3 / Control of Permanent Magnet Synchronous Machines

In this chapter, we consider the basic principals of control of permanent magnet synchronous machines. In the event that the PMSM is equipped with damper windings on the rotor, it is possible to create a PMSM which can be connected and operated directly from the three phase utility grid. In this case, the PMSM is acting as a traditional synchronous machine, and will have a steady-state speed corresponding to the applied frequency. However, it is much more common that the PMSM is driven from a device known as an inverter, in which the applied voltages are currents are explicit functions of the electrical rotor position. In this mode of operation, the machine characteristics are much more similar to a permanent magnet dc machine than a synchronous machine and so under these conditions the device is referred to a brushless dc machine. This is the mode of operation considered in this work.

Inverters can be configured to appear either as a voltage source or as a current source. As we saw in Chapter 2, however, that operation from a voltage source results in large over currents at low speeds; thus this mode of operation is only appropriate for very low power machines. Using the inverter to control the currents results in much more desirable operating characteristics and so will be the configuration considered here.

This chapter will proceed as follows. First, Section 3.1 will set forth the overall architecture of brushless dc machine control. Next, operating constraints which impact the control are discussed in Section 3.2. Section 3.3 addresses the question of how to pick the best possible current command to achieve control objectives. Next, Section 3.4 and 3.5 discuss two possible control methodologies, each of which can ensure that actual current is equal to the desired current. A case study comparing the controls is set forth in Section 3.6.

3.1 CONTROL STRATEGIES

Herein, we will consider the drive architecture depicted in Fig. 3.1-1. Therein, the dc source may be a transformer rectifier or a battery. The output voltage of this source is denoted v_{dc} and is the supply voltage for an inverter, whose ac terminals are connected to the PMSM.

It is assumed herein that it is our objective to control the PMSM in such a way that the electromagnetic torque produced is equal to a desired value denoted T_e^*. Speed or position controls will be at a higher level and determine T_e^* such that the desired speed or position trajectory is obtained; this control is highly application dependent and not considered here.

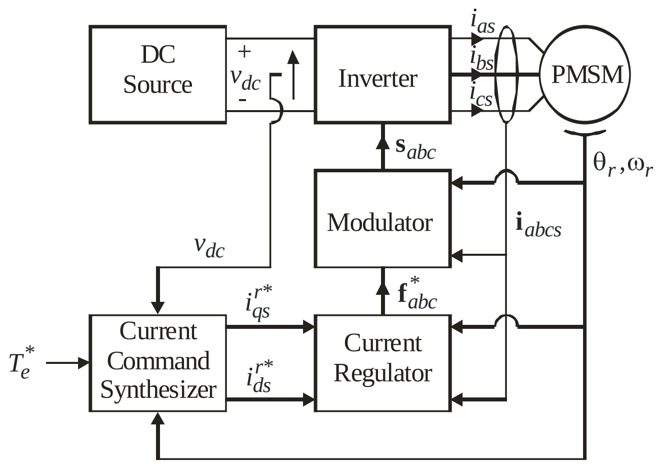

Figure 3.1-1. Control architecture.

Based upon the torque command T_e^*, the dc voltage v_{dc}, and the electrical rotor speed ω_r, the current command synthesizer determines the desired value of q- and d-axis currents, i_{qs}^{r*} and i_{ds}^{r*}, which will achieve the control objective (i.e. $T_e = T_e^*$) while at the same time ensuring that all operating constraints are met.

The current regulator attempts to force the actual q- and d-axis currents towards the commanded values. The inputs to the regulator are the q- and d-axis current command, the measured phase currents, which are assumed herein to be equal to the actual phase currents $\mathbf{i}_{abcs}$, and the electrical rotor position and speed, θ_r and ω_r. Depending upon the type of current regulator, it will generator either a phase variable voltage or phase variable current command to the modulator (generically described as $\mathbf{f}_{abc}^*$), which will in turn generator gating signals to the power semiconductors which are denoted $\mathbf{s}_{abc}$ in Fig. 3.3-1.

While the focus of this work will be the current command synthesizer and the current regulator which are particular to the brushless dc motor application; it is instructive to briefly consider a somewhat more detailed view of the inverter / modulator. Figure 3.3-2 depicts a three-phase fully controlled inverter, typical of the variety used to drive 3-phase brushless dc machines. Therein, the line-to-bottom rail voltages are denoted v_{ar}, v_{br}, and v_{cr}. The gating signals to each phase are denoted s_{xy}, where $x \in ['a','b','c']$ and denotes phase; $y \in ['u','l']$ to denote upper or lower position. The output of the modulator has been denoted $s_{abc} = [s_a \quad s_b \quad s_c]^T$. Upon neglecting deadtime logic (the logic used to prevent both the upper and lower switch of a phase from being turned on at the same time) we have that

$$s_{xu} = \bar{s}_{xl} = s_x \qquad (3.1\text{-}1)$$

where the overbar denotes logical complement. Note that upon neglecting forward semiconductor drops when s_x is high, $v_{xg} = v_{dc}$, and when s_x is low, $v_{xg} = 0$.

In this chapter, we will focus our attention on the two parts of the control particular to the brushless dc machine, the current command synthesizer and the current regulator. As alluded to earlier, the supervisory control determining the torque command is highly application specific and so will not be considered here; the modulator is very generic, and is treated in power electronics texts such as [1].

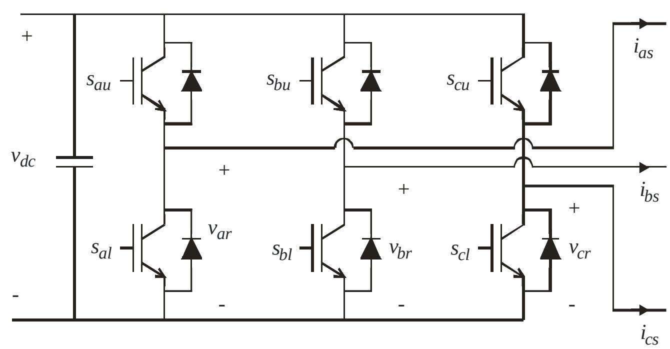

Figure 3.1-2. Inverter.

3.2 OPERATING CONSTRAINTS

The first control block we will consider will be the current command synthesizer. However, before we can do this is appropriate to identify all pertinent operating constraints. The first constraint arises from thermal limits on the machine and semiconductor limits of the inverter. In particular, we require that the rms phase current must be less than a given limit for the machine, i.e.

$$i_s \le i_{s,mx} \qquad (3.2\text{-}1)$$

The second limit is related to the inverter output voltage capabilities. For a given dc rail voltage, it can be shown that the largest rms phase voltage (of the fundamental component) that can be produced is [1]

$$v_{s,mx} = \frac{1}{\sqrt{6}} v_{dc} \qquad (3.2\text{-}2)$$

Since the output voltage is limited, we must operate the machine such that

$$v_s \le v_{s,mx} \tag{3.2-3}$$

There is a turn a limit on the dc voltage; this is driven by the inverter semiconductors and the machine winding insulation.

The final operating constraint that must be met is related to the d-axis current. In particular, we will find that in order to meet (3.2-3), it is often desirable to inject a negative d-axis current. However, an overly negative d-axis current can permanently demagnetize the permanent magnet and so must be avoided. For this reason, we will limit the d-axis current command to be above a pre-set threshold,

$$i_{ds}^{r*} > i_{ds,mn}^{r} \tag{3.2-4}$$

In (3.2-4), $i_{ds,mn}^{r}$ is a negative number.

This is the last of the operating constraints. We are now prepared to address the problem of current command synthesis.

3.3 CURRENT COMMAND SYNTHESIS

In this section, we consider the problem of current command synthesis, which is to say the problem of computing the desired currents so that the desired torque is achieved subject to satisfaction of all operating constraints. In this work, we will concentrate on non-salient machines, though some comments on salient machines will be made towards the conclusion of this section. There are two main operating modes, maximum torque per amp mode (also referred to as the constant torque region), and flux weakening mode (also referred to as the constant power region). The motivation for maximum torque per amp mode begins with the observation that the power into the machine may be expressed [2]

$$P_{in} = \frac{3}{2}\left(v_{qs}^r i_{qs}^r + v_{ds}^r i_{ds}^r\right) \tag{3.3-1}$$

Substitution of the q- and d-axis voltage equations (2.4-1) and (2.4-2) into (3.3-1) yields

$$P_{in} = \frac{3}{2}\left((r_s i_{qs}^r + \omega_r L_d i_{ds}^r + \omega_r \lambda_m)\, i_{qs}^r + (r_s i_{ds}^r - \omega_r L_q i_{qs}^r)\, i_{ds}^r\right) \tag{3.3-2}$$

Recall from (2.4-3) that the torque may be expressed

$$T_e = \frac{3}{2}\frac{P}{2}(\lambda_m i_{qs}^r - (L_q - L_d) i_{qs}^r i_{ds}^r) \tag{3.3-3}$$

Comparing (3.3-3) and (3.3-2) we have that

$$P_{in} = \frac{3}{2} r_s \left((i_{qs}^r)^2 + (i_{ds}^r)^2 \right) + \frac{2}{P} \omega_r T_e \tag{3.3-4}$$

which reduces to

$$P_{in} = 3 r_s i_s^2 + \frac{2}{P} \omega_r T_e \tag{3.3-5}$$

Thus, input power can be minimized (and efficiency maximized) by determining a q- and d-axis current command which maximizes the torque produced for a given rms phase current i_s. Controlling the machine in such a fashion is referred to as maximum torque per amp mode. While highly desirable, at a sufficiently high speed this mode is no longer achievable because eventually the voltage required will be in excess of that available. This leads to flux weakening mode.

3.3.1 Current Command Synthesis for Non-salient Machines

Let us now consider the motor operation of a non-salient machine. In particular, we will first turn our attention towards the maximum torque per amp operating mode.

Maximum Torque Per Amp Mode

Since the machine is non-salient, torque may be expressed

$$T_e = \frac{3}{2} \frac{P}{2} \lambda_m i_{qs}^r \tag{3.3-6}$$

Thus, to achieve a desired torque T_e^* the desired q-axis current command may be expressed

$$i_{qs}^{r*} = \frac{4 \min\left(\left| T_e^* \right|, T_{e,mx1} \right)}{3 \lambda_m P} \operatorname{sgn}(T_e^*) \tag{3.3-7}$$

where $T_{e,mx1}$ is the maximum allowed torque in this mode. A value for $T_{e,mx1}$ will be established in the subsequent discussion.

Since d-axis current will increase the phase current i_s, but not contribute to torque, the d-axis current command is set to zero, i.e.

$$i_{ds}^{r*} = 0 \tag{3.3-8}$$

Thus, (3.3-7) and (3.3-8) are the basis of our current command synthesizer. However, there are some practical constraints. First, because of thermal constraints on the motor and because of current limitations of the inverter, we have the current limit (3.2-1). Since the d-axis current is zero, is readily shown that

$$i_{qs,mx1}^{r} = \sqrt{2} i_{s,mx} \tag{3.3-9}$$

It follows that the maximum torque we can achieve in this mode is given by

$$T_{e,mx1} = \frac{3}{2}\frac{P}{2}\lambda_m \sqrt{2} i_{s,mx} \tag{3.3-10}$$

There is a maximum speed to which maximum torque pr amp control can be used. This is because of voltage limitation as expressed in (3.2-3). To evaluate this limit, suppose that we achieve the desired current. It follows that

$$v_{qs}^{r} = r_s i_{qs}^{r*} + \omega_r \lambda_m \tag{3.3-11}$$

$$v_{ds}^{r} = -\omega_r L_{ss} i_{qs}^{r*} \tag{3.3-12}$$

The rms value of the fundamental component of the applied voltage may be expressed

$$\sqrt{2} v_s = \sqrt{\left(v_{qs}^{r}\right)^2 + \left(v_{ds}^{r}\right)^2} \tag{3.3-13}$$

Substitution of (3.3-11) and (3.3-12) into (3.3-13), and the result into (3.2-3) and solving for the speed where the voltage limitation is reached, we have

$$\omega_{r,bnd}\left(i_{qs}^{r*}\right) = \frac{-r_s \lambda_m i_{qs}^{r*} \pm \sqrt{\left[r_s \lambda_m i_{qs}^{r*}\right]^2 + \left[L_{ss}^2 i_{qs}^{r*2} + \lambda_m^2\right]\left[2v_s^2 - r_s^2 i_{qs}^{r*2}\right]}}{L_{ss}^2 i_{qs}^{r*2} + \lambda_m^2} \tag{3.3-14}$$

where the negative and positive root are the negative and positive boundary of the valid maximum torque per amp mode.

Flux Weakening Mode

In flux weakening mode, the voltage limitation drives the inverter operation. As it turns out, if the rotor speed exceeds the limit given by (3.3-14), it may still be possible to achieve the desired torque by commanding a negative d-axis current. To see this note that substituting the steady-state q- and d-axis voltage equations into (3.3-13) yields

$$2v_{s,mx}^2 = (r_s i_{qs}^{r*} + \omega_r L_{ss} i_{ds}^{r*} + \omega_r \lambda_m)^2 + (r_s i_{ds}^{r*} - \omega_r L_{ss} i_{qs}^r)^2 \qquad (3.3\text{-}15)$$

Solving (3.3-15) for the desired d-axis current yields a d-axis current command of

$$i_{ds}^{r*} = \frac{-\lambda_m L_{ss}\omega_r^2 + \sqrt{2z^2 v_{s,mx}^2 - \left(r_s \omega_r \lambda_m + z^2 i_{qs}^{r*}\right)^2}}{z^2} \qquad (3.3\text{-}16)$$

where

$$z = \sqrt{r_s^2 + \omega_r^2 L_{ss}^2} \qquad (3.3\text{-}17)$$

In essence, by injecting the d-axis current predicted by (3.3-16) the voltage required by the machine is reduced, as the expense of additional losses.

However, there are several limitations on the use of (3.3-16). First of all, the fact that we are injecting d-axis current can lower the maximum q-axis current we can command without exceeding the current magnitude limit. From (2.4-16)

$$2i_{s,mx}^2 = i_{qs}^{r*2} + i_{ds}^{r*2} \qquad (3.3\text{-}18)$$

which may be re-arranged as

$$i_{ds}^{r*2} = -\sqrt{2i_{s,mx}^2 - i_{qs}^{r*2}} \qquad (3.3\text{-}19)$$

Substitution of (3.3-19) into (3.3-15) yields

$$\begin{aligned} 2v_{s,mx}^2 = &\left(r_s i_{qs,lm2}^r - \omega_r L_{ss}\sqrt{2i_{s,mx}^2 - i_{qs,lm2}^{r2}} + \omega_r \lambda_m\right)^2 \\ &+ \left(-r_s\sqrt{2i_{s,mx}^2 - i_{qs,lm2}^{r2}} + \omega_r L_{ss} i_{qs,lm2}^r\right)^2 \end{aligned} \qquad (3.3\text{-}20)$$

where $i_{qs,lm2}^r$ denotes the second limit on the q-axis current (the first one being given by (3.3-9)). By algebraic and trigonometric manipulation, (3.3-20) may be solved to yield

$$\left(i_{qs,mn2}^r, i_{qs,mx2}^r\right) = \sqrt{2}i_{s,mx}\cos\left(\text{angle}(r_s + j\omega_r L_{ss}) \pm \text{acos}\left(\frac{2v_{s,mx}^2 - 2z^2 i_{s,mx}^2 - \omega_r^2\lambda_m^2}{2\omega_r \lambda_m z\sqrt{2}i_{s,mx}}\right)\right) \qquad (3.3\text{-}21)$$

where z is defined by (3.3-17) and $i_{qs,mn2}^r$ and $i_{qs,mx2}^r$ are the two solutions for $i_{qs,lm2}^r$. The corresponding torque limits may be expressed:

$$T_{e,mn2} = \frac{3}{2}\frac{P}{2}\lambda_m i_{qs,mn2}^r \tag{3.3-22}$$

$$T_{e,mx2} = \frac{3}{2}\frac{P}{2}\lambda_m i_{qs,mx2}^r \tag{3.3-23}$$

The d-axis current constraint (3.2-4) adds another limit on the q-axis current and torque. The expression given by (3.3-16) yields a negative d-axis current. If this command becomes to negative, it can permanently demagnetize the permanent magnet. Setting $i_{ds}^{r*} = i_{ds,mn}^r$ in (3.3-16) yields the q-axis current limits

$$\left(i_{qs,mn3}^r, i_{qs,mx3}^r\right) = \frac{-r_s\lambda_m\omega_r \mp \sqrt{2z^2 v_s^2 - \left(z^2 i_{ds,mn}^r + \lambda_m L_{ss}\omega_r^2\right)^2}}{z^2} \tag{3.3-24}$$

and the corresponding torque limits

$$T_{e,mn3} = \frac{3}{2}\frac{P}{2}\lambda_m i_{qs,mn3}^r \tag{3.3-25}$$

and

$$T_{e,mx3} = \frac{3}{2}\frac{P}{2}\lambda_m i_{qs,mx3}^r \tag{3.3-26}$$

One final constraint arises from the requirement that the d-axis current command given by (3.3-16) is a real number. This yields the q-axis current limit

$$\left(i_{qs,mn4}^r, i_{qs,mx4}^r\right) = \frac{\mp\sqrt{2}v_s z - r_s\omega_r\lambda_m}{z^2} \tag{3.3-27}$$

with corresponding torque limits are

$$T_{e,mn4} = \frac{3}{2}\frac{P}{2}\lambda_m i_{qs,mm4}^r \tag{3.3-28}$$

$$T_{e,mx4} = \frac{3}{2}\frac{P}{2}\lambda_m i_{qs,mx4}^r \tag{3.3-29}$$

It should be noted that for positive speeds and torque commands, this constraint is normally dominated by other constraints, but is included for completeness.

Combining the limitations on the torque commands yields:

$$T_{e,mn} = \max(-T_{e,mx1}, T_{e,mn2}, T_{e,mn3}, T_{e,mn4}) \tag{3.3-30}$$

$$T_{e,mx} = \min(T_{e,mx1}, T_{e,mx2}, T_{e,mx3}, T_{e,mx4}) \quad (3.3\text{-}31)$$

From which the q-axis current command is calculated as

$$i_{qs}^{r*} = \frac{4\min(\max(T_e^*, T_{e,mn}), T_{e,mx})}{3P\lambda_m} \quad (3.3\text{-}32)$$

Summary

It is now appropriate to summarize the current command synthesizer for non-salient machines. The first step is to determine the mode, whether it be maximum torque per amp, or flux weakening. This is done by evaluating (3.3-14). In particular, if the speed satisfies the boundaries imposed by (3.3-14) operation is in the maximum torque per amp mode, and the q- and d-axis current commands are given by (3.3-7) and (3.3-8), respectively. In (3.3-7), $T_{e,mx1}$ is determined by (3.3-10). If operation is in the flux weakening mode, then the q- and d-axis current commands are given by (3.3-32) and (3.3-16). Note that not all limits are always defined (for example, involving the square root of a negative number); in this case such limits are removed from consideration in evaluating (3.3-31) and (3.3-32).

Example 3.3-1. Let us consider the operating limits of a PMSM machine. For our example machine, $r_s = 2.6\,\Omega$, $L_{ss} = 12.4\,\text{mH}$, $\lambda_m = 0.286\,\text{Vs}$, $P = 4$, $i_{s,mx} = 3.3\,\text{A}$, $v_{s,mx} = 230/\sqrt{3}\,\text{V}$, and $i_{d,mn} = -2.33\,\text{A}$. These parameters are loosely based on a 2 Hp (at 2000 rpm) commercial PMSM. Figure 3.3-1 depicts the torque capability versus speed, indicating the boundary between modes as well as the active constraints. As can be seen, the maximum torque is initial constant, begins to fall gradually as flux weakening mode is first entered, and then more rapidly as the d-axis current limit comes into play. Figure 3.3-2 illustrates output power versus speed. In the maximum torque per amp mode, the maximum obtainable torque is initially constant and so power increases linearly with speed. As flux weakening mode is entered, the power levels off (the constant power regime), and then begins to rapidly fall as the d-axis current limit is encountered.

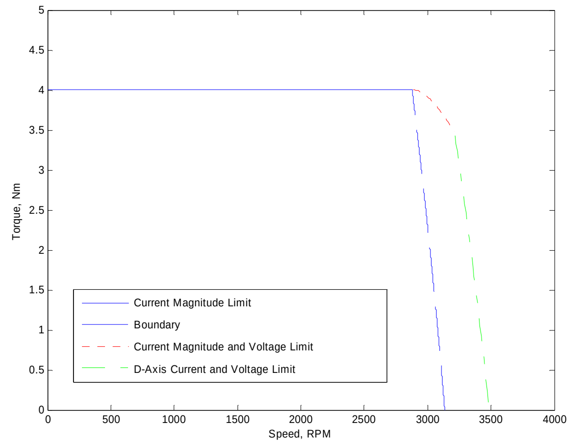

Figure 3.3-1. Torque capability.

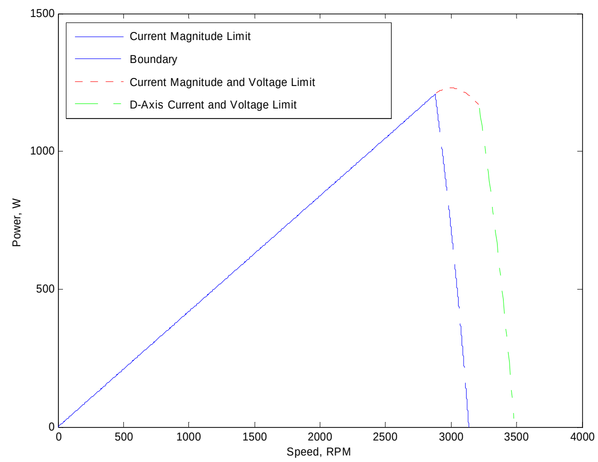

Figure 3.3-2. Power Capability.

3.3.2 Current Command Synthesis for Salient Machines

Although the emphasis of this work is on the simpler case of current command synthesis for non-salient machines, we will at least consider maximum torque per amp mode for salient machines. In the case of salient machines, both the q- and d-axis currents contribute to the average torque. Thus, when sufficient voltage is available it is convenient to formulate the q- and d-axis current command such the desired torque is obtained with as little loss as possible. To this end, recall that from (2.4-3), in the case of a salient machine, the electromagnetic torque may be expressed

$$T_e = \frac{3}{2}\frac{P}{2}(\lambda_m i_{qs}^r - (L_q - L_d) i_{qs}^r i_{ds}^r) \tag{3.3-27}$$

Thus, the amount of d-axis current required to achieve a desired torque may be expressed

$$i_{ds}^{r*} = g_d(i_{qs}^{r*}, T_e^*) = \frac{\frac{3}{2}\frac{P}{2}\lambda_m i_{qs}^{r*} - T_e^*}{\frac{3}{2}\frac{P}{2}(L_q - L_d) i_{qs}^{r*}} \tag{3.3-28}$$

Recall that

$$2 i_s^2 = i_{qs}^{r2} + i_{ds}^{r2} \tag{3.3-29}$$

Substitution of (3.3-28) into (3.3-29) yields

$$2 i_s^2 = \left(i_{qs}^{r*}\right)^2 + \left(\frac{\frac{3}{2}\frac{P}{2}\lambda_m i_{qs}^{r*} - T_e^*}{\frac{3}{2}\frac{P}{2}(L_q - L_d) i_{qs}^{r*}}\right)^2 \tag{3.3-30}$$

Setting the partial derivative of the left-hand side of (3.3-30) with respect to i_{qs}^{r*} equal to zero yields

$$\left(i_{qs}^{r*}\right)^4 + \frac{T_e^*\left(\frac{3}{2}\frac{P}{2}\lambda_m i_{qs}^{r*} - T_e^*\right)}{\left(\frac{3}{2}\frac{P}{2}\left(L_q - L_d\right)\right)^2} = 0 \tag{3.3-31}$$

which may be solved implicitly for the optimal q-axis current command. From a practical viewpoint, it is convenient to solve (3.3-31) numerically over a range of points spanning the practical torque range, and then determining the coefficients a_j and n_j so that

$$i_{qs}^{r*} = g_q(T_e^*) = \sum_{j=1}^{J} a_j \left(T_e^*\right)^{n_j} \tag{3.3-32}$$

is an adequate approximation to (3.3-31) . This may be done using a variety of curve-fitting techniques. Once the q-axis current command is calculated, the d-axis command is found using (3.3-28).

In order to find the boundaries of this mode of operation, let us first examine the effects of current limit. In particular, from (3.3-1) we have

$$2i_{s,mx}^2 = g_q(T_{e,mx1}^*)^2 + g_d(T_{e,mx1}^*, g_q(T_{e,mx1}^*))^2 \tag{3.3-33}$$

which may be solved numerically for the maximum torque achievable in this mode.

In order to establish the maximum speed to which maximum torque per-amp control can be used, (3.3-28) and (3.32) are used in conjunction with (2.4-17). Thus,

$$2v_{s,mx}^2 = \left(r_s g_q(T_e^*) + \omega_{r,bnd} L_d g_d(T_e^*, g_q(T_{e,mx}^*)) + \omega_{r,bnd}\lambda_m\right)^2 + \left(r_s g_d(T_e^*, g_q(T_e^*)) - \omega_{r,bnd} L_q g_q(T_e^*)\right)^2 \tag{3.3-34}$$

For a given value of torque command T_e^*, the boundary speed between maximum torque per amp and flux weakening controls can be found by solving (3.3-34) for $\omega_{r,bnd}$. By varying T_e^* and determining corresponding values of $\omega_{r,bnd}$, it is possible to approximate an explicit solution for the boundary speed as

$$\omega_{r,bnd} = h(T_e^*) = b_0 + \sum_{j=1}^{J} b_j \left(T_e^*\right)^{n_j} \tag{3.3-35}$$

It should be noted that when in the maximum torque per amp mode, the amount of negative d-axis current injected is typically not excessive. However, this is a supposition which must be checked for each individual machine. To this end, when commanding maximum torque, the corresponding q- and d-axis currents are found using (3.3-32) and (3.3-28), respectfully. If the resulting value of d-axis current satisfies (3.2-4) then the matter need not be considered further for maximum torque per amp mode. If the limit is reached, it results in an additional operating mode.

3.4 DIRECT CURRENT REGULATION

Now that we have addressed of the problem of determining the currents we want, we can focus on the related problem of obtaining those desired currents. Thus our attention focuses on the design of the current regulator block in Fig. 3.1-1. There are two main strategies to accomplish this depending upon the type of modulator. In modulators which attempt to make the inverter emulate a current source – including hysteretic, delta, and delta-hysteretic modulators we will utilize what will be referred to as a direct current regulator since we attempt to regulate the current with a current source. Both strategies make use of the concept of integral feedback in the rotor (a synchronous) reference frame, an idea set forth in [3]. In indirect current regulation, we will attempt to regulate current with a voltage source. Indirect current regulation is the topic of Section 3.5.

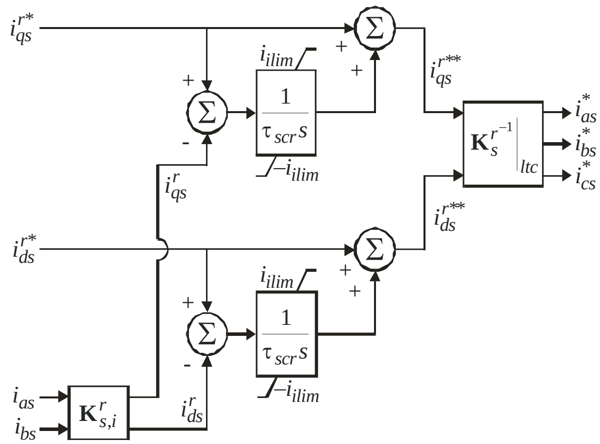

Figure 3.4-1. Direct current regulator.

Fig. 3.4-1 illustrate the direct current regulator. Therein, the first step is the transformation of the measured phase currents i_{as} and i_{bs} to the rotor reference frame using the modified transformation $\mathbf{K}_{s,i}^r$ as set forth in Chapter 2 of [2]:

$$\mathbf{i}_{qds}^r = \mathbf{K}_{s,i}^r \mathbf{i}_{abs} \tag{3.4-1}$$

$$\mathbf{K}_{s,i}^r = \frac{2}{\sqrt{3}} \begin{bmatrix} \cos(\theta_r - \pi/6) & \sin(\theta_r) \\ \sin(\theta_r - \pi/6) & -\cos(\theta_r) \end{bmatrix} \tag{3.4-2}$$

The use of the modified transformation is to reduce sensor requirements.

Next, modified q- and d- axis current commands are computed. In particular,

$$i_{qs}^{r**} = i_{qs}^{r*} + \int (i_{qs}^{r*} - i_{qs}^r)/\tau_{scr} dt \tag{3.4-3}$$

$$i_{ds}^{r**} = i_{ds}^{r*} + \int (i_{ds}^{r*} - i_{ds}^r)/\tau_{scr} dt \tag{3.4-4}$$

In (3.4-3) and (3.4-4), the first term is a feedforward term corresponding to the original current command. The second term is an integral feed back term used to ensure accurate tracking. Note that the output of the integral term is limited to $\pm i_{ilim}$.

Finally, the modified q-and d-axis commands are translated back to phase variable commands for use by the modulator. In particular,

$$i_{abc}^{*} = \mathbf{K}_s^{r^{-1}}\Big|_{ltc} i_{qds}^{r**} \tag{3.4-5}$$

where '*ltc*' denotes left two columns, i.e.

$$\mathbf{K}_s^{r^{-1}}\Big|_{ltc} = \begin{bmatrix} \cos(\theta_r) & \sin(\theta_r) \\ \cos(\theta_r - 2\pi/3) & \sin(\theta_r - 2\pi/3) \\ \cos(\theta_r + 2\pi/3) & \sin(\theta_r + 2\pi/3) \end{bmatrix} \tag{3.4-6}$$

In order to analyze the performance of the converter, let us assume that the inverter performance is such that the actual q-axis current is equal to the modified command (the q-axis command to the modulator) plus some error:

$$i_{qs}^{r} = i_{qs}^{r**} + \Delta i_{qs}^{r} \tag{3.4-7}$$

Assuming that the current limits are not reached, from (3.4-7) and the block diagram (Fig 3.4-1) it can be readily shown in the frequency domain that

$$i_{qs}^{r} = i_{qs}^{r*} + \frac{\tau_{scr} s}{\tau_{scr} s + 1} \Delta i_{qs}^{r} \tag{3.4-8}$$

where s denotes the Laplace operator. Observe that at dc ($s \to 0$), the measured value of q-axis current will to the commanded value.

In terms of setting the parameters τ_{scr} and i_{ilim} , it should be stated that the philosophy behind this control is that it is primarily feedforward; the integral term is only to null out errors associated with imperfect tracking on the part of the modulator. Thus, for example, if a hysteresis modulator is used with hysteresis level h , than i_{ilim} should be only somewhat bigger than h . The time constant τ_{scr} is generally made a small as can be practically achieved, given the limitations of sampling time of digital computations and settings of the modulator.

3.5 INDIRECT CURRENT REGULATION

A potential disadvantage of the hysteretic, delta, or delta-hysteretic modulators used in conjunction with direct current regulation is that the switching frequency, and hence switching losses, cannot be directly controlled. Voltage source based modulation strategies such as space-vector and sine-triangle modulation strategies offer fixed switching frequency. Using these strategies, the inverter appears as a three phase voltage source. This calls for a significant change in the current regulation strategy.

Figure 3.5-1 illustrates one possible control architecture which was inspired by [4]. Therein LPF denotes a low pass filter and FC a feedback control block (such as a proportional (P), or proportional plus integral (PI) control).

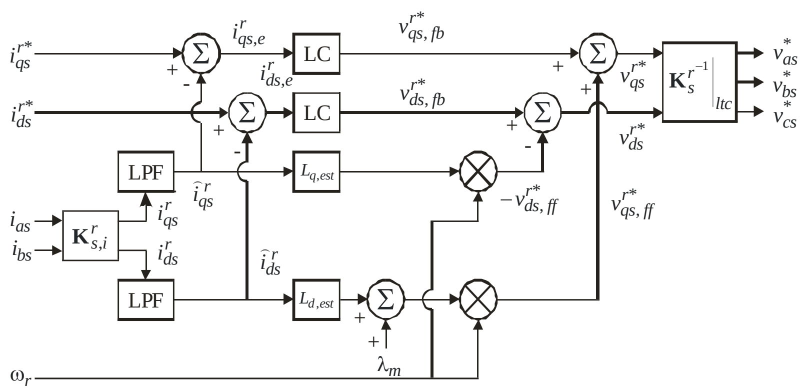

Figure 3.5-1. Indirect current control.

In this strategy, the measured q- and d-axis currents are obtained in the same way as they are in the case of the direct current regulation based control. However, in this case it will be necessary to filter the q- and d-axis current to prevent switching frequency current ripple from feeding through the control. Thus, in the frequency domain we have

$$\hat{i}_{qs}^r = F(s)i_{qs}^r \tag{3.5-1}$$

$$\hat{i}_{ds}^r = F(s)i_{ds}^r \tag{3.5-2}$$

where $F(s)$ is the low pass filter transfer function.

Next, the q- and d- axis current errors are found in accordance with

$$i_{qs,e}^r = i_{qs}^{r*} - \hat{i}_{qs}^r \tag{3.5-3}$$

$$i_{ds,e}^r = i_{ds}^{r*} - \hat{i}_{ds}^r \tag{3.5-4}$$

These errors are the input to a feedback control block FC, whose transfer function will be denoted $C(s)$. The output of these linear controls is the feedback portion of our control. In particular, in the frequency domain,

$$v_{qs,fb}^{r*} = C(s) i_{qs,e}^{r} \tag{3.5-5}$$

$$v_{ds,fb}^{r*} = C(s) i_{ds,e}^{r} \tag{3.5-6}$$

This portion of our control is primarily responsible for insuring that the desired currents are equal to the commanded currents.

Figure 3.5-2 depicts a generic PI class control which is both simple and effective. Therein, K is the gain, τ is the integrator time constant, and i_{ilim} is a limit on the integrator used to prevent signals. In addition, u denotes a generic input (for example $i_{qs,e}^{r}$) and y a generic output (for example, $v_{qs,fb}^{r*}$).

The feedforward portion of the control improves dynamic response by attempting to decouple the q- and d-axis dynamics. In particular, we have

$$v_{qs,ff}^{r*} = \omega_r L_{d,est} \hat{i}_{ds}^{r} + \omega_r \lambda_{m,est} \tag{3.5-7}$$

$$v_{ds,ff}^{r*} = -\omega_r L_{q,est} \hat{i}_{qs}^{r} \tag{3.5-8}$$

where $L_{d,est}$, $L_{q,est}$, and $\lambda_{m,est}$ are the estimated values of the machine parameters. The mechanism by which this term improves the performance will be explained towards the end of this section.

To complete the control, the total q- and d-axis voltage commands are then formed by summing the feedforward and feedback terms, thus

$$v_{qs}^{r*} = v_{qs,ff}^{r*} + v_{qs,fb}^{r*} \tag{3.5-9}$$

$$v_{ds}^{r*} = v_{ds,ff}^{r*} + v_{ds,fb}^{r*} \tag{3.5-10}$$

The q- and d-axis voltage commands are then transformed to phase variables commands and achieved by the inverter's modulator.

In order to understand the operation of the inverter, let us make the following assumptions: (*i*) at the frequency range of interest the low-pass filter can be ignored so that $\hat{i}_{qs}^{r} = i_{qs}^{r}$ and $\hat{i}_{ds}^{r} = i_{ds}^{r}$, (*ii*) the inverter obtains the desired q- and d-axis voltage command to within a finite error, thus $v_{qs}^{r} = v_{qs}^{r*} + \Delta v_{qs}^{r}$ and $v_{ds}^{r} = v_{ds}^{r*} + \Delta v_{ds}^{r}$, (*iii*) the estimated values of parameters are equal to the actual parameters, and (*iv*) the linear control block used is the PI control depicted in Fig. 3.5-2.

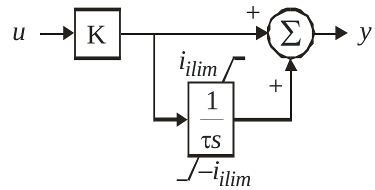

Figure 3.5-2. PI Control.

Upon making these assumptions, in the frequency domain we may write that the q- and d-axis voltages will be given by

$$v_{qs}^r = \underbrace{\omega_r L_d i_{ds}^r + \omega_r \lambda_m}_{v_{qs,ff}^{r*}} + \underbrace{K\left(1+\frac{1}{\tau s}\right)\left(i_{qs}^{r*} - i_{qs}^r\right)}_{v_{qs,fb}^{r*}} + \underbrace{\Delta v_{qs}^r}_{\text{inverter error}} \tag{3.5-11}$$

$$v_{ds}^r = \underbrace{-\omega_r L_q i_{qs}^r}_{v_{ds,ff}^{r*}} + \underbrace{K\left(1+\frac{1}{\tau s}\right)\left(i_{ds}^{r*} - i_{ds}^r\right)}_{v_{ds,fb}^{r*}} + \underbrace{\Delta v_{ds}^r}_{\text{inverter error}} \tag{3.5-12}$$

Recall from (2.4-1) and (2.4-2) that

$$v_{qs}^r = r_s i_{qs}^r + \omega_r L_d i_{ds}^r + \omega_r \lambda_m \tag{3.5-13}$$

$$v_{ds}^r = r_s i_{ds}^r - \omega_r L_q i_{qs}^r \tag{3.5-14}$$

Equating (3.5-11) to (3.5-13) and (3.5-12) to (3.5-14) and manipulating, we have

$$i_{qs}^r = \frac{K(\tau s+1)}{(r_s+K)\tau s+K} i_{qs}^{r*} + \frac{\tau s}{(r_s+K)\tau s+K}\Delta v_{qs}^r \tag{3.5-15}$$

$$i_{ds}^r = \frac{K(\tau s+1)}{(r_s+K)\tau s+K} i_{ds}^{r*} + \frac{\tau s}{(r_s+K)\tau s+K}\Delta v_{ds}^r \tag{3.5-16}$$

As can be seen, under the assumed conditions, the q- and d-axis will act independently (because of the feedforward portion of our control) and exhibit a first order response. At dc ($s \to 0$) it is readily seen that the q- and d-axis currents will approach their commanded values. However, this only hold if the integrator and other limits do not come into play, and the other assumed conditions hold as well. Design in this context is readily achieved since the pole of the response may be readily placed by choosing K and τ appropriately. When placing the pole, it must be remembered that the LPF has been ignored in our analysis; this filter should have a bandwidth an order of magnitude beyond the bandwidth of this control loop.

In terms of comparing indirect and direct current regulation, it should be mentioned that the direct approach is more straightforward, less susceptible to degradation because of parameter variations, and yields the best dynamic performance. The primary advantage of indirect current regulation is simply that a fixed switching frequency modulator can be used.

3.6 CASE STUDY

3.7 ACKNOWLEDGEMENTS

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3.8 REFERENCES

[1] P.C. Krause, O. Wasynczuk, S.D. Sudhoff, *Analysis of Electric Machinery and Drive Systems, 2nd Edition*, IEEE-Press, Piscataway, NJ through Wiley-Interscience.
[2] *Techniques for Analysis and Design of Electromechanical Systems*, S.D. Sudhoff, S. P. Pekarek, monograph in preparation for publication.
[3] T.M. Rowan, R.J. Kerkman, "A New Synchronous Current Regulator and an Analysis of Current-Regulated Inverters," *IEEE Transactions on Industry Applications*, Vol. IA-22, No. 4, 1986, pp. 678-690.
[4] O. Wasynczuk, S.D. Sudhoff, T.D. Tran, D.H. Clayton, and H.J. Hegner, "A Voltage Control Strategy for Current-Regulated PWM Inverters," *IEEE Transactions on Power Electronics*,Vo. 11, No. 1, January 1996, pp. 7-15.