EE595S: Class Lecture Notes
Chapter 15: Brushless DC Motor Drives

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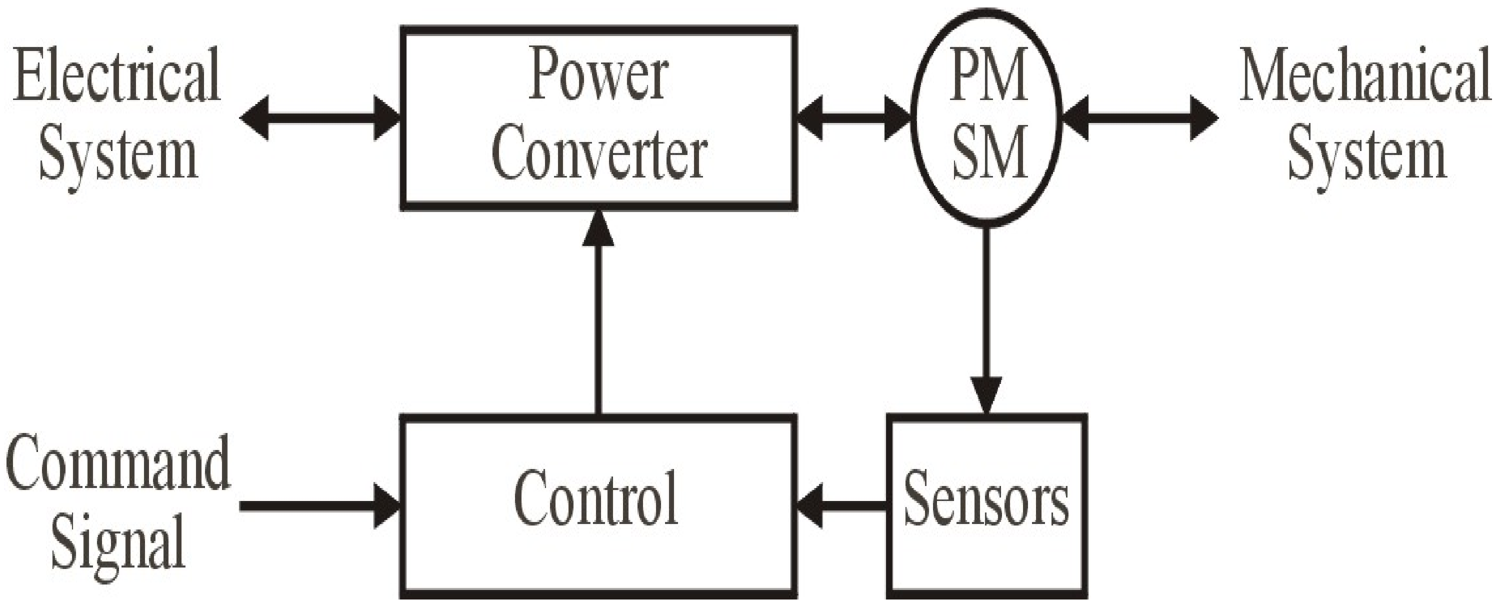
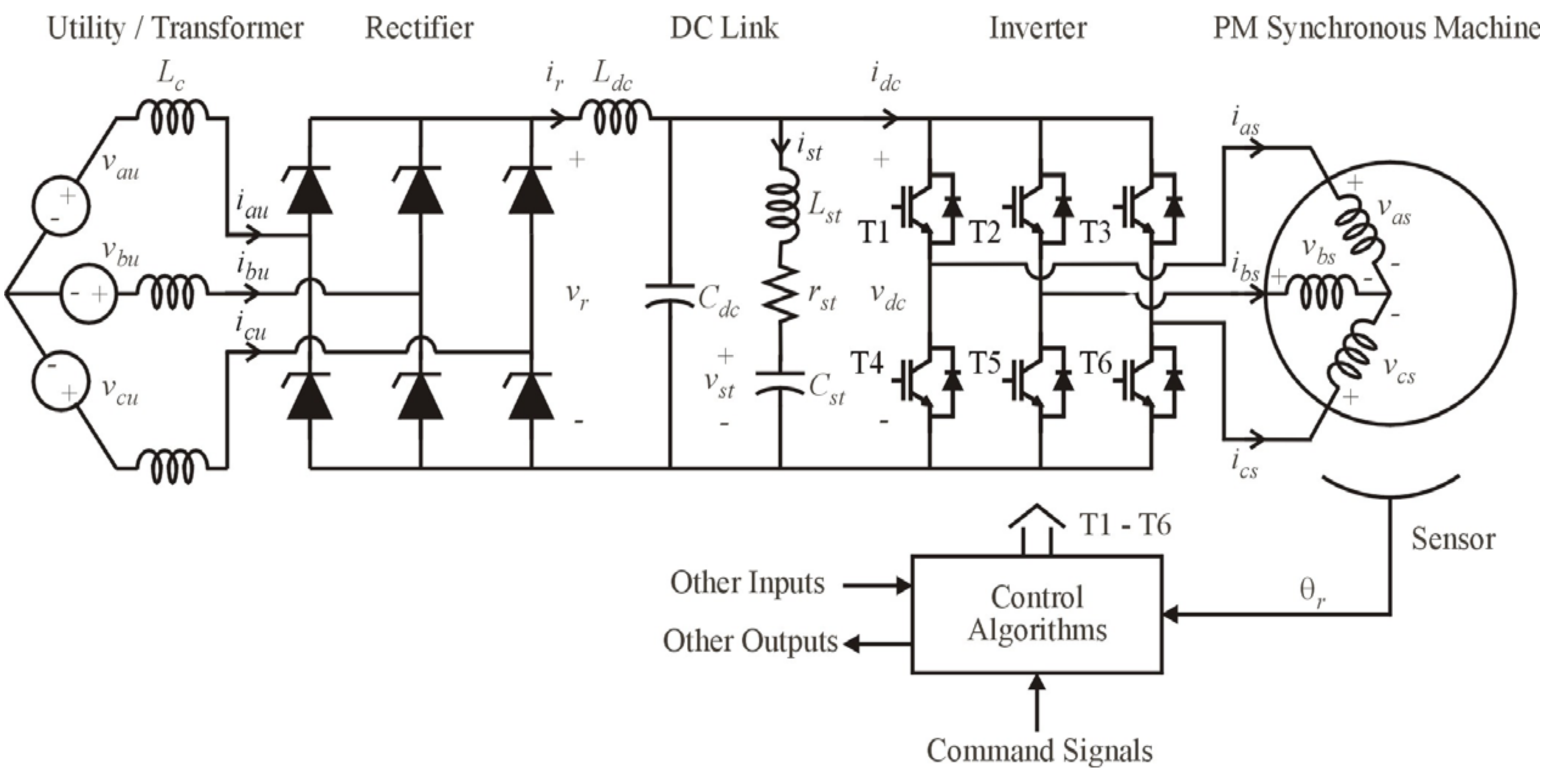
15.1 Architecture



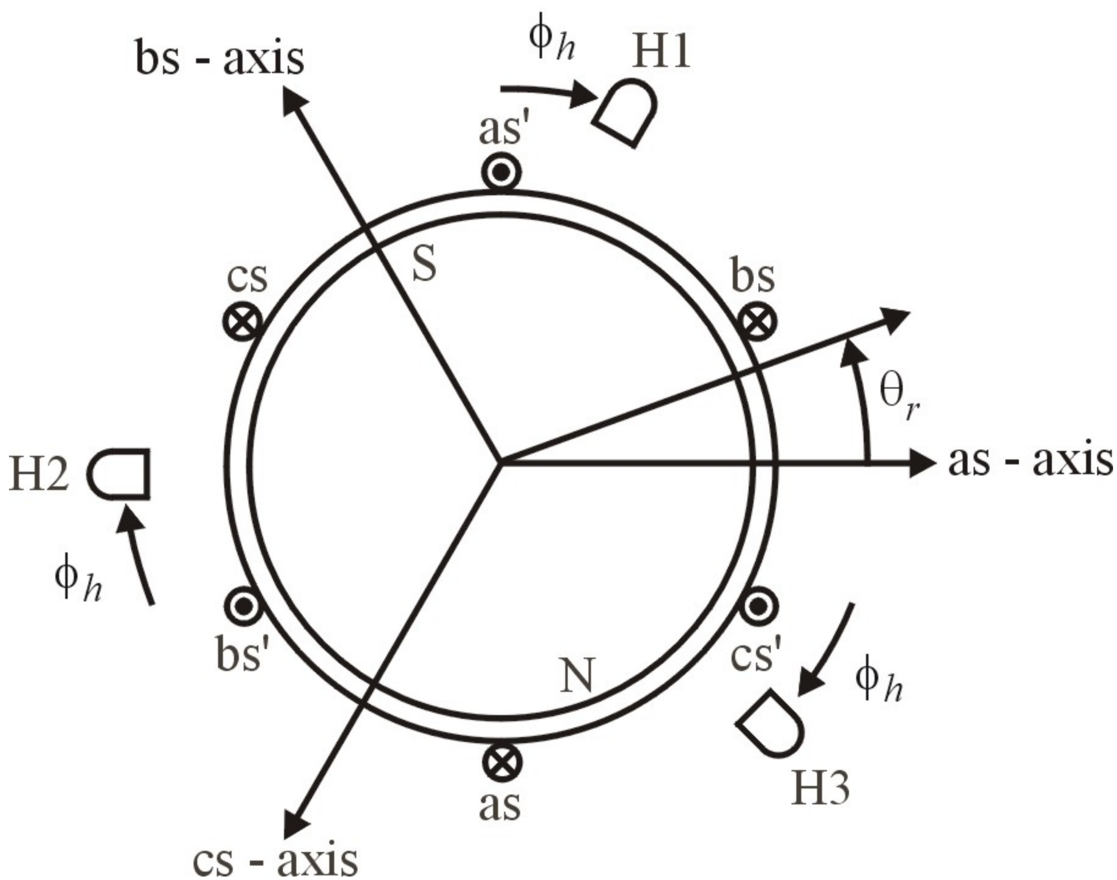
15.2 Topology



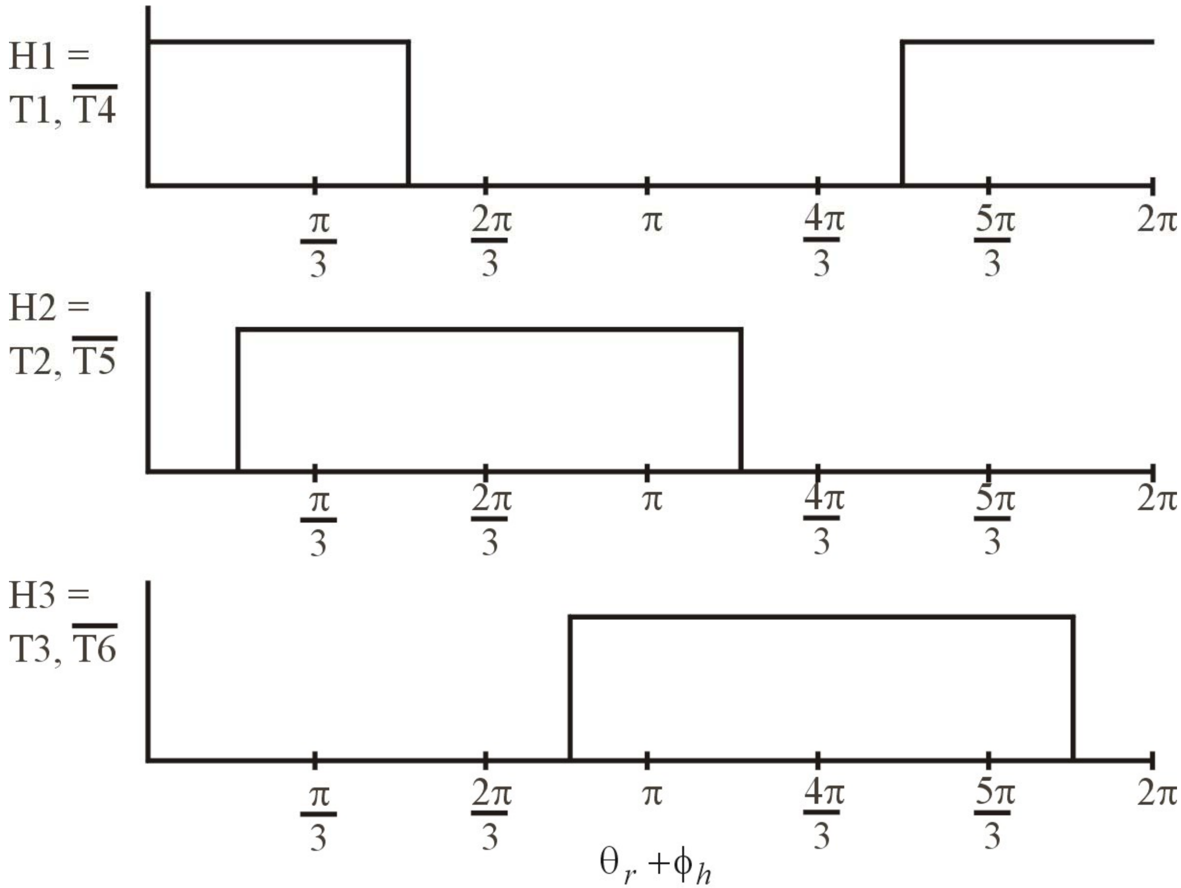
15.3 Equivalence of VSI Schemes to Idealized Source

- 180 VSI
- Duty Cycle Modulation
- Sine-Triangle Modulation
- Sine-Triangle Modulation (with 3rd)
- Space-Vector Modulation

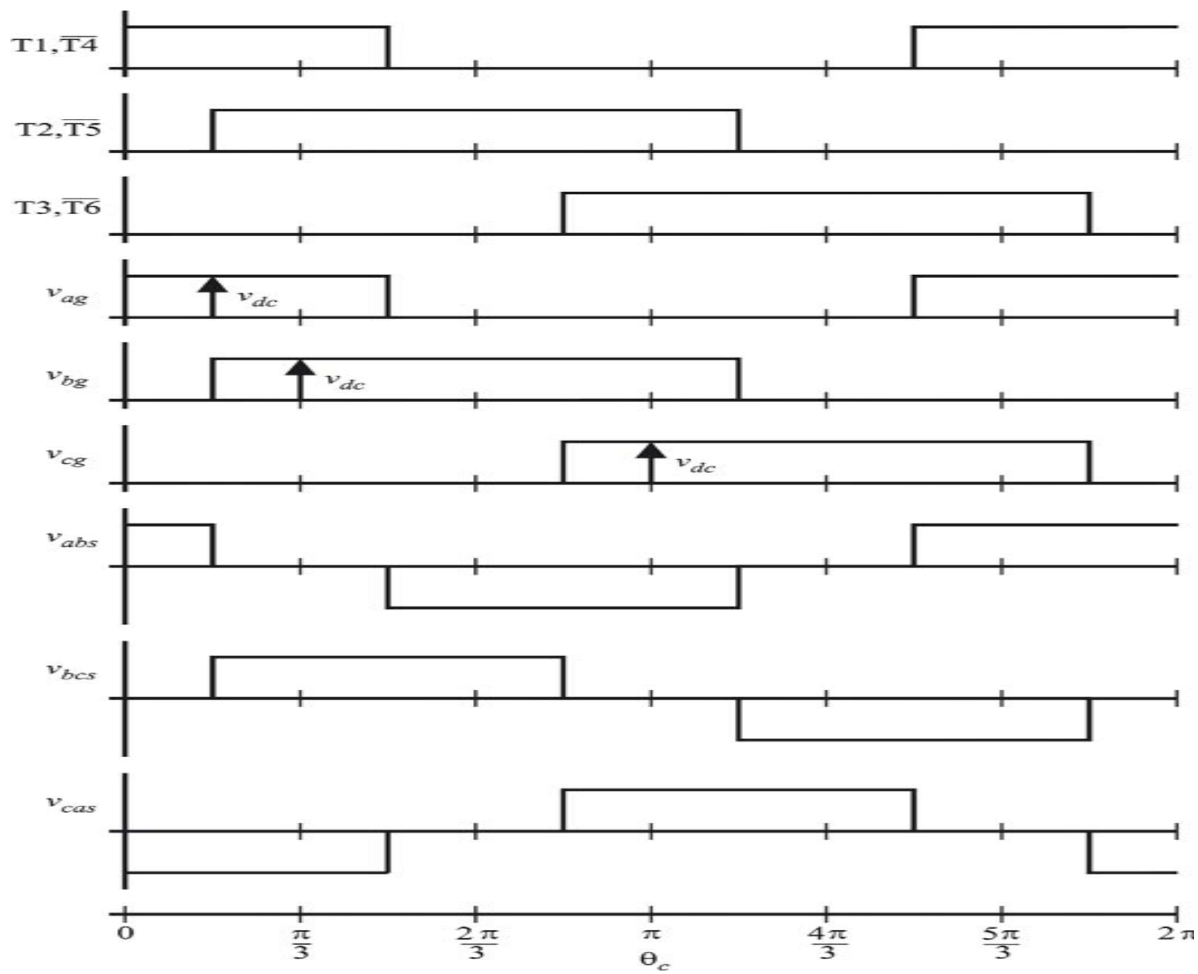
180 VSI Operation



180 VSI Operation



180 VSI Operation



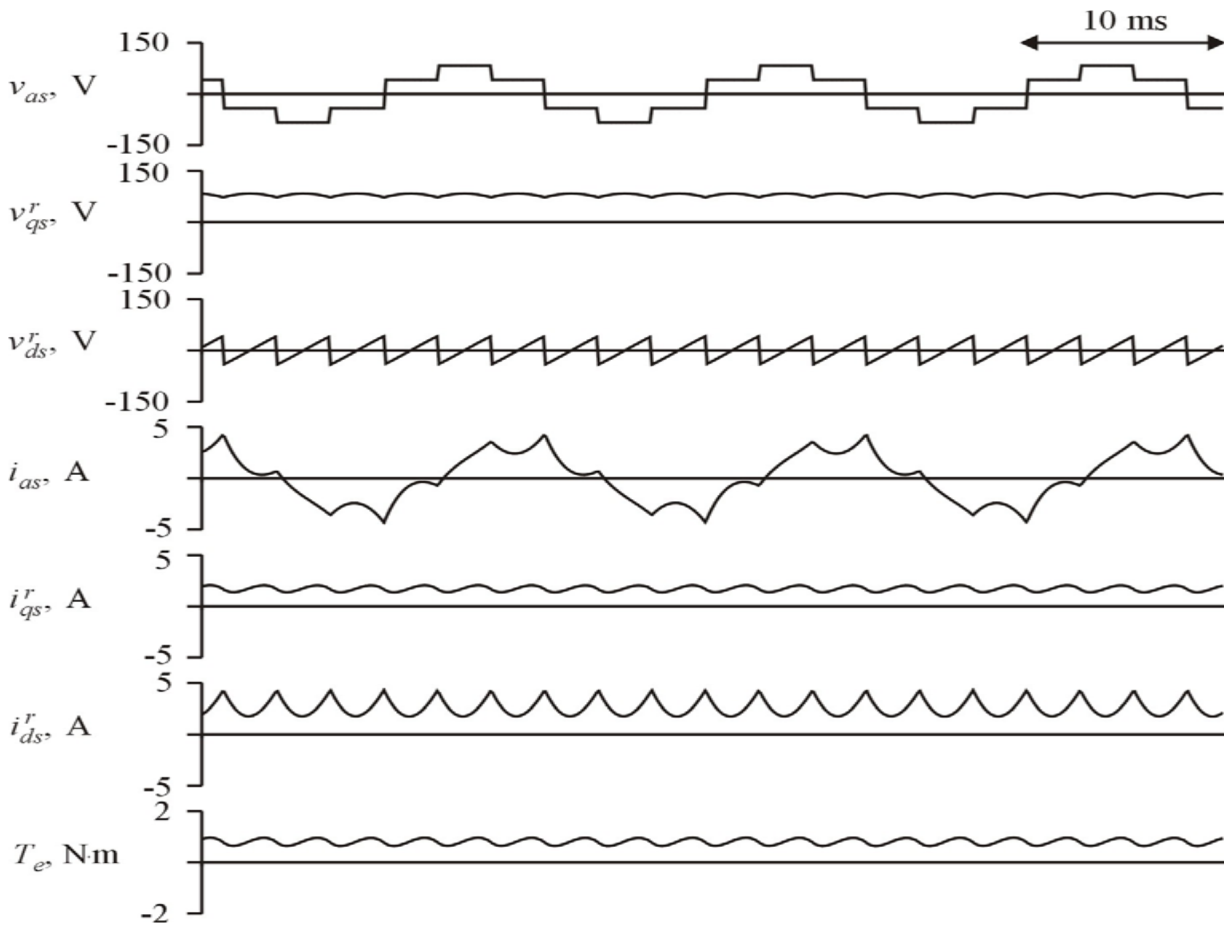
180 VSI Operation

- Comparing the two figures
  - $\theta_c = \theta_r + \phi_h$ (15.3-1)
- Now recall
  - $\bar{v}_{qs}^{c} = \frac{2}{\pi}\bar{v}_{dc}$ (15.3-2)
  - $\bar{v}_{ds}^{c} = 0$ (15.3-3)

180 VSI Operation

- Using the frame-to-frame transformation
  - $\bar{v}_{qs}^{r} = \frac{2}{\pi}\bar{v}_{dc}\cos\phi_h$ (15.3-4)
  - $\bar{v}_{ds}^{r} = -\frac{2}{\pi}\bar{v}_{dc}\sin\phi_h$ (15.3-5)
- Thus
  - $v_s = \frac{1}{\sqrt{2}}\frac{2}{\pi}\bar{v}_{dc}$ (15.3-6)
  - $\phi_v = \phi_h$ (15.3-7)

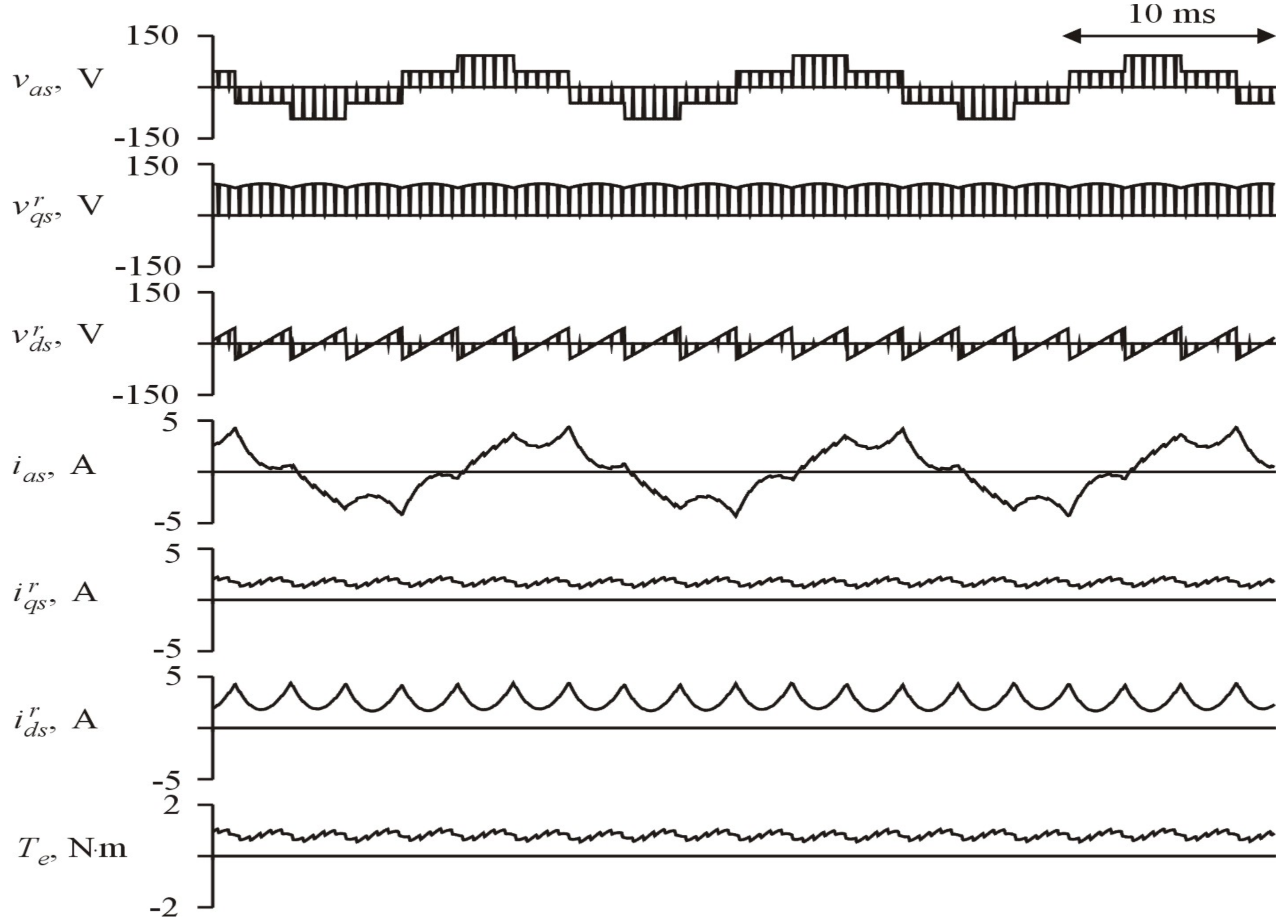
180 VSI Operation



Duty Cycle Modulation

- In this case
  - $\bar{v}_{qs}^{c} = \frac{2}{\pi} d\, \bar{v}_{dc}$ (15.3-8)
  - $\bar{v}_{ds}^{c} = 0$ (15.3-9)
- Thus
  - $\bar{v}_{qs}^{r} = \frac{2}{\pi} d\, \bar{v}_{dc} \cos\phi_h$ (15.3-10)
  - $\bar{v}_{ds}^{r} = -\frac{2}{\pi} d \bar{v}_{dc} \sin\phi_h$ (15.3-11)
- So
  - $v_s = \frac{1}{\sqrt{2}} \frac{2}{\pi} d\, \bar{v}_{dc}$ (15.3-12)
  - $\phi_v = \phi_h$ (15.3-7)

Duty Cycle Modulation



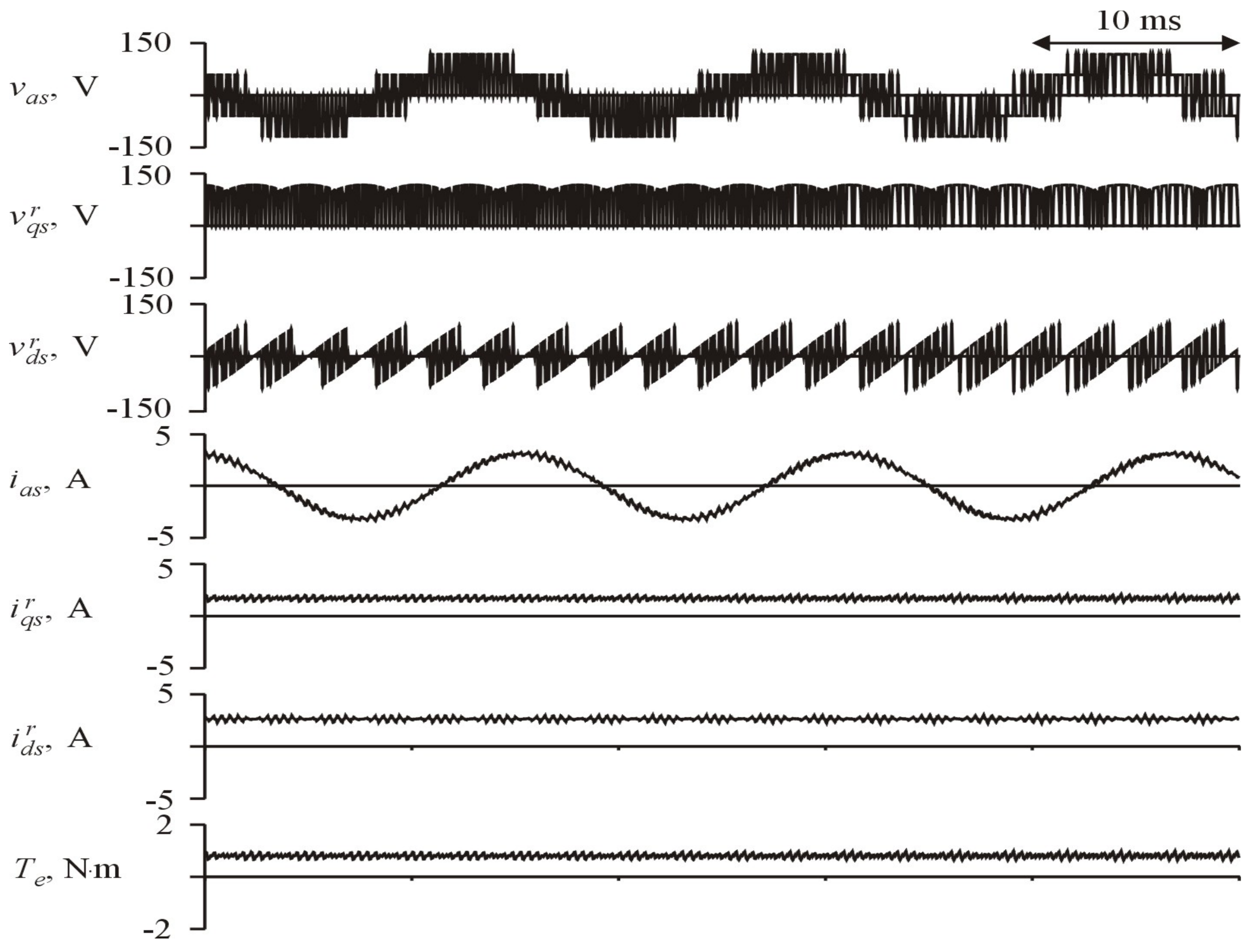
Sine-Triangle Modulation

- In this case, converter angle calculated as
  - $\theta_c = \theta_r + \phi_v$ (15.3-13)
- Now recall
  - $$\bar{v}_{qs}^{c} = \begin{cases} \frac{1}{2} d \bar{v}_{dc} & 0 < d \le 1 \\ \frac{2}{\pi} \bar{v}_{dc} f(d) & d > 1 \end{cases} \quad (15.3\text{-}14)$$
  - $\bar{v}_{ds}^{c} = 0$ (15.3-15)
  - $$f(d) = \frac{1}{2}\sqrt{1-\left(\frac{1}{d}\right)^2} + \frac{1}{4} d \left(\pi - 2\arccos\left(\frac{1}{d}\right)\right) \quad d > 1 \quad (15.3\text{-}16)$$

Sine-Triangle Modulation

- Converting to the rotor reference frame
  - $$\bar{v}_{qs}^{r} = \begin{cases} \frac{1}{2}\bar{v}_{dc}\, d\cos\phi_v & d \le 1 \\ \frac{2}{\pi}\bar{v}_{dc}\, f(d)\cos\phi_v & d > 1 \end{cases} \qquad (15.3\text{-}17)$$
- $$\bar{v}_{ds}^{r} = \begin{cases} -\frac{1}{2}\bar{v}_{dc}\, d\sin\phi_v & d \le 1 \\ -\frac{2}{\pi}\bar{v}_{dc}\, f(d)\sin\phi_v & d > 1 \end{cases} \qquad (15.3\text{-}18)$$

Sine-Triangle Modulation



Sine Triangle Modulation with 3rd Harmonic Injection

- Following our earlier work, we will have
  - $\bar{v}_{qs}^{r} = \frac{1}{2} d \bar{v}_{dc} \cos\phi_v \quad 0 \le d \le 2/\sqrt{3}$ (15.3-19)
  - $\bar{v}_{ds}^{r} = -\frac{1}{2} d \bar{v}_{dc} \sin\phi_v \quad 0 \le d \le 2/\sqrt{3}$ (15.3-20)

Space Vector Modulation

- In this case we have
  - $$\bar{v}_{qs}^{r} = \begin{cases} v_{qs}^{r*} & v_{spk}^{*} < v_{dc}/\sqrt{3} \\ \dfrac{\bar{v}_{dc}}{\sqrt{3}} \dfrac{v_{qs}^{r*}}{v_{spk}^{*}} & v_{spk}^{*} \geq v_{dc}/\sqrt{3} \end{cases} \qquad (15.3\text{-}21)$$
  - $$\bar{v}_{ds}^{r} = \begin{cases} v_{ds}^{r*} & v_{spk}^{*} < v_{dc}/\sqrt{3} \\ \dfrac{\bar{v}_{dc}}{\sqrt{3}} \dfrac{v_{ds}^{r*}}{v_{spk}^{*}} & v_{spk}^{*} \geq v_{dc}/\sqrt{3} \end{cases} \qquad (15.3\text{-}22)$$
  - $$v_{spk}^{*} = \sqrt{\left(v_{qs}^{r*}\right)^2 + \left(v_{ds}^{r*}\right)^2} \qquad (15.3\text{-}23)$$

Modulation Summary

- For all VSI strategies, we have
  - $\bar{v}_{qs}^{r} = \bar{v}_{dc} m \cos\phi_v$ (15.3-24)
  - $\bar{v}_{ds}^{r} = -\bar{v}_{dc} m \sin\phi_v$ (15.3-25)
- Where
  - $$m = \begin{cases} \frac{2}{\pi} & 180^{\text{o}} \text{ voltage-source modulation} \\ \frac{2}{\pi} d & \text{duty-cycle modulation} \\ \frac{1}{2} d & \text{sine-triangle modulation } (d \le 1) \\ \frac{2}{\pi} f(d) & \text{sine-triangle modulation } (d \ge 1) \\ \frac{1}{2} d & \text{third-harmonic modulation } (d \le 2/\sqrt{3}) \\ \frac{v_{spk}^{*}}{v_{dc}} & \text{space-vector modulation } \left(v_{spk}^{*} \le v_{dc}/\sqrt{3}\right) \\ \frac{1}{\sqrt{3}} & \text{space-vector modulation } \left(v_{spk}^{*} \ge v_{dc}/\sqrt{3}\right) \end{cases}$$ (15.3-26)

Modulation Summary

- Note, for space vector modulation
  - $\phi_v = angle(v_{qs}^{r*} - jv_{ds}^{r*})$ (15.3-27)

15.4 Average-Value Analysis of VSI Drives

- Average-value rectifier model
  - $\bar{v}_r = v_{ro}\cos\alpha - r_r \bar{i}_r - l_r p \bar{i}_r$ (15.4-1)
- Where
  - $v_{r0} = \begin{cases} \dfrac{3\sqrt{3}}{\pi}\sqrt{2}E & \text{three phase rectifier} \\ \dfrac{2}{\pi}\sqrt{2}E & \text{single phase rectifier} \end{cases}$ (15.4-2)
  - $r_r = \begin{cases} \dfrac{3}{\pi}\omega_{eu}L_c & \text{three phase rectifier} \\ \dfrac{2}{\pi}\omega_{eu}L_c & \text{single phase rectifier} \end{cases}$ (15.4-3)
  - $l_r = \begin{cases} 2L_c & \text{three phase rectifier} \\ L_c & \text{single phase rectifier} \end{cases}$ (15.4-4)

Working Out the Details…

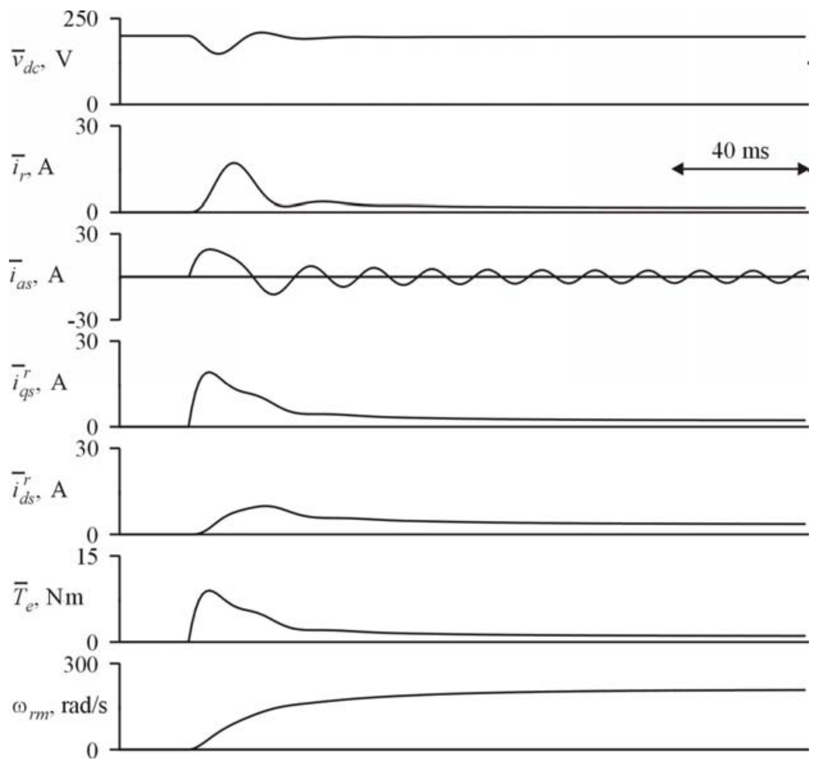
Final Average Value Model

- Nonlinear Average Value Model (NLAM)
  - ➢

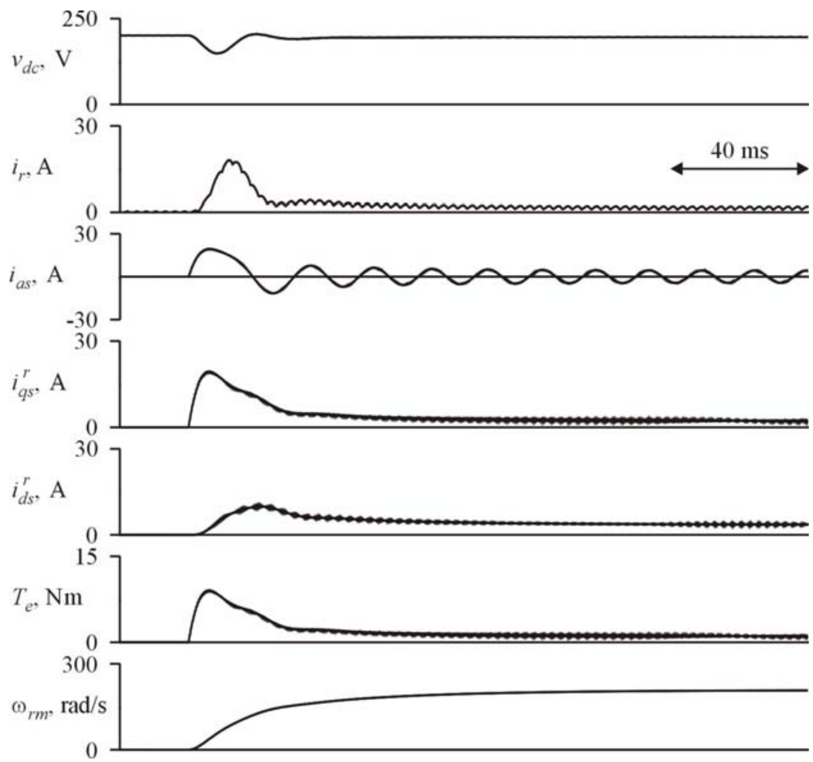
$$p\begin{bmatrix}\bar{i}_r\\ \bar{v}_{dc}\\ \bar{i}_{st}\\ \bar{v}_{st}\\ \bar{i}_{qs}^r\\ \bar{i}_{ds}^r\\ \omega_r\end{bmatrix}=\begin{bmatrix}-\frac{r_{rl}}{L_{rl}} & -\frac{1}{L_{rl}} & 0 & 0 & 0 & 0 & 0\\ \frac{1}{C_{dc}} & 0 & -\frac{1}{C_{dc}} & 0 & 0 & 0 & 0\\ 0 & \frac{1}{L_{st}} & -\frac{r_{st}}{L_{st}} & -\frac{1}{L_{st}} & 0 & 0 & 0\\ 0 & 0 & \frac{1}{C_{st}} & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & -\frac{r_s}{L_q} & 0 & -\frac{\lambda_m'}{L_q}\\ 0 & 0 & 0 & 0 & 0 & -\frac{r_s}{L_d} & 0\\ 0 & 0 & 0 & 0 & \frac{3}{2}\left(\frac{P}{2}\right)^2\frac{1}{J}\lambda_m' & 0 & 0\end{bmatrix}\begin{bmatrix}\bar{i}_r\\ \bar{v}_{dc}\\ \bar{i}_{st}\\ \bar{v}_{st}\\ \bar{i}_{qs}^r\\ \bar{i}_{ds}^r\\ \omega_r\end{bmatrix}+\begin{bmatrix}\frac{1}{L_{rl}}v_{r0}\cos\alpha\\ -\frac{3}{2}\frac{m}{C_{dc}}(\cos\phi_v\ \bar{i}_{qs}^r-\sin\phi_v\,\bar{i}_{ds}^r)\\ 0\\ 0\\ \frac{1}{L_q}\bar{v}_{dc}m\cos\phi_v-\frac{L_d}{L_q}\omega_r\bar{i}_{ds}^r\\ -\frac{1}{L_d}\bar{v}_{dc}m\sin\phi_v+\frac{L_q}{L_d}\omega_r\bar{i}_{qs}^r\\ \frac{P}{2}\frac{1}{J}(\frac{3}{2}\frac{P}{2}(L_d-L_q)\bar{i}_{qs}^r\bar{i}_{ds}^r-T_L)\end{bmatrix}$$

(15.4-21)

NLAM Startup Prediction Figure 15.6-2



Detailed Startup Prediction
Figure 15.6-1



Steady-State Performance of VSI Drives

- For Steady State Conditions
  - $0 = v_{r0} \cos\alpha - \bar{V}_{dc} - r_{rl} \bar{I}_r$ (15.5-1)
- From Our Earlier Work
  - $\bar{i}_{dc} = \frac{3}{2} m(\bar{i}_{qs}^r \cos\phi_v - \bar{i}_{ds}^r \sin\phi_v)$ (15.4-14)
- Thus
  - $0 = \bar{I}_r - \bar{I}_{st} - \frac{3}{2} m(\bar{I}_{qs}^r \cos\phi_v - \bar{I}_{ds}^r \sin\phi_v)$ (15.5-2)
- The Steady-State Stabilizing Current is Zero So
  - $0 = \bar{I}_r - \frac{3}{2} m(\bar{I}_{qs}^r \cos\phi_v - \bar{I}_{ds}^r \sin\phi_v)$ (15.5-3)

Steady-State Performance of VSI Drives

- Combining (15.5-3) with (15.5-1) yields
  - $\bar{V}_{dc} = v_{r0}\cos\alpha - \frac{3}{2} r_{rl} m(\bar{I}_{qs}^{r}\cos\phi_v - \bar{I}_{ds}^{r}\sin\phi_v)$ (15.5-4)
- From the Machine Voltage Equations
  - $0 = \bar{V}_{dc} m\cos\phi_v - r_s \bar{I}_{qs}^{r} - \omega_r L_d \bar{I}_{ds}^{r} - \omega_r \lambda_m'$ (15.5-5)
  - $0 = -\bar{V}_{dc} m\sin\phi_v - r_s \bar{I}_{ds}^{r} + \omega_r L_q \bar{I}_{qs}^{r}$ (15.5-6)

Steady-State Performance of VSI Drives

- Solving (15.5-5) and (15.5-6) for the Currents
  - $$\bar{I}_{qs}^{r} = \frac{r_s(\bar{V}_{dc} m \cos\phi_v - \omega_r \lambda_m') + \omega_r L_d \bar{V}_{dc} m \sin\phi_v}{r_s^2 + \omega_r^2 L_d L_q} \quad (15.5\text{-}7)$$
  - $$\bar{I}_{ds}^{r} = \frac{\omega_r L_q(\bar{V}_{dc} m \cos\phi_v - \omega_r \lambda_m') - r_s \bar{V}_{dc} m \sin\phi_v}{r_s^2 + \omega_r^2 L_d L_q} \quad (15.5\text{-}8)$$

Steady-State Performance of VSI Drives

- Substitution of (15.5-7) and (15.5-8) into (15.5-4) and Solving Yields
  - $$\bar{V}_{dc} = \frac{(r_s^2 + \omega_r^2 L_d L_q) v_{r0} \cos\alpha + \frac{3}{2} r_{rl} m \omega_r \lambda_m' (r_s \cos\phi_v - \omega_r L_q \sin\phi_v)}{r_s^2 + \omega_r^2 L_d L_q + \frac{3}{2} m^2 r_{rl} r_s + \frac{3}{4} r_{rl} \omega_r (L_d - L_q) m^2 \sin 2\phi_v} \quad (15.5\text{-}9)$$

- Caution: This Expression Only Valid if DC Current (15.5-14) Is Positive

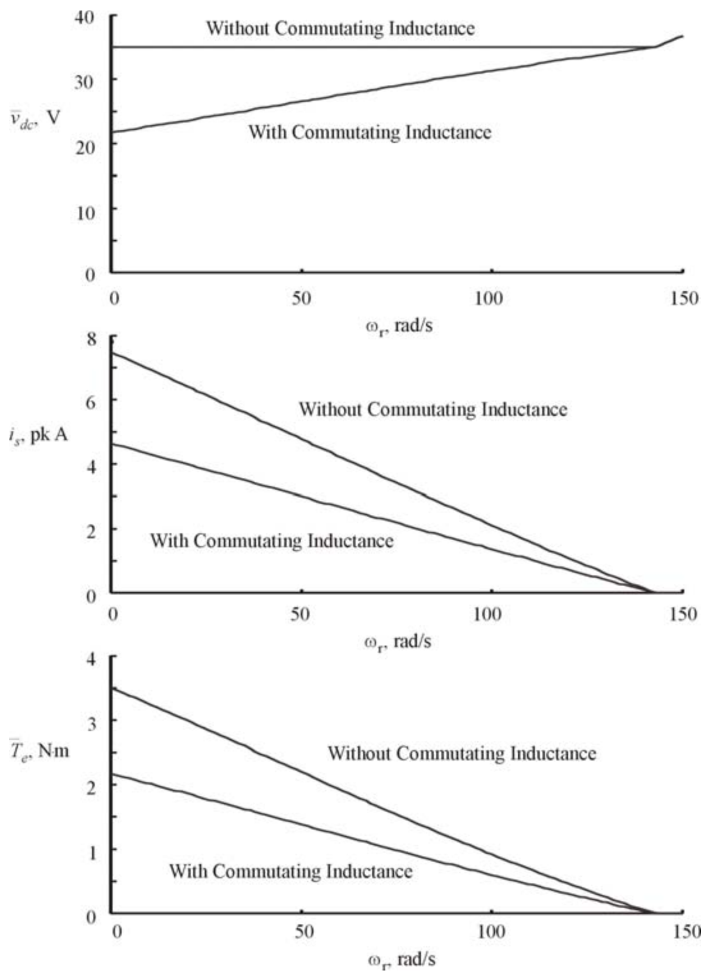
Steady-State Performance of VSI Drives

- Suppose the DC Current from (15.5-14) is Negative
  - ➢This Can't Happen
  - ➢Thus
    - $\bar{I}_{qs}^{r}\cos\phi_v - \bar{I}_{ds}^{r}\sin\phi_v = 0$ (15.5-10)
  - ➢Substitution of (15.5-7) and (15.5-8) Into (15.5-10) Yields
    - $\bar{V}_{dc}\Big|_{\bar{I}_{dc}=0} = \dfrac{\omega_r \lambda_m'(r_s\cos\phi_v - \omega_r L_q \sin\phi_v)}{m(r_s + \frac{1}{2}\omega_r(L_d - L_q)\sin 2\phi_v)}$ (15.5-11)

Steady-State Performance of VSI Drives

- Solution Algorithm
  1. Calculate DC Voltage Using (15.5-9)
  2. Calculate QD Currents Using (15.5-7); (15.5-8)
  3. Calculate DC Current Using (15.5-14)
  4. If DC Current >= 0, Goto 7
  5. Calculate DC Voltage Using (15.5-11)
  6. Goto 2
  7. Done

Steady-State Performance of VSI Drives



15.6 Transient Performance of VSI Drives

- Linearizing (15.4-21) We Have

$$
p\begin{bmatrix}\Delta\bar{i}_r & \Delta\bar{v}_{dc} & \Delta\bar{i}_{st} & \Delta\bar{v}_{st} & \Delta\bar{i}_{qs}^r & \Delta\bar{i}_{ds}^r & \Delta\omega_r\end{bmatrix}^T =
$$

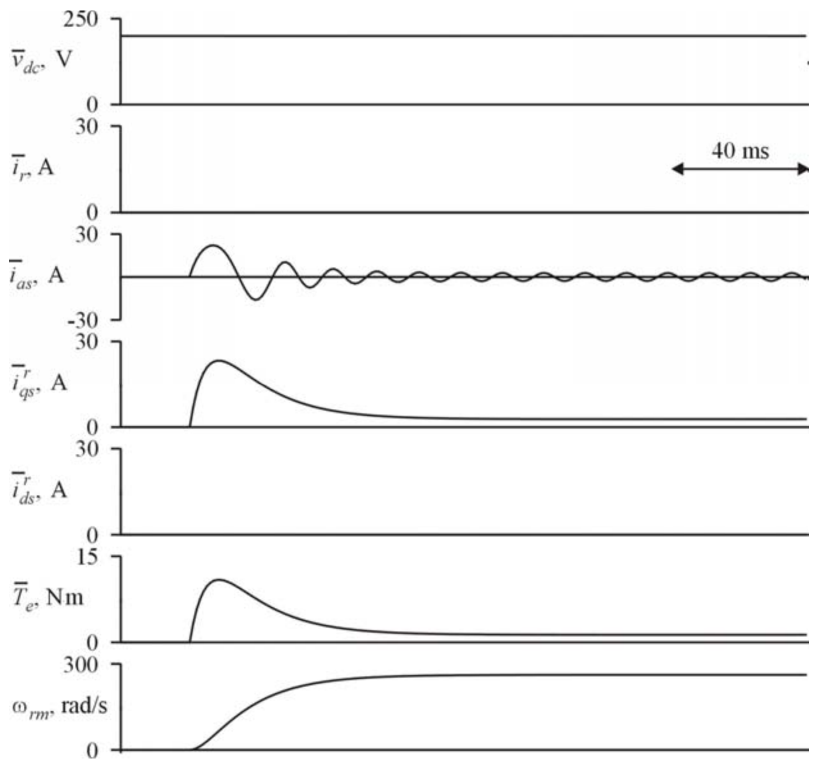
$$
\begin{bmatrix}
-\frac{r_{rl}}{L_{rl}} & -\frac{1}{L_{rl}} & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{C_{dc}} & 0 & -\frac{1}{C_{dc}} & 0 & -\frac{3}{2}\frac{m_0}{C_{dc}}\cos\phi_{v0} & \frac{3}{2}\frac{m_0}{C_{dc}}\sin\phi_{v0} & 0 \\
0 & \frac{1}{Lst} & -\frac{r_{st}}{Lst} & -\frac{1}{Lst} & 0 & 0 & 0 \\
0 & 0 & \frac{1}{Cst} & 0 & 0 & 0 & 0 \\
0 & \frac{m_0}{L_q}\cos\phi_{v0} & 0 & 0 & -\frac{r_s}{L_q} & -\frac{L_d}{L_q}\omega_{r0} & -\frac{L_d}{L_q}\bar{i}_{ds0}^r - \frac{\lambda_m'}{L_q} \\
0 & -\frac{m_0}{L_d}\sin\phi_{v0} & 0 & 0 & +\frac{L_q}{L_d}\omega_{r0} & -\frac{r_s}{L_d} & \frac{L_q}{L_d}\bar{i}_{qs0}^r \\
0 & 0 & 0 & 0 & \frac{3}{2}\left(\frac{P}{2}\right)^2\frac{1}{J}(\lambda_m' + (L_d - L_q)\bar{i}_{ds0}^r) & \frac{3}{2}\left(\frac{P}{2}\right)^2\frac{1}{J}(L_d - L_q)\bar{i}_{qs0}^r & 0
\end{bmatrix}
\begin{bmatrix}\Delta\bar{i}_r \\ \Delta\bar{v}_{dc} \\ \Delta\bar{i}_{st} \\ \Delta\bar{v}_{st} \\ \Delta\bar{i}_{qs}^r \\ \Delta\bar{i}_{ds}^r \\ \Delta\omega_r\end{bmatrix}
$$

$$
+\begin{bmatrix}
\frac{v_{r0}}{L_{rl}} & \frac{\cos\alpha_0}{L_{rl}} & 0 & 0 & 0 \\
0 & 0 & -\frac{3}{2}\frac{(\cos\phi_{v0}\bar{i}_{qs0}^r - \sin\phi_{v0}\bar{i}_{ds0}^r)}{C_{dc}} & \frac{3}{2}\frac{m_0(\sin\phi_{v0}\bar{i}_{qs0}^r + \cos\phi_{v0}\bar{i}_{ds0}^r)}{C_{dc}} & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{\bar{v}_{dc0}}{L_q}\cos\phi_{v0} & -\frac{\bar{v}_{dc0}m_0}{L_q}\sin\phi_{v0} & 0 \\
0 & 0 & -\frac{\bar{v}_{dc0}}{L_d}\sin\phi_{v0} & -\frac{\bar{v}_{dc0}m_0}{L_d}\cos\phi_{v0} & 0 \\
0 & 0 & 0 & 0 & -\frac{P}{2}\frac{1}{J}
\end{bmatrix}
\begin{bmatrix}\Delta\cos\alpha \\ \Delta v_{ro} \\ \Delta m \\ \Delta\phi_v \\ \Delta T_L\end{bmatrix}
\qquad (15.6\text{-}1)
$$

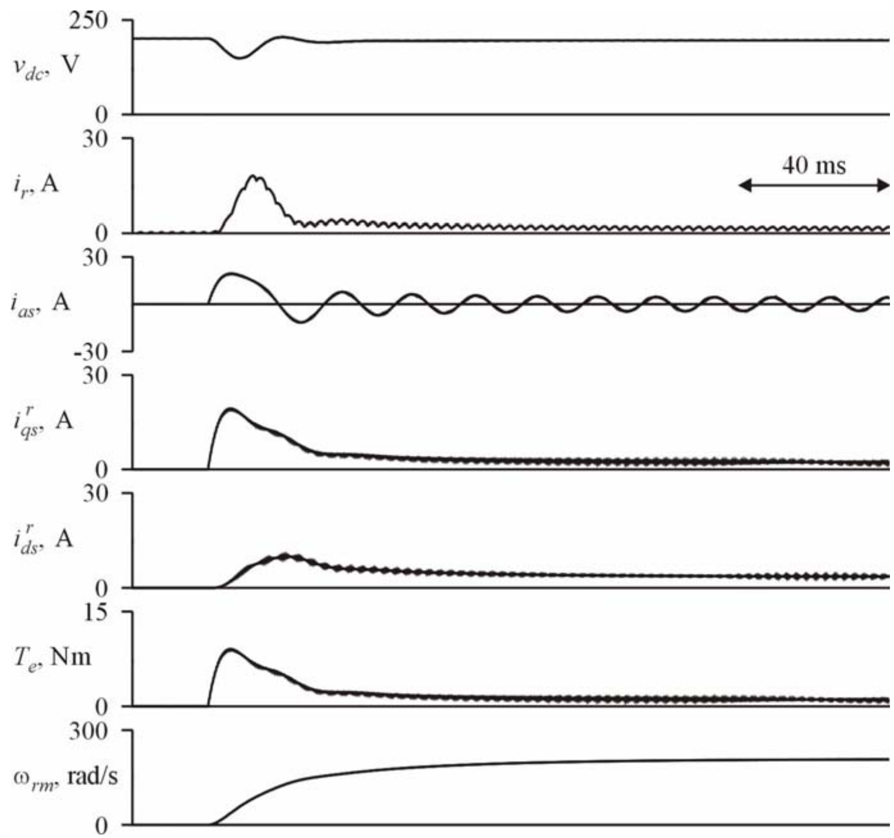
Transient Performance of VSI Drives

- In (15.6-1), Note that
  - $x = x_0 + \Delta x$ (15.6-2)

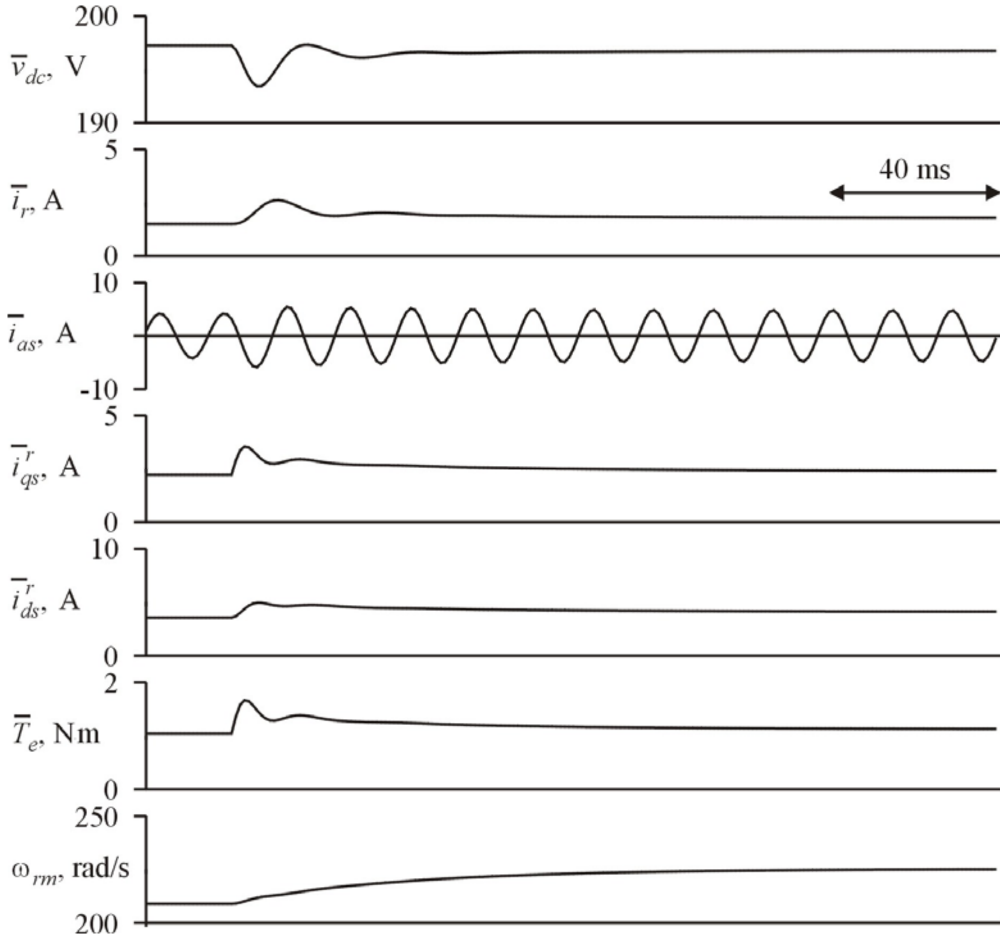
Predictions of Linearized Model Startup



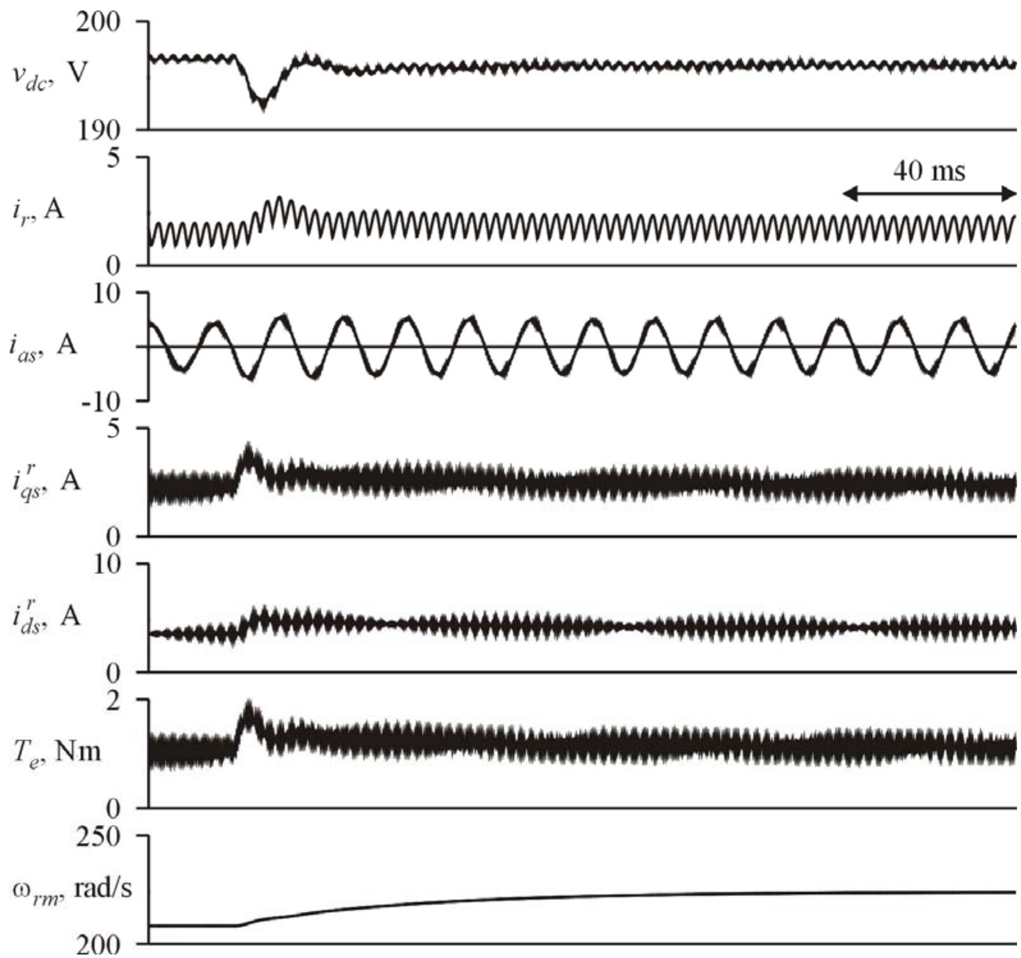
Predictions of Detailed Model Startup



Predictions of Linearized Model Step Change in Duty Cycle



Predictions of Detailed Model Step Change in Duty Cycle



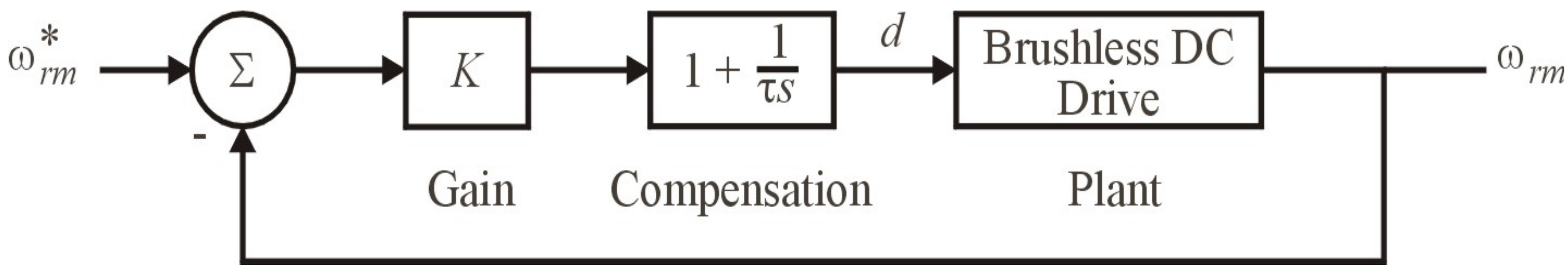
15.8 Case Study: VSI Based Speed Control

- Parameters

E	$85.5\ V$	C_{dc}	$1000\ \mu F$	L_d	$11.4\ mH$
ω_{eu}	$377\ rad/s$	J	$5\ mN{\cdot}m{\cdot}s^2$	λ'_m	$0.156\ Vs$
L_c	$5\ mH$	r_s	$2.98\,\Omega$	P	4
L_{dc}	$5\ mH$	L_q	$11.4\ mH$		

- Design Objectives
  - Load is Inertial
  - No Steady-State Error
  - Phase Margin of 60^{o} at Nominal Speed of 200 rad/s (mechanical)

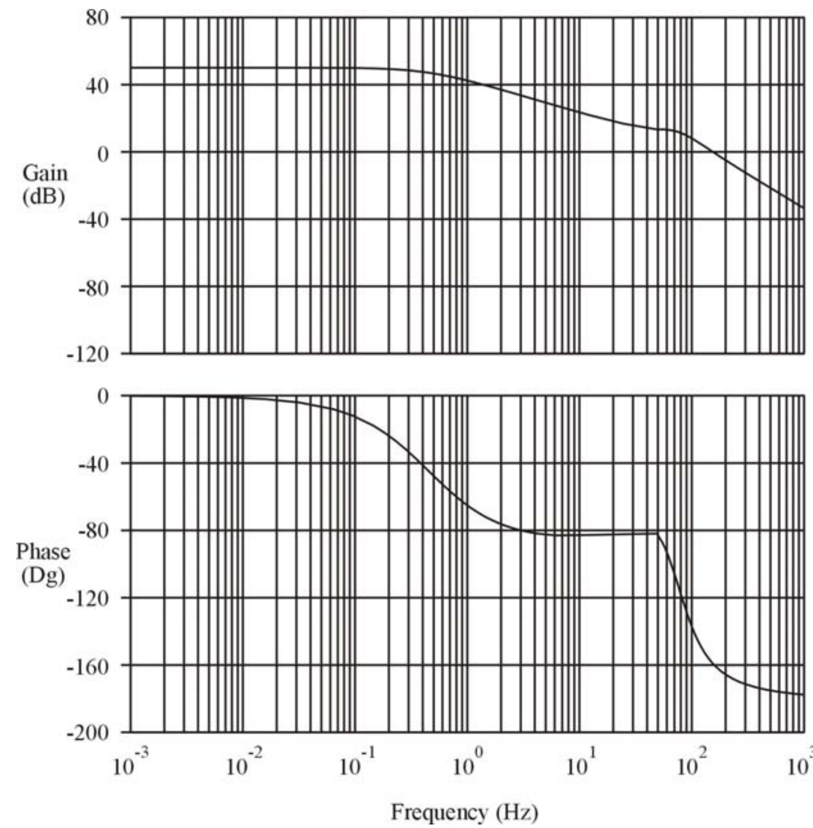
Control Law



- Mathematically
  - $d = K(\omega^*_{rm} - \omega_{rm}) + \frac{K}{\tau}\int(\omega^*_{rm} - \omega_{rm})dt$ (15.8-1)

Starting Point: Open Loop Frequency Response

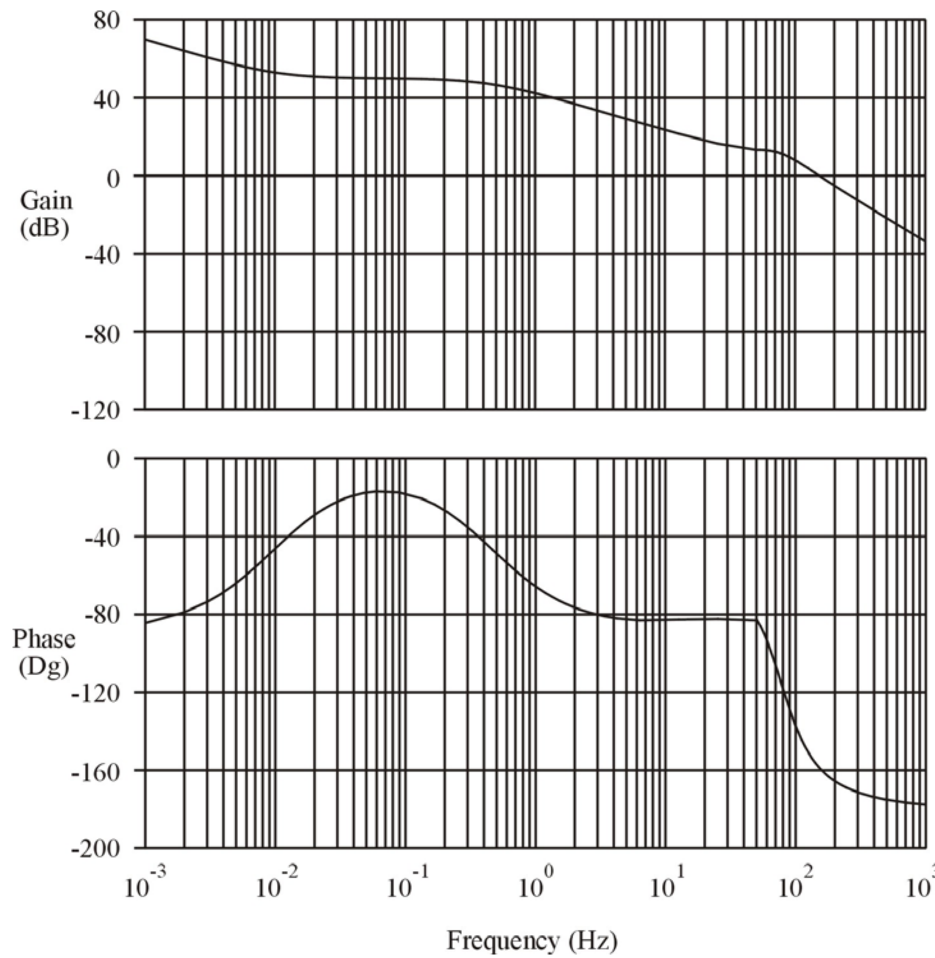
- Observe
  - Gain Margin is Infinite
  - Phase Margin is 20°



Selection of τ

- Integral Feedback Will Decrease Phase By 90^o at Frequencies Less Than $1/(2\pi\tau)$
- We Will Select Breakpoint Frequency at 0.01 Hz for τ of 16 s

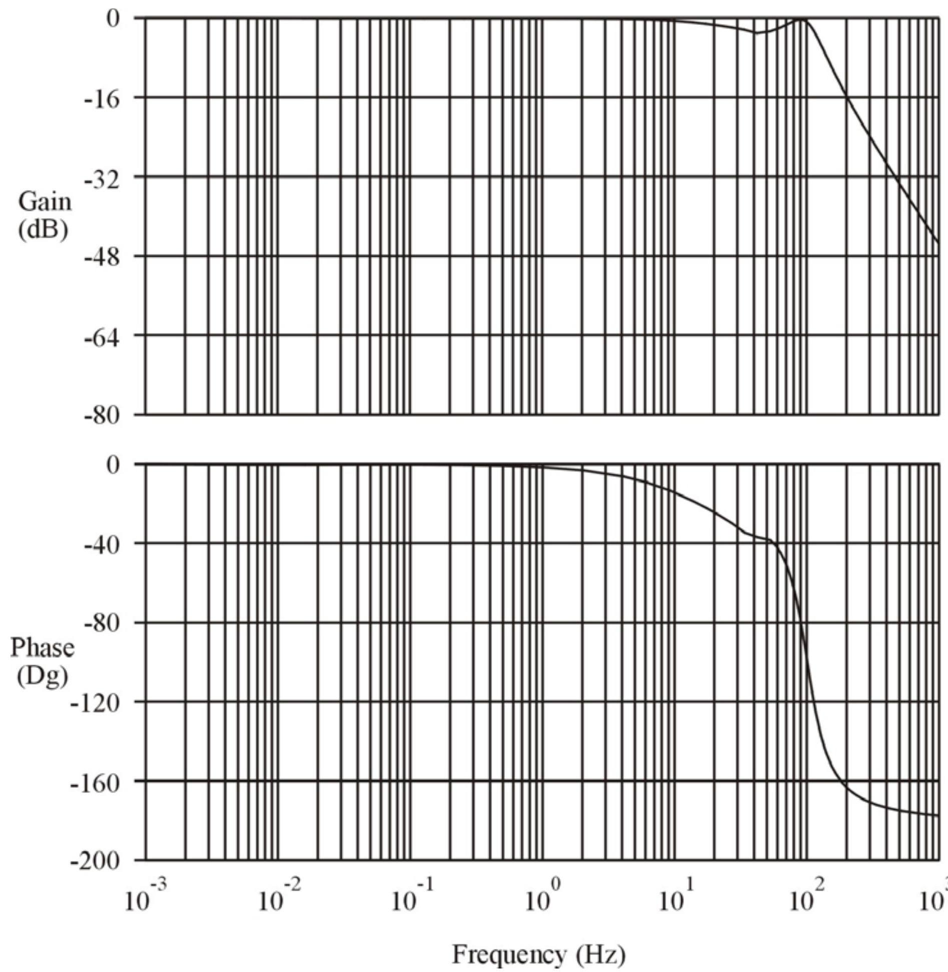
Compensated Frequency Response



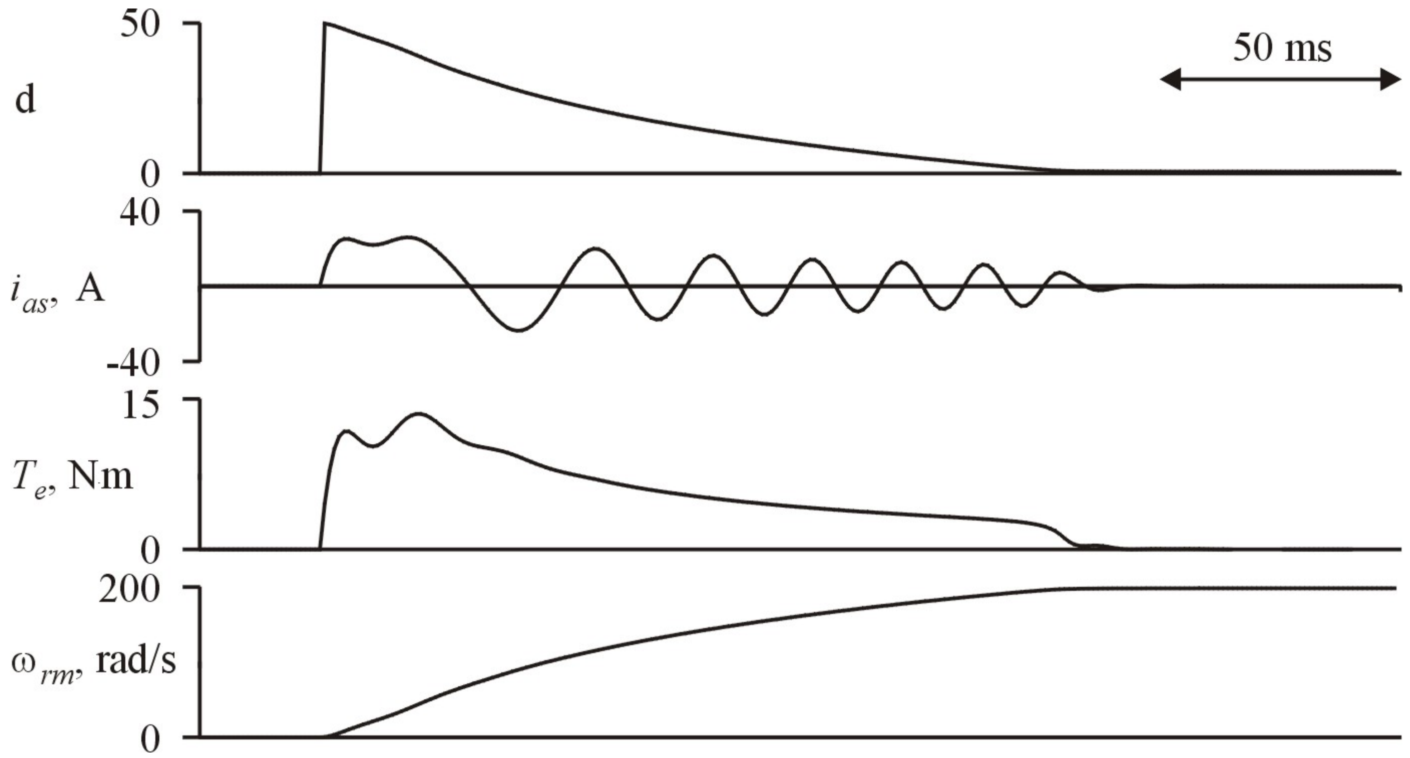
Selection of K

- Select K to Achieve Phase Margin of 60^o
- Thus, We Choose K=0.25 (-12 dB)

Compensated Frequency Response



Time-Domain Response



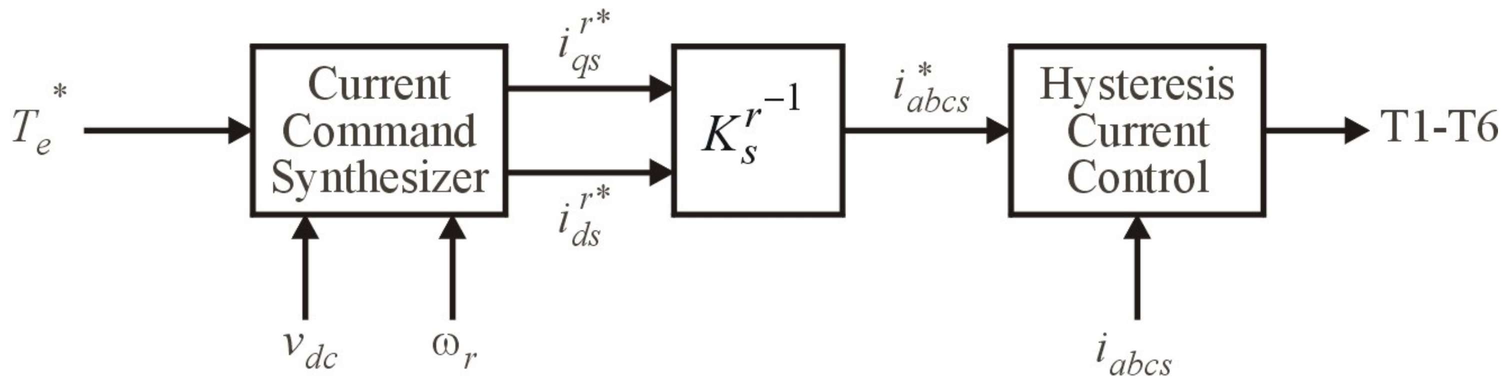
Critique of Design

- Drive Becomes Quite Overmodulated
- Bandwidth is Not Nearly 100 Hz
- Significant Overcurrent (Rated is 2.6 A, rms)
- The Speed is not 200 rad/s At End of Study

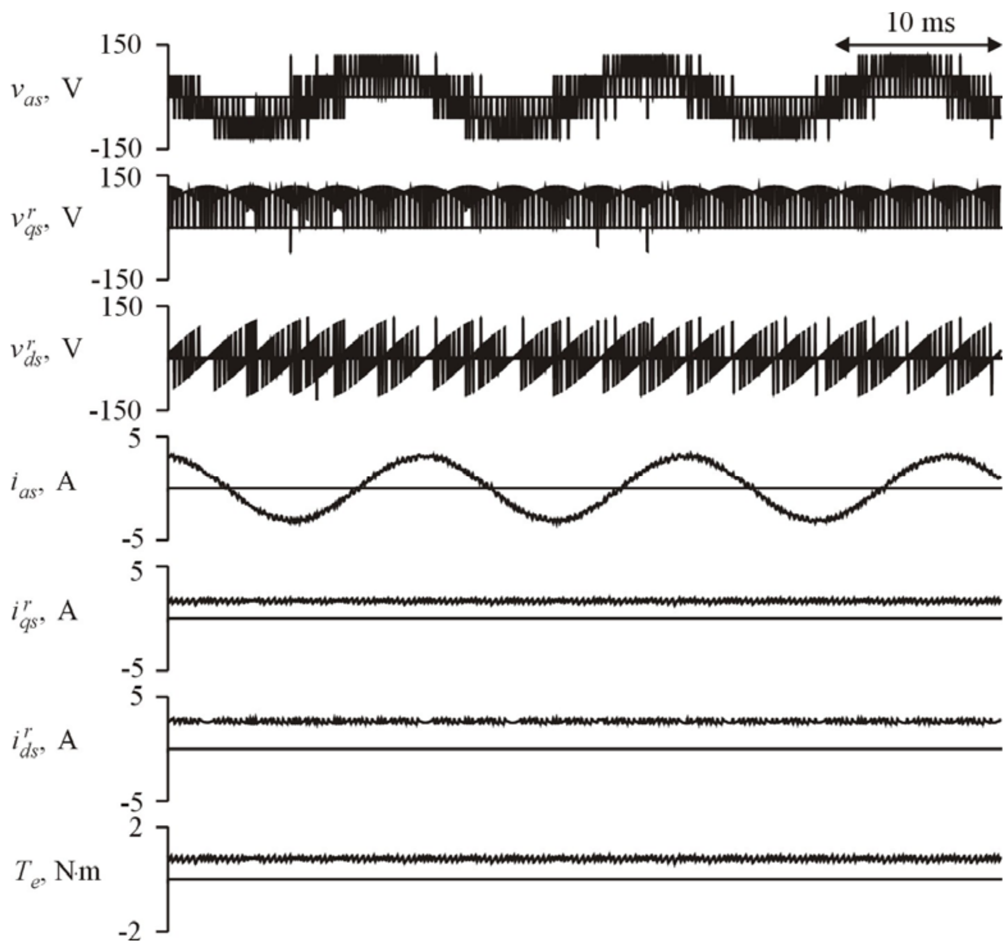
15.9 Current-Regulated Drives

- Current Based Inverter Current Regulation
  - ➢Advantages
    - Torque Control Bandwidth
    - Robustness w.r.t. Machine Parameters
    - Robust w.r.t. Faults
    - Control design with lower order system
  - ➢Disadvantage
    - Switching Frequency Cannot Be Directly Controlled

Hysteresis Current-Regulated Drive



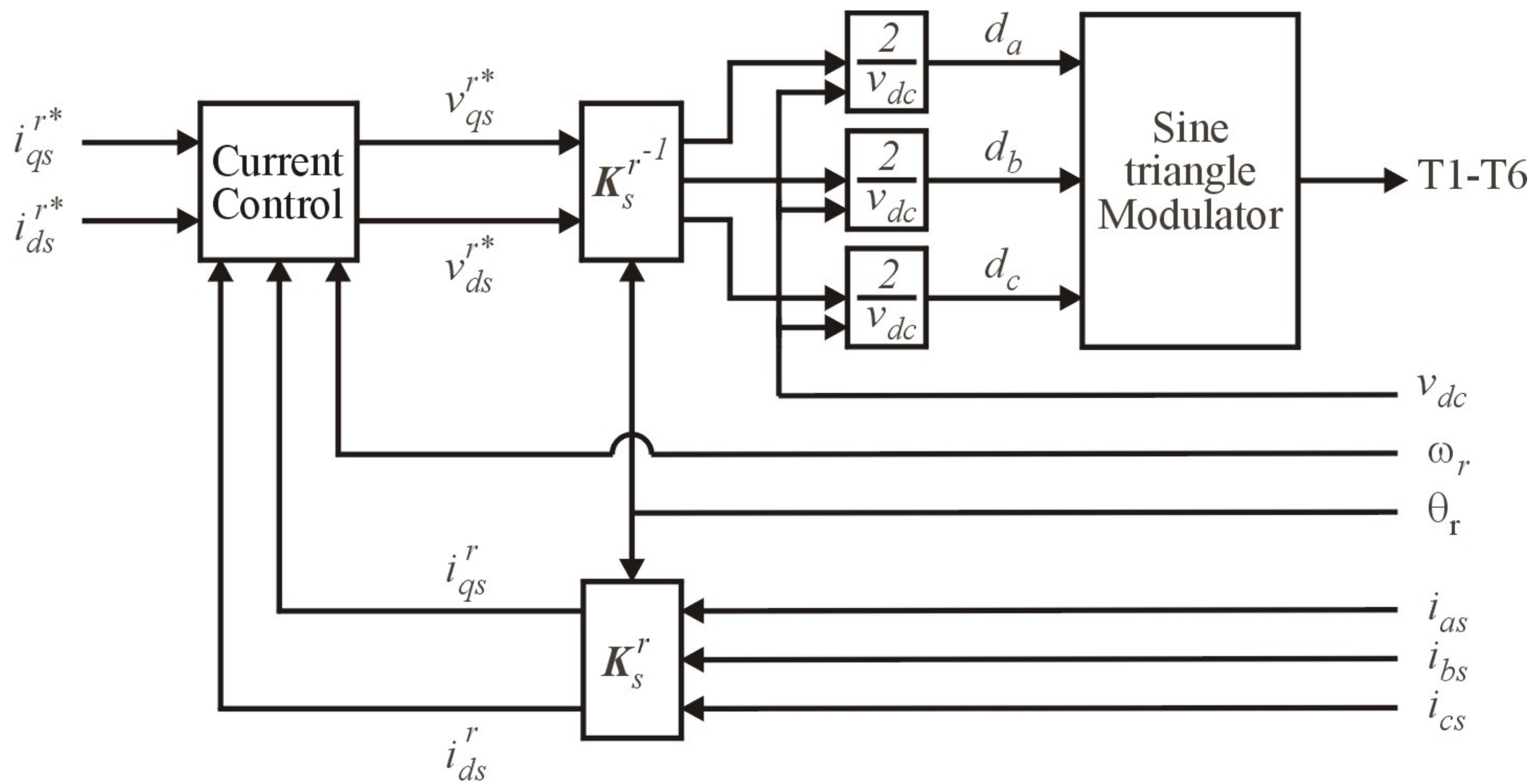
Performance of Hysteresis Controlled Drive



Voltage Source Based Current Regulation

- Advantages
  - ➢Controlled Switching Frequency
  - ➢Can Offer Excellent Waveform Quality
- Disadvantage
  - ➢Lower Control Bandwidth

Sine Triangle Current Regulator



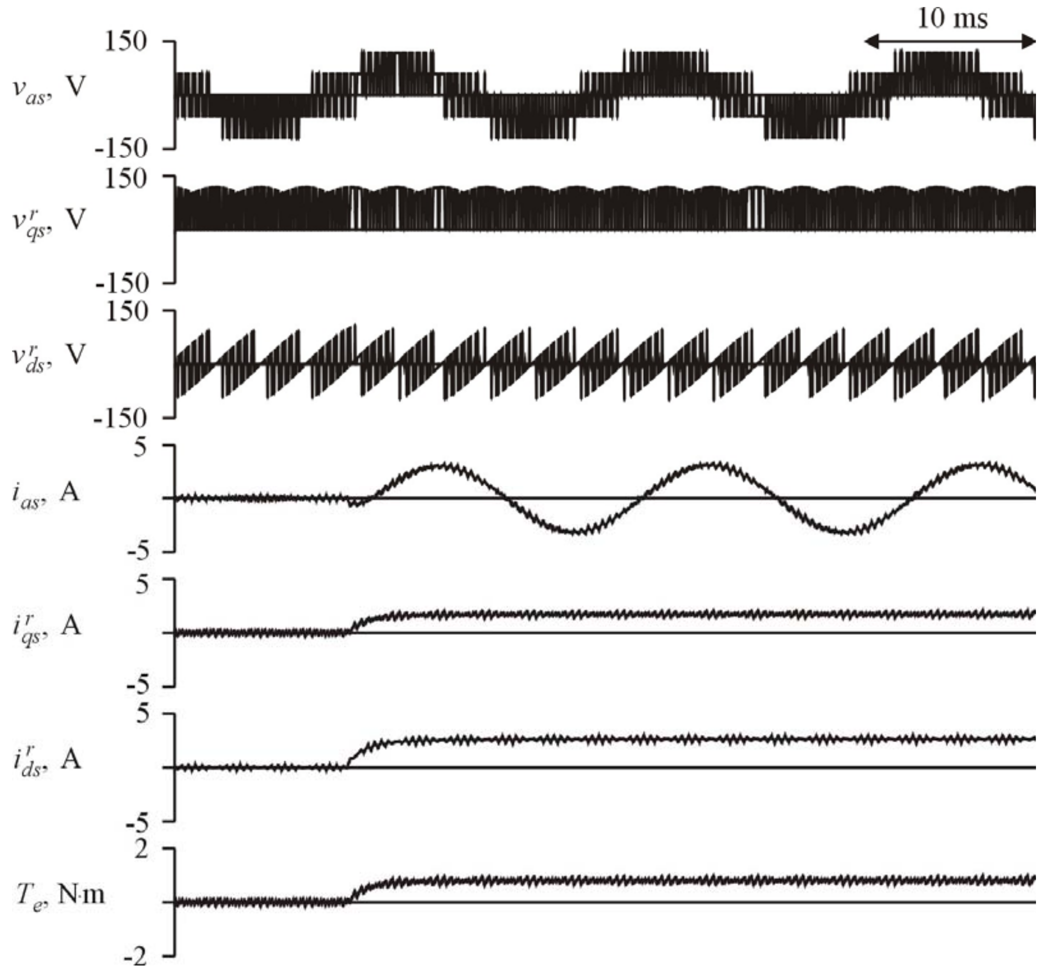
Example 15B

- Suppose We Utilize
  - $$v_{qs}^{r*} = \omega_r (L_{ss} i_{ds}^r + \lambda_m') + \left(K_p + \frac{K_i}{s} \right)(i_{qs}^{r*} - i_{qs}^r) \qquad (15.B\text{-}1)$$
  - $$v_{ds}^{r*} = -\omega_r L_{ss} i_{qs}^r + \left(K_p + \frac{K_i}{s} \right)(i_{ds}^{r*} - i_{ds}^r) \qquad (15.B\text{-}2)$$
- Place Poles At *s*=-200, *s*=-1000

Example 15B

- Results
  - $K_i = 2280\,\Omega / s$ (15B-4)
  - $K_p = 10.7\,\Omega$ (15B-5)

Performance During Step Change In Current



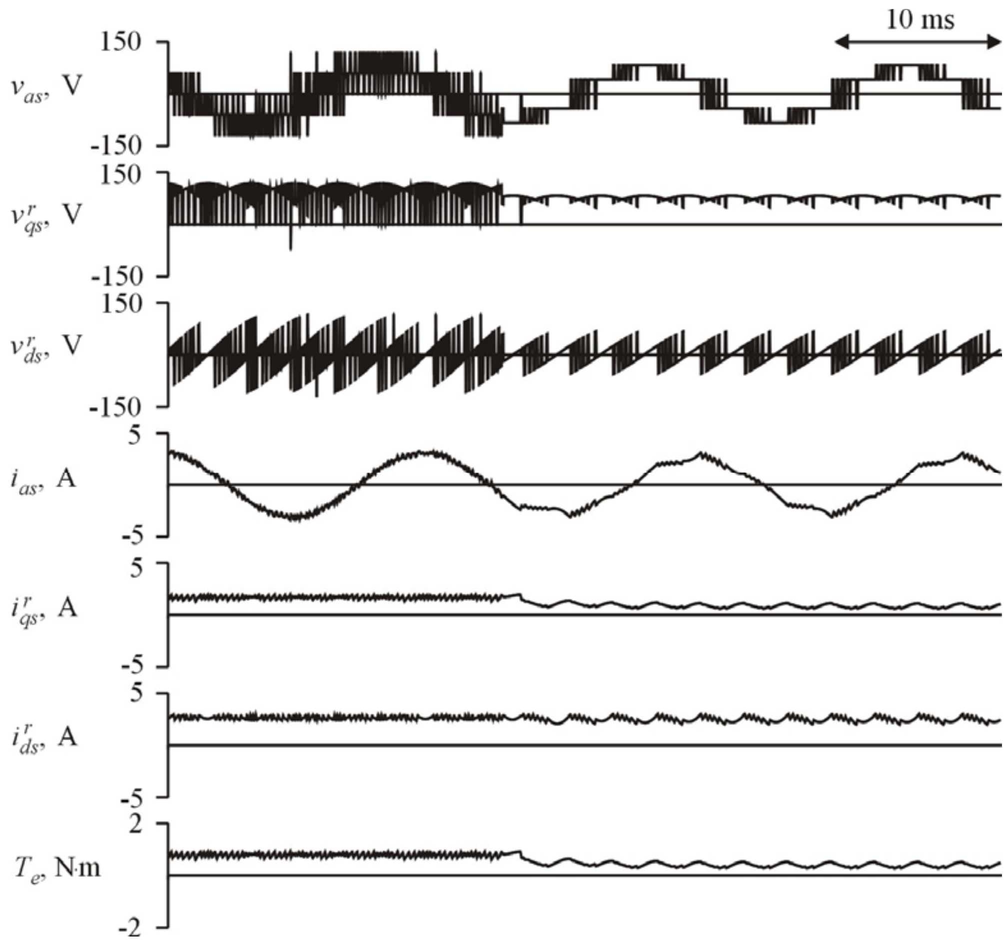
15.10 Voltage Limitations of Current-Source Inverter Drives

- Lets Assume
  - $\bar{i}_{qs}^{r} = i_{qs}^{r*}$ (15.10-1)
  - $\bar{i}_{ds}^{r} = i_{ds}^{r*}$ (15.10-2)
- Then
  - $\bar{v}_{qs}^{r} = r_s i_{qs}^{r*} + \omega_r L_d i_{ds}^{r*} + \omega_r \lambda_m'$ (15.10-3)
  - $\bar{v}_{ds}^{r} = r_s i_{ds}^{r*} - \omega_r L_q i_{qs}^{r*}$ (15.10-4)

15.10 Voltage Limitations of Current-Source Inverter Drives

- Recall That
  - $$v_s = \frac{1}{\sqrt{2}}\sqrt{\left(\bar{v}_{qs}^r\right)^2 + \left(\bar{v}_{ds}^r\right)^2} \qquad (15.10\text{-}5)$$
- Thus
  - $$v_s = \frac{1}{\sqrt{2}}\sqrt{(r_s i_{qs}^{r*} + \omega_r L_d i_{ds}^{r*} + \lambda_m' \omega_r)^2 + (r_s i_{ds}^{r*} - \omega_r L_q i_{qs}^{r*})^2} \qquad (15.10\text{-}6)$$
- Also Recall
  - $$v_s = \frac{1}{\sqrt{6}}\bar{v}_{dc}$$ (15.10-7) (w/o Harmonics)
  - $$v_s = \frac{\sqrt{2}}{\pi}\bar{v}_{dc}$$ (15.10-8) (w Harmonics)

Response to Step Decrease in DC Voltage



15.11 Current Command Synthesis (Non-Salient Machines)

- Recall
  - $T_e = \frac{3}{2}\frac{P}{2}\lambda'_m i^r_{qs}$ (15.11-1)
- Thus
  - $i^{r*}_{qs} = \frac{2}{3}\frac{2}{P}\frac{1}{\lambda'_m}T_e{}^*$ (15.11-2)
  - $i^{r*}_{ds} = 0$ (15.11-3)
- Advantages

- But..

15.11 Current Command Synthesis (Non-Salient Machines)

- Voltage Requirement
  - $v_s = \frac{1}{\sqrt{2}}\sqrt{(r_s i_{qs}^{r*} + \omega_r L_{ss} i_{ds}^{r*} + \lambda_m' \omega_r)^2 + (r_s i_{ds}^{r*} - \omega_r L_{ss} i_{qs}^{r*})^2}$ (15.11-4)
- If Not Enough Voltage
  - $i_{ds}^{r*} = \frac{-\lambda_m' L_{ss} \omega_r^2 + \sqrt{2z^2 v_s^2 - (r_s \omega_r \lambda_m' + z^2 i_{qs}^{r*})^2}}{z^2}$ (15.11-5)
  - $z = \sqrt{r_s^2 + \omega_r^2 L_{ss}^2}$ (15.11-6)
- This is Referred to as Flux Weakening
- Warning:

15.11 Current Command Synthesis (Salient Machines)

- Recall
  - $$T_e = \frac{3}{2}\frac{P}{2}(\lambda'_m i^r_{qs} + (L_d - L_q) i^r_{qs} i^r_{ds}) \qquad (15.11\text{-}7)$$
- Thus
  - $$i^{r*}_{ds} = \frac{T_e}{\frac{3}{2}\frac{P}{2}(L_d - L_q)} \frac{1}{i^{r*}_{qs}} - \frac{\lambda'_m}{L_d - L_q} \qquad (15.11\text{-}8)$$
- Also Recall
  - $$i_s = \frac{1}{\sqrt{2}}\sqrt{(i^{r*}_{qs})^2 + (i^{r*}_{ds})^2} \qquad (15.11\text{-}9)$$

15.11 Current Command Synthesis (Salient Machines)

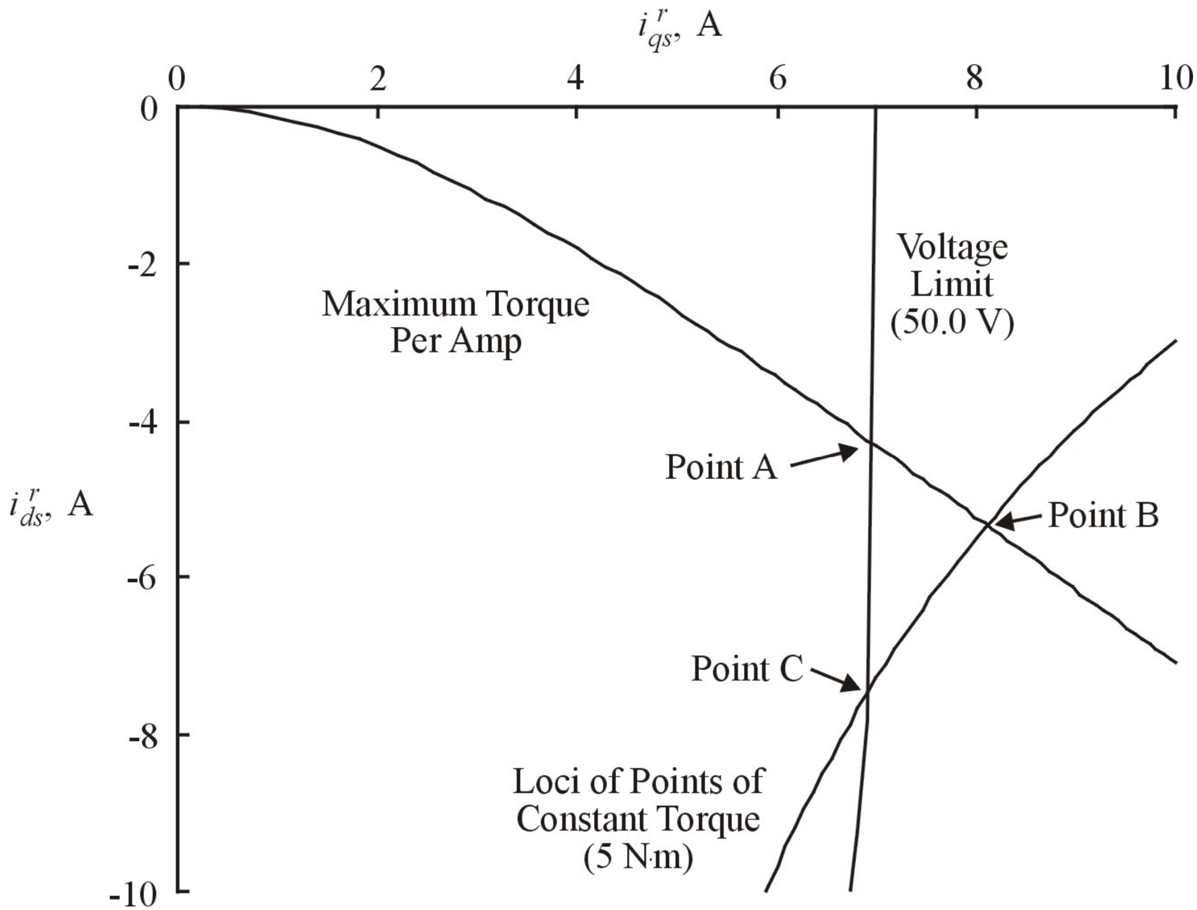
- Strategy: Minimize Fundamental Component of Current
- Why?

15.11 Current Command Synthesis (Salient Machines)

- Result

- In this Case, What If Inadequate Voltage

15.11 Current Command Synthesis (Salient Machines)



15.12 Average-Value Modeling of Current Regulated Drives

- We Begin With
  - $\bar{i}_{qs}^{r} = i_{qs}^{r*}$ (15.12-1)
  - $\bar{i}_{ds}^{r} = i_{ds}^{r*}$ (15.12-2)
- Ignoring Stator Transients
  - $\bar{v}_{qs}^{r*} = r_s i_{qs}^{r*} + \omega_r L_d i_{ds}^{r*} + \lambda_m' \omega_r$ (15.12-3)
  - $\bar{v}_{ds}^{r} = r_s i_{ds}^{r*} - \omega_r L_q i_{qs}^{r*}$ (15.12-4)
- Thus
  - $P = \frac{3}{2}[r_s (i_{qs}^{r*} + i_{ds}^{r*})^2 + \omega_r (L_d - L_q) i_{qs}^{r*} i_{ds}^{r*} + \omega_r \lambda_m' i_{qs}^{r}]$ (15.12-5)

15.12 Average-Value Modeling of Current Regulated Drives

- Since
  - $\bar{i}_{dc} = \dfrac{P}{\bar{V}_{dc}}$ (15.12-6)
- We Have
  - $\bar{i}_{dc} = \frac{3}{2}\frac{1}{\bar{V}_{dc}}[r_s(i_{qs}^{r*} + i_{ds}^{r*})^2 + \omega_r(L_d - L_q)i_{qs}^{r*}i_{ds}^{r*} + \omega_r\lambda_m' i_{qs}^{r}]$ (15.12-7)
- Again Assuming Our Currents Are Equal to Commanded Values
  - $T_e = \frac{3}{2}\frac{P}{2}(\lambda_m' i_{qs}^{r*} + (L_d - L_q)i_{qs}^{r*}i_{ds}^{r*})$ (15.12-8)

15.12 Average-Value Modeling of Current Regulated Drives

- Resulting State Model
  - $$p\begin{bmatrix} \bar{i}_r \\ \bar{v}_{dc} \\ \bar{i}_{st} \\ \bar{v}_{st} \\ \omega_r \end{bmatrix} = \begin{bmatrix} -\frac{r_{rl}}{L_{rl}} & -\frac{1}{L_{rl}} & 0 & 0 & 0 \\ \frac{1}{C_{dc}} & 0 & -\frac{1}{C_{dc}} & 0 & 0 \\ 0 & \frac{1}{L_{st}} & -\frac{r_{st}}{L_{st}} & -\frac{1}{L_{st}} & 0 \\ 0 & 0 & \frac{1}{C_{st}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{i}_r \\ \bar{v}_{dc} \\ \bar{i}_{st} \\ \bar{v}_{st} \\ \omega_r \end{bmatrix} \qquad (15.12\text{-}9)$$
    $$+\begin{bmatrix} \dfrac{v_{r0}\cos\alpha}{L_{rl}} \\ -\frac{1}{C_{dc}}\frac{3}{2}\frac{1}{\overline{\overline{V}}_{dc}}[r_s(i_{qs}^{r*}+i_{ds}^{r*})^2+\omega_r(L_d-L_q)i_{qs}^{r*}i_{ds}^{r*}+\omega_r\lambda_m' i_{qs}^{r*}] \\ 0 \\ 0 \\ \frac{P}{2}\frac{1}{J}[\frac{3}{2}\frac{P}{2}(\lambda_m' i_{qs}^{r*}+(L_d-L_q)i_{qs}^{r*}i_{ds}^{r*})-T_L] \end{bmatrix}$$

15.3 Case Study: Current-Regulated Inverter-Based Seed Control

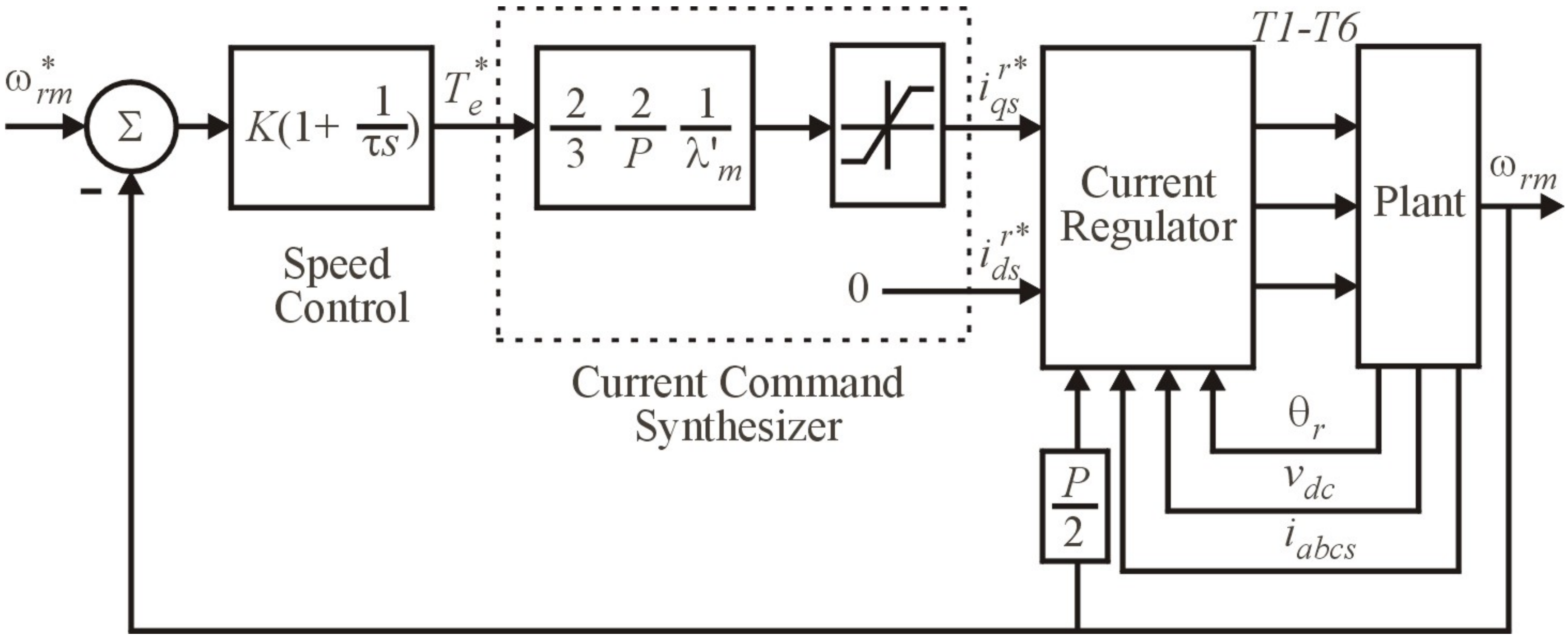
- Chief Assumption
  - $T_e = T_e^*$ (15.13-1)
- We'll Use
  - $T_e^* = K_p(1+\frac{1}{\tau s})(\omega_{rm}^* - \omega_{rm})$ (15.13-2)
- Thus We Obtain
  - $\frac{\omega_{rm}}{\omega_{rm}^*} = \frac{\frac{K}{J\tau}(\tau s+1)}{s^2+\frac{K}{J}s+\frac{K}{J\tau}}$ (15.13-3)

15.3 Case Study: Current-Regulated Inverter-Based Seed Control

- Let Us Place Poles At
  - s=-5, s=-50
- This Yields
  - K_p=0.257 Nms, τ=0.22 s

- Other Design Aspects
  - Use Sine Triangle Current Regulator From Example 15 B (s=-200, s=-1000)
  - Limit Currents to +/- 3.68 A
  - Inject All Current into Q-Axis

15.3 Case Study: Current-Regulated Inverter-Based Seed Control



15.3 Case Study: Current-Regulated Inverter-Based Seed Control