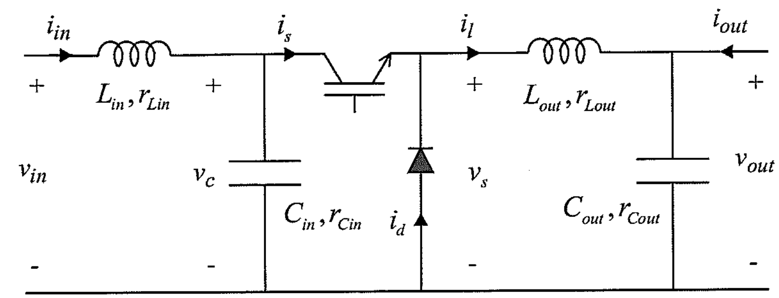
State Space Averaging (SSA)

Application of SSA to Buck Conv.

$$\hat{x} = \begin{bmatrix} \hat{i}_{in} \\ \hat{v}_{Cin} \\ \hat{i}_{l} \\ \hat{v}_{Cout} \end{bmatrix}$$

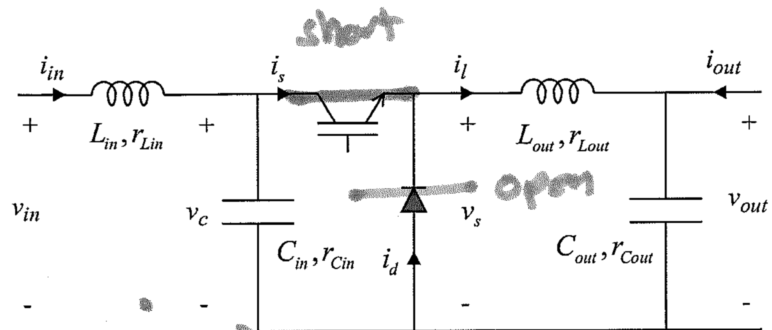
$$\hat{u} = \begin{bmatrix} \hat{v}_{in} \\ \hat{v}_{out} \end{bmatrix}$$

$$\hat{y} = \begin{bmatrix} \hat{i}_{in} \\ \hat{v}_{T,out} \end{bmatrix}$$



Application of SSA to Buck Conv.

- On State Model:



$- r_{Lin} i_{in}$

$$p i_{in} = (V_{in} - V_c)\frac{1}{L_{in}}$$

$$= \left(V_{in} - \left(r_{cin}(i_{in} - i_l) + V_{cin}\right) - r_{Lin} i_{in}\right)\frac{1}{L_{in}}$$

$$p i_{in} = -\frac{r_{cin} + r_{Lin}}{L_{in}} i_{in} - \frac{1}{L_{in}} V_{cin} + \frac{r_{cin}}{L_{in}} i_l + \frac{V_{in}}{L_{in}}$$

$$p V_{cin} = \frac{1}{C_{in}}(i_{in} - i_l) = \frac{1}{C_{in}} i_{in} - \frac{1}{C_{in}} i_l$$

$$p i_l = (V_c - V_{out} - r_{Lout} i_l)\frac{1}{L_{out}}$$

$$= \left(r_{cin}(\underline{i_{in}}_{3} - \underline{i_l}) + V_{c_{in}}{}_{3} - V_{out} - r_{Lout}\underline{i_l}\right)\frac{1}{L_{out}}$$

$$= \frac{r_{cin}}{L_{out}} i_{in} + \frac{1}{L_{out}} V_{cin} - \underline{\frac{(r_{cin} + r_{Lout})}{L_{out}}} i_l - \frac{1}{L_{out}} V_{out}$$

Application of SSA to Buck Conv.

$$p\,v_{cout} = \frac{v_{out} - v_{cout}}{r_{cout} C_{out}} = -\frac{1}{r_{cout} C_{out}} v_{cout} + \frac{1}{r_{cout} C_{out}} v_{out}$$

$$A_1 = \begin{array}{cccc} i_{in} & v_{cin} & i_L & v_{cout} \end{array}$$
$$\begin{bmatrix} -\frac{r_{cin} + r_{Lin}}{L_{in}} & -\frac{1}{L_{in}} & \frac{r_{cin}}{L_{in}} & 0 \\ \frac{1}{C_{in}} & 0 & -\frac{1}{C_{in}} & 0 \\ \frac{r_{cin}}{L_{out}} & \frac{1}{L_{out}} & -\frac{r_{cin} + r_{Lout}}{L_{out}} & 0 \\ 0 & 0 & 0 & -\frac{1}{r_{cout} C_{out}} \end{bmatrix}$$

$$B_1 = \begin{array}{cc} v_{in} & v_{out} \end{array}$$
$$\begin{bmatrix} \frac{1}{L_{in}} & 0 \\ 0 & 0 \\ 0 & -\frac{1}{L_{out}} \\ 0 & \frac{1}{r_{cout} C_{out}} \end{bmatrix}$$

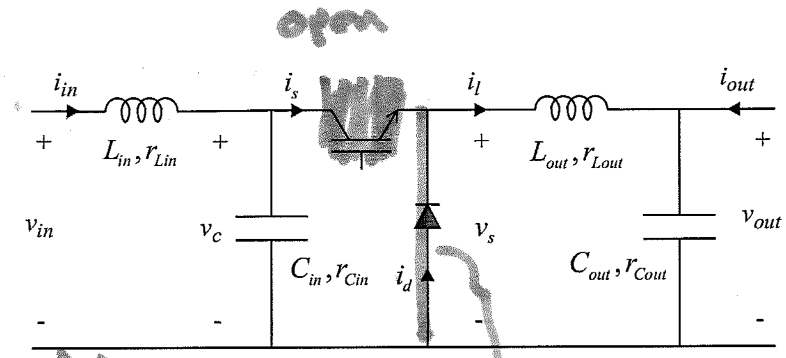
Application of SSA to Buck Conv.

$$y = \begin{bmatrix} i_{in} \\ v_{t,out} \end{bmatrix} \qquad v_{t,out} = r_{cout} i_e + v_{cout}$$

$$y = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & r_{cout} & 1 \end{bmatrix}}_{C_1 = C_2} x + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}}_{D_1 = D_2} u$$

Application of SSA to Buck

- Off State Model



$$p\, i_{in} = (v_{in} - v_c - r_{Lin}\, i_{in}) \frac{1}{L_{in}}$$

$$= \left(v_{in} - (r_{Cin}\, i_{in} + v_{Cin}) - r_{Lin}\, i_{in}\right) \frac{1}{L_{in}}$$

$$= -\frac{r_{Cin} + r_{Lin}}{L_{in}}\, i_{in} - \frac{v_{Cin}}{L_{in}} + \frac{1}{L_{in}} v_{in}$$

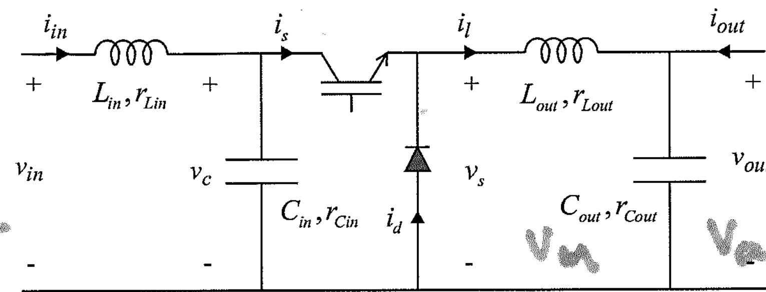
$$p\, v_{Cin} = \frac{1}{C_{in}}\, i_{in}$$

$$p\, i_l = (-v_{out} - r_{Lout}\, i_l) \frac{1}{L_{out}} = -\frac{r_{Lout}}{L_{out}}\, i_l - \frac{1}{L_{out}} v_{out}$$

$$p\, v_{Cout} = \frac{1}{C_{out}} \left(\frac{v_{out} - v_{Cout}}{r_{Cout}} \right)$$

Application of SSA to Buck

- Off State Model



$$A_2 = \begin{array}{c} \begin{array}{cccc} i_{in} & v_{Cin} & i_l & v_{Cout} \end{array} \\ \begin{bmatrix} -\frac{r_{Cin} + r_{Lin}}{L_{in}} & -\frac{1}{L_{in}} & 0 & 0 \\ \frac{1}{C_{in}} & 0 & 0 & 0 \\ 0 & 0 & -\frac{r_{Lout}}{L_{out}} & 0 \\ 0 & 0 & 0 & -\frac{1}{r_{Cout} C_{out}} \end{bmatrix} \end{array}$$

$$B_2 = \begin{array}{c} \begin{array}{cc} v_{in} & v_{out} \end{array} \\ \begin{bmatrix} \frac{1}{L_{in}} & 0 \\ 0 & 0 \\ 0 & -\frac{1}{L_{out}} \\ 0 & \frac{1}{r_{Cout} C_{out}} \end{bmatrix} \end{array}$$

Application of SSA to Buck

ECE61016 Power Electronics Converters and Systems

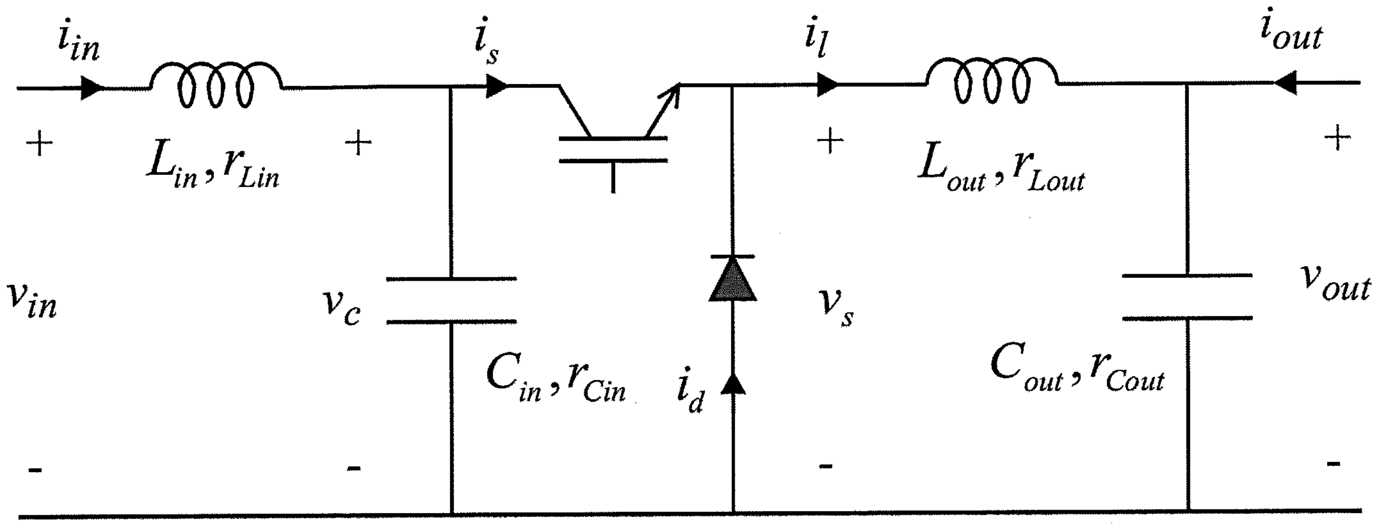
Lecture Set 3D: Buck Converter Control

S.D. Sudhoff

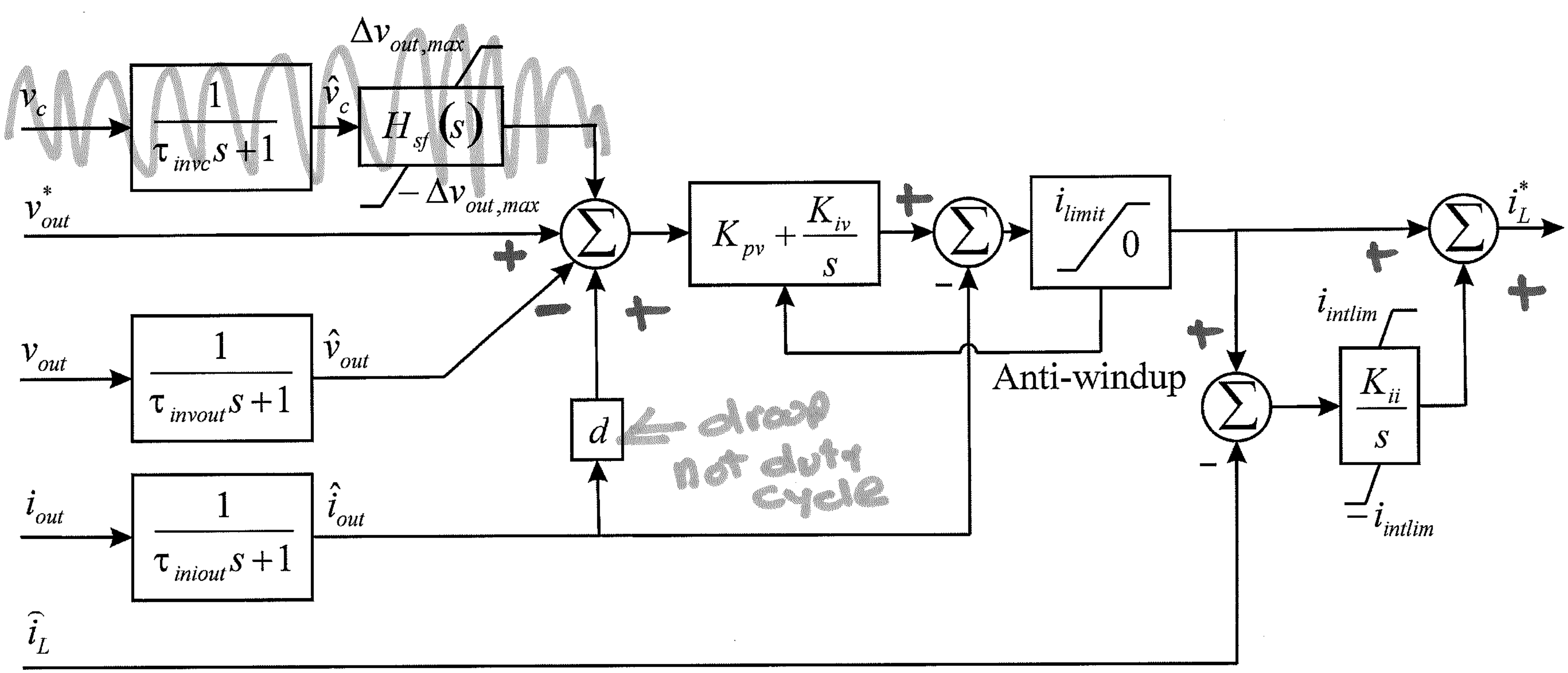
Fall 2016

Buck Converter Control

- Topology



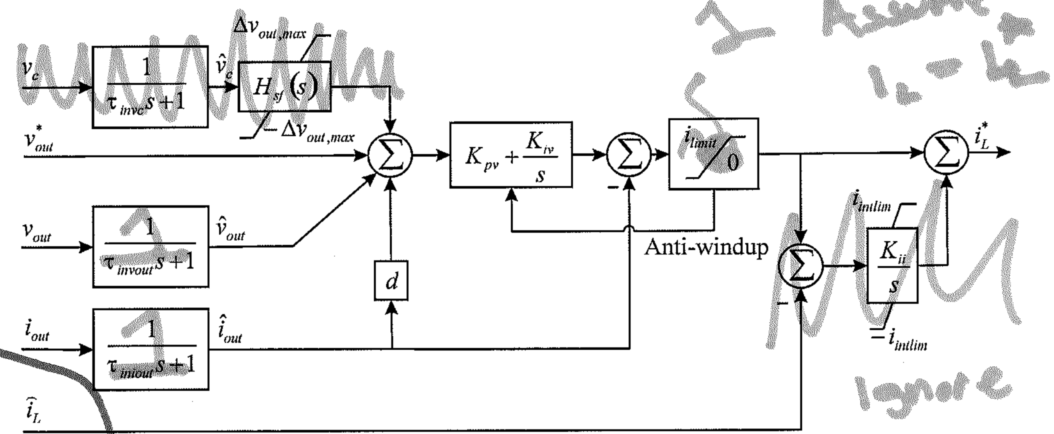
Buck Converter Control

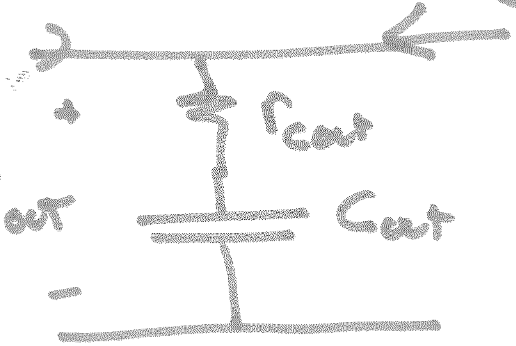


Analysis of Voltage Control Loop

$i_L^* = i_L$

$i_L = \left(K_{pv} + \frac{K_{iv}}{s}\right)\left(v_{out}^* - v_{out} + d\, i_{out}\right) - i_{out}$





$V_{out} = \left(r_{Cout} + \frac{1}{C_{out}s}\right)\left(i_L + i_{out}\right)$

Analysis of Voltage Control Loop

$$V_{out} = \left(r_{cout} + \frac{1}{C_{out} S}\right)\left(K_{pv} + \frac{K_{iv}}{S}\right)\left(\hat{V_{out}} - V_{out} + d\, i_{out}\right)$$

$$V_{out} C_{out} S^2 = \left(r_{cout} C_{out} S + 1\right)\left(K_{pv} S + K_{iv}\right)\left(\hat{V_{out}} - V_{out} + d\, i_{out}\right)$$

$$V_{out}\Big[\left(C_{out} + r_{cout} C_{out} K_{pv}\right) S^2 + \left(r_{cout} C_{out} K_{iv} + K_{pv}\right) S + K_{iv}\Big]$$

$$= \left(r_{cout} C_{out} S + 1\right)\left(K_{pv} S + K_{iv}\right)\left(\hat{V_{out}} + d\, i_{out}\right)$$

<u>AT DC</u>

$$V_{out} \cancel{K_{iv}} = \cancel{K_{iv}} \left(\hat{V_{out}} + d\, i_{out}\right)$$

Characteristic Polynomial

- Thus we have

$$\text{CP: } s^2 + \frac{r_{cout} C_{out} K_{iv} + k_{pv}}{C_{out}(1 + r_{cout} k_{pv})} s + \frac{K_{iv}}{(\,\bullet\,)}$$

Desired Characteristic Polynomial

- What we want

DCP: $(s - s_1)(s - s_2)$

$$s^2 - (s_1 + s_2)s + s_1 s_2$$

$$s_1 s_2 = \frac{K_{iv}}{C_{out}(1 + r_{cout} K_{pv})}$$

$$K_{iv} = s_1 s_2 C_{out}(1 + r_{cout} K_{pv})$$

Equating Coefficients

$$-(S_1 + S_2) = \frac{r_{C_{out}} C_{out} \overbrace{S_1 S_2 C_{out} (1 + r_{C_{out}} K_{pv})}^{K_{iv}} + K_{pv}}{C_{out}(1 + r_{C_{out}} K_{pv})}$$

Analysis of Voltage Control Loop

- Thus, in order to get poles at $s = s_1, s_2$
- Set gains to

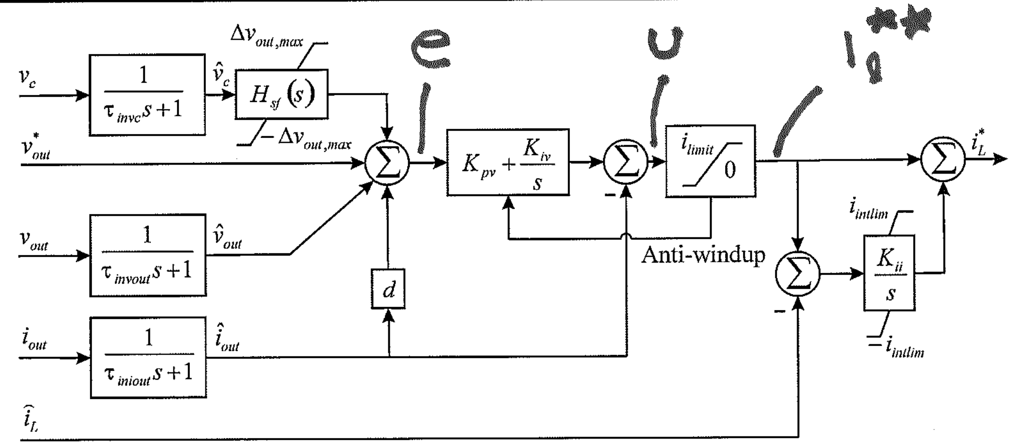
$$K_{iv} = s_1 s_2 C_{out}(1 + r_{cout} K_{pv})$$

Evaluate Second

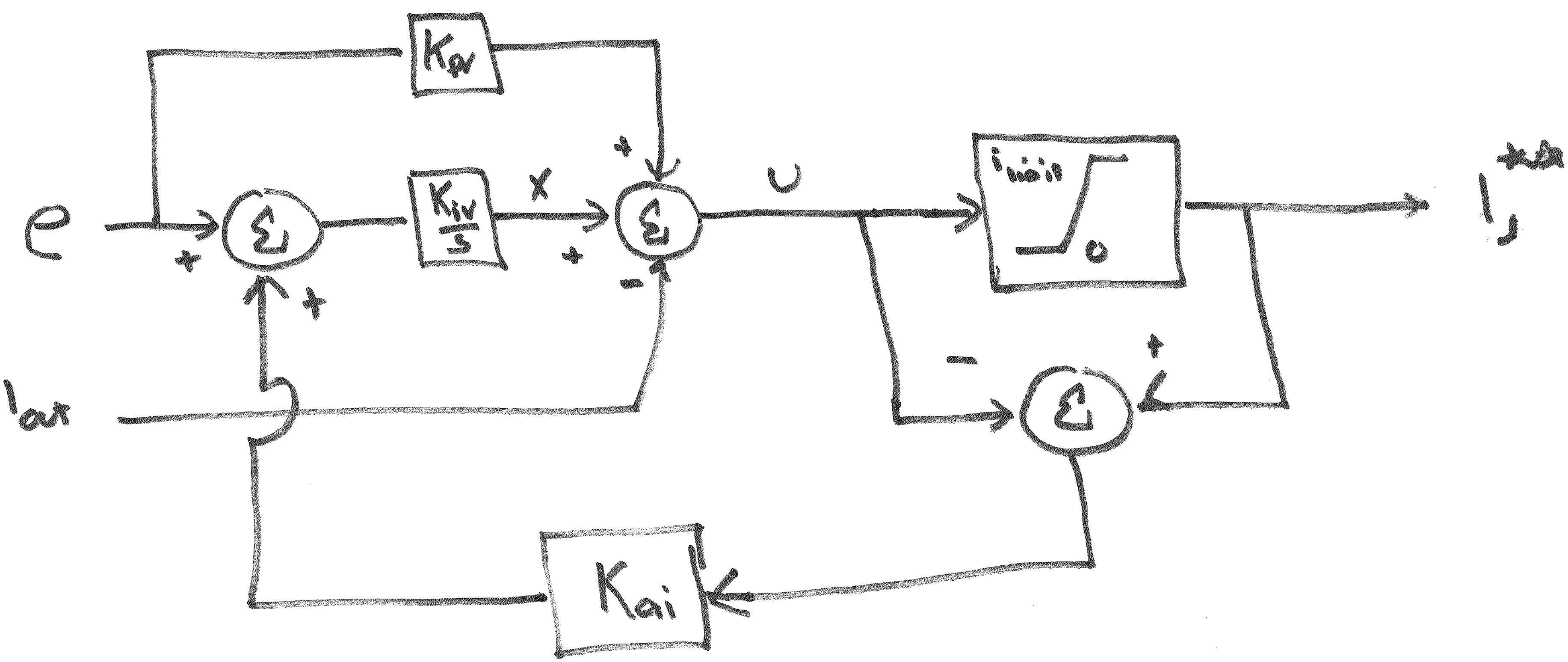
$$K_{pv} = -\frac{C_{out}}{\dfrac{1}{s_1 + s_2 + s_1 s_2 r_{cout} C_{out}} + C_{out} r_{cout}}$$

Evaluate First

Anti-Windup



v_c
$\frac{1}{\tau_{invc}s+1}$
$\hat{v}_c$
$\Delta v_{out,max}$
$H_{sf}(s)$
$-\Delta v_{out,max}$
v_{out}^*
v_{out}
$\frac{1}{\tau_{invout}s+1}$
$\hat{v}_{out}$
i_{out}
$\frac{1}{\tau_{iniout}s+1}$
$\hat{i}_{out}$
$\hat{i}_L$
d
e
$K_{pv}+\frac{K_{iv}}{s}$
u
i_{limit}
0
Anti-windup
i_q^{**}
i_{intlim}
$\frac{K_{ii}}{s}$
$-i_{intlim}$
i_L^*



e
K_{pv}
$\frac{K_{iv}}{s}$
x
u
i_{limit}
0
i_q^{**}
i_{out}
K_{ai}

Anti-Windup

- Assume a large constant e
- $i_q^{sat} = i_{limit}$; $i_{out} = -i_{limit}$

$-i_{out}$

$$U = K_{pv} e + \frac{K_{iv}}{s}\left[e + K_{ai}\left(i_{limit} - U\right)\right] + i_{limit}$$

$$Us = K_{pv} e s + K_{iv}\left[e + K_{ai}\left(i_{limit} - U\right)\right] + s\, i_{limit}$$

$$\left[s + K_{iv} K_{ai}\right] U = \left[K_{pv} s + K_{iv}\right] e + \left[K_{iv} K_{ai} + s\right] i_{limit}$$

$$U = \frac{K_{pv} s + K_{iv}}{s + K_{iv} K_{ai}} e + i_{limit}$$

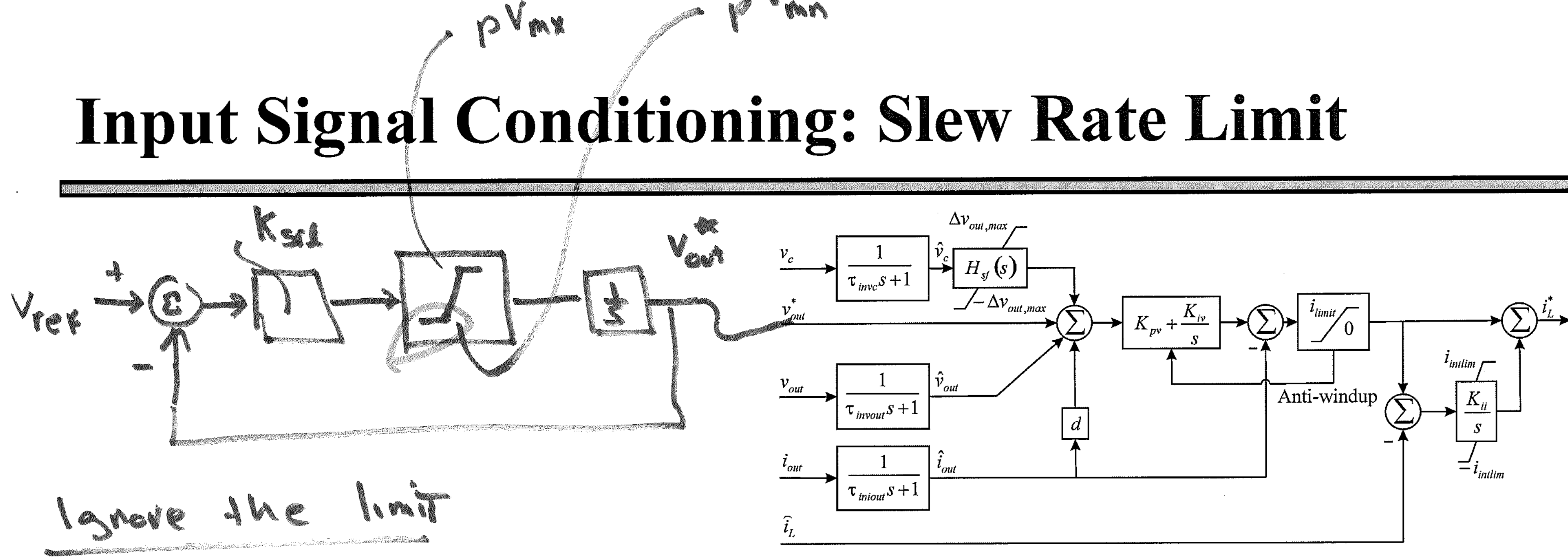
$$\boxed{\tau = \frac{1}{K_{iv} K_{ai}}}$$

In steady state

$$U = \frac{e}{K_{ai}} + i_{limit}$$

Anti-Windup

Input Signal Conditioning: Slew Rate Limit



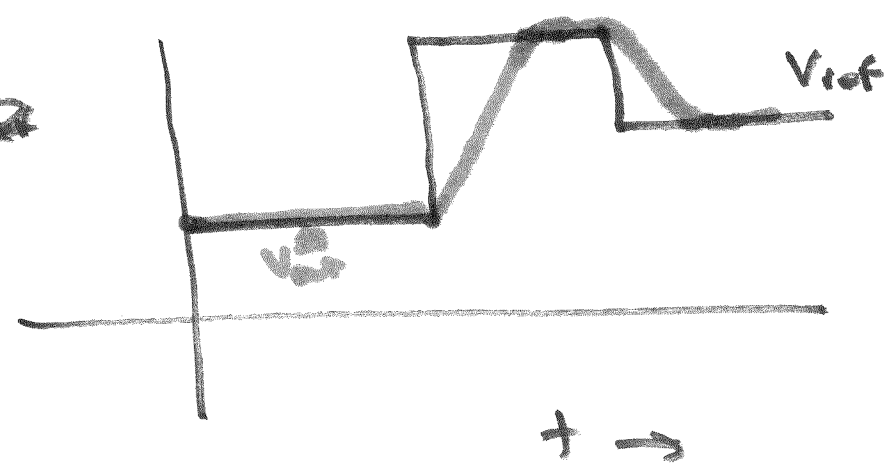
Ignore the limit

$$V^*_{out} = \frac{K_{srl}}{s}\left(V_{ref} - V^*_{out}\right)$$

$$\left[s + K_{srl}\right] V^*_{out} = K_{srl} V_{ref}$$

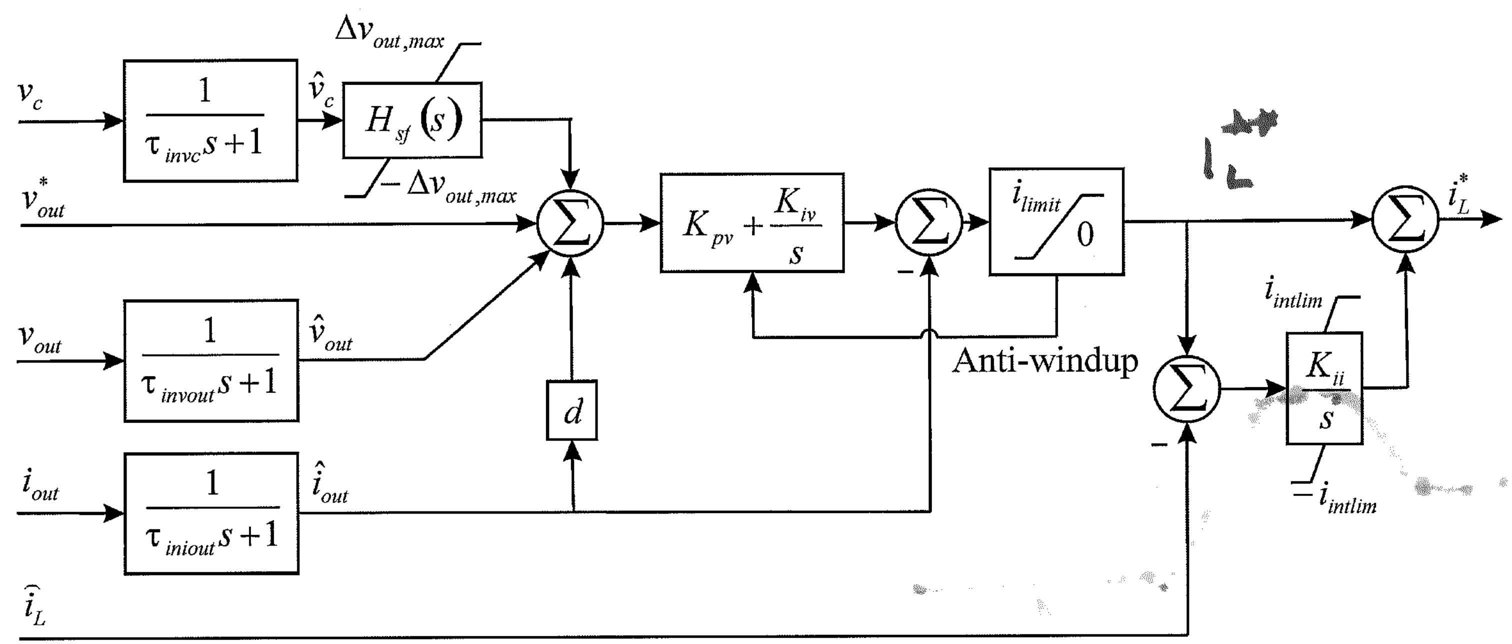
$$V^*_{out} = \frac{K_{srl}}{s + K_{srl}} V_{ref}$$

$$\left(\tau = \frac{1}{K_{srl}}\right.$$



Analysis of Current Control Loop

- Control:



- Hysteresis Model: $i_L = i_L^* + \Delta$

Analysis of Current Control Loop

$$I_L^{\wedge} = I_L^{**} + \frac{K_{ii}}{s}\left(I_L^{**} - I_L\right) = I_L - \Delta$$

$$s I_L^{\wedge *} + K_{ii}\left(I_L^{\wedge *} - I_L\right) = s\left(I_L - \Delta\right)$$

$$\left(s + K_{ii}\right) I_L^{\wedge *} = I_L\left(K_{ii} + s\right) - s\Delta$$

$$I_L = I_L^{\wedge *} + \frac{s\Delta}{K_{ii} + s}$$

Analysis of Current Control Loop

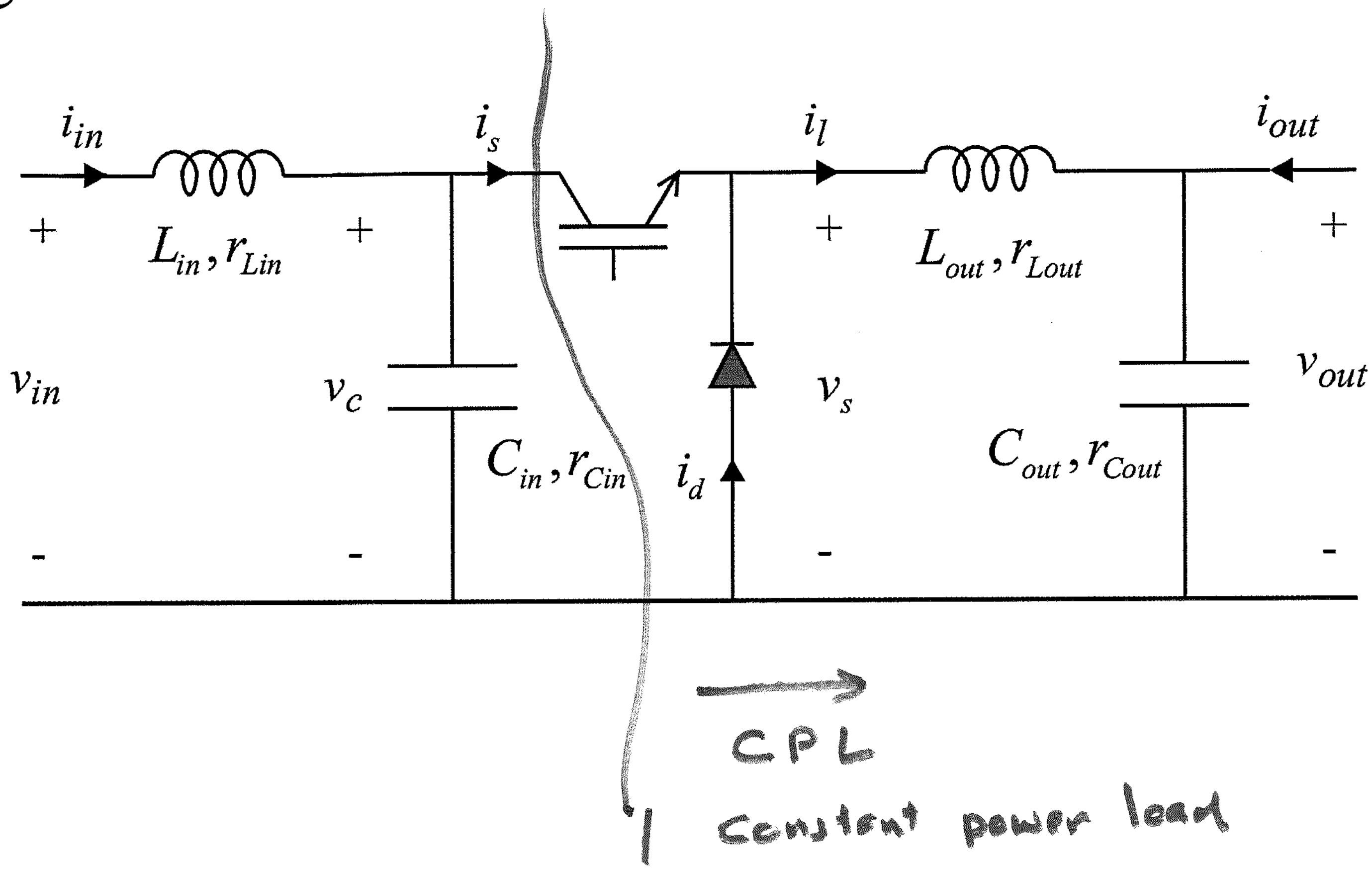
- Thus,

$$i_l = i_l^{**} + \frac{s\Delta}{s + K_{ii}}$$

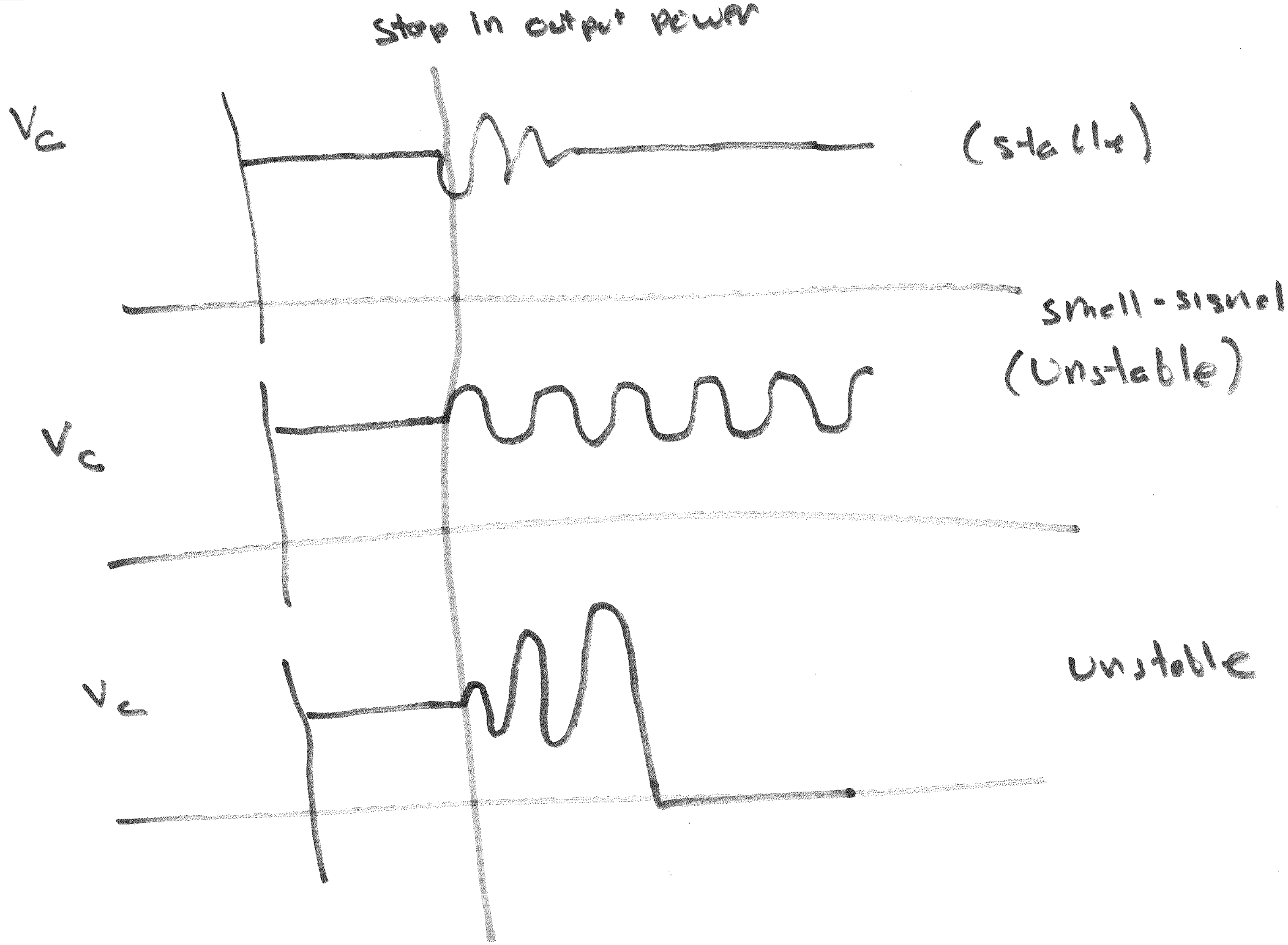
- Implications

Input Filter Stability Considerations

- Stability must also be considered in the input filter design



Behavior of Stable and Unstable Operating Points



Operating Point Determination

$$X = \underbrace{X_o}_{\text{nominal steady-state value}} + \underbrace{\Delta X}_{\text{perturbation}}$$

$$-V_{in} + L_{in}\frac{di_{in}}{dt} + r_{Lin}\, i_{in} + V_c = 0$$

$$-(V_{ino} + \Delta V_{in}) + L_{in}\frac{d}{dt}(I_{no} + \Delta i_{in}) + r_{Lin}(I_{no} + \Delta I_{in}) + V_{co} + \Delta V_c = 0$$

$$-V_{ino} + r_{Lin} I_{no} + V_{co} = 0$$