EE595S: Class Lecture Notes
Chapter 3: Analysis and Design of Permanent Magnet Synchronous Machines

Current Regulated Brushless DC Motor Drives

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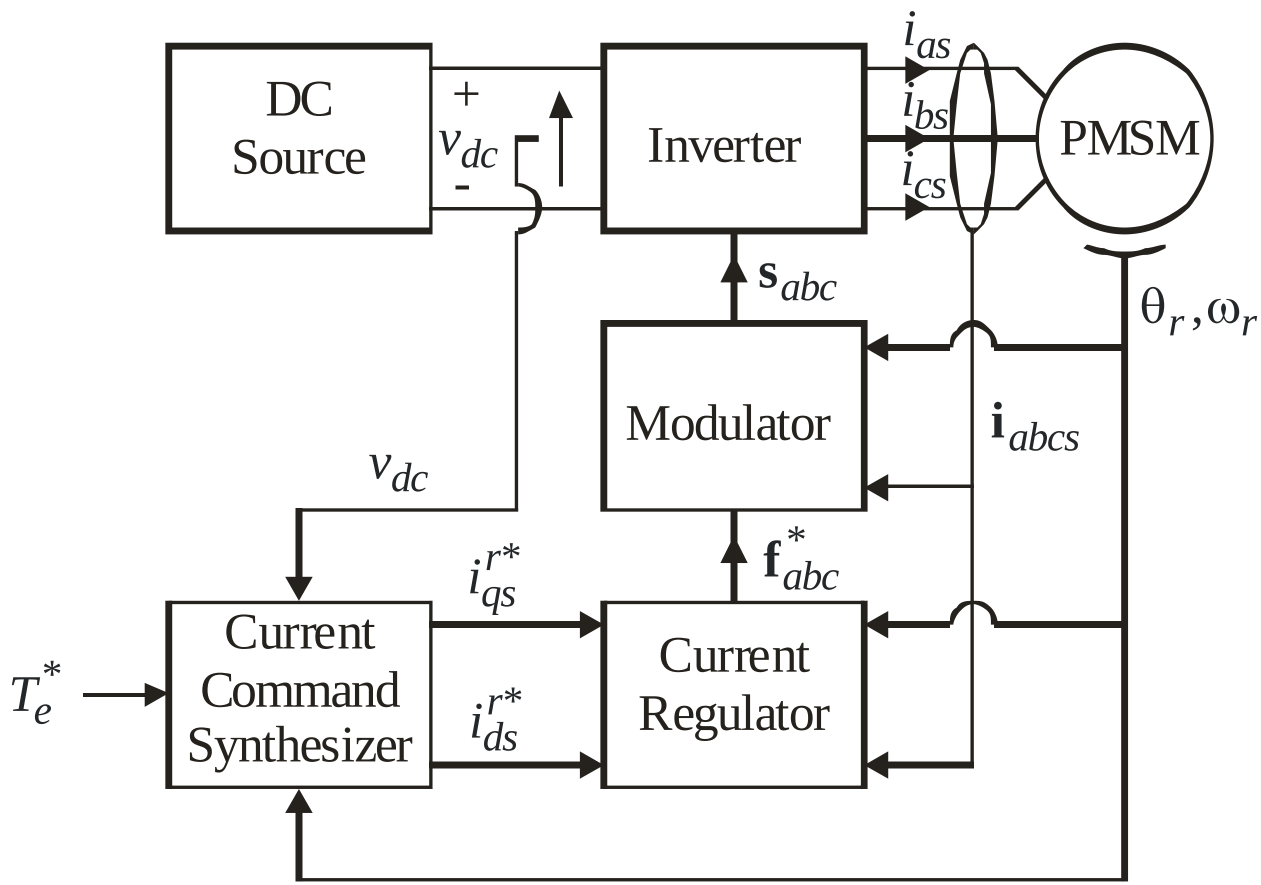
3.1 Control Architecture



3.2 Operating Constraints

3.2 Operating Constraints - Summary

- Current Limit: $i_s \le i_{s,mx}$
- Voltage Limit: $v_s \le v_{s,mx} \quad v_{s,mx} = \frac{1}{\sqrt{6}} v_{dc}$
- D-Axis Current Limit: $i_{ds}^{r*} > i_{ds,mn}^{r}$

3.3 Current Command Synthesis

- Maximum torque per amp control

3.3 Current Command Synthesis

- Thus we have

$$P_{in} = 3r_s i_s^2 + \frac{2}{P}\omega_r T_e \qquad (3.3\text{-}5)$$

3.3.1 Current Command Synthesis for Non-Salient Machines

- *Maximum Torque Per Amp Mode*

$$i_{qs}^{r*} = \frac{4\min\left(\left|T_e^*\right|, T_{e,mx1}\right)}{3\lambda_m P}\operatorname{sgn}(T_e^*) \qquad (3.3\text{-}7)$$

$$i_{ds}^{r*} = 0 \qquad (3.3\text{-}8)$$

$$T_{e,mx1} = \frac{3\sqrt{2}}{2}\frac{P}{2} i_{s,mx} \qquad (3.3\text{-}10)$$

$$\omega_{r,bnd}\left(i_{qs}^{r*}\right) = \frac{-r_s\lambda_m i_{qs}^{r*} \pm \sqrt{\left[r_s\lambda_m\, i_{qs}^{r*}\right]^2 + \left[L_{ss}^2 i_{qs}^{r*2} + \lambda_m^2\right]\left[2v_s^2 - r_s^2 i_{qs}^{r*2}\right]}}{L_{ss}^2 i_{qs}^{r*2} + \lambda_m^2} \qquad (3.3\text{-}14)$$

3.3.1 Current Command Synthesis for Non-Salient Machines

- Derivation of Maximum Torque Per Amp Mode

3.3.1 Current Command Synthesis for Non-Salient Machines

- Derivation of Maximum Torque Per Amp Mode (Cont.)

3.3.1 Current Command Synthesis for Non-Salient Machines

- *Flux Weakening*

$$i_{ds}^{r*} = \frac{-\lambda_m L_{ss}\omega_r^2 + \sqrt{2z^2 v_{s,mx2}^2 - \left(r_s \omega_r \lambda_m + z^2 i_{qs}^{r*}\right)^2}}{z^2} \qquad (3.3\text{-}16)$$

$$z = \sqrt{r_s^2 + \omega_r^2 L_{ss}^2} \qquad (3.3\text{-}17)$$

3.3.1 Current Command Synthesis for Non-Salient Machines

- Derivation of Flux Weakening

3.3.1 Current Command Synthesis for Non-Salient Machines

- Derivation of Flux Weakening (Cont.)

3.3.1 Current Command Synthesis for Non-Salient Machines

- Current Limit Restriction on Flux Weakening

$$2v_{s,mx}^2 = \left(r_s i_{qs,lm2}^r - \omega_r L_{ss}\sqrt{2i_{s,mx}^2 - i_{qs,lm2}^{r2}} + \omega_r \lambda_m \right)^2 + \left(-r_s\sqrt{2i_{s,mx}^2 - i_{qs,lm2}^{r2}} + \omega_r L_{ss} i_{qs,lm2}^r \right)^2 \quad (3.3\text{-}20)$$

$$\left(i_{qs,mn2}^r, i_{qs,mx2}^r\right) = \sqrt{2} i_{s,mx} \cos\left(\text{angle}(r_s + j\omega_r L_{ss}) \mp \text{acos}\left(\frac{2v_{s,mx}^2 - 2z^2 i_{s,mx}^2 - \omega_r^2 \lambda_m^2}{2\omega_r \lambda_m z \sqrt{2} i_{s,mx}} \right) \right) \quad (3.3\text{-}21)$$

$$T_{e,mx2} = \frac{3}{2}\frac{P}{2}\lambda_m i_{qs,mx2}^r \quad (3.3\text{-}22) \qquad T_{e,mn2} = \frac{3}{2}\frac{P}{2}\lambda_m i_{qs,mn2}^r \quad (3.3\text{-}23)$$

3.3.1 Current Command Synthesis for Non-Salient Machines

- Derivation of Current Limit Restriction on Flux Weakening:

3.3.1 Current Command Synthesis for Non-Salient Machines

- Derivation of Current Limit Restriction on Flux Weakening:

3.3.1 Current Command Synthesis for Non-Salient Machines

- D-axis Current Limit Restriction of Flux Weakening

$$\left(i_{qs,mn3}^{r}, i_{qs,mx3}^{r}\right) = \frac{-r_s\lambda_m\omega_r \mp \sqrt{2z^2v_s^2 - \left(z^2 i_{ds,mn}^{r} + \lambda_m L_{ss}\omega_r^2\right)^2}}{z^2} \quad (3.3\text{-}24)$$

$$T_{e,mn3} = \frac{3}{2}\frac{P}{2}\lambda_m i_{qs,mn3}^{r} \quad (3.3\text{-}25)$$

$$T_{e,mx3} = \frac{3}{2}\frac{P}{2}\lambda_m i_{qs,mx3}^{r} \quad (3.3\text{-}26)$$

3.3.1 Current Command Synthesis for Non-Salient Machines

- Derivation of D-Axis Current Limit Restriction on Flux Weakening

$$i_{ds}^{r*} = \frac{-\lambda_m L_{ss}\omega_r^2 + \sqrt{2z^2 v_{s,mx2}^2 - \left(r_s\omega_r\lambda_m + z^2 i_{qs}^{r*}\right)^2}}{z^2} \qquad (3.3\text{-}16)$$

3.3.1 Current Command Synthesis for Non-Salient Machines

- Derivation of D-Axis Current Limit Restriction on Flux Weakening (Cont.)

3.3.1 Current Command Synthesis for Non-Salient Machines

- Real number restriction on q-axis current

$$\left(i_{qs,mn4}^{r}, i_{qs,mx4}^{r}\right) = \frac{\mp\sqrt{2}v_s z - r_s \omega_r \lambda_m}{z^2} \qquad (3.3\text{-}27)$$

$$T_{e,mn4} = \frac{3}{2}\frac{P}{2}\lambda_m i_{qs,mm4}^{r} \qquad (3.3\text{-}28)$$

$$T_{e,mx4} = \frac{3}{2}\frac{P}{2}\lambda_m i_{qs,mx4}^{r} \qquad (3.3\text{-}29)$$

3.3.1 Current Command Synthesis for Non-Salient Machines

- Derivation of real number restriction on q-axis current

$$i_{ds}^{r*} = \frac{-\lambda_m L_{ss}\omega_r^2 + \sqrt{2z^2 v_{s,mx2}^2 - \left(r_s \omega_r \lambda_m + z^2 i_{qs}^{r*}\right)^2}}{z^2} \qquad (3.3\text{-}16)$$

3.3.1 Current Command Synthesis for Non-Salient Machines

- Derivation of real number restriction on q-axis current

3.3.1 Current Command Synthesis for Non-Salient Machines

- Combined representation of current commands in flux-weakening mode:

$$T_{e,mn} = \max(-T_{e,mx1}, T_{e,mn2}, T_{e,mn3}, T_{e,mn4}) \qquad (3.3\text{-}30)$$

$$T_{e,mx} = \min(T_{e,mx1}, T_{e,mx2}, T_{e,mx3}, T_{e,mx4}) \qquad (3.3\text{-}31)$$

$$i_{qs}^{r*} = \frac{4\min(\max(T_e^*, T_{e,mn}), T_{e,mx})}{3P\lambda_m} \qquad (3.3\text{-}32)$$

3.3.1 Current Command Synthesis for Non-Salient Machines

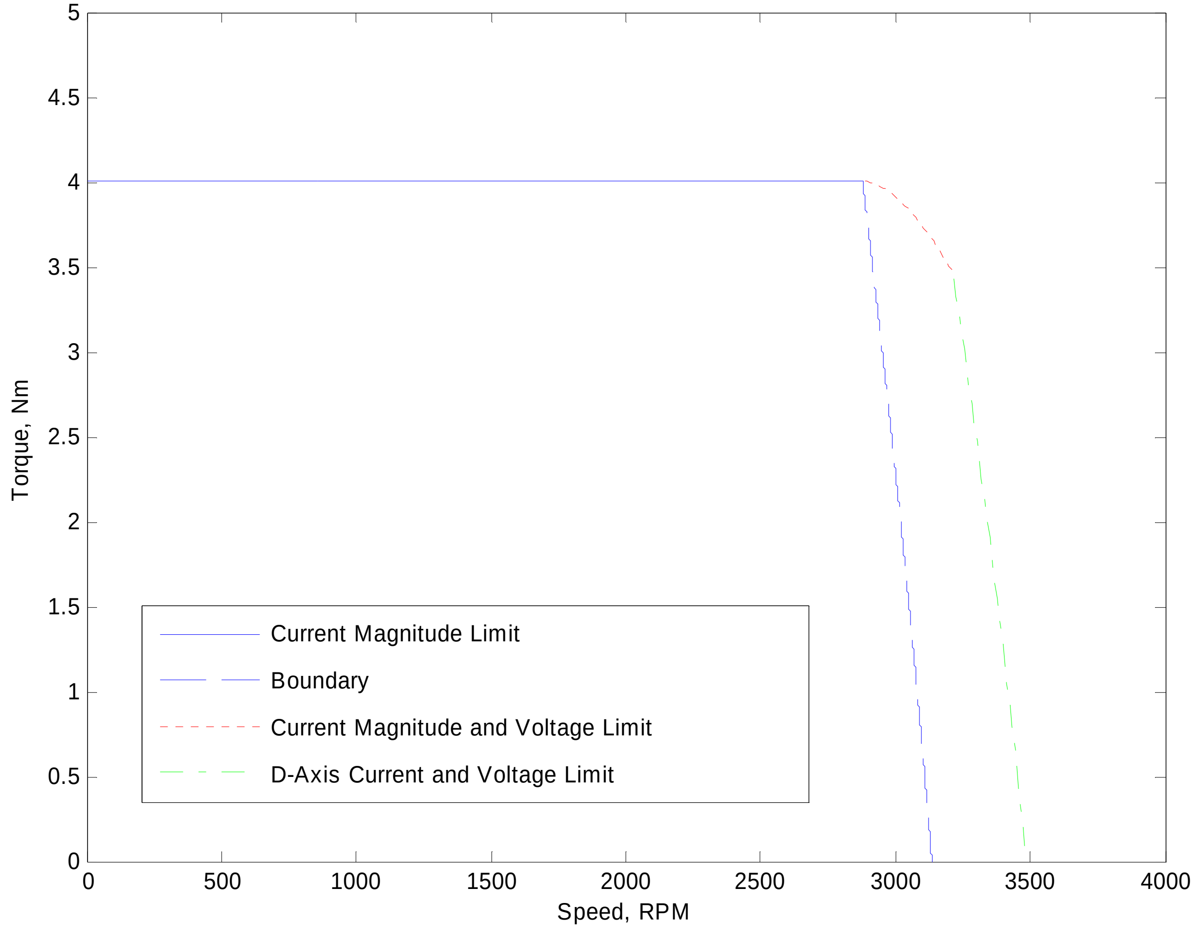
- Example: consider a machine in which

$$r_s = 2.6\,\Omega \qquad L_{ss} = 12.4\,\text{mH} \qquad v_{s,mx} = 230/\sqrt{3}\,\text{V}$$

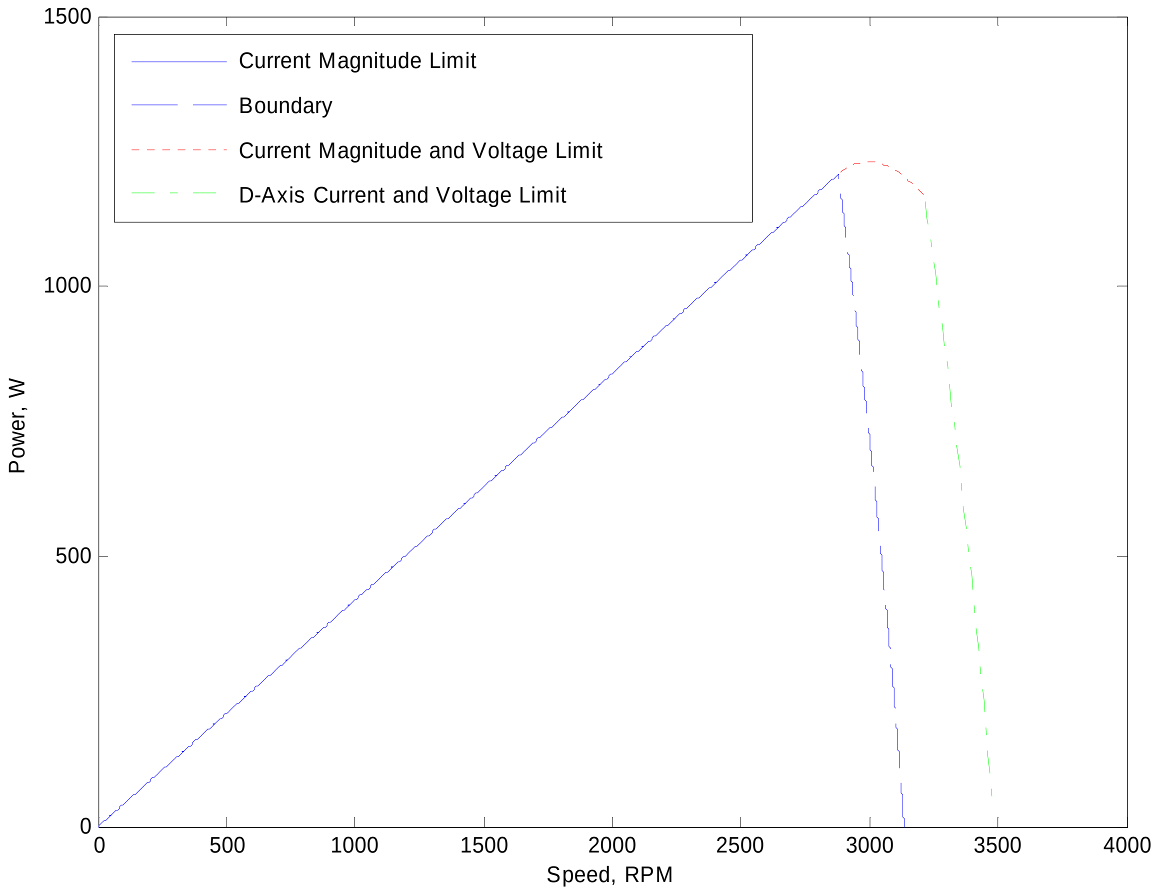
$$\lambda_m = 0.286\,\text{Vs} \qquad P = 4 \qquad i_{s,mx} = 3.3\,\text{A}$$

$$i_{d,mn} = -2.33\,\text{A}$$

3.3.1 Current Command Synthesis for Non-Salient Machines



3.3.1 Current Command Synthesis for Non-Salient Machines



3.3.2 Current Command Synthesis for Salient Machines

- *Maximum Torque Per Amp*

$$i_{qs}^{r*} = g_q(T_e^*) = \sum_{j=1}^{J} a_j \left(T_e^*\right)^{n_j} \qquad (3.3\text{-}32)$$

$$i_{ds}^{r*} = g_d(i_{qs}^{r*}, T_e^*) = \frac{\frac{3}{2}\frac{P}{2}\lambda_m i_{qs}^{r*} - T_e^*}{\frac{3}{2}\frac{P}{2}(L_q - L_d) i_{qs}^{r*}} \qquad (3.3\text{-}28)$$

$$\left(i_{qs}^{r*}\right)^4 + \frac{T_e^*\left(\frac{3}{2}\frac{P}{2}\lambda_m i_{qs}^{r*} - T_e^*\right)}{\left(\frac{3}{2}\frac{P}{2}\left(L_q - L_d\right)\right)^2} = 0 \qquad (3.3\text{-}31)$$

3.3.2 Current Command Synthesis for Salient Machines

- Derivation of maximum torque per amp control:

$$T_e = \frac{3}{2}\frac{P}{2}(\lambda_m i_{qs}^r - (L_q - L_d) i_{qs}^r i_{ds}^r) \qquad (3.3\text{-}27)$$

3.3.2 Current Command Synthesis for Salient Machines

- Derivation of maximum torque per amp control (continued)

3.3.2 Current Command Synthesis for Salient Machines

- Current limit on maximum torque per amp control

$$2i_{s,mx}^2 = g_q(T_{e,mx1}^*)^2 + g_d(T_{e,mx1}^*, g_g(T_{e,mx1}^*))^2 \qquad (3.3\text{-}33)$$

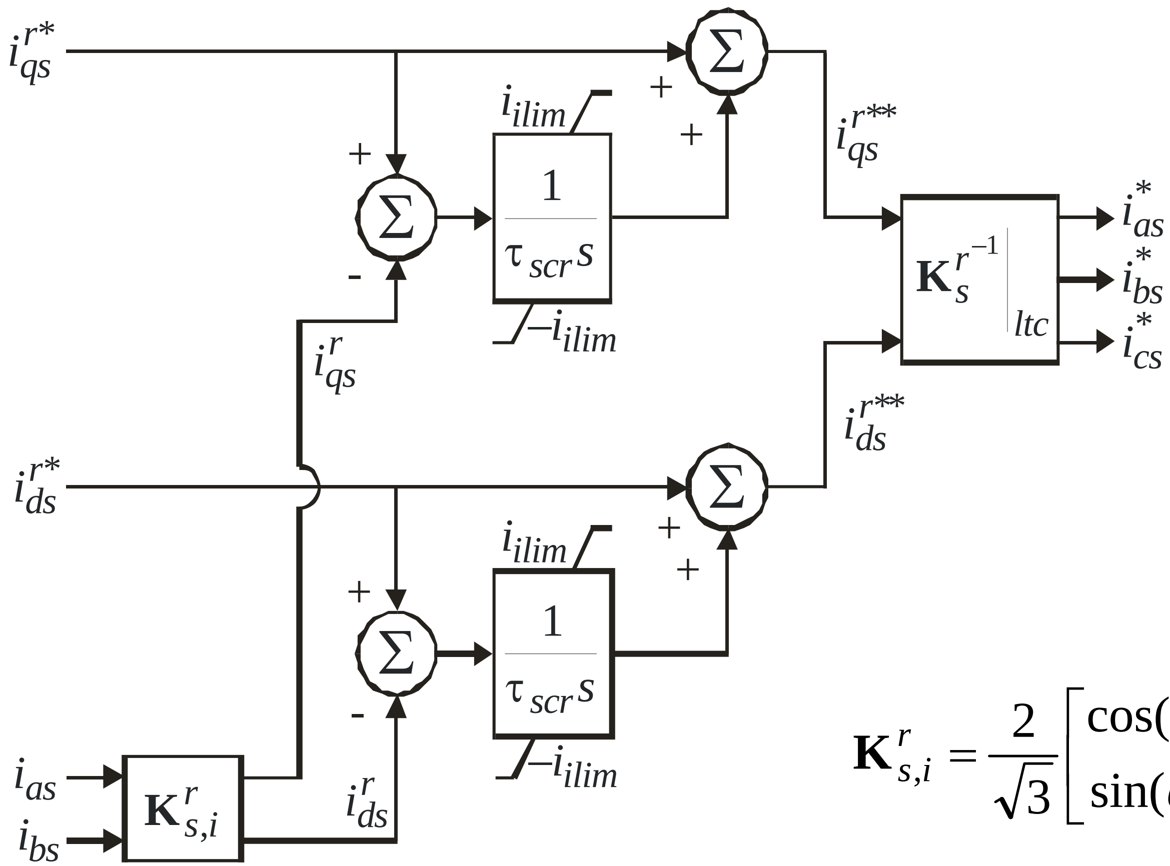
3.3.2 Current Command Synthesis for Salient Machines

- Speed limit on maximum torque per amp control

$$2v_{s,mx}^2 = \left(r_s g_q(T_{e,mx}^*) + \omega_{r,bnd} L_d g_d(T_{e,mx}^*, g_q(T_{e,mx}^*)) + \omega_{r,bnd}\right)^2 + \left(r_s g_d(T_{e,mx}^*, g_q(T_{e,mx}^*)) + \omega_{r,bnd} L_q g_q(T_{e,mx}^*)\right) \tag{3.3-34}$$

$$\omega_{r,bnd} = h(T_e^*) = b_0 + \sum_{j=1}^{J} b_j \left(T_e^*\right)^{n_j} \tag{3.3-35}$$

3.4 Direct Current Regulation



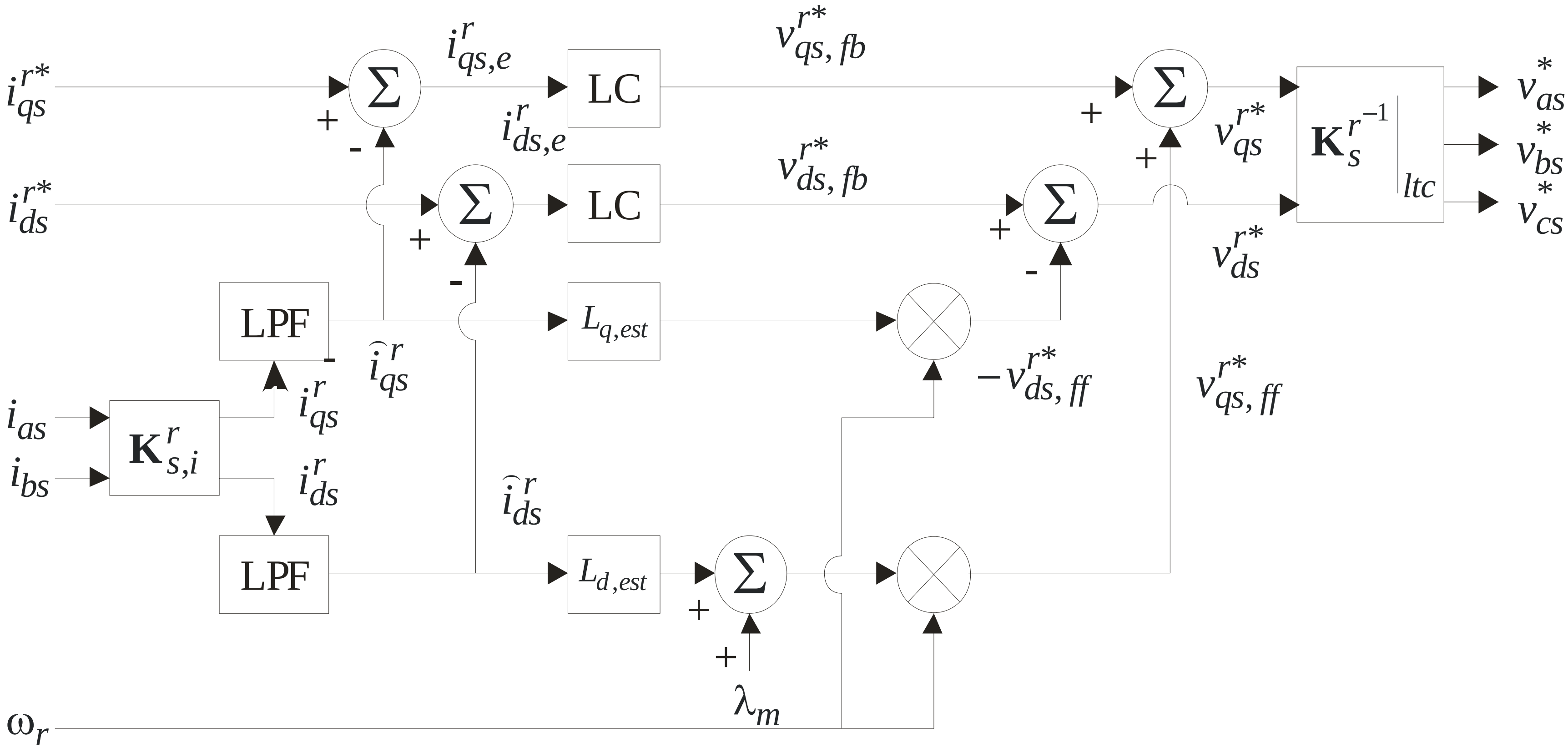
$$\mathbf{K}_{s,i}^r = \frac{2}{\sqrt{3}} \begin{bmatrix} \cos(\theta_r - \pi/6) & \sin(\theta_r) \\ \sin(\theta_r - \pi/6) & -\cos(\theta_r) \end{bmatrix}$$

3.4 Direct Current Regulation

- Analysis

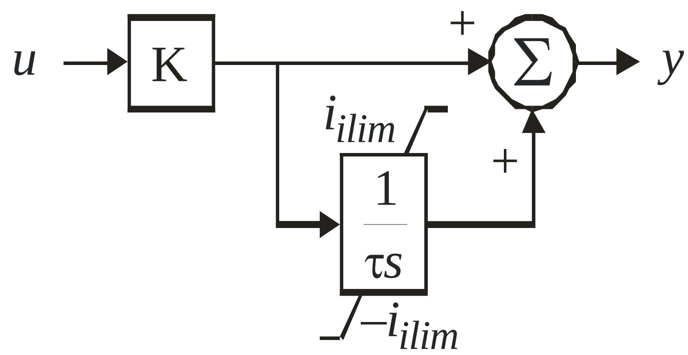
$$i_{qs}^{r} = i_{qs}^{r*} + \frac{\tau_{scr}s}{\tau_{scr}s+1}\Delta i_{qs}^{r} \qquad (3.4\text{-}8)$$

3.5 Indirect Current Regulation



3.5 Indirect Current Regulation

- Consider the following



- With appropriate assumptions,

$$i_{qs}^{r} = \frac{K(\tau s + 1)}{(r_s + K)\tau s + K} i_{qs}^{r*} + \frac{\tau s}{(r_s + K)\tau s + K} \Delta v_{qs}^{r} \qquad (3.\ 5\text{-}15)$$

3.5 Indirect Current Regulation

- Derivation

3.5 Indirect Current Regulation

- Derivation (Cont.)