doteq 1-x$$

$$(1+x)^n\doteq 1+nx$$

$$e^x\doteq 1+x$$

$$\ln(1+x)\doteq x$$

$$\sin x\doteq x$$

$$\cos x\doteq 1-\frac{x^2}{2}$$

$$\tan x\doteq x$$