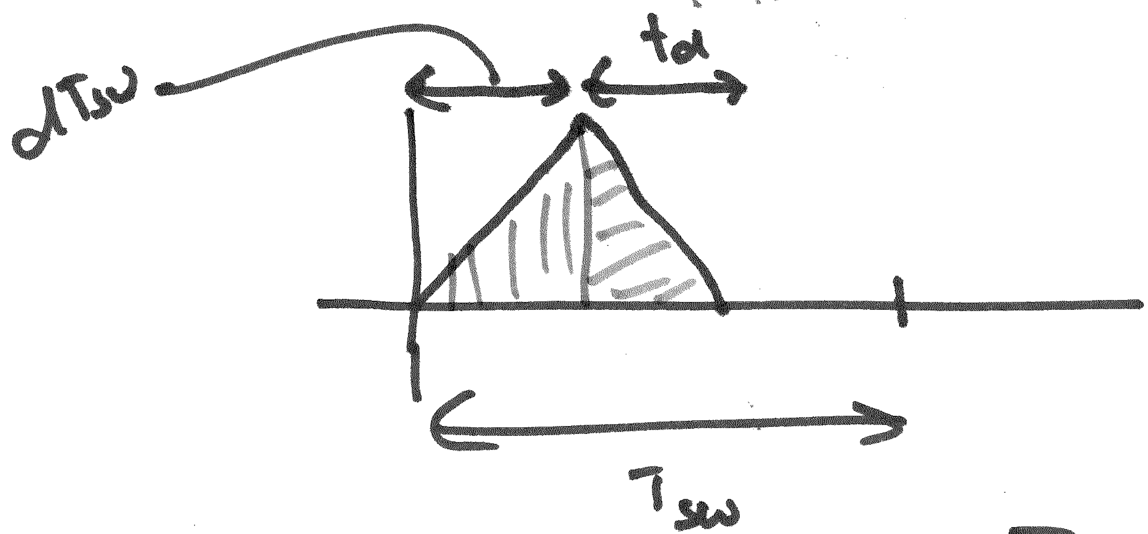
Discontinuous Current (DC) Buck Operation

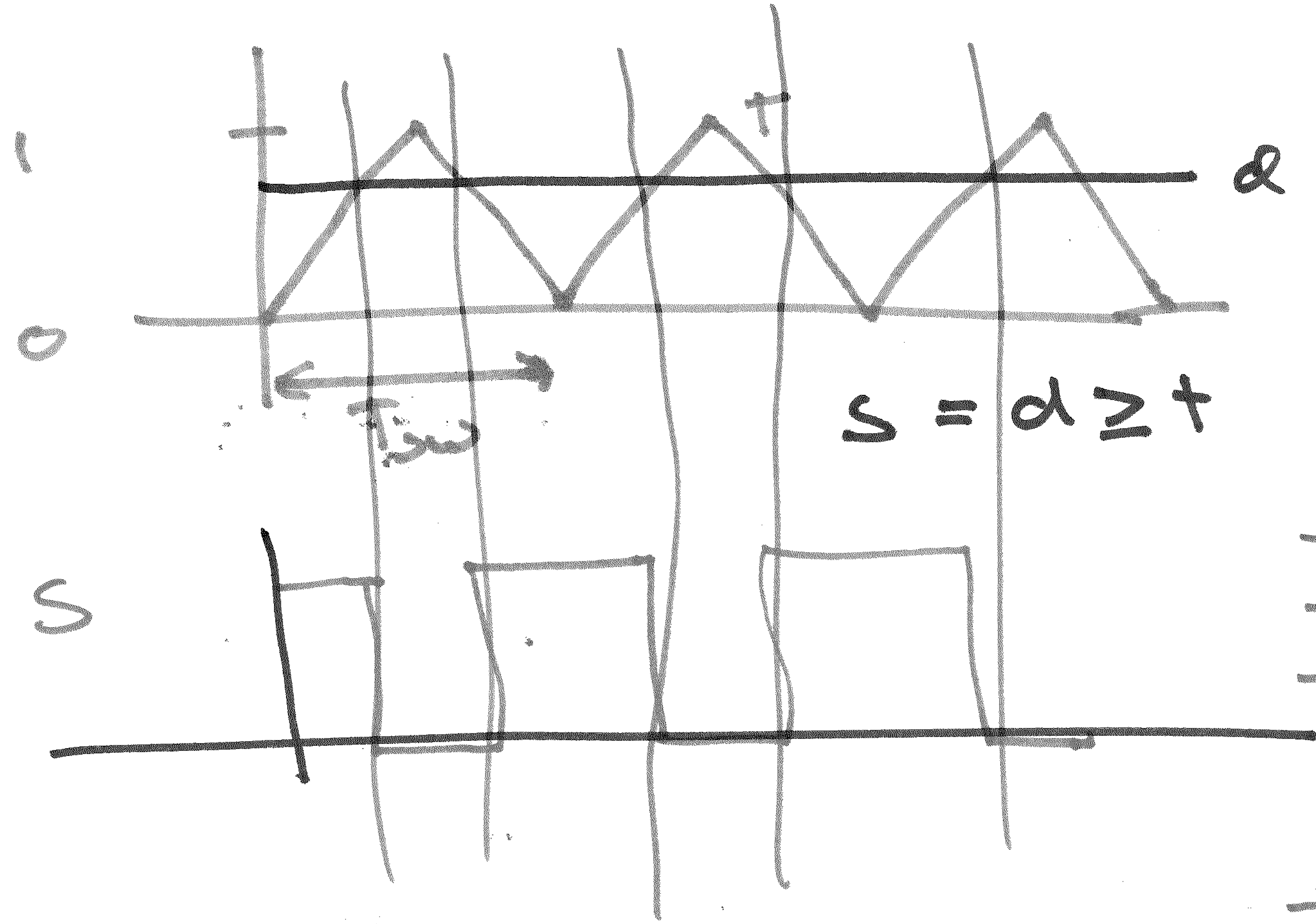
- Now, the average current may be given by

$$\bar{i}_l = \frac{1}{T_{sw}}\left(\frac{1}{2}dT_{sw}i_{mx} + \frac{1}{2}t_d i_{mx}\right)$$



$$\bar{I}_l = \frac{1}{T_{sw}}\int_0^{T_{sw}} i_l\,dt = \frac{1}{T_{sw}}\left[\frac{1}{2}dT_{sw}\,i_{mx} + \frac{1}{2}t_d\,i_{mx}\right]$$

Duty Cycle Modulation



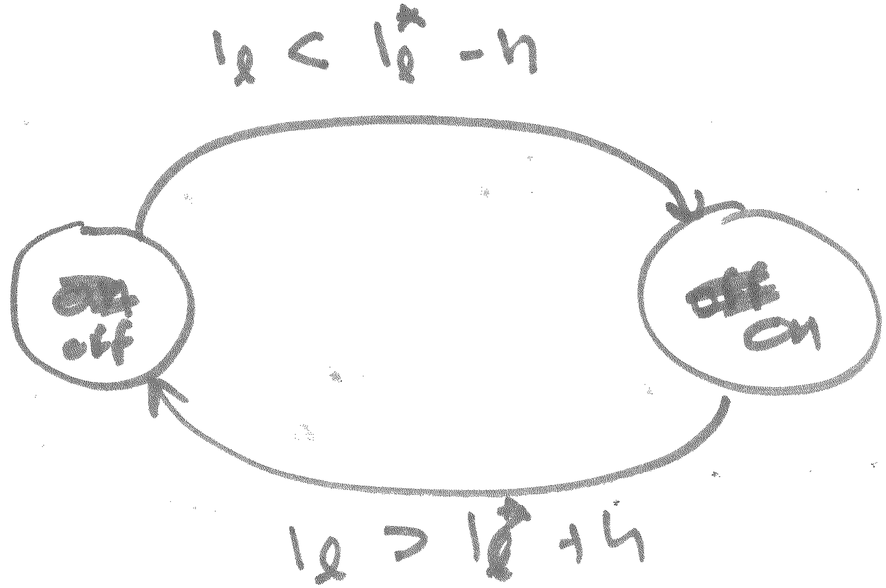
$s = \alpha \geq t$

→ simple
→ periodic
→ switching freq is fixed
→ transients are not well controlled

→ voltage source control

$V_s \approx \alpha V_C$

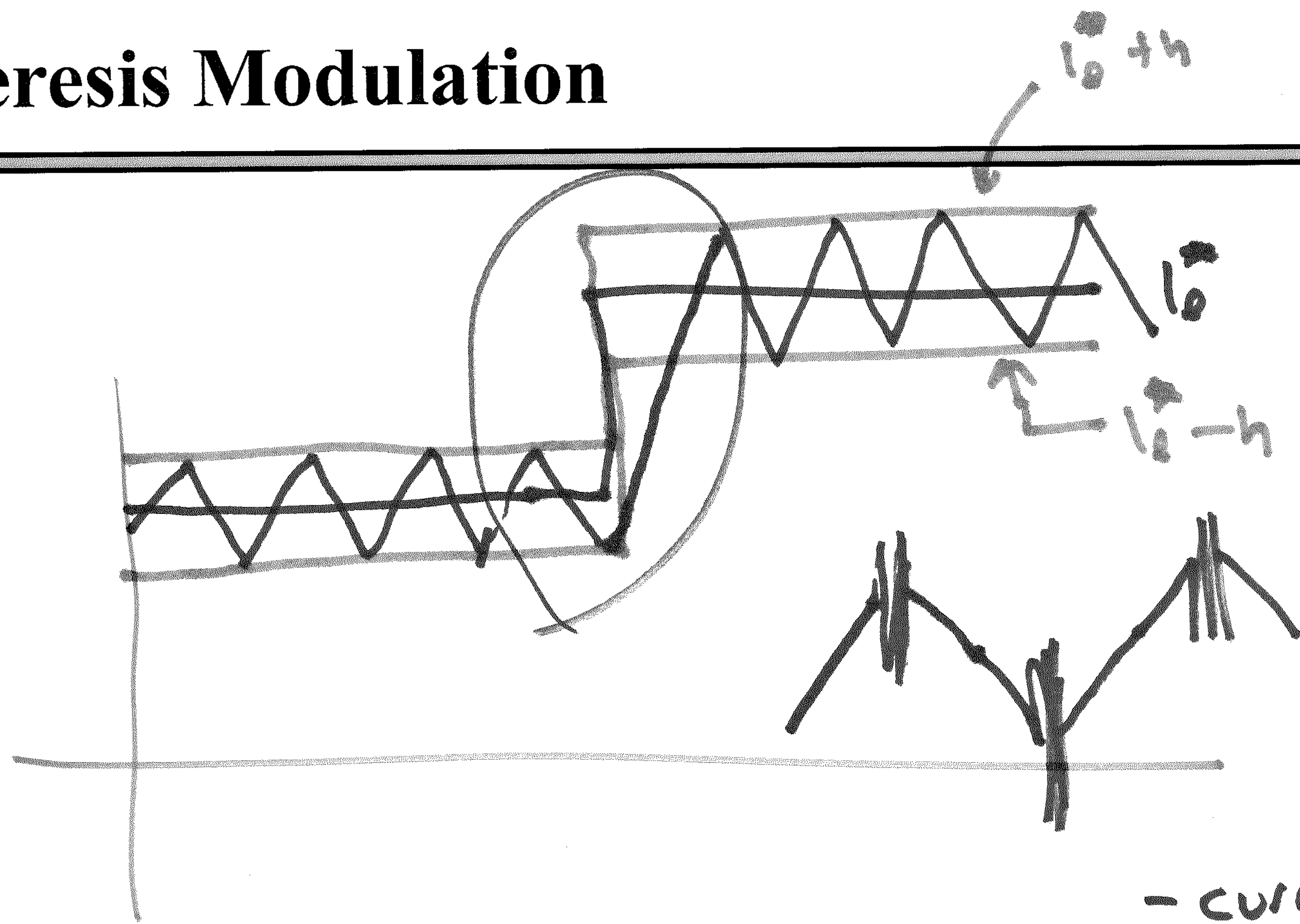
Hysteresis Modulation

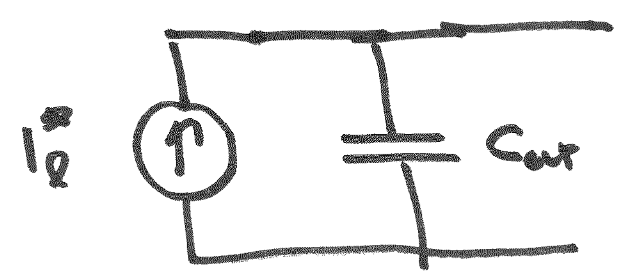


→ current control

$i_\ell \approx i_\ell^*$

Hysteresis Modulation



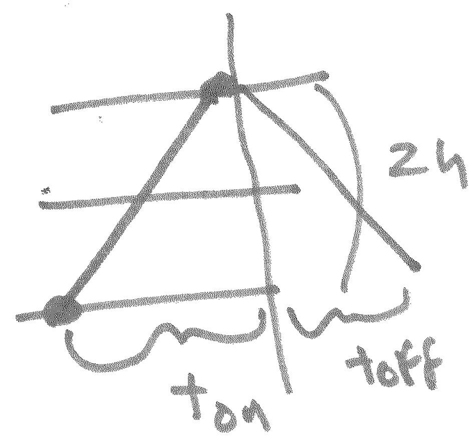


- current source
- robust to faults
- need a <u>good</u> current sensor
- Variable switching frequency

Hysteresis Modulation

- On time

$$-V_c + V_{fsw} + L_{out}\underbrace{\cancel{\frac{di_l}{dt}}}_{\frac{2h}{t_{on}}} + r_{Lout}\overbrace{\cancel{i_l}}^{i_l^*} + V_{out} = 0$$



$$\Downarrow$$

- Thus

$$t_{on} = \frac{2hL_{out}}{v_{in} - v_{fsw} - r_{Lout}i_l^* - v_{out}}$$

Hysteresis Modulation

- Off time

$$-V_{fd} + L_{out}\underbrace{\cancel{\frac{di_l}{dt}}}_{-\frac{2h}{t_{off}}} + r_{Lout}\overbrace{\cancel{i_l}}^{i_l^*} + V_{out} = 0$$

$$\Downarrow$$

- Thus

$$t_{off} = \frac{2hL_{out}}{v_{fd} + r_{Lout}i_l^* + v_{out}}$$

Hysteresis Modulation

- On time: $t_{on} = \dfrac{2hL_{out}}{v_{in} - v_{fsw} - r_{Lout} i_l^* - v_{out}}$
- Off time: $t_{off} = \dfrac{2hL_{out}}{v_{fd} + r_{Lout} i_l^* + v_{out}}$
- Now $f_{sw} = \dfrac{1}{T_{sw}} = \dfrac{1}{t_{on} + t_{off}}$

$$\Downarrow$$

- So

$$f_{sw} = \frac{1}{2hL_{out}} \frac{\left(v_{fd} + r_{Lout} i_l^* + v_{out}\right)\left(v_{in} - v_{fsw} - r_{Lout} i_l^* - v_{out}\right)}{v_{in} + v_{fd} - v_{fsw}}$$

ECE61016 Power Electronic Converters and Systems

Lecture Set 3B: Buck Converter Design

S.D. Sudhoff

Fall 2016

Overview

- Inductor sizing
- Capacitor sizing
- Conduction losses
- Switching losses
- Design constraints
- Design metrics
- Design fitness

Ferrite Core DC Inductor Size & Loss

- Sizing model for ferrite core dc inductor

$$K_J = 1$$

$$E_{mi} = \frac{1}{2} L_{inc} i_{pk}^2$$

peak <u>dc</u> current over operating points of interest

$$M = c_M E_{mi} \prod_{k=1}^{K_M} \left(J_{pk} E_{mi}^{1/3} + b_{M,k} \right)^{n_{M,k}}$$

current density corresponding to i_{pk}

$$P_{dc} = c_P K_J^2 E_{mi}^{1/3} \prod_{k=1}^{K_P} \left(J_{pk} E_{mi}^{1/3} + b_{P,k} \right)^{n_{P,k}}$$

dc loss corresponding to i_{pk}

Sudhoff, S.D.; Shane, G.M.; Suryanarayana, H., "Magnetic-Equivalent-Circuit-Based Scaling Laws for Low-Frequency Magnetic Devices," *Energy Conversion, IEEE Transactions on* , vol.28, no.3, pp.746,755, Sept. 2013

Ferrite Core DC Inductor Size & Loss

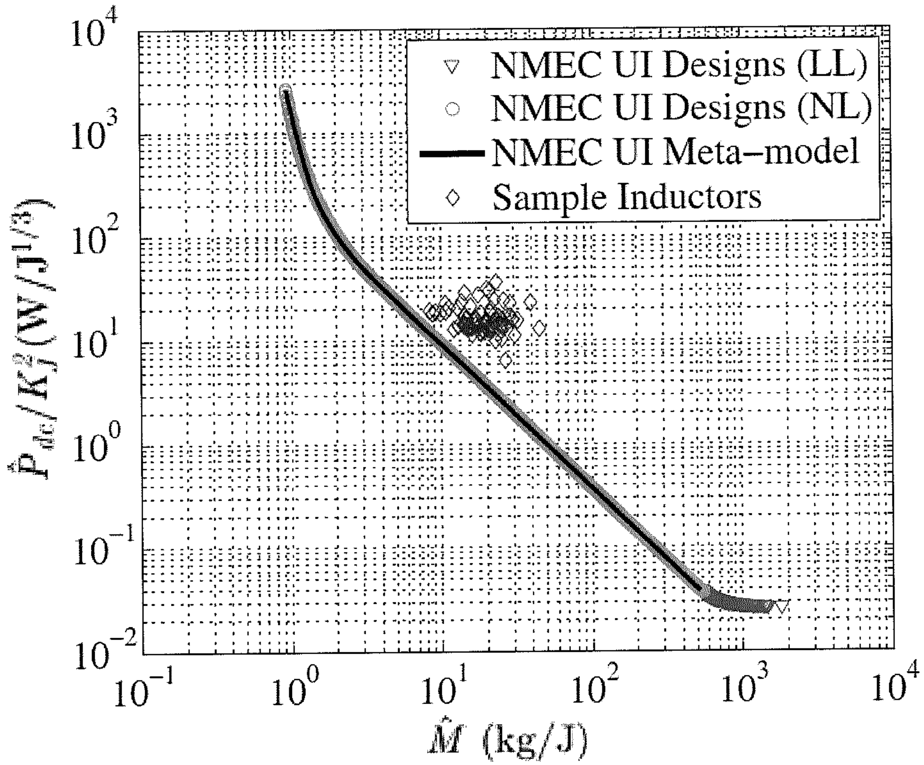
TABLE I
METAMODEL PARAMETERS

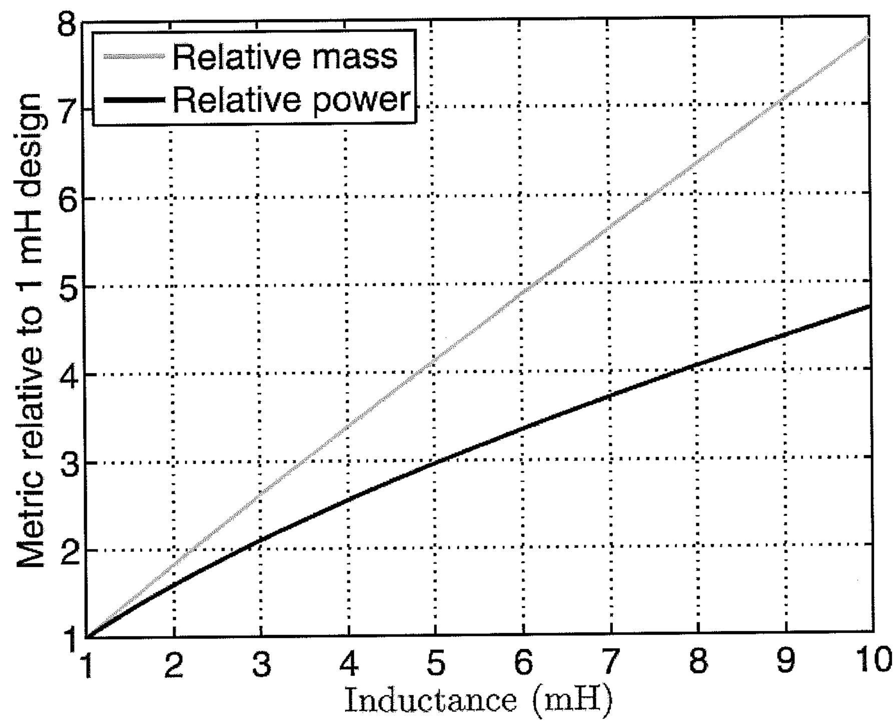
k	$b_{M,k}$	$n_{M,k}$	$b_{P,k}$	$n_{P,k}$
1	0	0.24700	0	0.54482
2	100.05	0.24673	$1.1658 \cdot 10^{3}$	0.25254
3	100.05	-1.3215	$1.1658 \cdot 10^{3}$	0.17114
4	$3.4677 \cdot 10^{6}$	-1.2423	$1.1659 \cdot 10^{3}$	0.24906
5	$8.2537 \cdot 10^{6}$	2.4809	$5.1412 \cdot 10^{4}$	-0.15241
6	$7.3079 \cdot 10^{7}$	-2.0633	$4.4344 \cdot 10^{5}$	0.52755
7	$1.0430 \cdot 10^{8}$	1.5530	$1.2330 \cdot 10^{6}$	-0.59614

$c_M = 5.9851$; $c_P = 1.1021 \cdot 10^{-5}$

Sudhoff, S.D.; Shane, G.M.; Suryanarayana, H., "Magnetic-Equivalent-Circuit-Based Scaling Laws for Low-Frequency Magnetic Devices," *Energy Conversion, IEEE Transactions on* , vol.28, no.3, pp.746,755, Sept. 2013

Ferrite Core DC Inductor Size & Loss





Sudhoff, S.D.; Shane, G.M.; Suryanarayana, H., "Magnetic-Equivalent-Circuit-Based Scaling Laws for Low-Frequency Magnetic Devices," *Energy Conversion, IEEE Transactions on* , vol.28, no.3, pp.746,755, Sept. 2013

Electrolytic Capacitor Size & Loss

- Behavioral Sizing Model

$$C = C_0\left(\alpha_C + \frac{1-\alpha_C}{1+(f_{sw}/f_C)^{\eta_C}}\right)$$

$$M_C = \beta_C C_0 v_{C,b}^{3/2}$$

$$r_C = \frac{\gamma_C}{C_0 v_{C,b}}$$

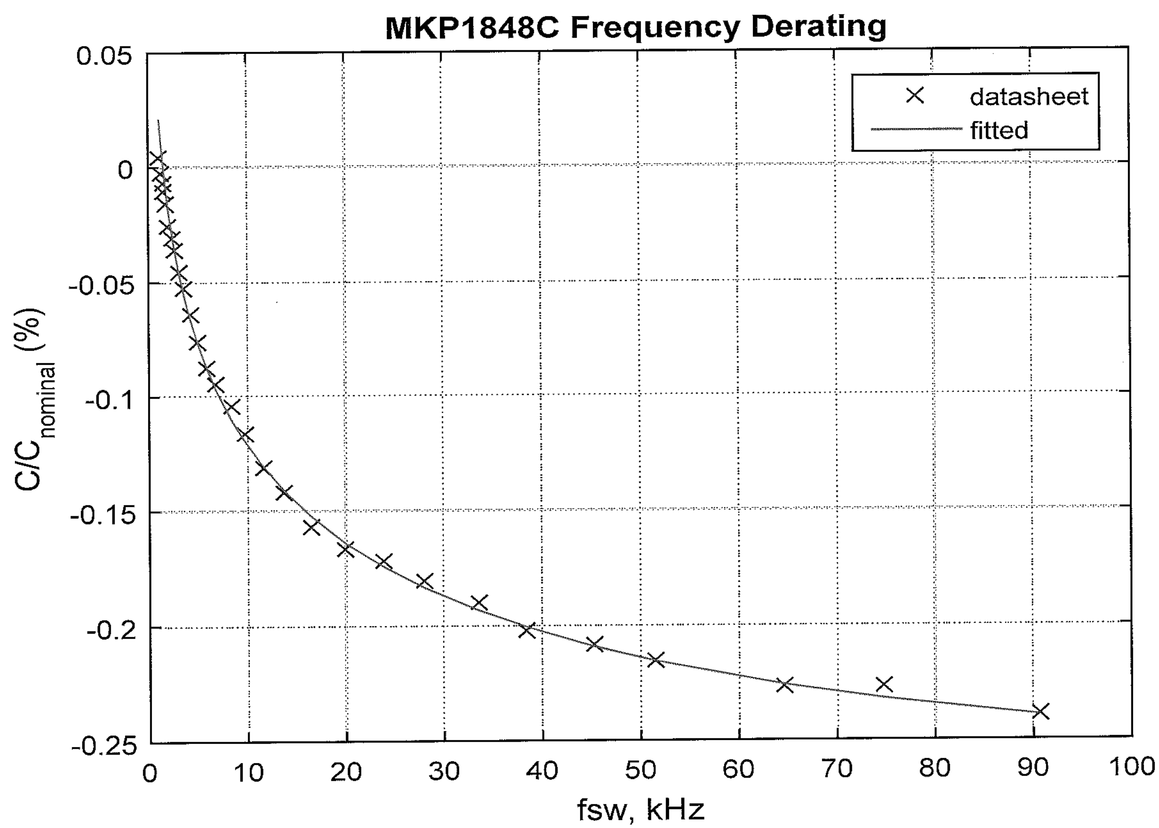
<u>Evox Rifa PEH200 Output Capacitor</u>

α_C	$1.436\cdot10^{-19}$	Effective capacitance model parameter
f_C	8746.2 Hz	Effective capacitance model parameter
n_C	1.9255	Effective capacitance model parameter
β_C	$3.3578\cdot10^{-2}$ kg/(F·V$^{3/2}$)	Effective mass model parameter
γ_C	$2.694\cdot10^{-2}$ Ω·F·V	Effective ESR model parameter

S.Wang, "Automated design approaches for power electronics converters," Ph.D. dissertation, School of Electrical and Computer Engineering, Purdue University, West Lafayette, IN, USA, Aug. 2010.

Polypropylene Capacitor Size & Loss

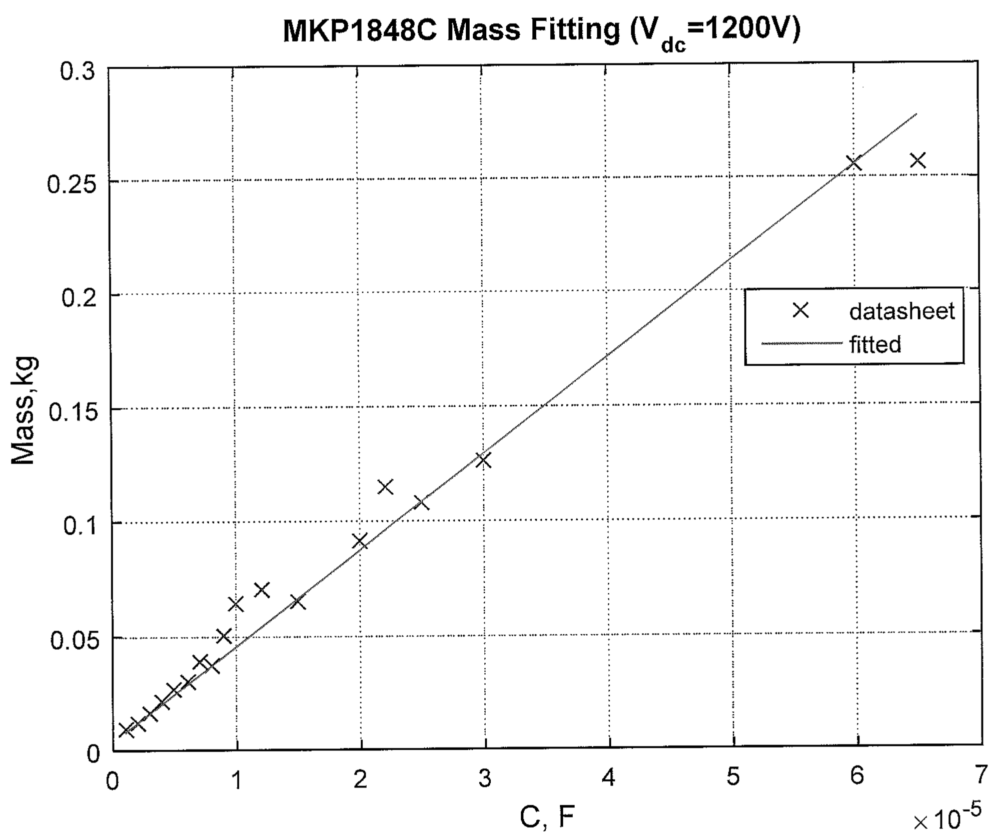
- Consider VISHA MKP1848C (500-1200 V)



Vishay Intertechnology, Inc, “MKP1848C DC-Link Datasheet.” Available: vishay.com

Polypropylene Capacitor Size & Loss

- Mass



$$M_C = \alpha_m C + \beta_m$$

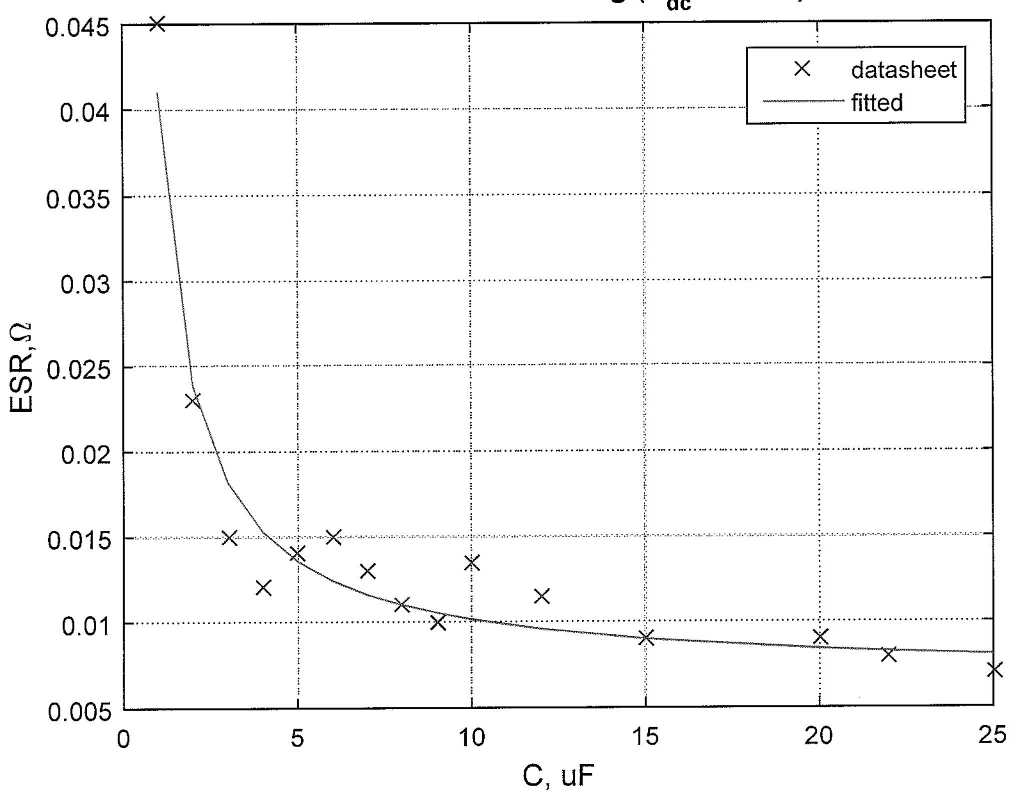
$$\alpha_m = 4.21 \cdot 10^3 \text{ kg/F}$$

$$\beta_m = 3.3 \cdot 10^{-3} \text{ kg}$$

Vishay Intertechnology, Inc, "MKP1848C DC-Link Datasheet." Available: vishay.com

Polypropylene Capacitor Size & Loss

- Loss



$$P_{esr} = R_{esr} I^2_{c,rms}$$

$$R_{esr} = \frac{\alpha_{esr}}{C} + \beta_{esr}$$

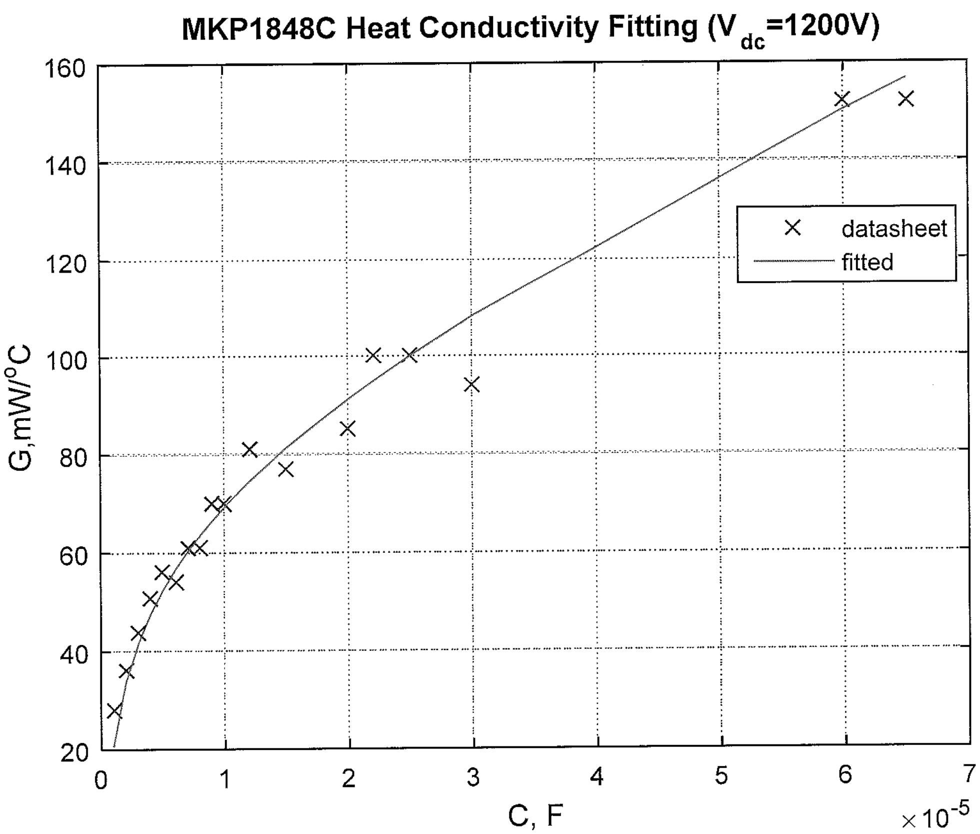
$$\alpha_{esr} = 3.43 \cdot 10^{-8}\ \Omega\mathrm{F}$$

$$\beta_m = 6.7 \cdot 10^{-3}\ \Omega$$

Vishay Intertechnology, Inc, "MKP1848C DC-Link Datasheet." Available: vishay.com

Polypropylene Capacitor Size & Loss

- Thermal Conductivity



$$T_{Ccase} = T_A + \frac{P_{esr}}{G_{Ccase}}$$

$$G_{Ccase} = \alpha_G \ln\left(\frac{C}{1\text{F}}\right) + \beta_G C + \gamma_G$$

$$\alpha_G = 1.73 \cdot 10^{-2} \frac{W}{^oC}$$

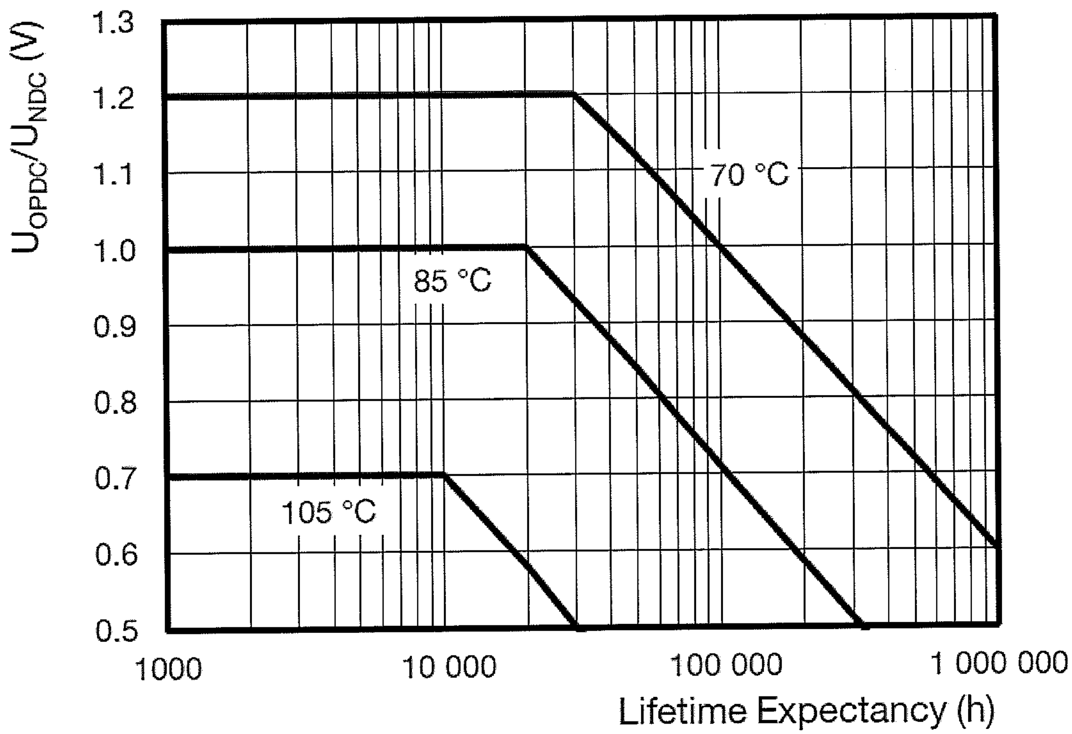
$$\beta_G = 1.00 \cdot 10^{3} \frac{W}{^oC\ F}$$

$$\gamma_G = 2.58 \cdot 10^{-1} \frac{W}{^oC}$$

Vishay Intertechnology, Inc, "MKP1848C DC-Link Datasheet." Available: vishay.com

Polypropylene Capacitor Size & Loss

- Lifetime



Lifetime expectancy
(typical)

Vishay Intertechnology, Inc, "MKP1848C DC-Link Datasheet." Available: vishay.com

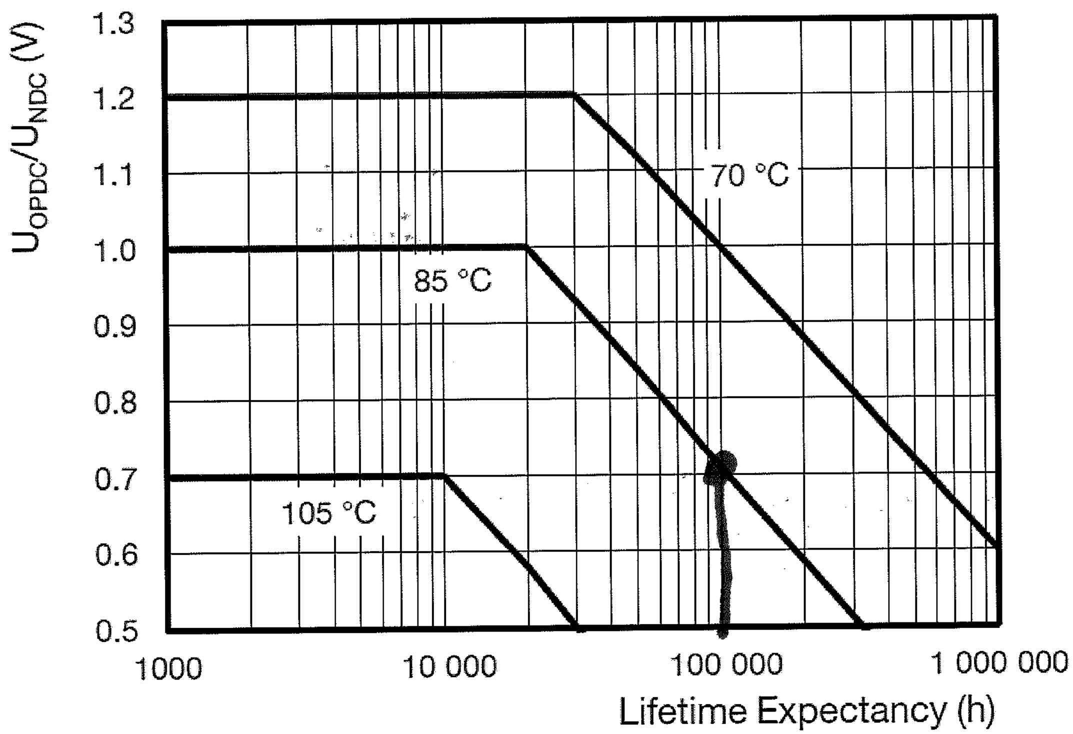
Polypropylene Capacitor Size & Loss

- Example. Suppose we are design a buck converter and selecting an input capacitor. The rms current into the capacitor so 10 A rms, and the nominal voltage is ~~600~~ 340 V. What is the voltage and minimum capacitance we can use assuming we want to operate on the 85°C curve and achieve a 100,000 h expected lifetime. Ambient temperature $T_A = 40°C$.

Polypropylene Capacitor Size & Loss

- Voltage

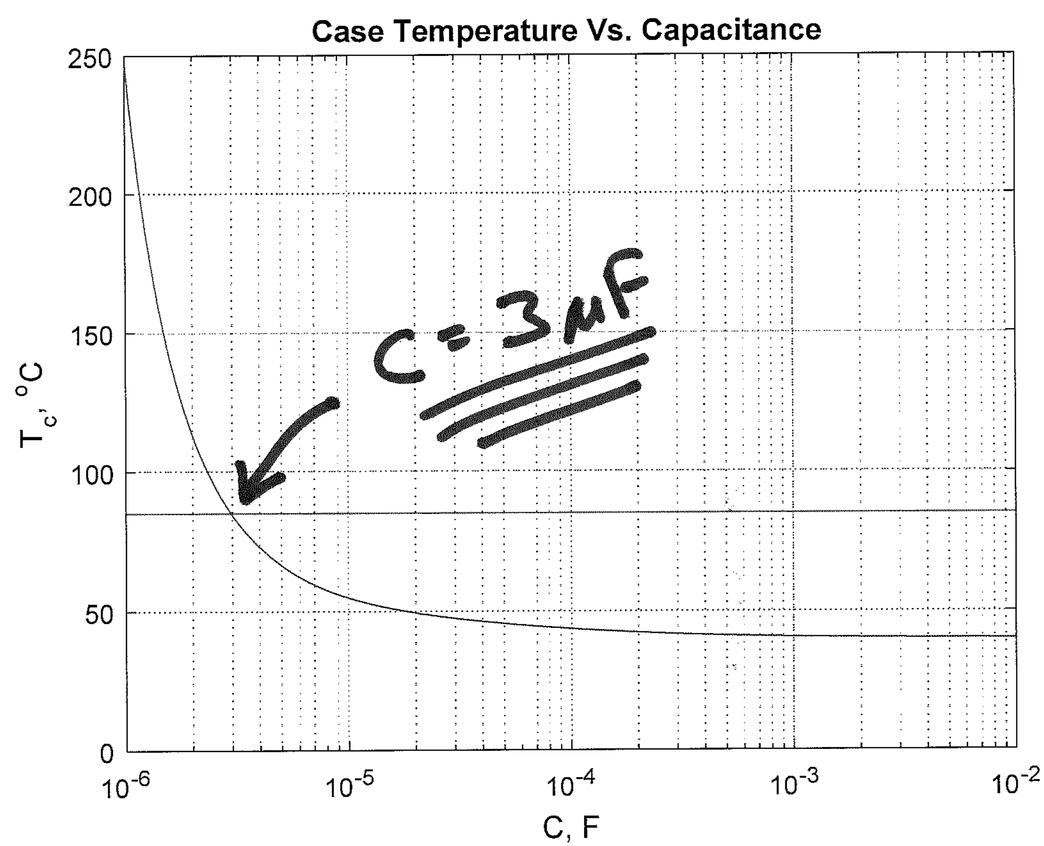
$U_{NDC} = \frac{840V}{0.7} = 1200V$



Lifetime expectancy
(typical)

Polypropylene Capacitor Size & Loss

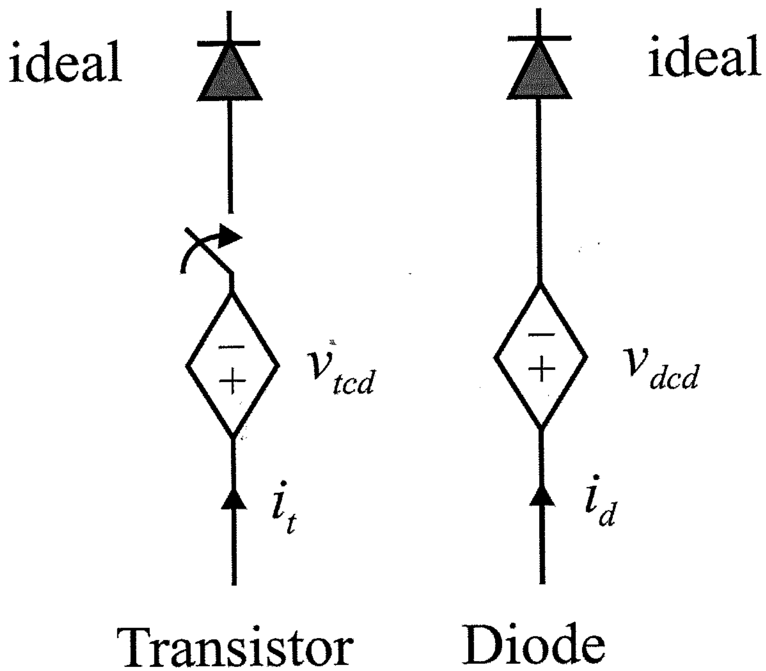
- Capacitance



$C \to G \to T$

$C \to R \to P \to T$

Representation of Conduction Losses



$$v_{xcd} = f_{xcd}(i_x, T_{xj})$$

$X = \{ 't', 'd' \}$

V_{fsw}

V_{fd}

B.T. Cassimere, *Computationally Efficient Modeling and Characterization of Power Semiconductor Losses*, PhD dissertation, Purdue University, May 2008

IGBT Conduction Drop

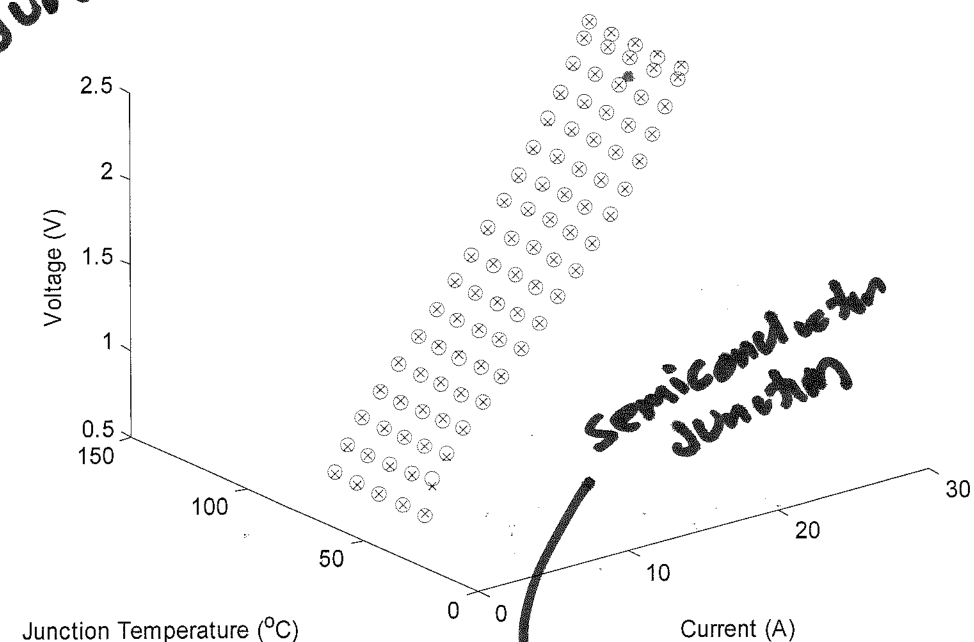
Fuji 7MBR30SA060 (30 A, 600 V)

$$v_{tcd} = \sum_{j=1}^{3} a_{t,j} T_{jt}^{b_{t,j}} i_t^{c_{t,j}}$$

Junction temp

Table 4.4 – IGBT Parameters

Parameter	Value	Unit
$a_{t,1}$	2.420	$\frac{\Omega}{A^{c-1} \circ C^{b}}$
$b_{t,1}$	-2.151×10^{-1}	
$c_{t,1}$	-8.605×10^{-2}	
$a_{t,2}$	-1.060	$\frac{\Omega}{A^{c-1} \circ C^{b}}$
$b_{t,2}$	-3.343×10^{-1}	
$c_{t,2}$	-4.552×10^{-1}	
$a_{t,3}$	9.634×10^{-2}	$\frac{\Omega}{A^{c-1} \circ C^{b}}$
$b_{t,3}$	9.764×10^{-2}	
$c_{t,3}$	7.266×10^{-1}	



Semiconductor Junction

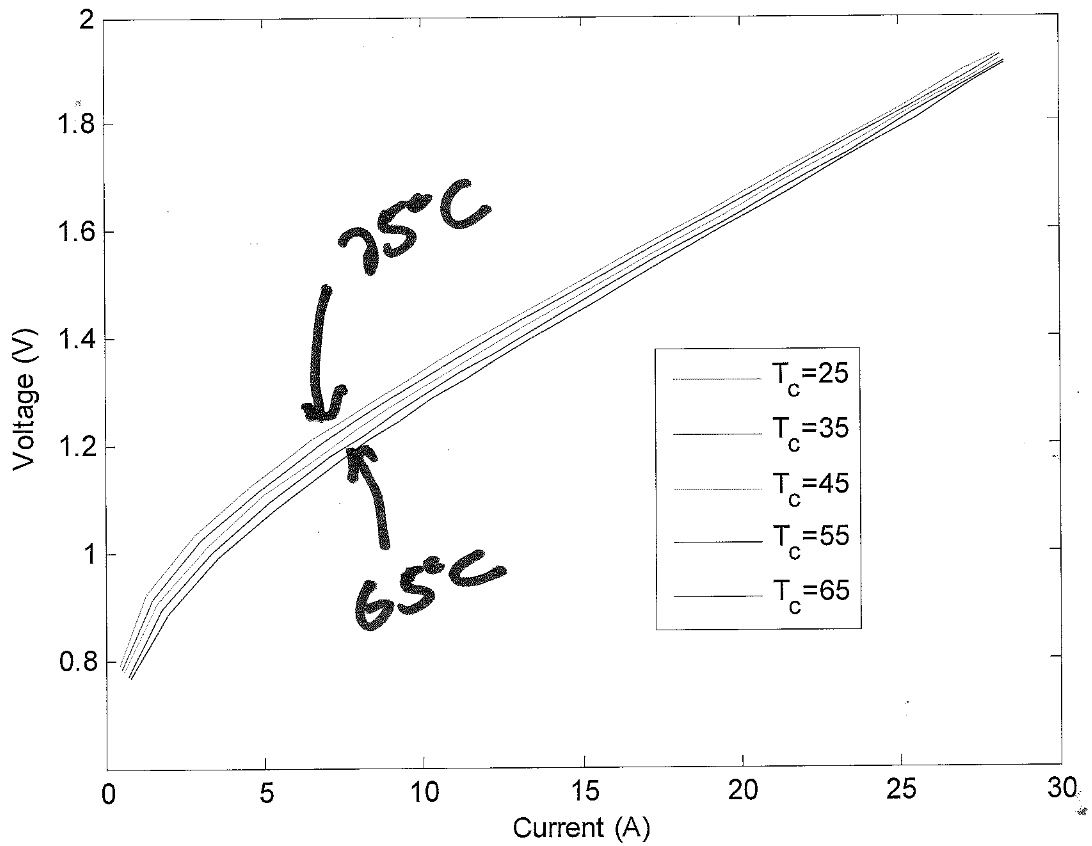
Case

$$T_j = R_{jc} P_j + T_c$$

B.T. Cassimere, *Computationally Efficient Modeling and Characterization of Power Semiconductor Losses*, PhD dissertation, Purdue University, May 2008

Diode Conduction Losses

Fuji 7MBR30SA060 (30 A, 600 V)



B.T. Cassimere, *Computationally Efficient Modeling and Characterization of Power Semiconductor Losses*, PhD dissertation, Purdue University, May 2008

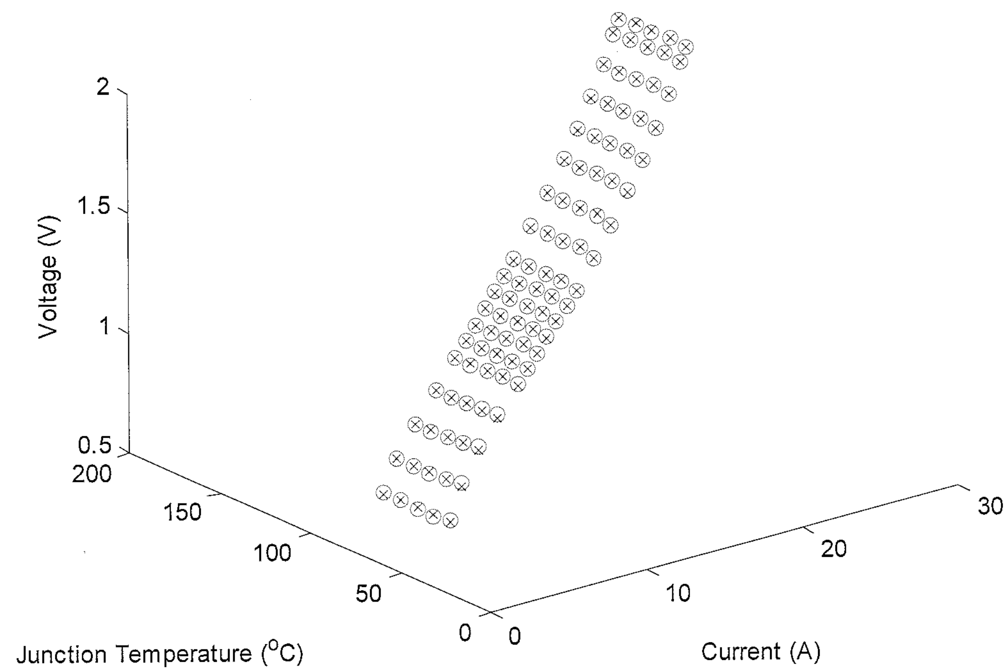
Diode Conduction Drop

Fuji 7MBR30SA060 (30 A, 600 V)

$$v_{dcd} = \sum_{j=1}^{3} a_{d,j} T_{jd}^{b_{d,j}} i_d^{c_{d,j}}$$

Table 4.5 - Diode Parameters

Parameter	Value	Unit
$a_{d,1}$	2.932×10^{-2}	$\frac{\Omega}{A^{c-1} \circ C^{b}}$
$b_{d,1}$	-2.472×10^{-2}	
$c_{d,1}$	1.058	
$a_{d,2}$	-2.482	$\frac{\Omega}{A^{c-1} \circ C^{b}}$
$b_{d,2}$	-2.155	
$c_{d,2}$	1.777	
$a_{d,3}$	1.382	$\frac{\Omega}{A^{c-1} \circ C^{b}}$
$b_{d,3}$	-1.398×10^{-1}	
$c_{d,3}$	1.261×10^{-1}	



B.T. Cassimere, *Computationally Efficient Modeling and Characterization of Power Semiconductor Losses*, PhD dissertation, Purdue University, May 2008

Semiconductor Conduction Losses

- Let's consider a simpler form

instantaneous loss

$$p_{x,cd} = \alpha_{x,cd} i + \beta_{x,cd} i^{\gamma_{x,cd}}$$

average loss

$$P_{x,cd} = \frac{1}{T_{sw}} \int_0^{T_{sw}} p_{x,cd}\, dt$$

suppose $i = mt + b$

<u>aside</u>

$$T = \int \alpha_{x,cd}(mt+b) + \beta_{x,cd}(mt+b)^{\gamma_{x,cd}}\, dt$$

$$= \frac{1}{2}\frac{\alpha_{x,cd}}{m}(mt+b)^2 + \frac{\beta_{x,cd}}{m(\gamma_{x,cd}+1)}(mt+b)^{\gamma_{x,cd}+1}$$

$$T = \frac{1}{2}\frac{\alpha_{x,cd}}{m} i^2 + \frac{\beta_{x,cd}}{m(\gamma_{x,cd}+1)} i^{\gamma_{x,cd}+1}$$

Semiconductor Conduction Losses

- For diode

$$P_{d,cd} = \frac{1}{T_{sw}} \int_0^{T_{sw}} p_{t,cd}\, dt$$

$$i = mt + b \longrightarrow m = \left(\frac{I_{mn} - I_{mx}}{(1-d)T_{sw}} \right)$$

$$P_{d,cd} = \frac{1}{T_{sw}} \int_0^{(1-d)T_{sw}} p_{d,cd}\, dt$$

$$= \frac{1}{\cancel{T_{sw}} \left(\frac{I_{mn} - I_{mx}}{(1-d)\cancel{T_{sw}}} \right)} \left[\frac{1}{2} \alpha_{d,cd} \left(I_{mn}^2 - I_{mx}^2 \right) + \frac{1}{\gamma_{d,cd}+1} \beta_{d,cd} \left(I_{mn}^{\gamma_{cd}+1} - I_{mx}^{\gamma_{cd}+1} \right) \right]$$

$$= \frac{1-d}{I_{mx} - I_{mn}} \left[\frac{1}{2} \alpha_{d,cd} \left(I_{mx}^2 - I_{mn}^2 \right) + \frac{1}{\gamma_{d,cd}+1} \beta_{d,cd} \left(I_{mx}^{\gamma_{cd}+1} - I_{mn}^{\gamma_{cd}+1} \right) \right]$$

IGBT Switching Losses

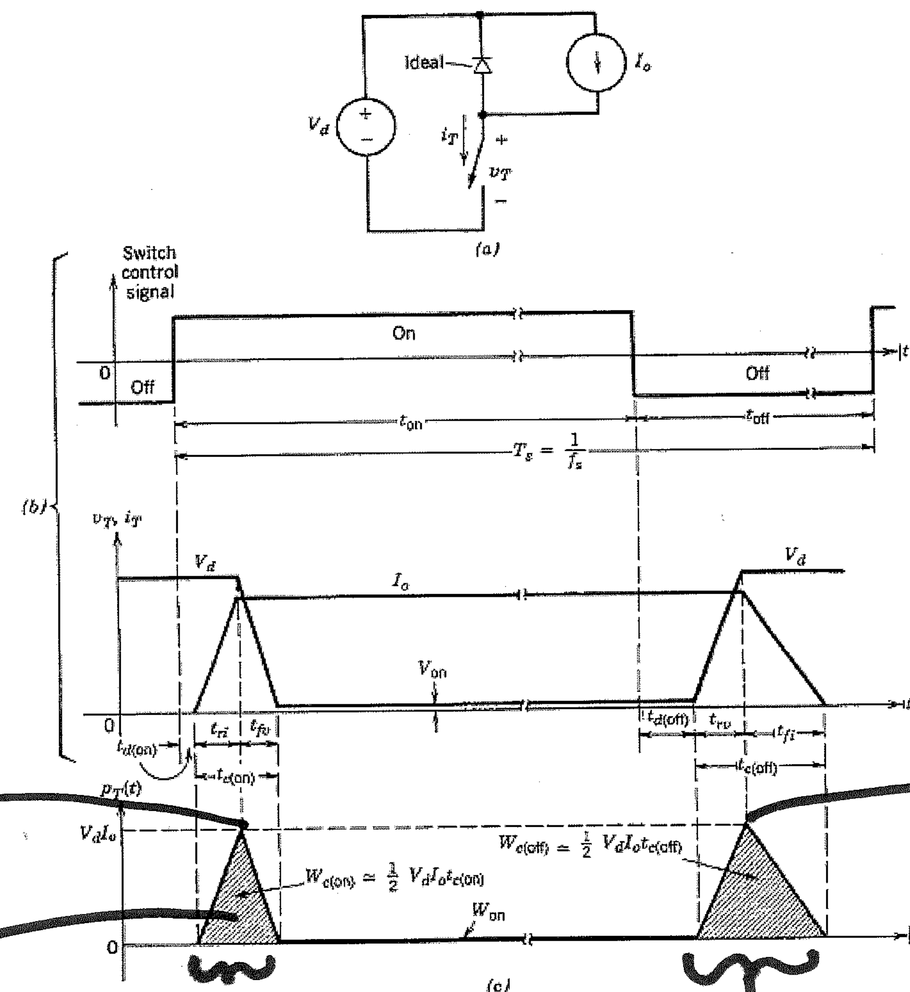
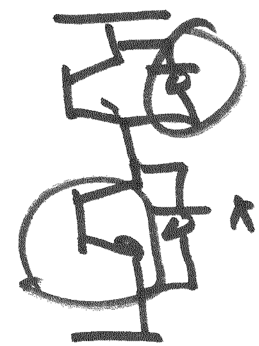


Figure 2-6 Generic switch switching characteristics (linearized): (*a*) simplified clamped-inductive-switching circuit, (*b*) switch waveforms, (*c*) instantaneous switch power loss.



$V_d I_o$

$E_{off} = \frac{1}{2} t_{off} V_d I_o$

$V_d I_o$

$\boxed{E_{on} = \frac{1}{2} t_{on} V_d I_o}$

*Mohan, Undeland, Robbins, *Power Electronics, Converters, Applications, and Design*, 2nd Edition

IGBT Switching Losses

- One approach

$$\mathrm{E}_{t,y}(i, v_{CE}) = \frac{v_{CE}}{v_{t,b}}\left(\alpha_{t,y} i_t^2 + \beta_{t,y} i_t + \gamma_{t,y}\right)$$

$$P_{t,on} = f_{sw}\,\mathrm{E}_{t,on}(i_{mn}, v_c)$$

$$P_{t,off} = f_{sw}\,\mathrm{E}_{t,off}(i_{mx}, v_c)$$

Shane, G.M.; Sudhoff, S.D., "Design Paradigm for Permanent-Magnet-Inductor-Based Power Converters," *Energy Conversion, IEEE Transactions on* , vol.28, no.4, pp.880,893, Dec. 2013

Power Diode Turn Off Losses

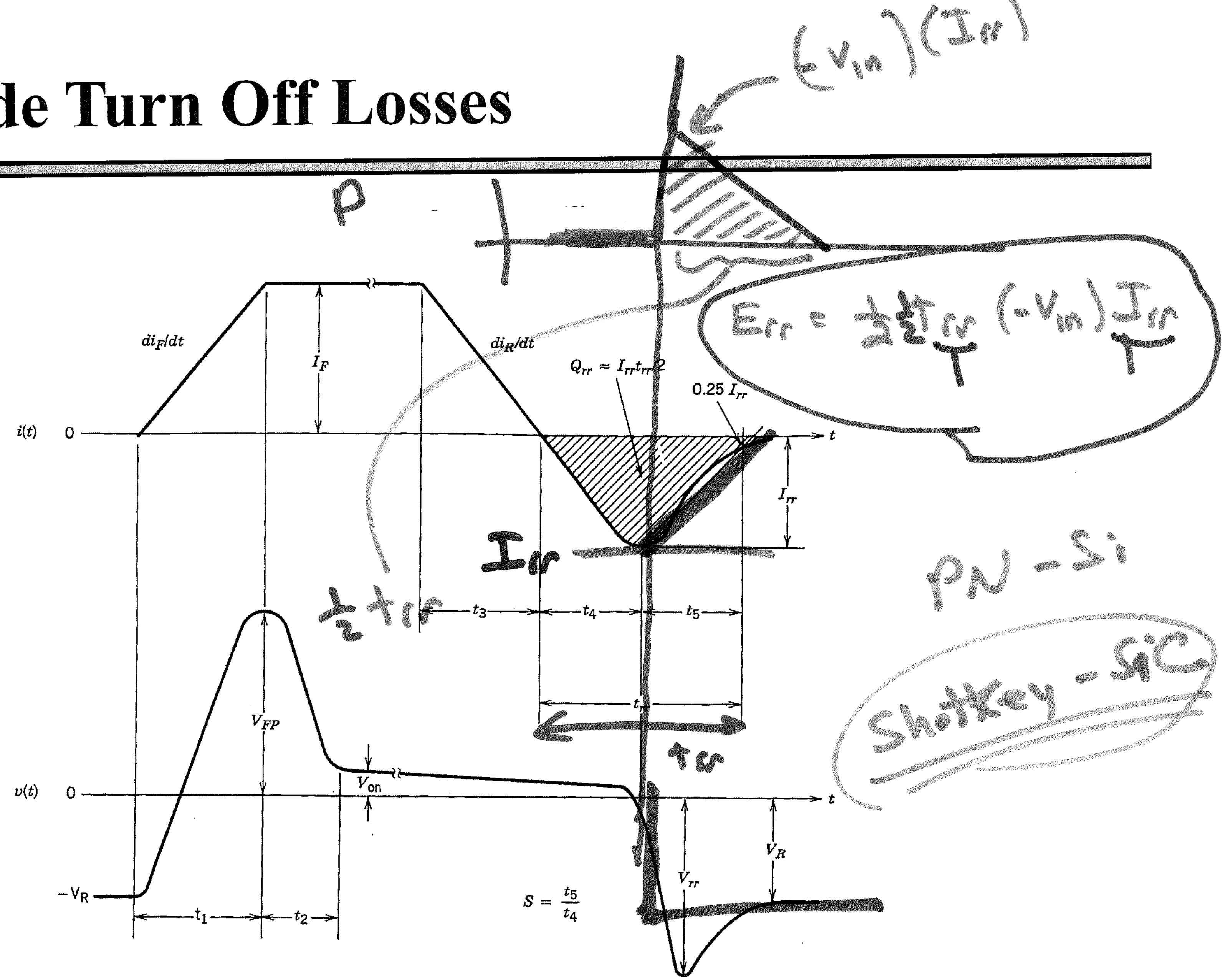


Figure 20-9 Voltage and current waveforms for a power diode driven by currents with a specified rate of rise during turn-on and a specified rate of fall during turn-off.

*Mohan, Undeland, Robbins, *Power Electronics, Converters, Applications, and Design,* 2nd *Edition*

Power Diode Turn Off Losses

- One approach

$$I_{rr0}(i_d) = \alpha_{d,I_{rr}} i_d + \beta_{d,I_{rr}} (i_d)^{\gamma_{d,I_{rr}}}$$

nominal I_{rr} at a base value of voltage $V_{d,b}$

$$t_{rr}(i_d) = \alpha_{d,t_{rr}} i_d + \beta_{d,t_{rr}} (i_d)^{\gamma_{d,t_{rr}}}$$

reverse recovery time

$$I_{rr}(i_d) = \frac{v_{d,off} \xrightarrow{} -V_{in}}{v_{d,b}} I_{rr0}(i_d)$$

actual reverse recovery current scaled for voltage

$$P_{d,rr} = \frac{v_{in}^2 I_{rr0}(i_{mn}) t_{rr}(i_{mn})}{4 v_{d,b} T_{sw}}$$

$$E_{rr} = \frac{1}{4} t_{rr} (-V_{in}) I_{rr}$$

$$P_{d,rr} = f_{sw} E_{rr} = \frac{E_{rr}}{T_{sw}}$$

Shane, G.M.; Sudhoff, S.D., "Design Paradigm for Permanent-Magnet-Inductor-Based Power Converters," *Energy Conversion, IEEE Transactions on* , vol.28, no.4, pp.880,893, Dec. 2013

Power Diode Turn Off Losses

- Explanation

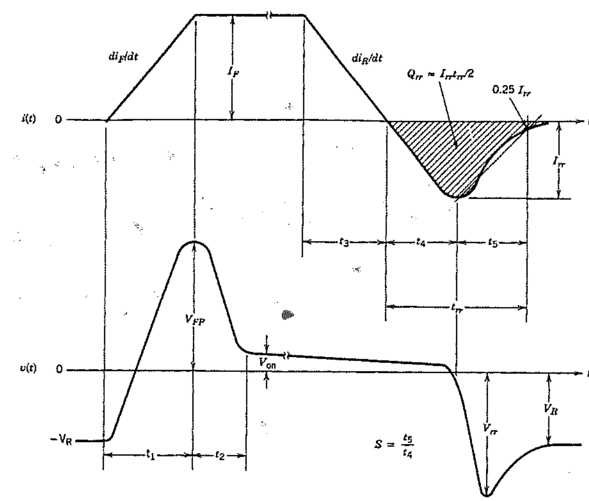
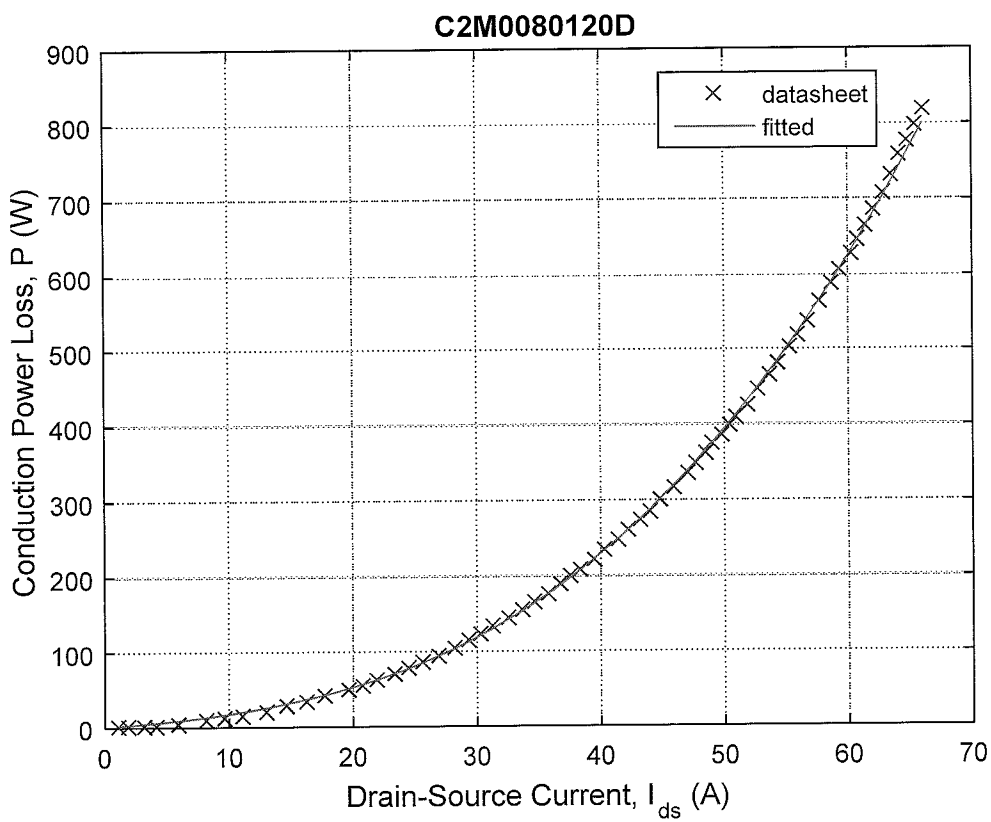


Figure 20-9 Voltage and current waveforms for a power diode driven by currents with a specified rate of rise during turn-on and a specified rate of fall during turn-off.

SiC MOSFETS

- Conduction loss (C2M0080120D – 1.2 kV, 36 A)



$$P_{t,cd} = \alpha_{t,cd} i_{ds} + \beta_{t,cd} i_{ds}^{\gamma_{t,cd}}$$

$$\alpha_{t,cd} = 1.30 \text{ V}$$

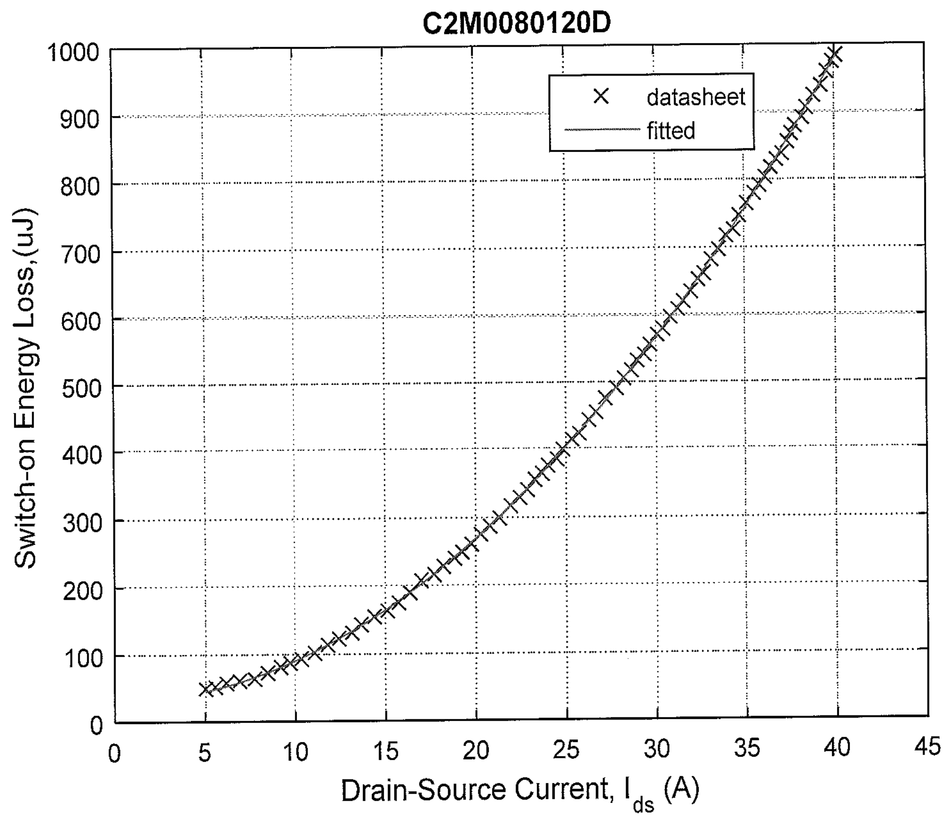
$$\beta_{t,cd} = 6.4 \cdot 10^{-3} V$$

$$\gamma_{t,cd} = 2.77$$

Junction at 150°C

SiC MOSFETS

- Turn on Loss (C2M0080120D – 1.2 kV, 36 A)



$$E_{t,on} = \left(\alpha_{t,on} i_{ds}^2 + \beta_{t,on} i_{ds} + \gamma_{t,on}\right) \frac{v_{CE}}{v_{t,b}}$$

$$\alpha_{t,on} = 0.585 \text{ J/A}^2$$

$$\beta_{t,on} = 0.375 \text{ Vs}$$

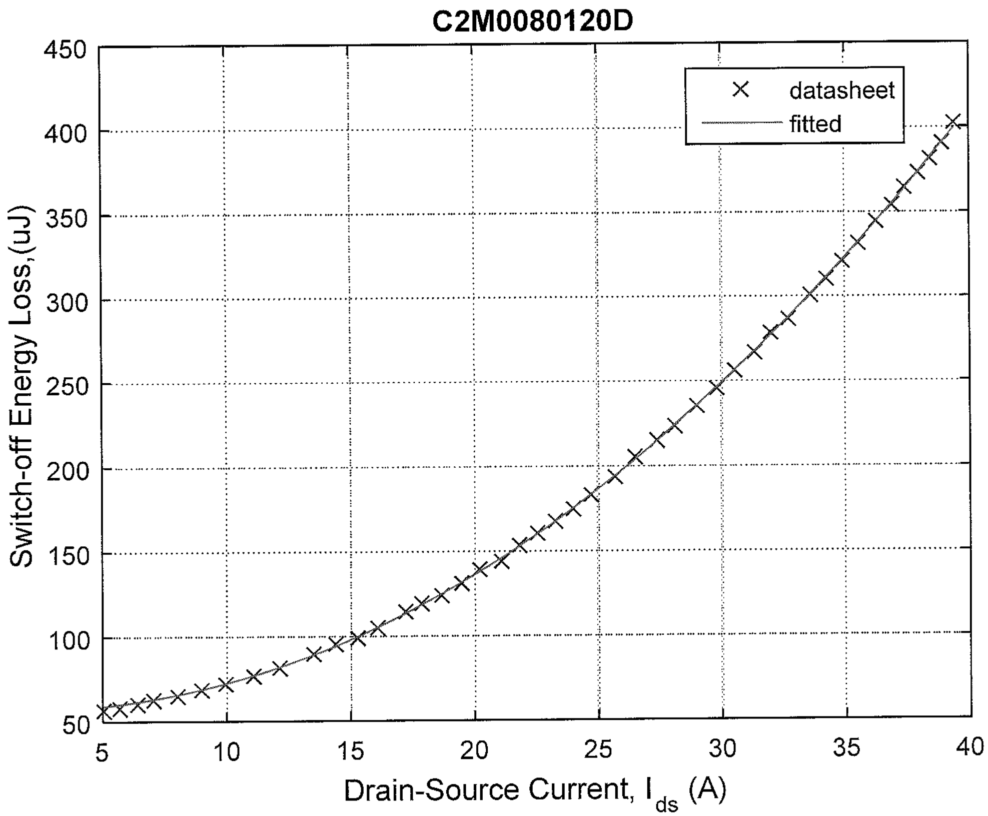
$$\gamma_{t,on} = 2.74 \cdot 10^{-5} \text{ J}$$

$$v_{t,b} = 800 \text{ V}$$

Junction at 150°C

SiC MOSFETS

- Turn off Loss (C2M0080120D – 1.2 kV, 36 A)



$$E_{t,off} = \left(\alpha_{t,off} i_{ds}^2 + \beta_{t,off} i_{ds} + \gamma_{t,off}\right)\frac{v_C}{v_{t,}}$$

$\alpha_{t,off} = 0.245\ \mathrm{J/A^2}$

$\beta_{t,off} = -0.994\ \mathrm{Vs}$

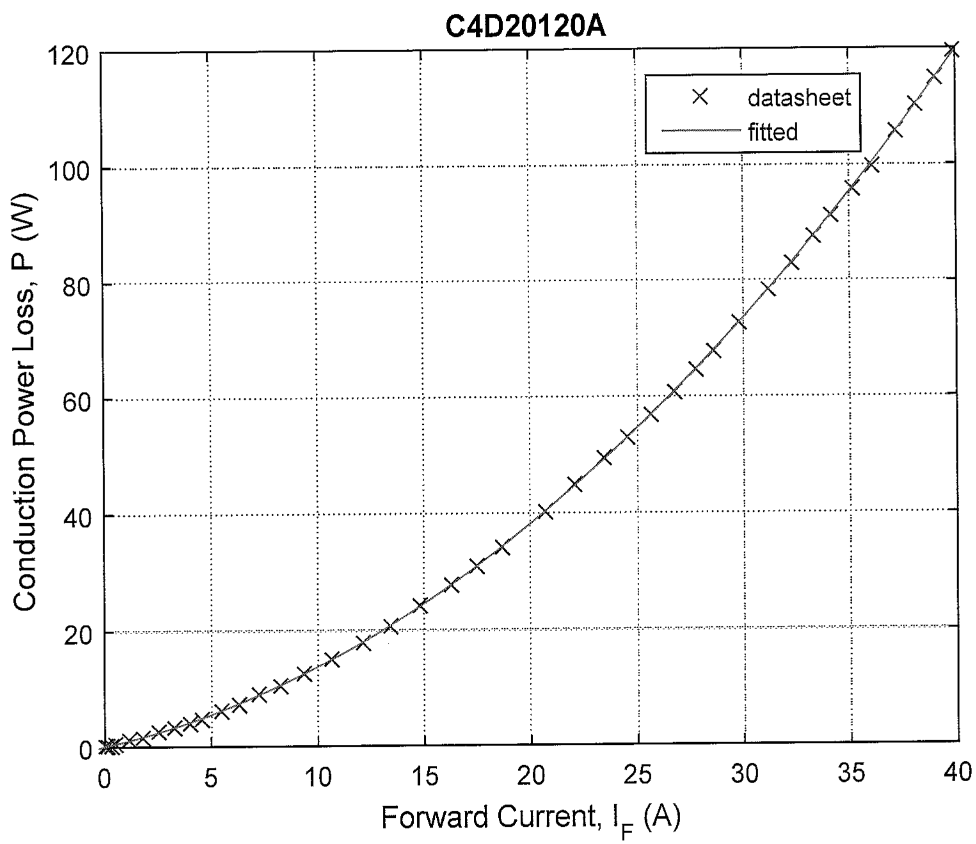
$\gamma_{t,off} = 5.75 \cdot 10^{-5}\ \mathrm{J}$

$v_{t,b} = 800\ \mathrm{V}$

Junction at 150°C

SiC DIODES

- Conduction loss (C4D2012A – 1.2 kV, ~25 A)



$$P_{d,cd} = \alpha_{d,cd} i_{ds} + \beta_{d,cd} i_{ds}^{\gamma_{d,cd}}$$

$$\alpha_{d,cd} = 0.845\ \mathrm{V}$$

$$\beta_{d,cd} = 5.04 \cdot 10^{-2} V$$

$$\gamma_{d,cd} = 2.01$$

Junction at 125°C

Steady-State Analysis

- Given v_{fsw}, and v_{fd} what is d?
- Recall

Know V_{in} V_{out} P_{out}

$$\bar{i}_l = -\bar{i}_{out}$$

$$\bar{i}_s = d\bar{i}_l$$

$$\bar{v}_{out} = \bar{v}_s - r_{Lout}\bar{i}_l$$

$$\bar{I}_l = \frac{P_{out}}{V_{out}}$$

$$\bar{v}_s = d(\bar{v}_c - v_{fsw}) - (1-d)v_{fd}$$

$$\bar{v}_c = \bar{v}_{in} - r_{Lin}\bar{i}_s$$

$$\begin{aligned}\bar{V}_{out} &= d(\bar{V}_c - V_{fsw}) - (1-d)V_{fd} - r_{Lout}\bar{I}_l \\ &= d(\bar{V}_{in} - r_{Lin} d\bar{I}_l - V_{fsw}) - (1-d)V_{fd} - r_{Lout}\bar{I}_l \\ &= d\left(\bar{V}_{in} - r_{Lin} d\frac{P_{out}}{V_{out}} - V_{fsw}\right) - (1-d)V_{fd} - r_{Lout}\frac{P_{out}}{V_{out}}\end{aligned}$$

Steady-State Analysis

$$0 = \underbrace{\left[-r_{Lin}\frac{P_{out}}{V_{out}}\right]}_{a} d^2 + \overbrace{\left[\bar{V}_{in} - V_{Fsw} + V_{Fd}\right]}^{b} d + \underbrace{\left[-V_{out} - V_{Fd} - r_{Lout}\frac{P_{out}}{V_{out}}\right]}_{c}$$

$$d = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad d \approx -\frac{c}{b}$$

why

$$\sqrt{1-x} \approx 1 - \tfrac{1}{2}x$$

$$d = \frac{-b \pm b\sqrt{1 - \frac{4ac}{b^2}}}{2a}$$

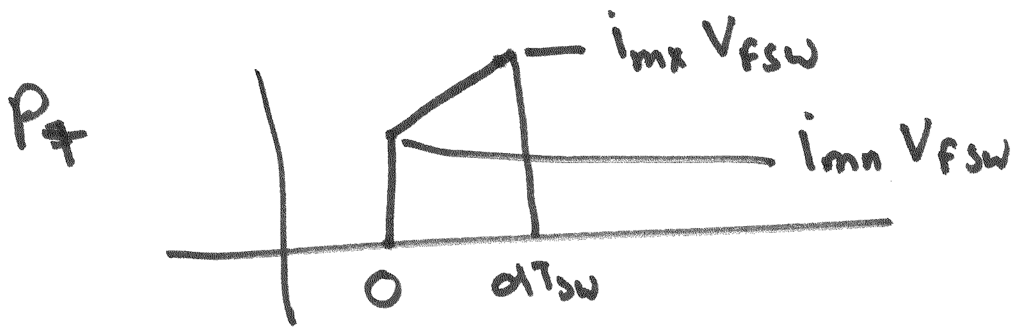
$$= \frac{\cancel{-b} + b\left(\cancel{1} - \frac{1}{2}\frac{4ac}{b^2}\right)}{2a} = -\frac{\cancel{\tfrac{1}{2}}\cancel{4}a c}{\cancel{2}b\cancel{a}}$$

Steady-State Analysis

- Thus

Steady-State Analysis

- Given d, what is v_{fsw}?



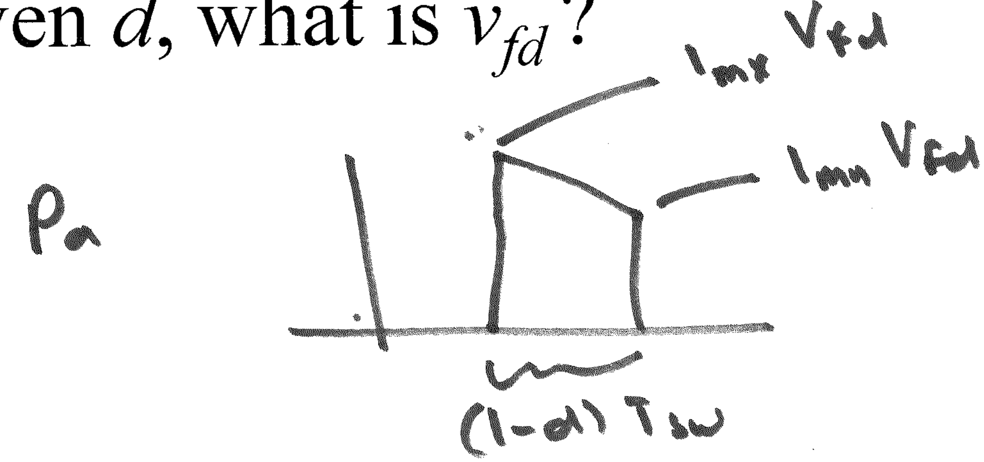
$$\overline{P}_+ = \frac{1}{\cancel{T_{sw}}}(d\cancel{T_{sw}})\frac{1}{2}(i_{mx} V_{fsw} + i_{mn} V_{fsw})$$

$$= d\, V_{fsw}\, \overline{I_\ell}$$

$$\boxed{V_{fsw} = \frac{\overline{P}_+}{d\, \overline{I_\ell}}}$$

Steady-State Analysis

- Given d, what is v_{fd}?



$$\bar{p}_d = \frac{1}{\cancel{T_{sw}}}(1-d)\cancel{T_{sw}}\,\underbrace{\cancel{\tfrac{1}{2}(I_{mx}+I_{mn})}}^{\bar{I}_\ell}\,V_{fd}$$

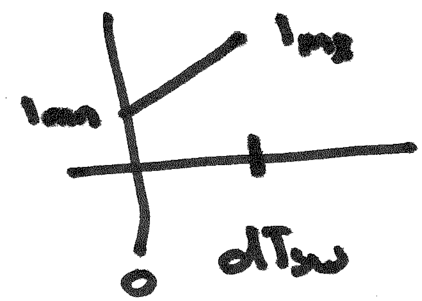
$$= (1-d)\,\bar{I}_\ell\,V_{fd}$$

$$V_{fd} = \frac{\bar{p}_d}{(1-d)\,\bar{I}_\ell}$$

total power loss in the diode

Semiconductor Conduction Losses

- For transistor



$$P_{t,cd} = \frac{1}{T_{sw}} \int_0^{dT_{sw}} p_{t,cd}\, dt$$

$$P_{t,cd} = \frac{1}{\cancel{T_{sw}}\left(\frac{I_{mx}-I_{mn}}{d\cancel{T_{sw}}}\right)} \left[\frac{1}{2}\alpha_{t,cd}\left(I_{mx}^2 - I_{mn}^2\right) + \frac{\beta_{t,cd}}{\gamma_{t,cd}+1}\left(I_{mx}^{\gamma_{t,cd}+1} - I_{mn}^{\gamma_{t,cd}+1}\right)\right]$$

$$m = \left(\frac{I_{mx}-I_{mn}}{dT_{sw}}\right)$$

$$P_{t,cd} = \frac{d}{I_{mx}-I_{mn}}\left[\frac{1}{2}\alpha_{t,cd}\left(I_{mx}^2 - I_{mn}^2\right) + \frac{1}{\gamma_{t,cd}+1}\beta_{t,cd}\left(I_{mx}^{\gamma_{t,cd}+1} - I_{mn}^{\gamma_{t,cd}+1}\right)\right]$$

Steady-State Analysis

- Iterative algorithm for computing d, v_{fsw}, and v_{fd}

Initialization

$K = 1$ (iteration count)

$e = 0$ (error)

$v_{fsw} = 0$

$v_{fd} = 0$

$\bar{i}_l = \frac{P_{out}}{V_{out}}$

$i_{in} = \frac{P_{out}}{V_{in}}$

$c_1 = 1$

Based on i_l, J_{out}, L_{out} $\longrightarrow r_{Lout}$

analysis is OK (flag)

Steady-State Analysis

while $(k=1)$ OR $[\,(e > e_{max})$ and
$(k \le k_{max})$ and
$(c_1 = 1)\,]$

Based on $\dot{i}_{in}$, J_{in}, L_{in} $\longrightarrow$ r_{Lin}

$$c_1 = \begin{cases} 1 & \text{if } b^2 - 4ac \ge 0 \\ 0 & \text{if not} \end{cases} \Bigg\} \text{ slide 32}$$

if c_1

Find d, $\dot{i}_{mn}$, $\dot{i}_{mx}$

Find $V_{fsw,new}$
$V_{fd,new}$

$$e = \sqrt{(V_{fsw,new} - V_{fsw})^2 + (V_{fd,new} - V_{fd})^2}$$

Steady-State Analysis

$$V_{fsw} = V_{fsw,new}$$

$$V_{fd} = V_{fd,new}$$

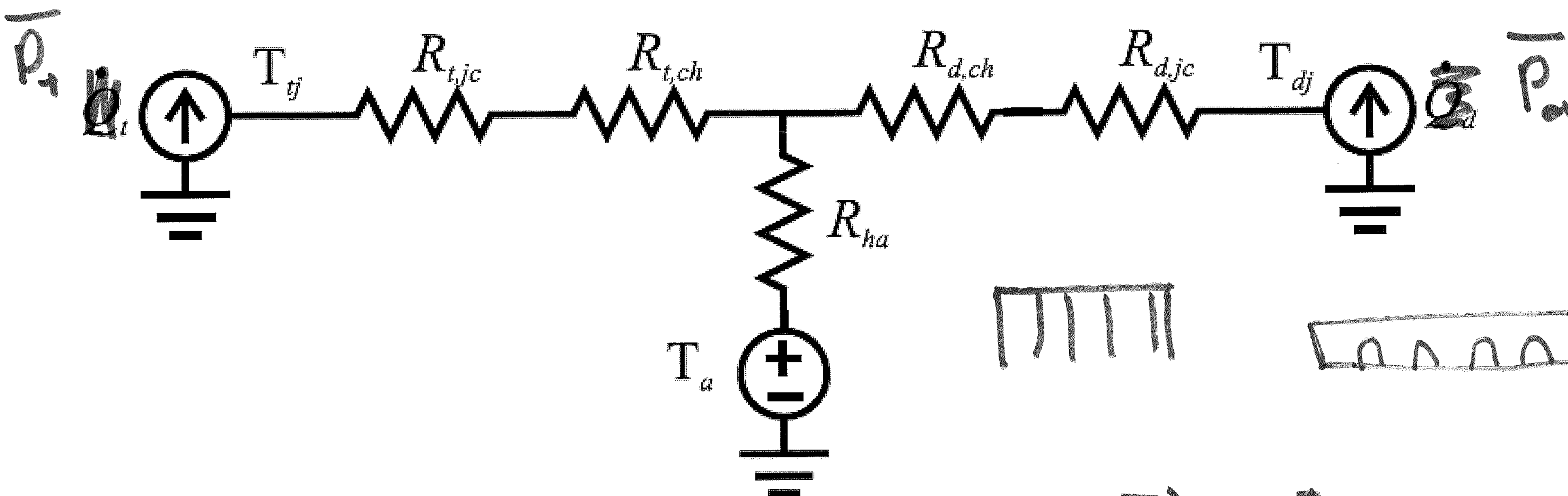
$$\overline{I_{in}} = d\,\overline{I_L}$$

$$k = k + 1$$

```
end % if
end % while
```

Heat Sink

- Simple thermal model



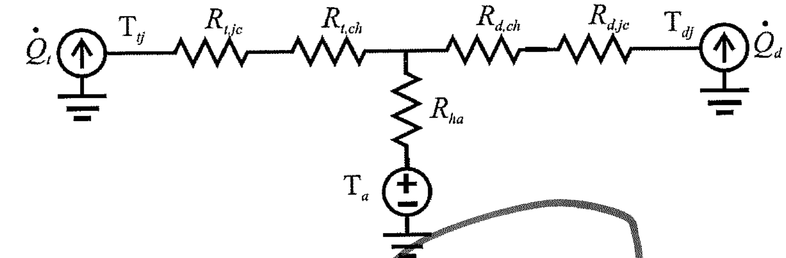
$$T_{tj} = (R_{t,jc} + R_{t,ch})\bar{P}_t + R_{ha}(\bar{P}_t + \bar{P}_d) + T_a$$

$$T_{dj} = (R_{d,jc} + R_{d,ch})\bar{P}_d + R_{ha}(\bar{P}_t + \bar{P}_d) + T_a$$

Heat Sink

if we are not modeling the semiconductor loss as a function of temp.

- Calculation of required thermal resistance



Suppose transistor hits limit

$$T_{tj,max} = (R_{t,jc} + R_{t,ch})\,\overline{P}_t + R_{ha}(\overline{P}_t + \overline{P}_d) + T_a$$

$$R_{ha,1} = \frac{T_{tj,max} - (R_{t,jc} + R_{t,ch})\,\overline{P}_t - T_A}{\overline{P}_t + \overline{P}_d}$$

$R_{ha} = \min(R_{ha,1},\ R_{ha,2})$

Suppose diode sets the limit

$$T_{dj,max} = (R_{d,jc} + R_{d,ch})\,\overline{P}_d + R_{ha}(\overline{P}_t + \overline{P}_d) + T_A$$

$$R_{ha,2} = \frac{T_{dj,max} - (R_{d,jc} + R_{d,ch})\,\overline{P}_d - T_A}{\overline{P}_t + \overline{P}_d}$$

Heat Sink

- Heat sink mass

$$M_H = \frac{a_1}{\left(\frac{R_{ha}}{R_b}\right)^{n_1}} + \frac{a_2}{\left(\frac{R_{ha}}{R_b}\right)^{n_2}}$$

$$R_b = 1^o C / W$$

Aluminum Plate Fin Heat Sink		
a_1	0.1516 kg	Heat sink mass model parameter
a_2	$7.5568 \cdot 10^{-5}$ kg	Heat sink mass model parameter
n_1	1.1688	Heat sink mass model parameter
n_2	5.5445	Heat sink mass model parameter

Shane, G.M.; Sudhoff, S.D., "Design Paradigm for Permanent-Magnet-Inductor-Based Power Converters," *Energy Conversion, IEEE Transactions on* , vol.28, no.4, pp.880,893, Dec. 2013

Some Sample Data

Microsemi APT80GP60JDQ3 IGBT

$\alpha_{t,cd}$	0.1542 V	Conduction loss model parameter
$\beta_{t,cd}$	0.3522 V	Conduction loss model parameter
$\gamma_{t,cd}$	1.3752	Conduction loss model parameter
$\alpha_{t,on}$	$8.656 \cdot 10^{-8}$ V·s/A	Turn-on loss model parameter
$\beta_{t,on}$	$1.645 \cdot 10^{-5}$ V·s	Turn-on loss model parameter
$\gamma_{t,on}$	$2.840 \cdot 10^{-4}$ J	Turn-on loss model parameter
$\alpha_{t,off}$	$9.304 \cdot 10^{-8}$ V·s/A	Turn-off loss model parameter
$\beta_{t,off}$	$1.192 \cdot 10^{-5}$ V·s	Turn-off loss model parameter
$\gamma_{t,off}$	$1.639 \cdot 10^{-4}$ J	Turn-off loss model parameter
$v_{t,b}$	400 V	Base voltage
v_{fsw}	1.25 V	Average forward voltage drop
T_{tj}	150 °C	Junction temperature
$R_{t,jc}$	0.27 °C/W	Thermal resistance junction-to-case
$R_{t,ch}$	0.3 °C/W	Thermal resistance case-to-heat sink

Powerex CS241250 Diode

$\alpha_{t,cd}$	0.4131 V	Conduction loss model parameter
$\beta_{t,cd}$	0.2799 V	Conduction loss model parameter
$\gamma_{t,cd}$	1.3553	Conduction loss model parameter
$\alpha_{d,trr}$	0.0031 μs/A	Reverse-recovery time model parameter
$\beta_{d,trr}$	0.2609 μs/A	Reverse-recovery time model parameter
$\gamma_{d,trr}$	0.1275	Reverse-recovery time model parameter
$\alpha_{d,Irr}$	$1.7636 \cdot 10^{-6}$	Reverse-recovery current magnitude model parameter
$\beta_{d,Irr}$	4.1159	Reverse-recovery current magnitude model parameter
$\gamma_{d,Irr}$	0.6493	Reverse-recovery current magnitude model parameter
$v_{d,b}$	600 V	Base voltage
v_{fd}	1.25 V	Average forward voltage drop
T_{dj}	150 °C	Junction temperature
$R_{d,jc}$	0.6 °C/W	Thermal resistance junction-to-case
$R_{d,ch}$	0.4 °C/W	Thermal resistance case-to-heat sink

Shane, G.M.; Sudhoff, S.D., "Design Paradigm for Permanent-Magnet-Inductor-Based Power Converters," *Energy Conversion, IEEE Transactions on* , vol.28, no.4, pp.880,893, Dec. 2013

Case Study Specifications

- Design a buck converter to the following specifications
- Input voltage: 400 V
- Output voltage: 200 V
- Output current: 40 A
- Transistor: Microsemi APT80GP60JDQ3
- Diode: Powerex CS241250

Case Study Specifications

- Peak-to-peak ripple specifications
  - Output voltage: 1% of average
  - Output inductor current: 20% of average
  - Input capacitor voltage: 5% of average
  - Input inductor current: 1% of average
- Continuous current mode
- Heat sink thermal resistance is positive
- Max semiconductor junction temperature: 150 ^{o}C
- Max inductor current density: 7.5 A/mm^2

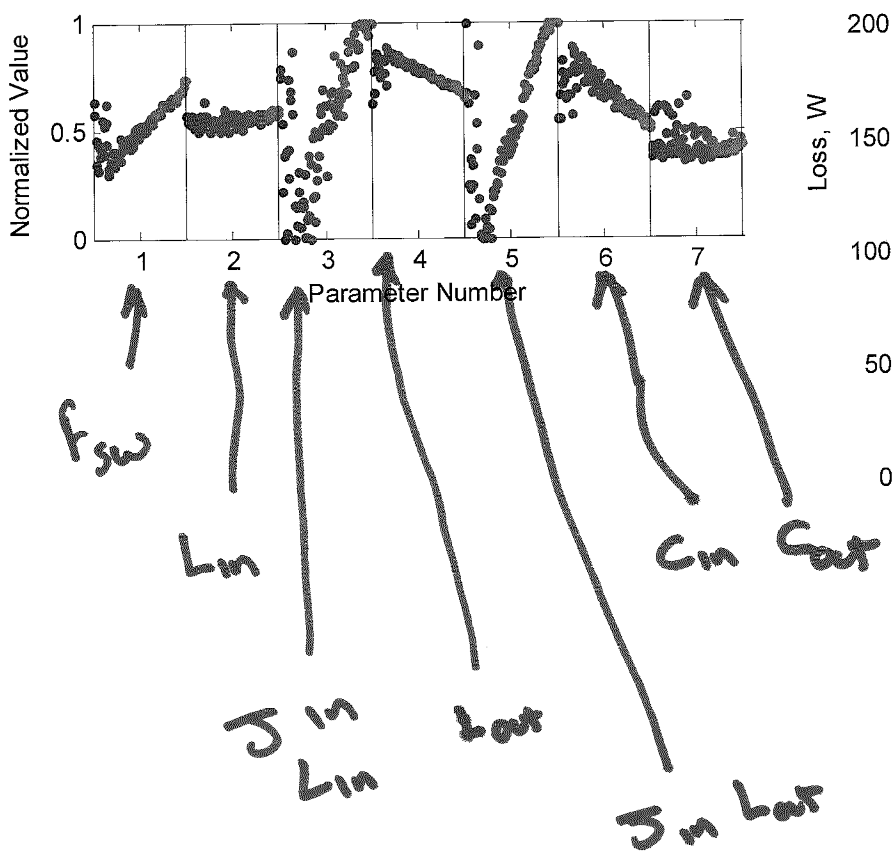
Case Study Degrees of Freedom

- Switching frequency
- Input inductor and current density
- Output inductor and current density
- Nominal input capacitor capacitance
- Nominal output capacitor capacitance

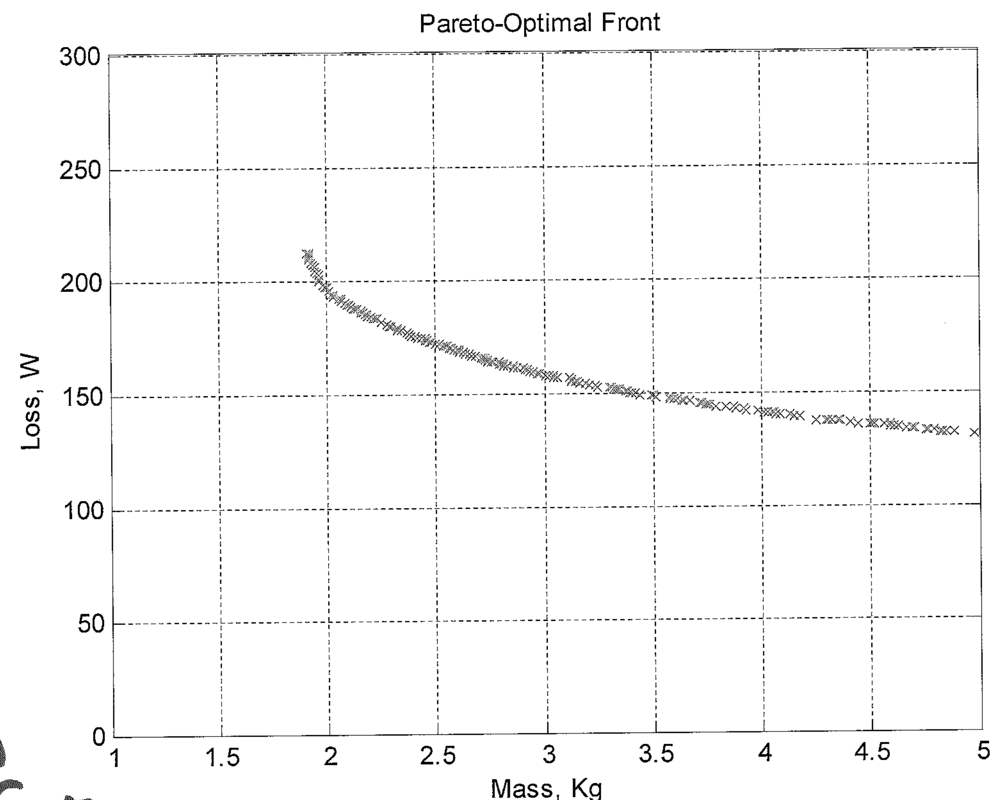
Case Study Metrics

- Mass
  - Includes sum of mass of input inductor, output inductor, input capacitor, output capacitor, heat sink
- Loss
  - Include sum of input inductor loss, output inductor loss, and semiconductor loss (conduction and switching in diode and IGBT)

Results

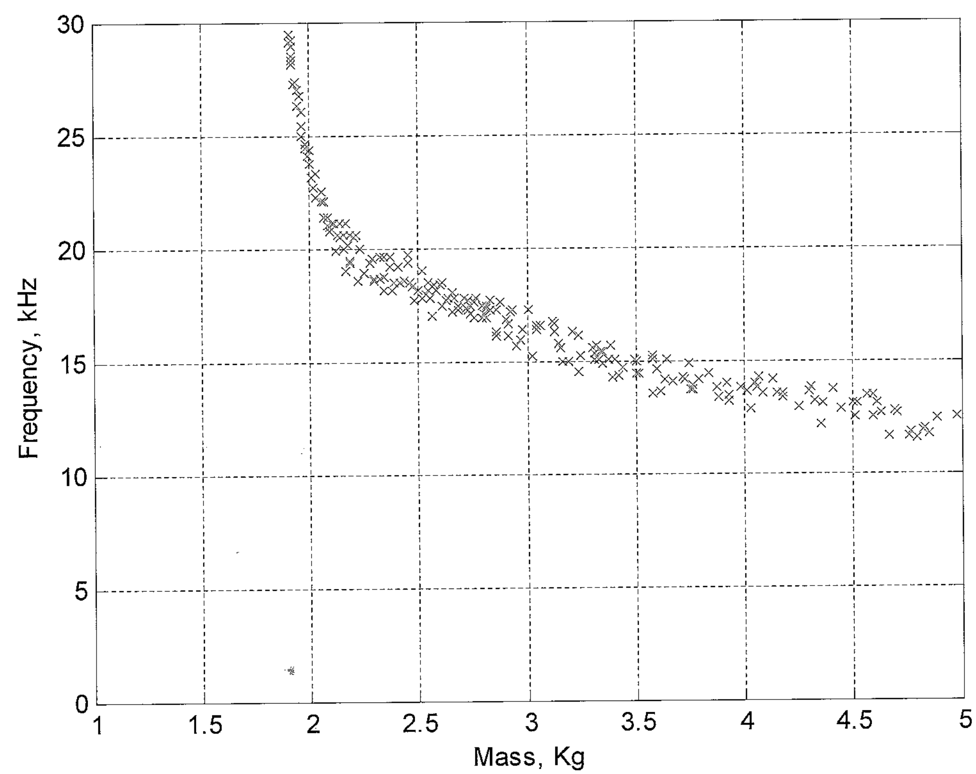


Normalized Value
1
0.5
0
1
2
3
4
5
6
7
Parameter Number
f_{sw}
L_{in}
J in L_{in}
L_{out}
J in L_{out}
C_{in}
C_{out}

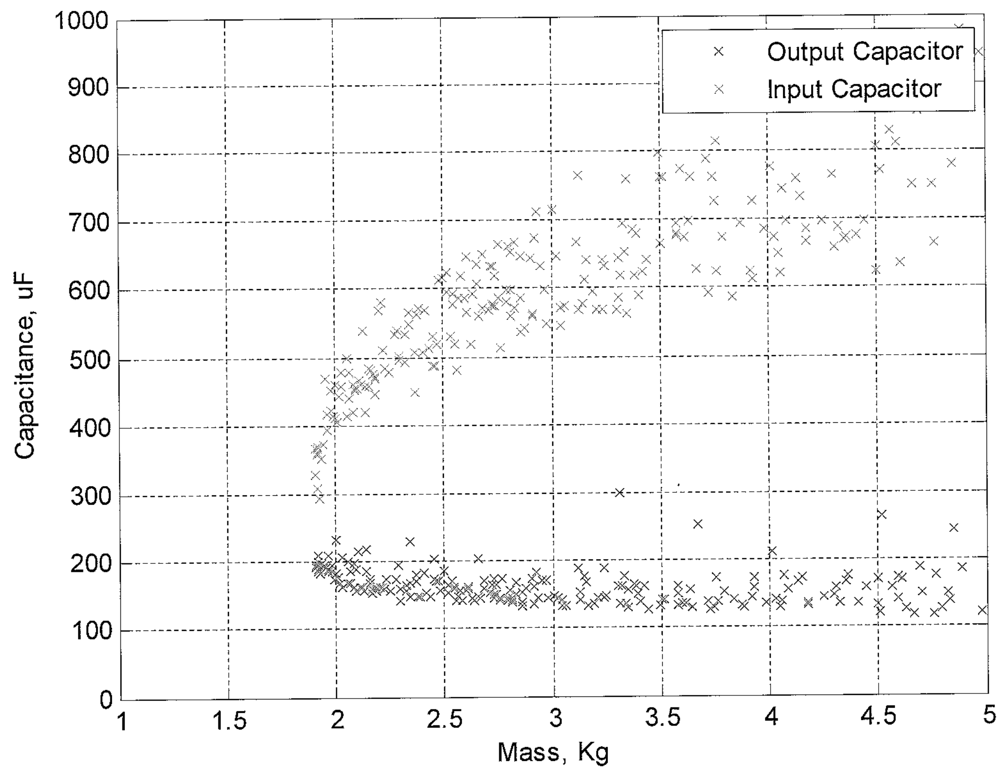


Pareto-Optimal Front
Loss, W
300
250
200
150
100
50
0
1
1.5
2
2.5
3
3.5
4
4.5
5
Mass, Kg

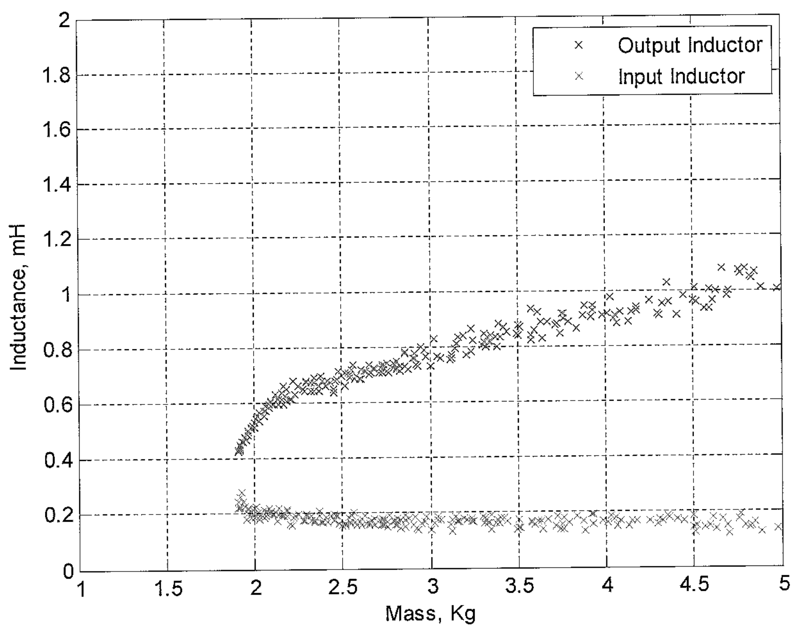
Switching Frequency



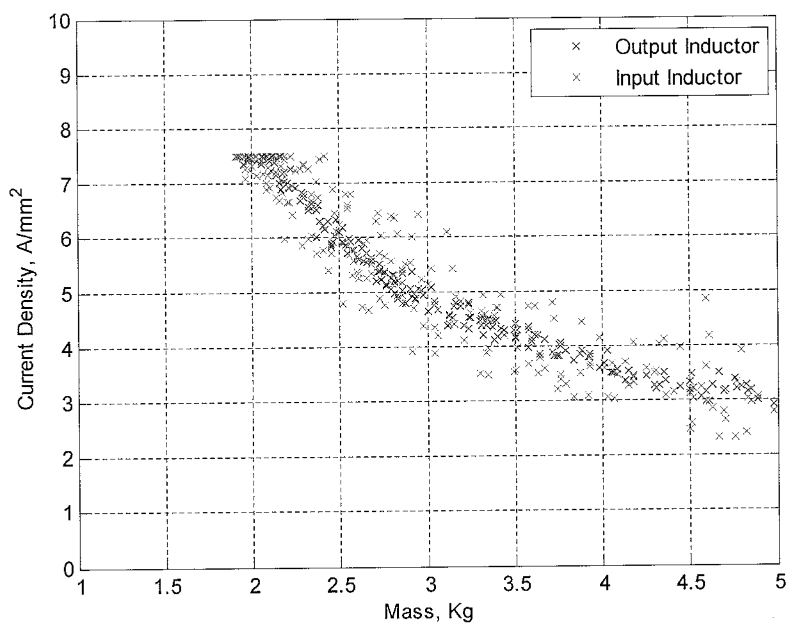
Capacitance



Inductance

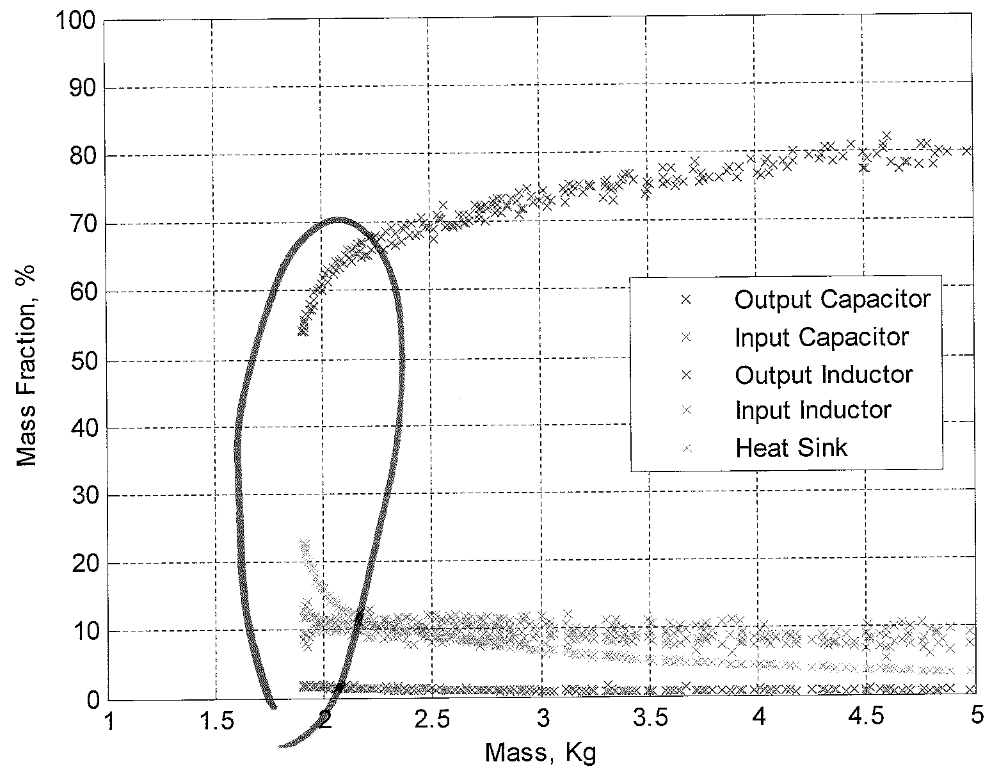


Output Inductor
Input Inductor
Inductance, mH
Mass, Kg
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
1
1.5
2
2.5
3
3.5
4
4.5
5

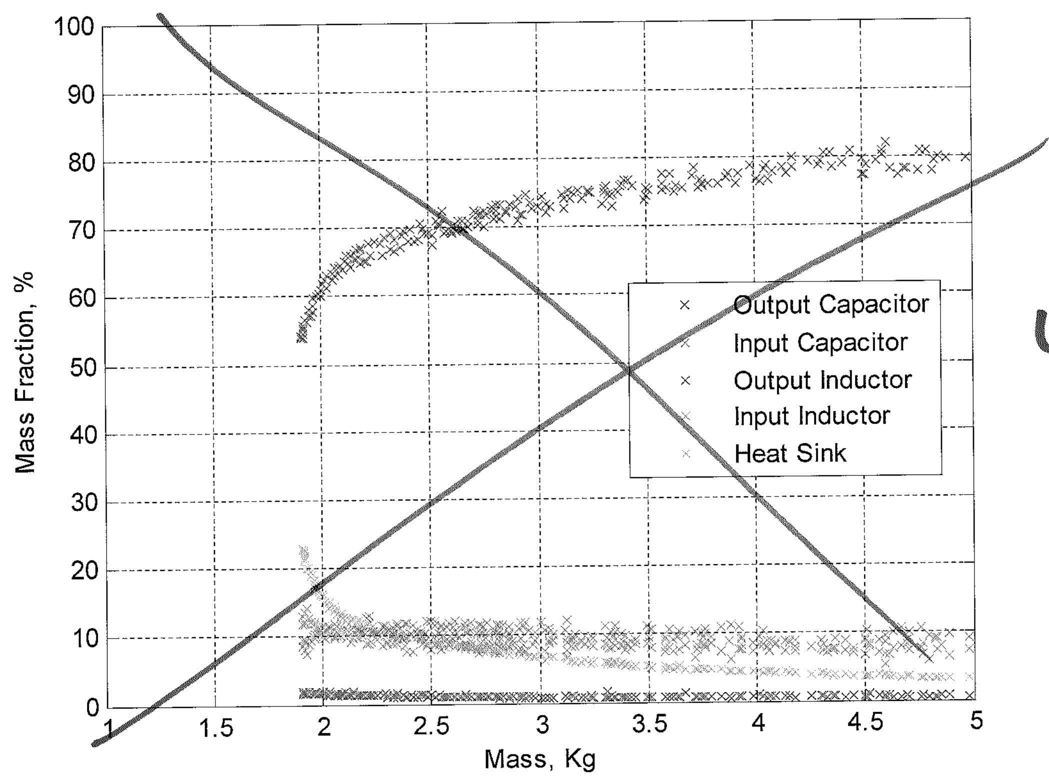


Output Inductor
Input Inductor
Current Density, A/mm2
Mass, Kg
10
9
8
7
6
5
4
3
2
1
0
1
1.5
2
2.5
3
3.5
4
4.5
5

Mass Fraction

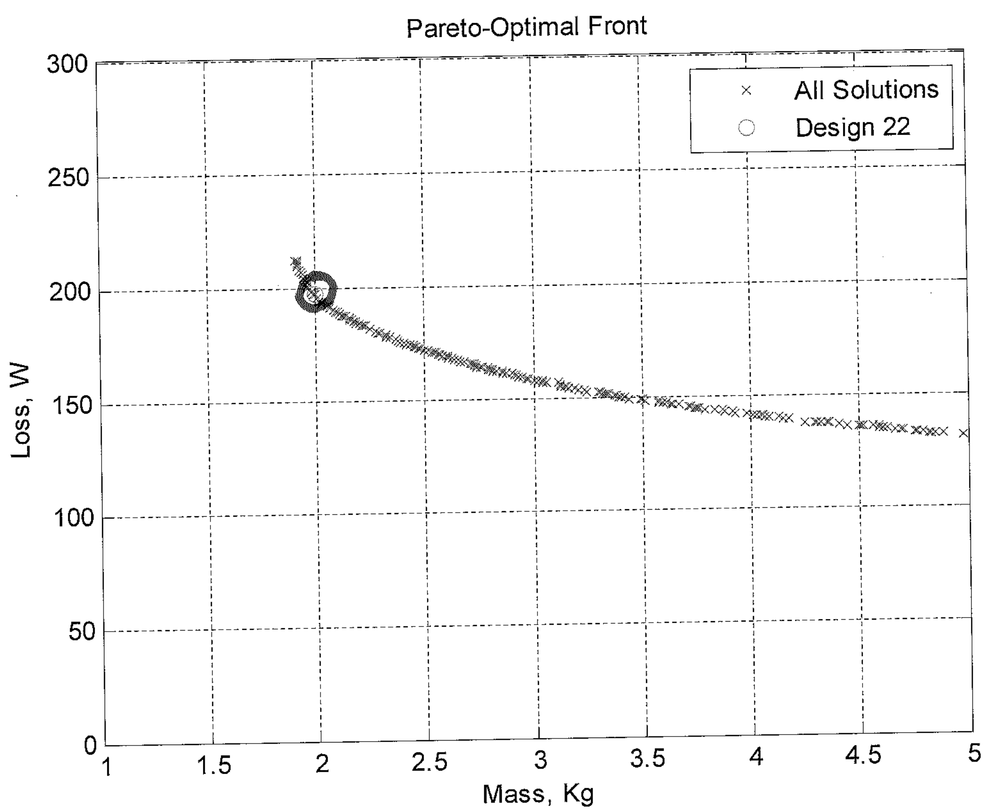


Losses



wrong plot

Design 10



M: 1.9968 kg

P: 196.7482 W

f_{sw}: 2.4342e+04 Hz

C_{out0}: 2.3202e-04 F

C_{in0}: 4.6120e-04 F

L_{out}: 5.1263e-04 H

L_{in}: 1.8283e-04 H

J_{Lout}: 7.4321e+06 A/m^2

J_{Lin}: 7.3145e+06 A/m^2

M_{Cout}: 0.0405 kg

M_{Cin}: 0.2276 kg

M_{Lout}: 1.1979 kg

M_{Lin}: 0.2040 kg

M_H: 0.3269 kg

P_{tcd}: 31.9893 W

P_{tsw}: 45.9400 W

P_{dcd}: 28.3253 W

P_{dsw}: 43.5020 W

P_{Lin}: 7.4516 W

P_{Lout}: 39.5399 W

Power Electronics Converters and Systems Time-Domain Simulation

Homework 3

S.D. Sudhoff

Fall 2016

Aspects of Assignment

- Problem 5: Repeat the case study in Lecture Set 3C. Include all the plots discussed in lecture. Report on a design with a mass near 4.6 kg as on slide 41.
- Problem 6: Repeat problem 5, but in this case assume that all switching losses are 25% of the value in Problem 5. Superimpose the Pareto-optimal front with that obtained in Problem 5. You do not need to report on a particular design.

ECE61016 Power Electronics Converters and Systems

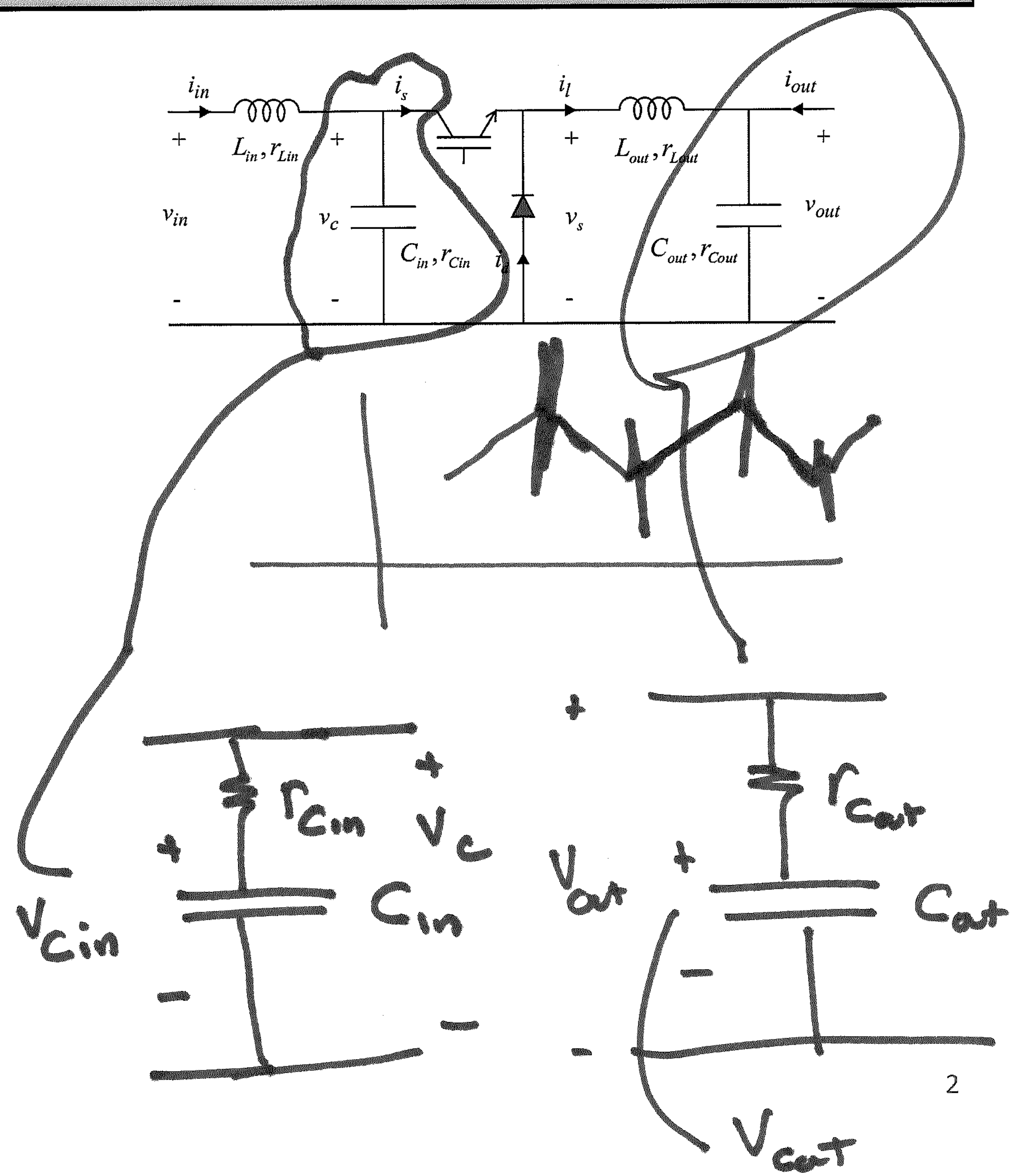
Lecture Set 3C: Simulation of Buck Converter

S.D. Sudhoff

Fall 2016

Waveform Level Buck Simulation

- Key Assumptions
  - Nearly idealy switches
    V_{fsw}, V_{fd}
  - Linear passives
- States:
  i_{in}
  V_{Cin}
  V_{Cout}
  i_{ℓ}



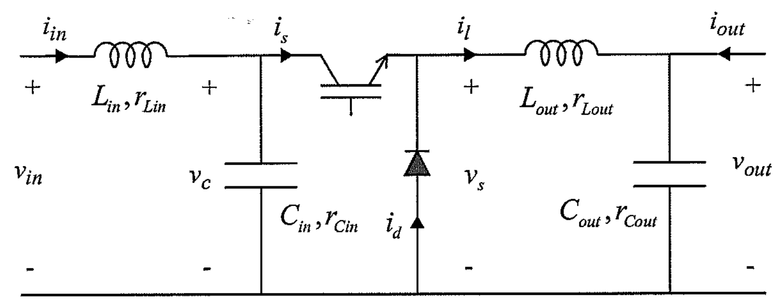
Buck Input Side Interface

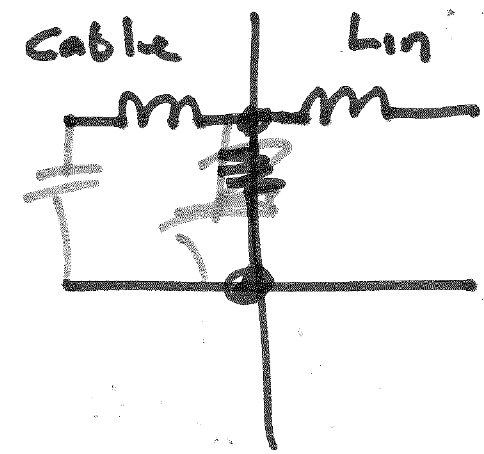
- What should be the model input?

  v_{in}

- What should be the model output?

  i_{in}

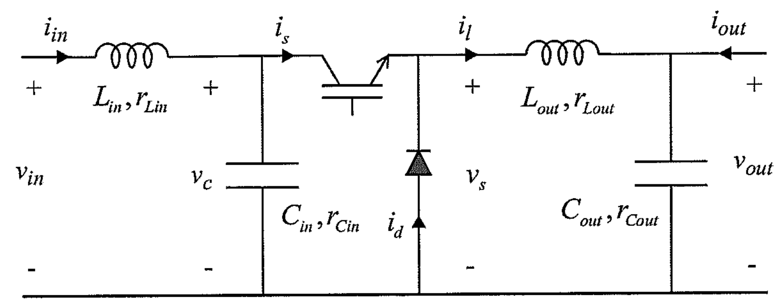


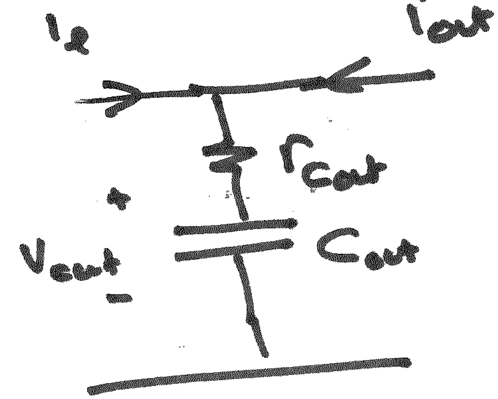


Buck Output Side Interface

- What should be the model input ?

V_{out}

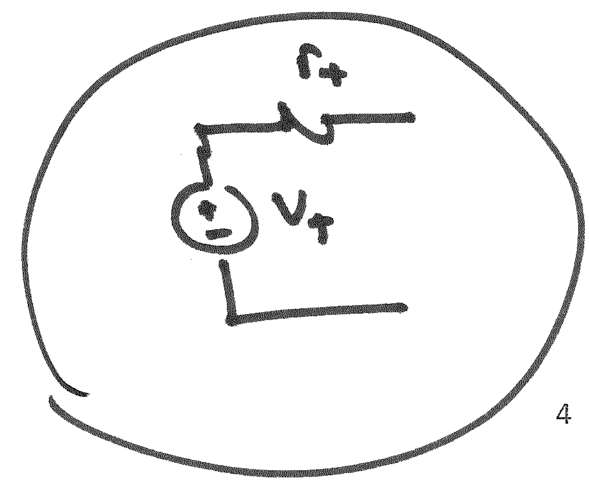




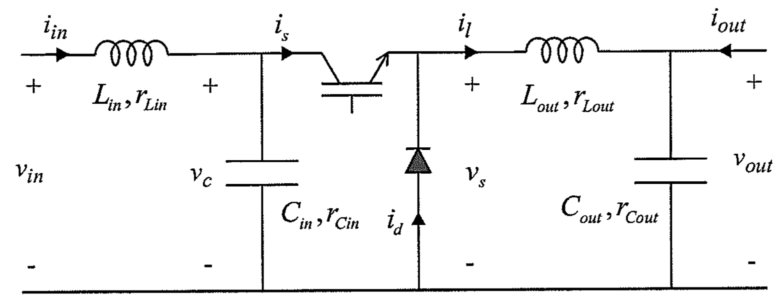
- What should be the model output?

$$V_{out} = V_{Cout} + r_{Cout}(i_l + i_{out})$$

$$= \underbrace{\left[V_{Cout} + r_{Cout} i_l\right]}_{V_T} + \underbrace{r_{Cout}}_{r_T} i_{out}$$



Buck Output Side Interface



Example 1

We're connected to R_{load}

$$\boxed{V_{out} = \frac{R_{load}}{R_{load} + r_+} V_+}$$

Example 2

We're ~~connected~~ connected to L

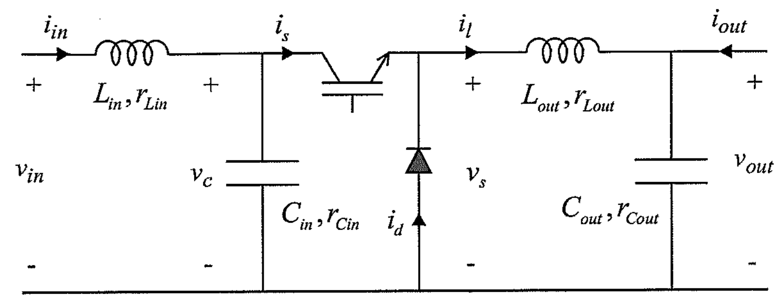
$$\boxed{V_{out} = r_+ i_{out} + V_+}$$

Switchpoint Quantities

Given x, u
px

- sort

- Calculation of i_s and v_c



$$I_s = \begin{cases} I_l & \text{switch on} \\ 0 & \text{switch off} \end{cases}$$

$$V_c = r_{cin}(i_{in} - I_s) + V_{cin}$$

Switchpoint Quantities

- Calculation of v_s
- Switch Open, $i_l > 0$

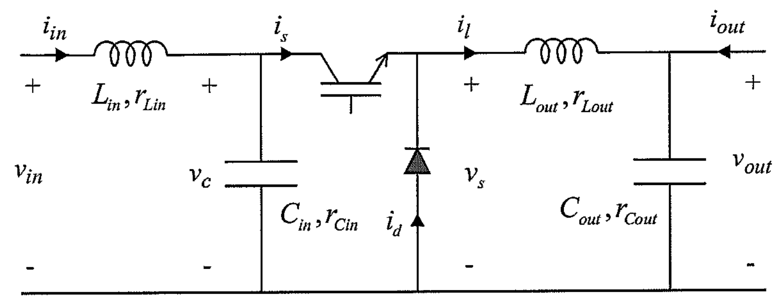
  $V_s = -V_{fd}$

- Switch Open, $i_l = 0$

  $V_s = \max(V_{out}, -V_{fd})$

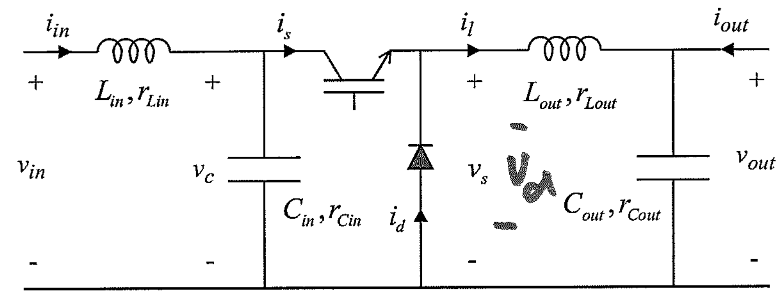
- Switch Open, $i_l < 0$

  $V_s = -\underbrace{R_{rb}}_{\text{artificial resistance}} i_l$



Switchpoint Quantities

- Calculation of v_s

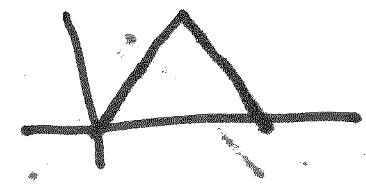


- Switch Closed, $i_l > 0$

  $V_s = \max(V_c - V_{fsw}, -V_{fd})$

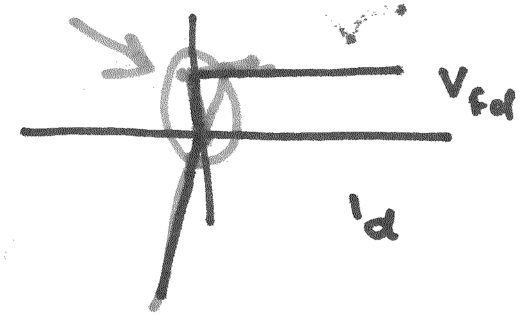
- Switch Closed, $i_l = 0$

  $V_s = \max(V_c - V_{fsw}, V_{out}, \overset{0}{\cancel{-V_{fd}}})$



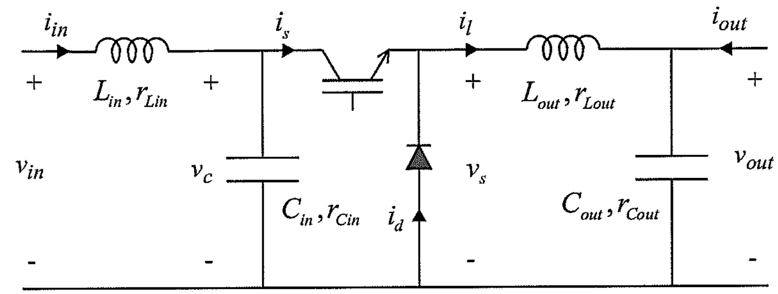
- Switch Closed, $i_l < 0$

  $V_s = \max(V_c - V_{fsw}, -R_{rb} i_l)$



State Derivatives

$$p i_{in} = \frac{v_{in} - v_c - r_{Lin} i_{in}}{L_{in}}$$



$$p v_{cin} = \frac{i_{in} - i_s}{C_{in}}$$

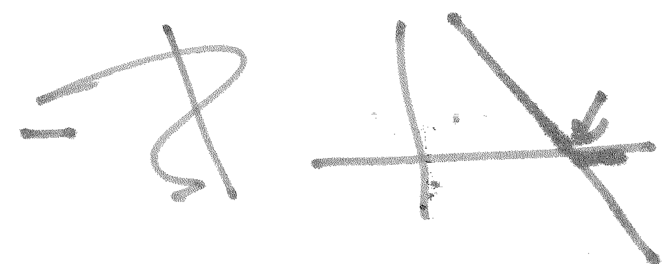
if ($v_{cin} \le 0$) AND ($p v_{cin} < 0$)

$p v_{cin} = 0$

end

dc electrolytic capacitor

$$p i_l = \frac{v_s - v_{out} - r_{Lout} i_l}{L_{out}}$$



if ($i_l \le 0$) AND ($p i_l < 0$)

$p i_l = 0$

end

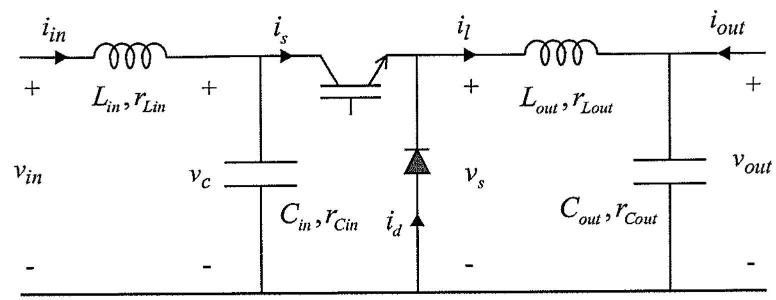
$$p v_{cout} = \frac{i_l + i_{out}}{C_{out}}$$

if ($v_{cout} \le 0$) AND ($p v_{cout} < 0$)

$p v_{cout} = 0$

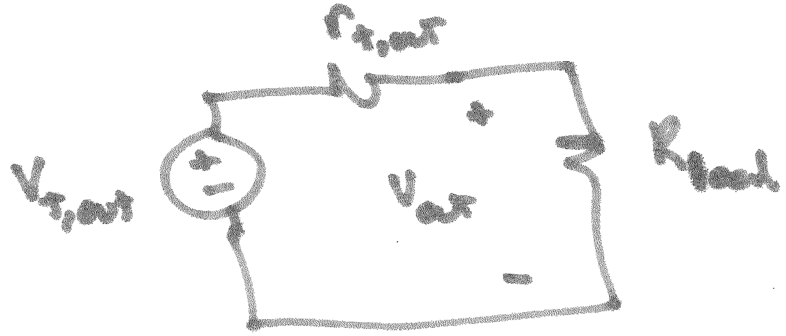
end

State Derivatives



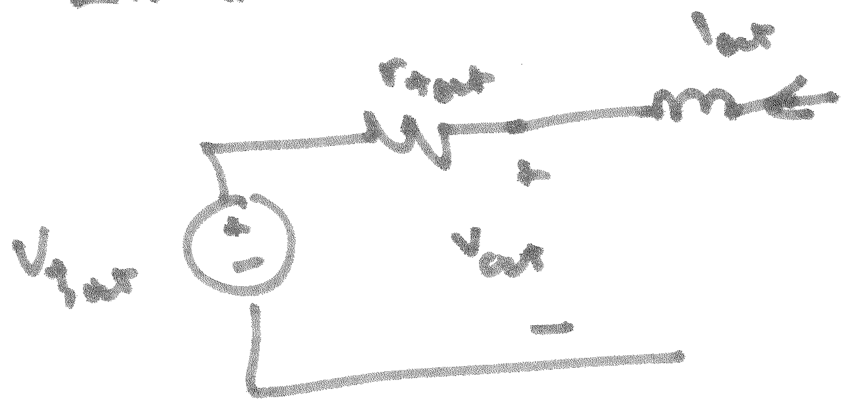
Load Modeling

Example → Resistive



$$V_{out} = \frac{R_{load}}{R_{load} + r_{T,out}} V_{T,out}$$

Example → Inductive

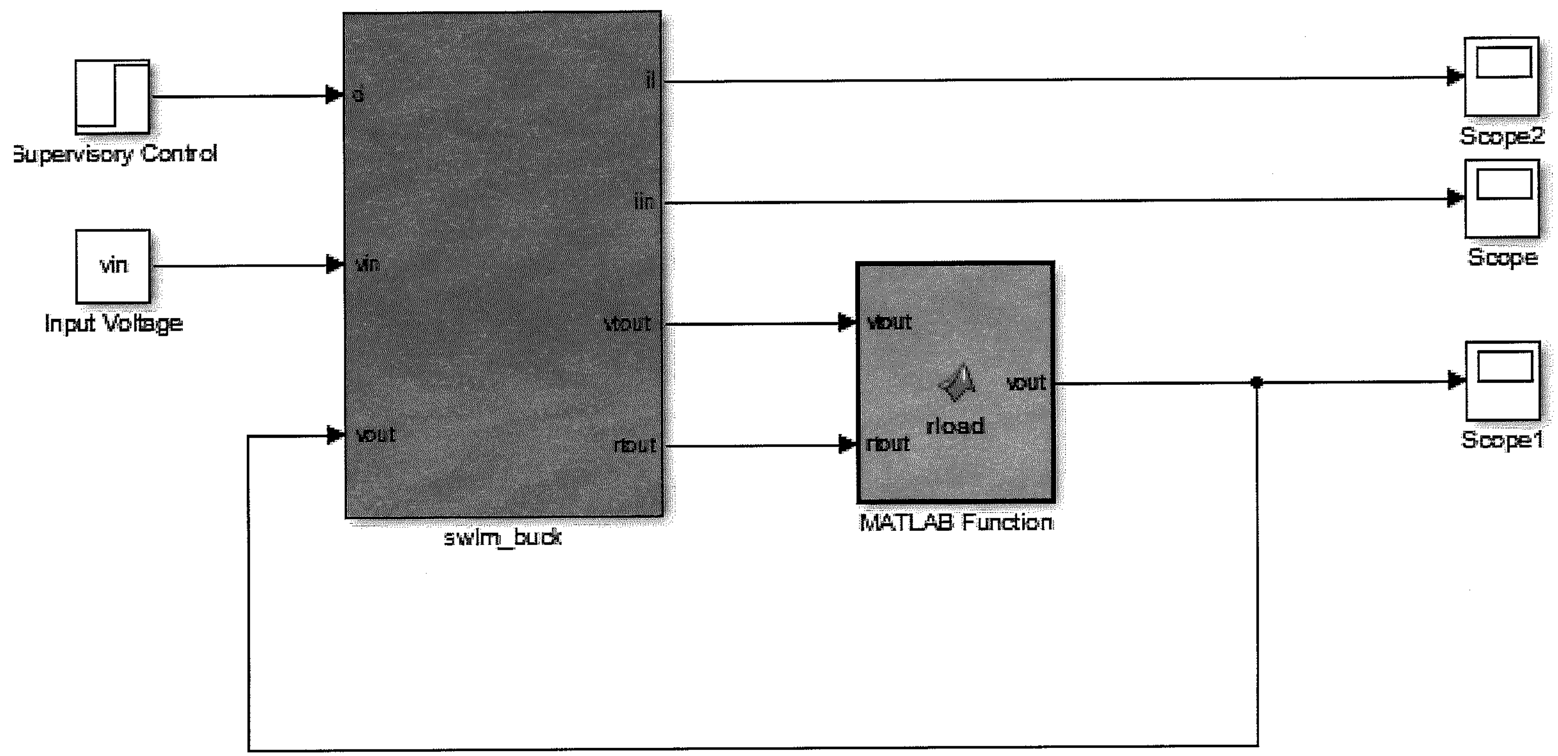


$$V_{out} = V_{T,out} + r_{T,out}\, i_{out}$$

Simulation Implementation - Files

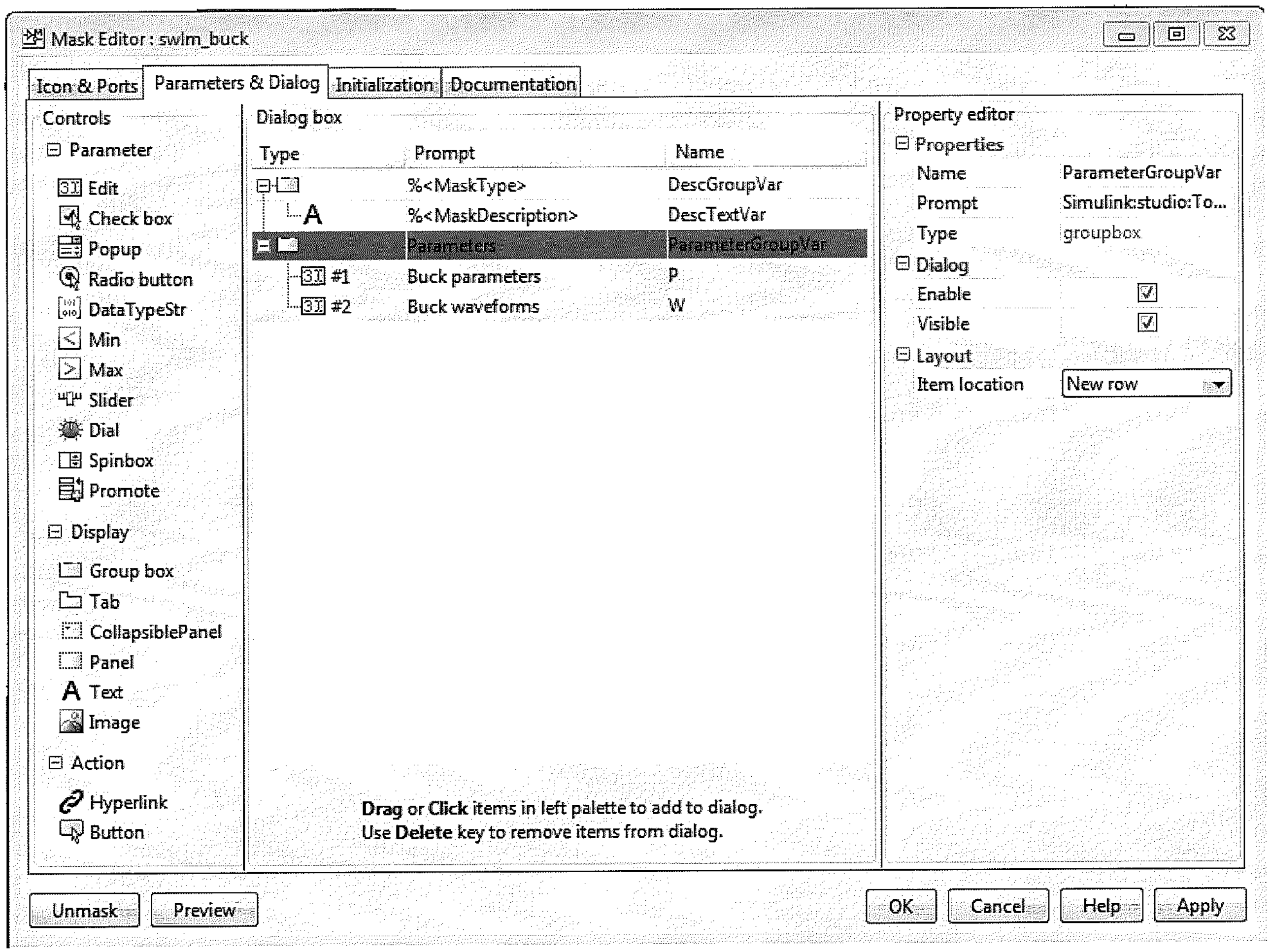
- swlm_buck.slx
- swlm_buck_setup.m
- swlm_buck_plot.m
- swlm_buck_study.m

Simulation Implementation



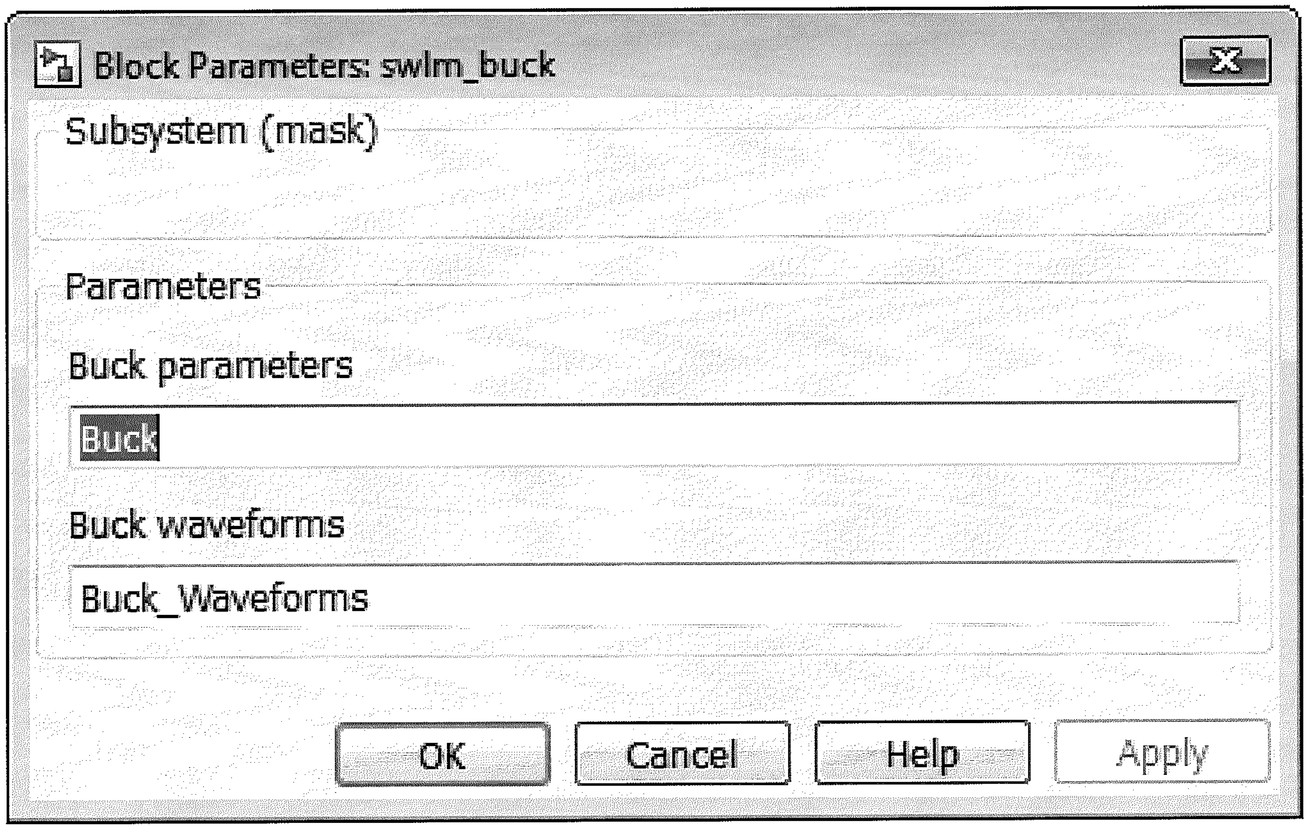
Simulation Implementation – The Mask

- Right clicking on subsystem and selecting ‘Mask’

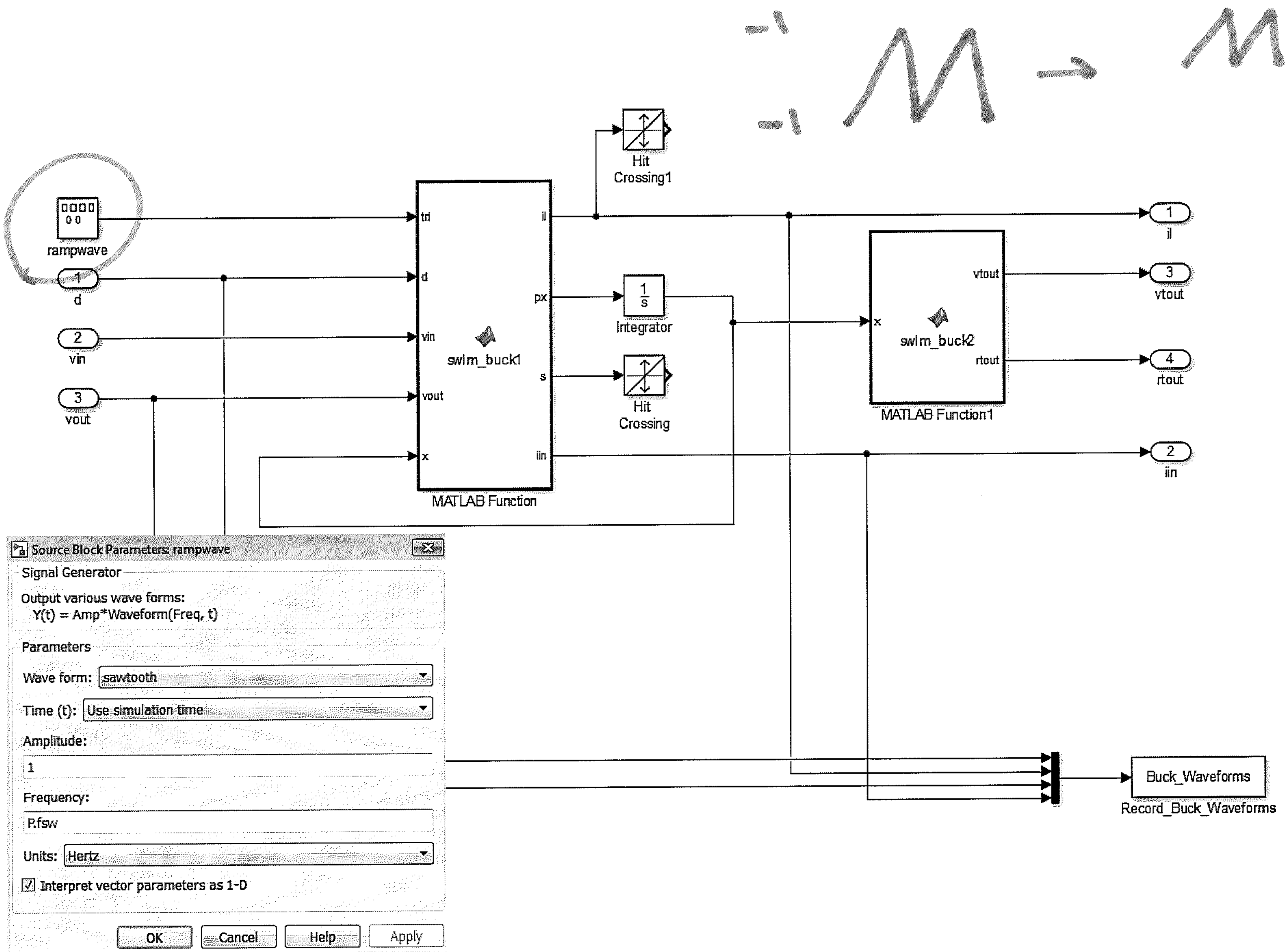


Simulation Implementation – Structures

- Double click on subsystem to set input parameter

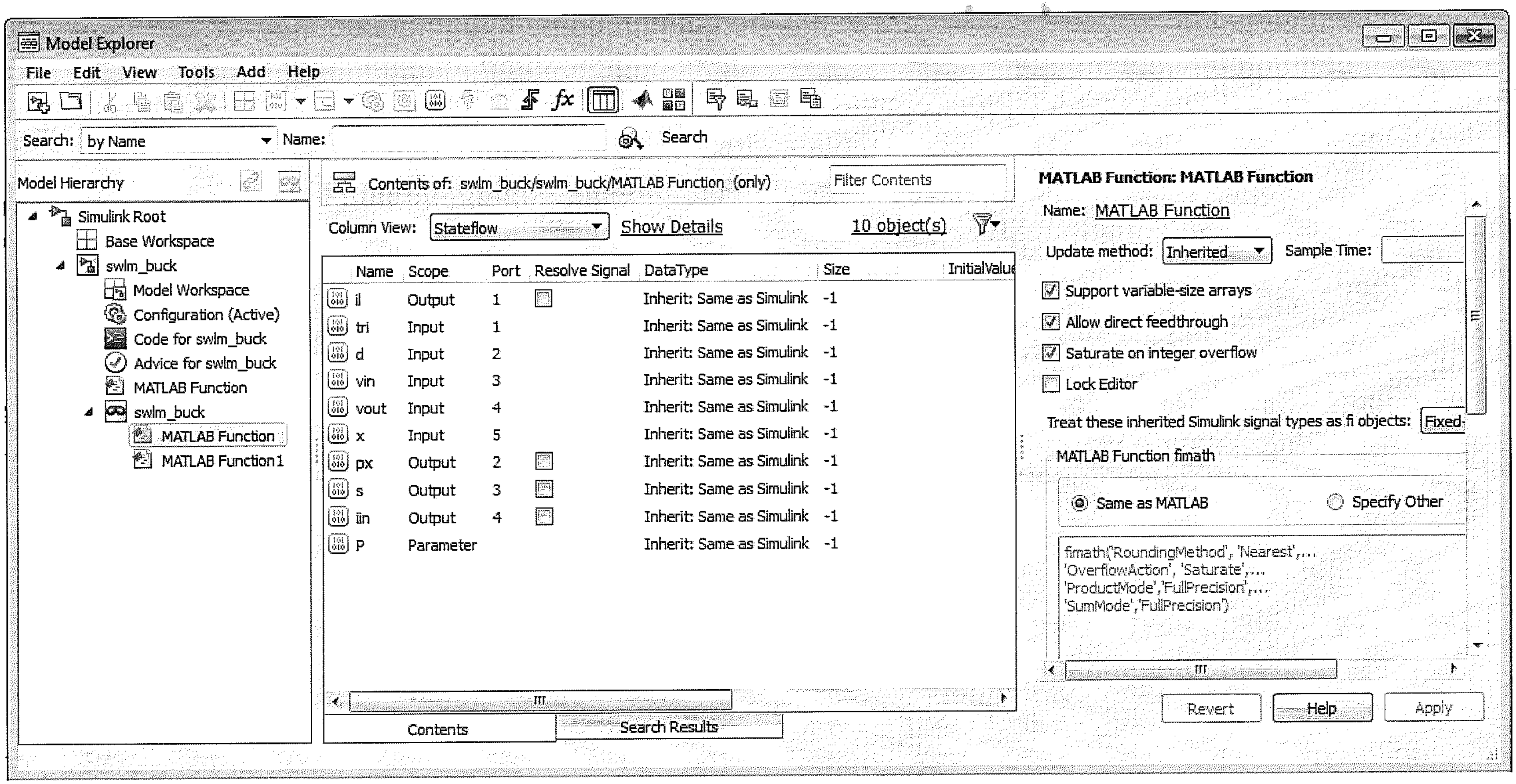


Simulation Issues – Ramp Wave



Simulation Issues – Function Structure

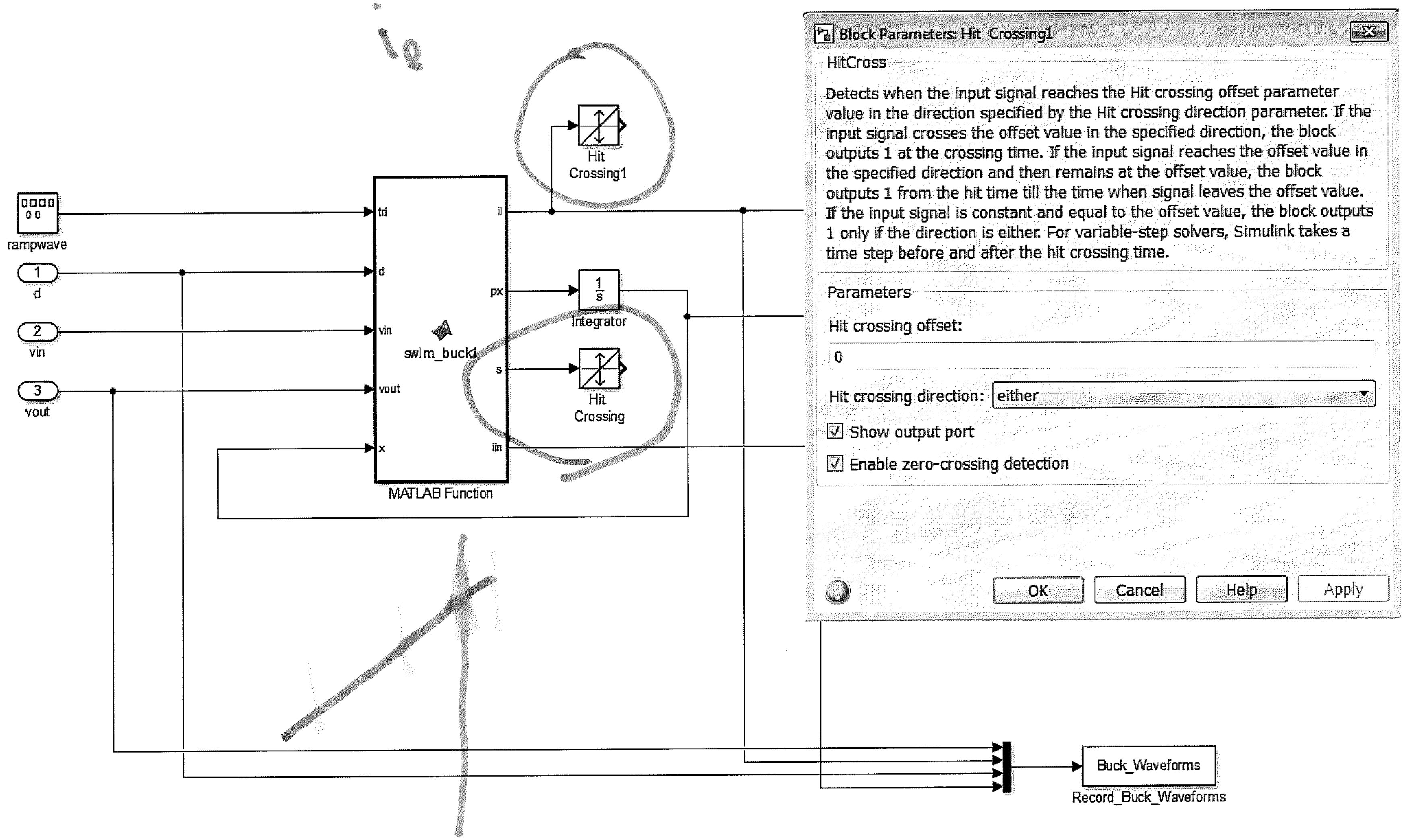
- Using Explore on the swlm_buck1



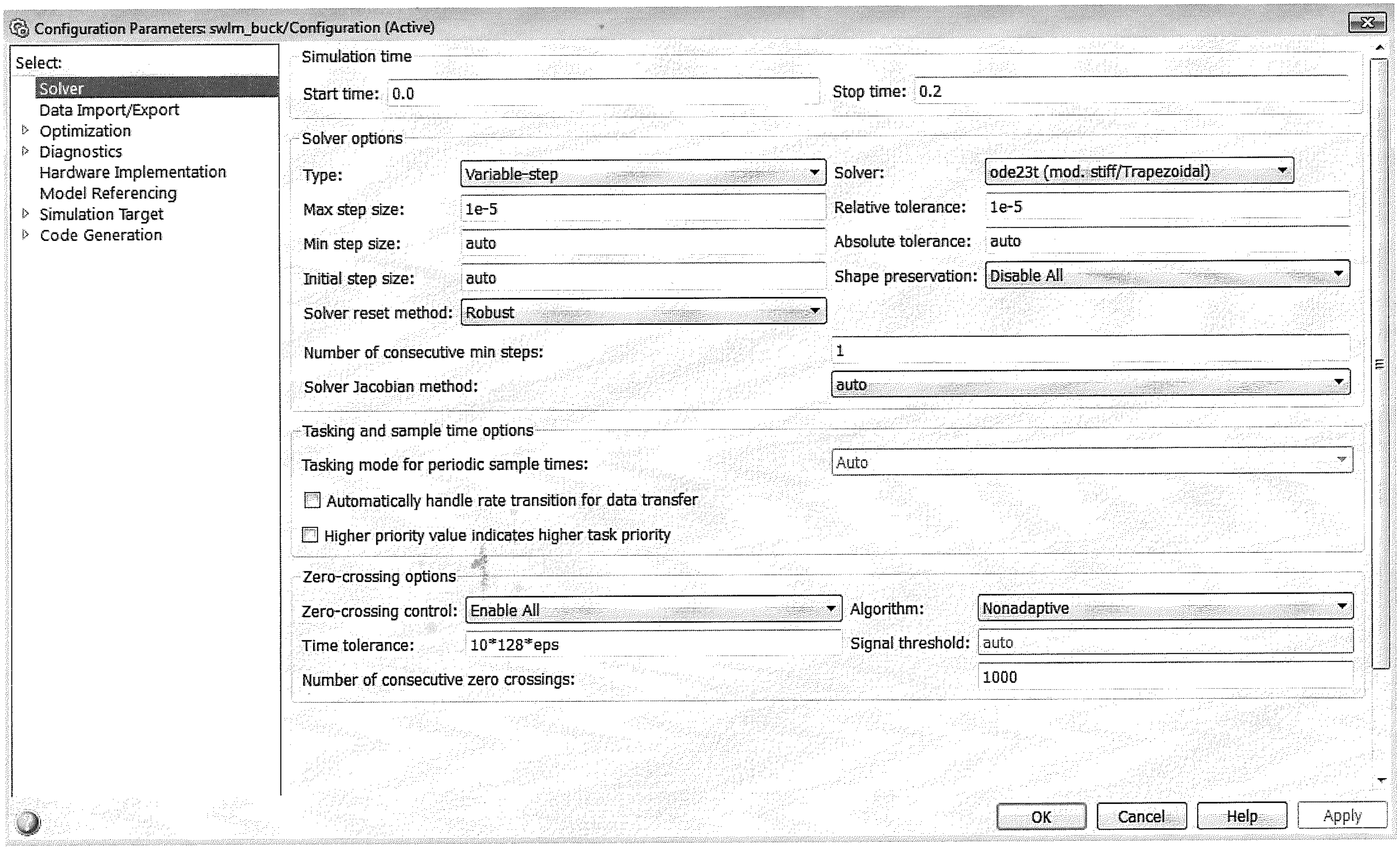
Simulation Issues – Function Structure

```
function [il,px,s,iin] = swlm_buck1(tri,d,vin,vout,x,P)
% This routine calculates the dynamics of a waveform level
% buck converter model
%
% Inputs:
% tri       = triange or sawtooth wave. should go from -1
to 1.
% d         = switch duty cycle (between 0 and 1)
% vin       = input voltage (V)
% vout      = output voltage (V)
```

Simulation Issues – Zero Cross Detection



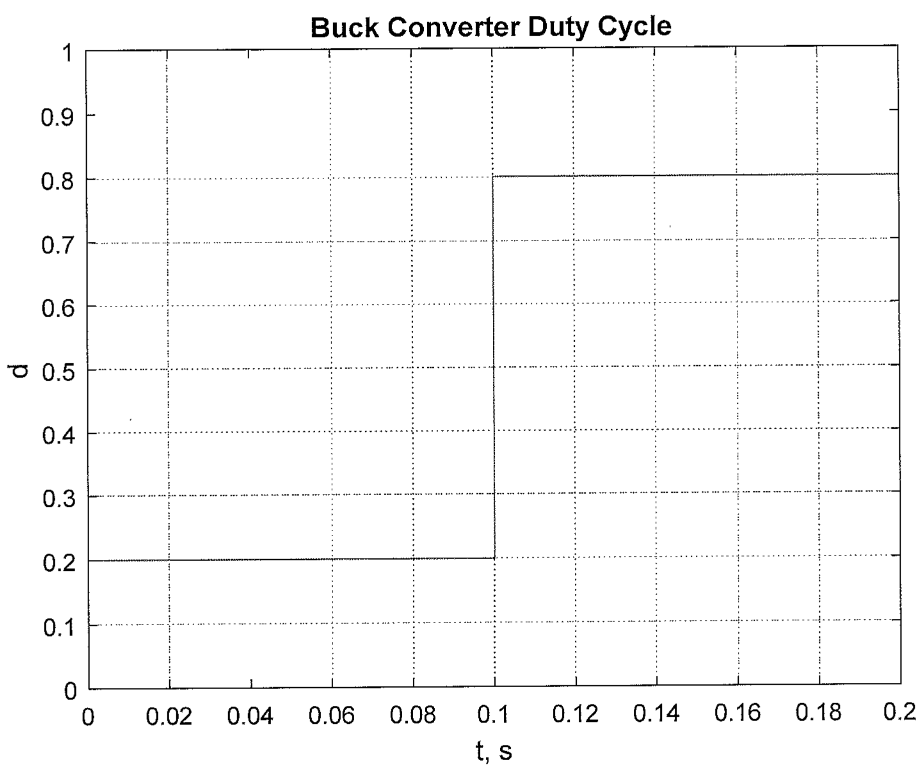
Simulation Issues – Zero Cross Detection

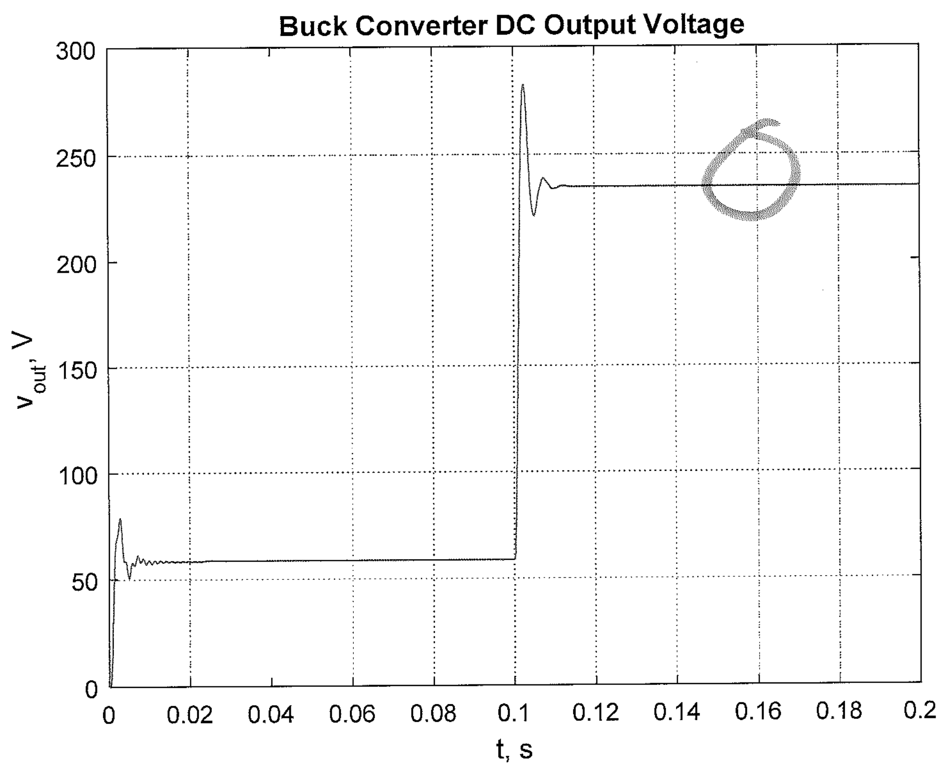


Sample Study

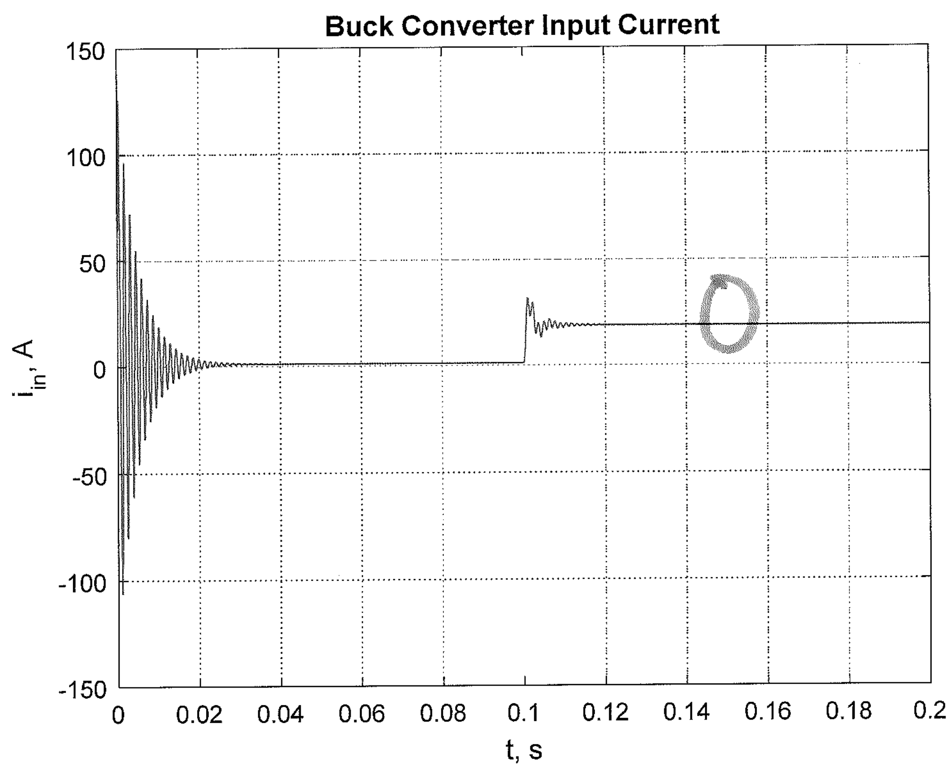
- Input voltage: 300 V
- Input inductor: 0.5 mH; 0.1 Ω
- Input capacitor: 100 uF, 0.1 Ω
- Output inductor: 5 mH, 0.1 Ω
- Output capacitor:100 uF, 0.1 Ω
- Switch forward drop: 1.5 V
- Diode forward drop: 1.0 V
- Reverse biased resistance: 1 kΩ
- Switching frequency: 10 kHz
- Initial conditions: Zero
- Duty cycle changes from 0.2 to 0.8 at $t = 0.1$ s
- Load: 10 Ω

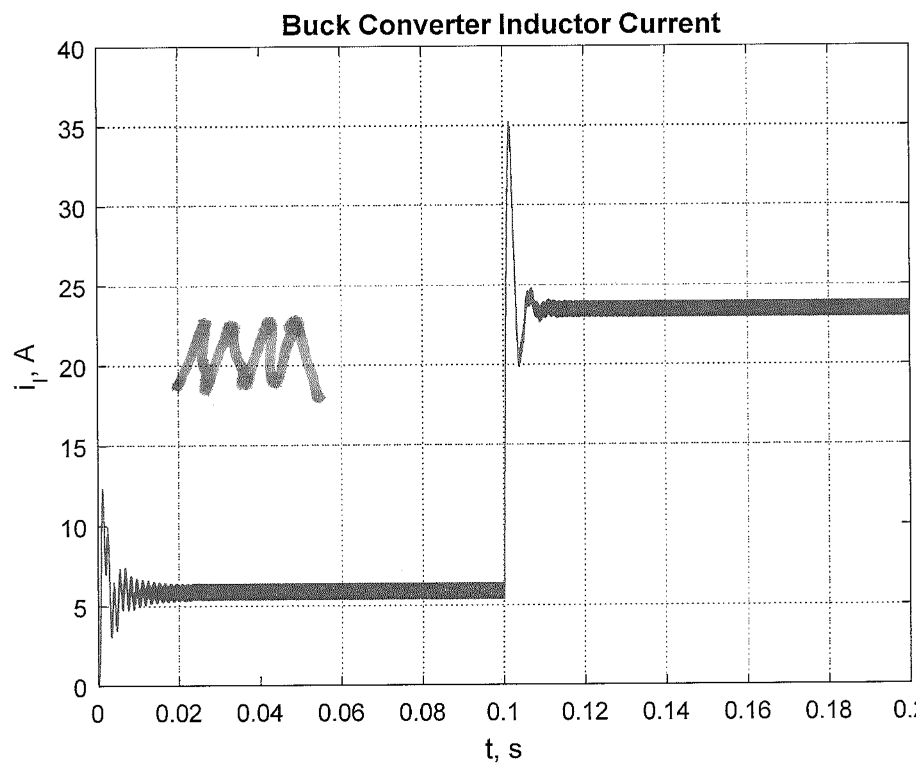
Sample Study





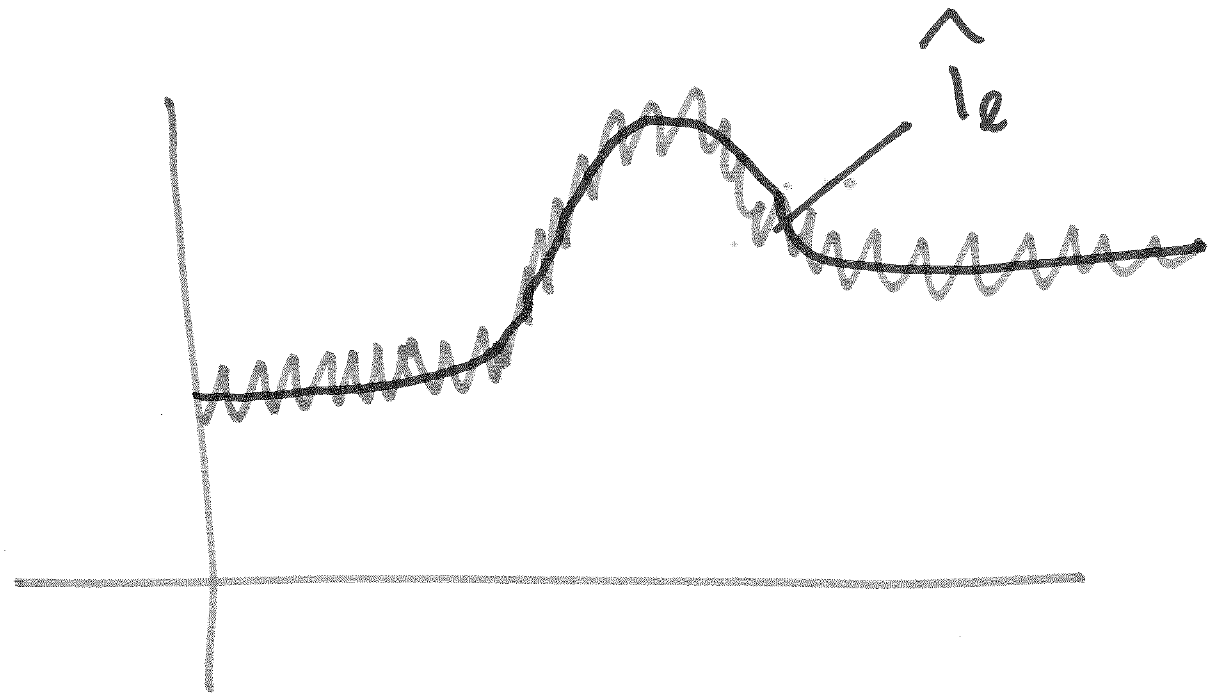
Sample Study





Average Value Modeling

- Also called Nonlinear Average Value Modeling (NLAM)
- The basic idea:



Advantages of Average Value Models

- Simulation Speed — can be much faster
  - → no effort in zero crossings
  - → larger time step (no switching)
  - → much larger time step because we can use advanced integration algorithms
  $$J = \frac{\partial f}{\partial x}$$
- Simulation Complexity
  - → simpler – no switching details
  - → complicated
- Control Design
  - → dc steady-state operating point.

Disadvantages of AVMs

- Less Information
  - ~~[illegible]~~
- Simulation Fidelity
  - dependent
  - multiple modes
  - heuristic aspect
- Model Development
  - can be difficult

Definition of Fast Average

- Formal Definition

my present time

$$\hat{x}(t) = \frac{1}{T_{sw}} \int_{t-T_{sw}}^{t} x(\tau)\,d\tau$$

1 Them

2 Beginning of the End

Fast average
dynamic average
windowed average
average

"average value modeling" $\Leftrightarrow$ "reduced-order modeling"

Model Bandwidth

- Consider a sinusoidal signal and its fast average:

$$x(t) = \cos(\omega_f t) \qquad \tilde{x} = 1\angle 0$$

$$\hat{x}(t) = \frac{1}{T_{sw}} \int_{t-T_{sw}}^{t} \cos(\omega_f \tau)\, d\tau \qquad \begin{array}{l} C' = -S \\ S' = C \end{array}$$

$$= \frac{1}{T_{sw}\omega_f}\left(\sin(\omega_f t) - \sin(\underbrace{\omega_f t}_{A} - \underbrace{\omega_f T_{sw}}_{B})\right)$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$= \frac{1}{T_{sw}\omega_f}\Big[\sin(\omega_f t) - \sin(\omega_f t)\cos(\omega_f T_{sw}) + \cos(\omega_f t)\sin(\omega_f T_{sw})\Big]$$

Model Bandwidth

$$\hat{x}(t) = \underbrace{\frac{1}{T_{sw}\omega_f} \sin(T_{sw}\omega_f)}_{A} \cos(\omega_f t)$$

$$+ \underbrace{\frac{1}{T_{sw}\omega_f}\left(1 - \cos(\omega_f T_{sw})\right)}_{B} \sin(\omega_f t)$$

Suppose $T_{sw}\omega_f << 1$

$\sin(T_{sw}\omega_f) \approx T_{sw}\omega_f$

$\cos(T_{sw}\omega_f) \approx 1$

$T_{sw}\omega_f = \cancel{\frac{1}{T_{sw}}}\ 2\pi \frac{F_F}{F_{sw}}$

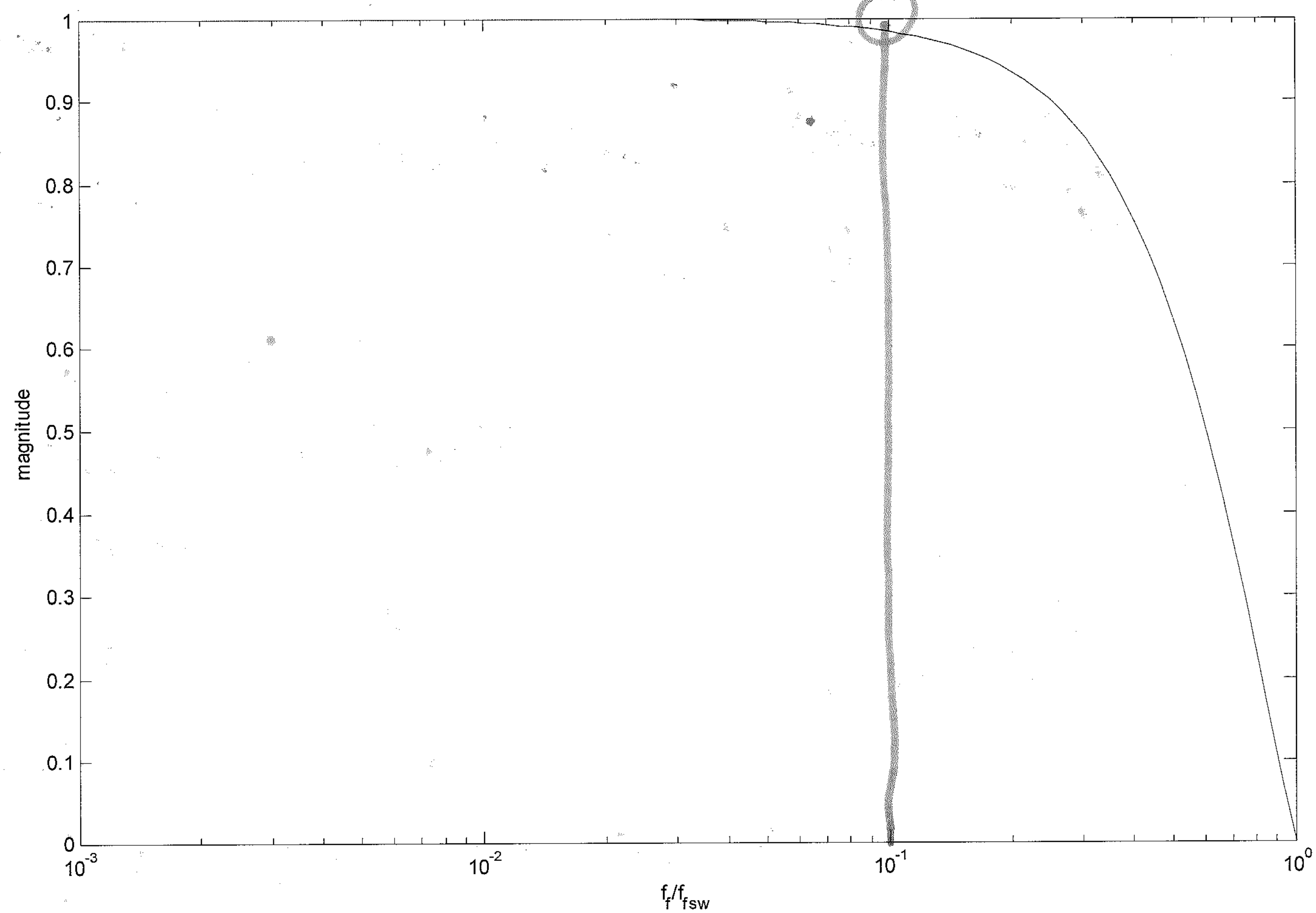
So $\hat{x}(t) = \cos(\omega_f t)$

in general

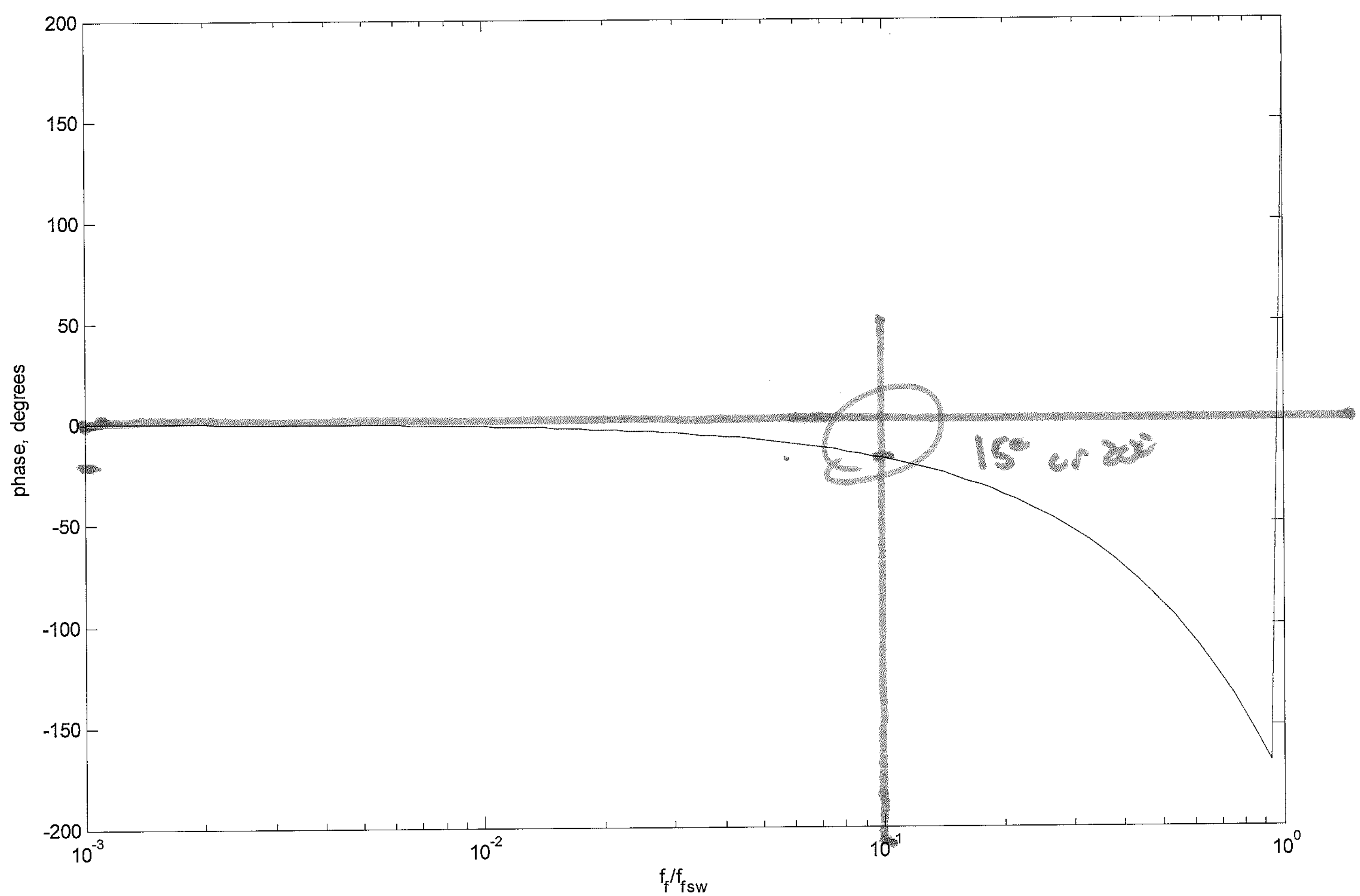
$$\hat{x}(t) = \underbrace{\sqrt{A^2 + B^2}}_{\text{magnitude}} \cos(\omega_f t + \phi)$$

$$\underbrace{\phi = \text{angle}(A - jB)}_{\text{phase}}$$

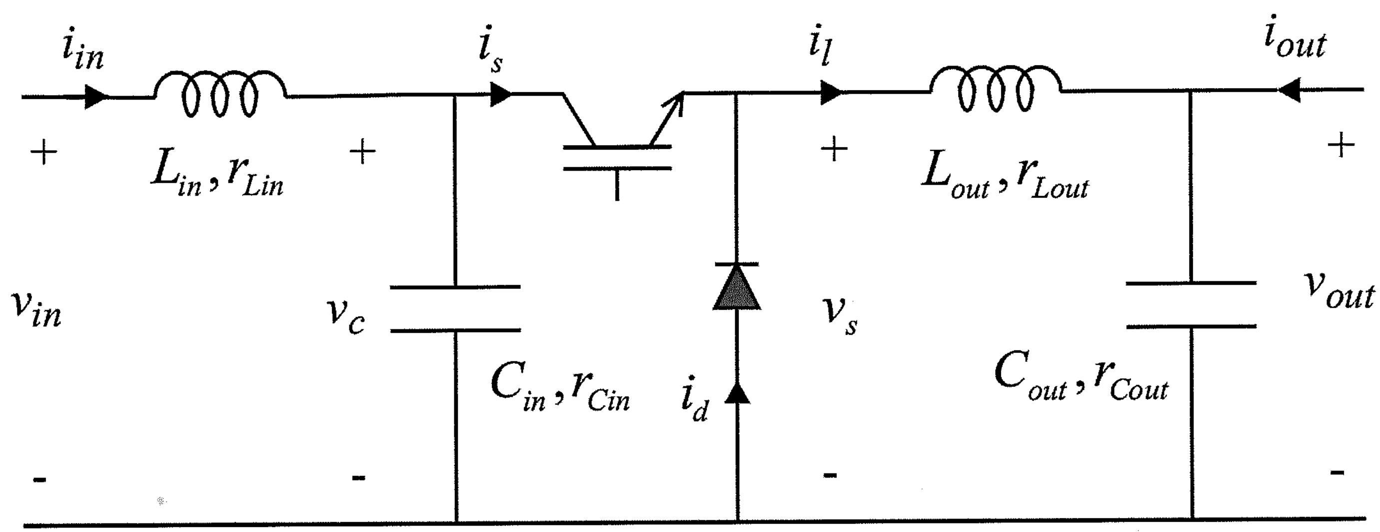
Model Bandwidth



Model Bandwidth



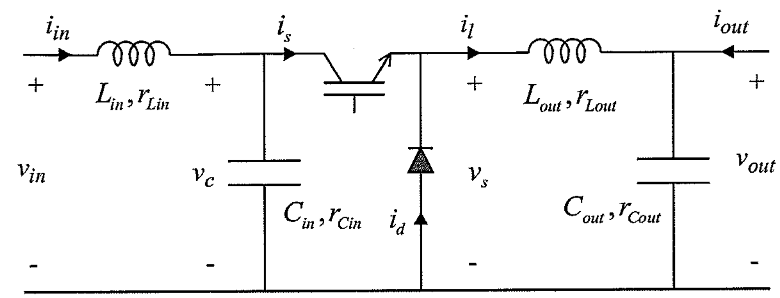
Consider our Buck Converter



- States: $\hat{i}_{in}$ $\hat{v}_{Cin}$ $\hat{i}_l$ $\hat{v}_{Cout}$
- Inputs: $\hat{v}_{in}$ $\hat{v}_{out}$
- Outputs: $\hat{i}_{in}$ $\hat{v}_{taug}$ r_q

Derivation of Buck AVM

$$\frac{1}{T_{sw}}\int_{t-T_{sw}}^{t} ax + by + c\frac{dy}{dt} + d \, dt = 0$$



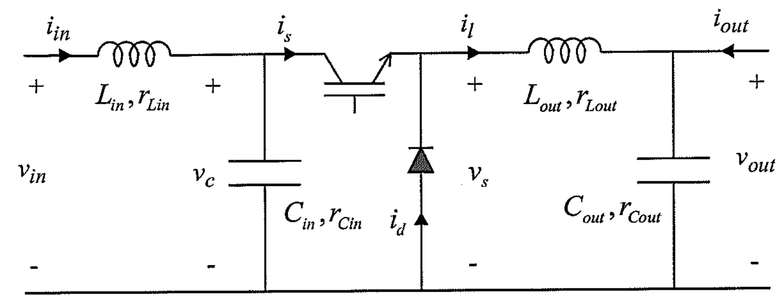
$$\frac{1}{T_{sw}}\int ax \, dt + \frac{1}{T_{sw}}\int by \, dt$$

$$+ \frac{1}{T_{sw}}\int c\frac{dy}{dt} \, dt + \frac{1}{T_{sw}}\int d \, dt = 0$$

$$a\hat{x} + b\hat{y} + c\frac{d}{dt}(\hat{x}) + d = 0$$

$\Rightarrow$ any linear or affine equation, algebraic or ODE, remains unchanged.

Derivation of Buck AVM



$$x = z^2$$

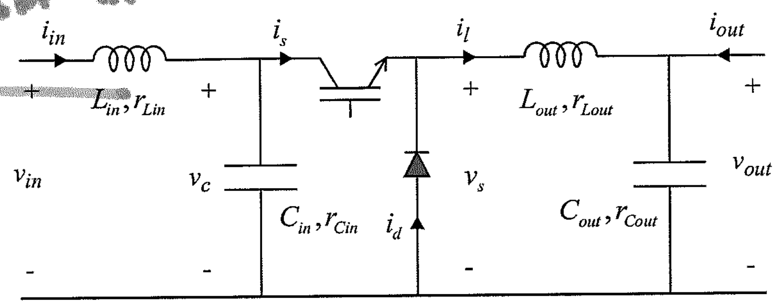
$$\frac{1}{T_{sw}} \int_{t-T_{sw}}^{t} x \, d\tau = \frac{1}{T_{sw}} \int_{t-T_{sw}}^{t} z^2 \, d\tau$$

$$\hat{x} = \widehat{z^2} \neq \hat{z}^2$$

Care is required

Derivation of Buck AVM

$$p\hat{i}_{in} = \frac{\hat{v}_{in} - [r_{Cin}(\hat{i}_{in} - \hat{i}_s) + \hat{v}_{cin}] - r_{Lin}\hat{i}_{in}}{L_{in}}$$



$$p\hat{v}_{cin} = \frac{\hat{i}_{in} - \hat{i}_s}{C_{in}}$$

$\hookrightarrow$ Limit as in WLM to prevent $v_{cin} < 0$

$$p\hat{i}_l = \frac{\hat{v}_s - \hat{v}_{out} - r_{Lout}\hat{i}_l}{L_{out}}$$

$\hookrightarrow$ Limit as in WLM

we still need $\hat{v}_s$ and $\hat{i}_s$

$$p\hat{v}_{Cout} = \frac{\hat{v}_{out} - \hat{v}_{Cout}}{r_{Cout} C_{out}}$$

$\hookrightarrow$ Limit as in WLM

Switch Voltage & Current – Cont. Mode

- Average switch current $\hat{i}_S = \hat{i}_l d$
- ~~Average input cap. voltage~~ $\cancel{\hat{v}_c = \hat{v}_{cin} + R_{Cin}\left(\hat{i}_{in} - \hat{i}_s\right)}$
- Average switch point voltage $\hat{v}_s = (\hat{v}_c - v_{fsw})d - v_{fd}(1-d)$
- Assumptions
- Checks:

$$\Delta i = i_{mx} - i_{mn} = \frac{d(1-d)\left(\hat{v}_c + v_{fd} - v_{fsw}\right)}{L_{out} f_{sw}}$$

$$\hat{i}_l - \Delta i / 2 > 0$$

Switch Voltage & Current – Discont. Mode

- Recall

$$i_{mx} = \frac{\left(\hat{v}_c - v_{fsw} - \hat{v}_{out}\right) dT_{sw}}{L_{out} + r_{Lout} dT_{sw}}$$

$$t_d = \frac{L_{out} i_{mx}}{v_{out} + v_{fd} + \frac{1}{2} r_{Lout} i_{mx}}$$

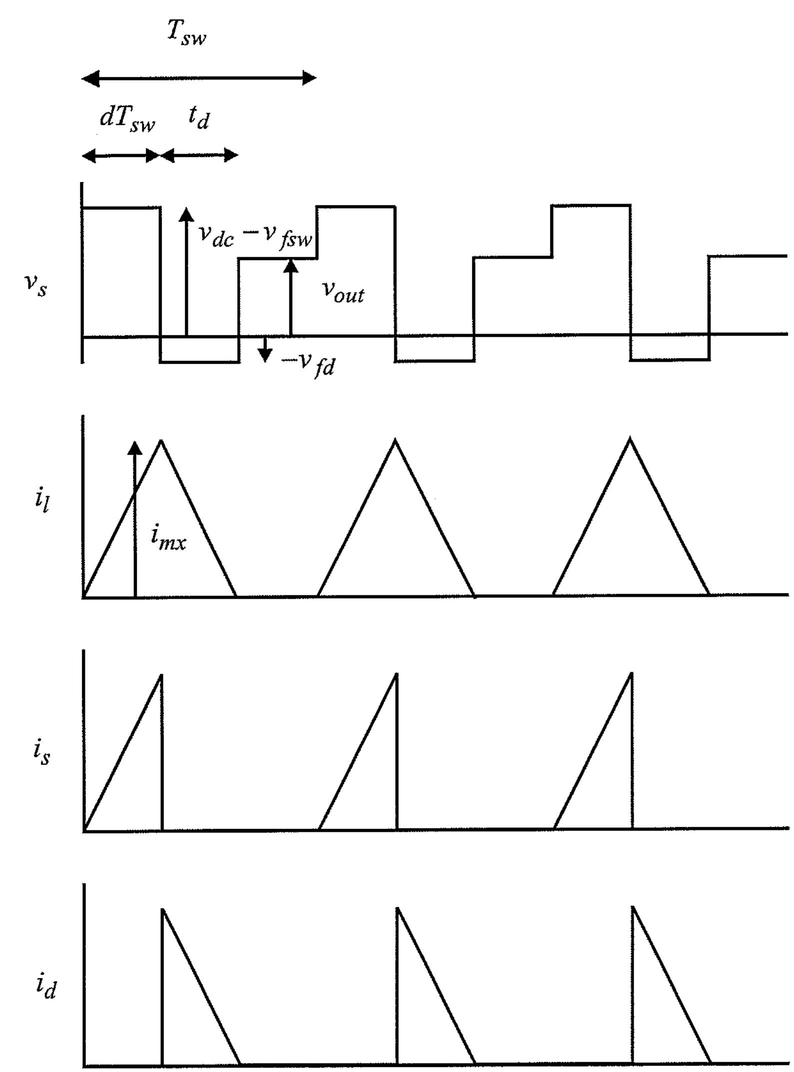
$$\boxed{\hat{i}_s = \frac{1}{2} d i_{mx}}$$

Switch Voltage & Current – Discont. Mode

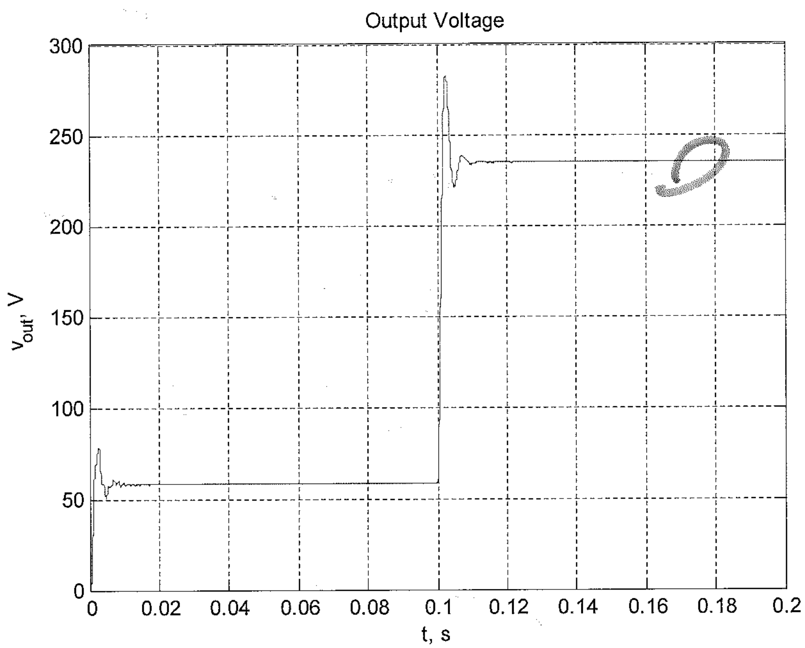
$$\hat{v}_s = \frac{1}{T_{sw}} \int_0^{T_{sw}} v_s \, d\tau$$

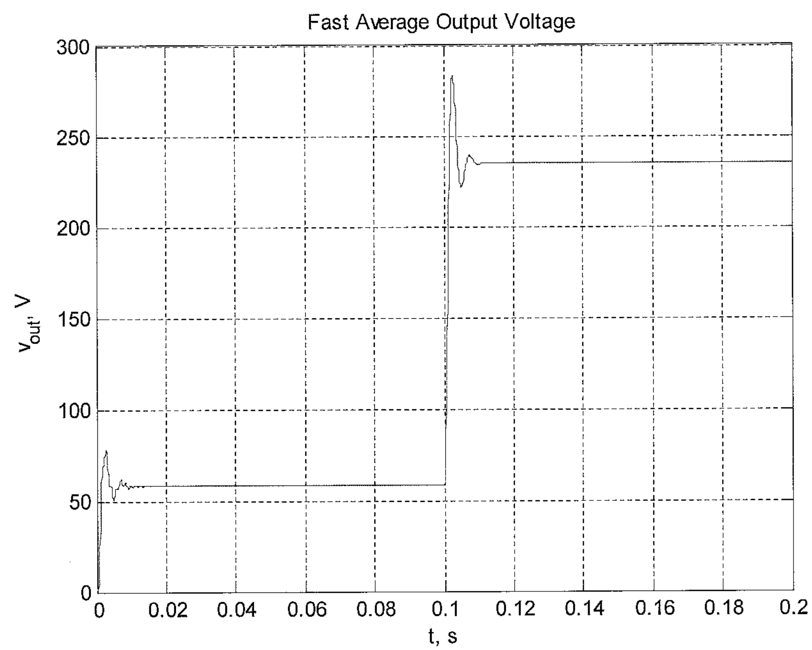
$$= \frac{1}{T_{sw}} \Big[(v_{dc} - v_{fsw})\, dT_{sw} - v_{fd}\, t_d + v_{out}(T_{sw} - dT_{sw} - t_d) \Big]$$

$$\boxed{\hat{v}_s = (v_{dc} - v_{fsw})\, d - v_{fd} \frac{t_d}{T_{sw}} + v_{out}\left(1 - d - \frac{t_d}{T_{sw}}\right)}$$

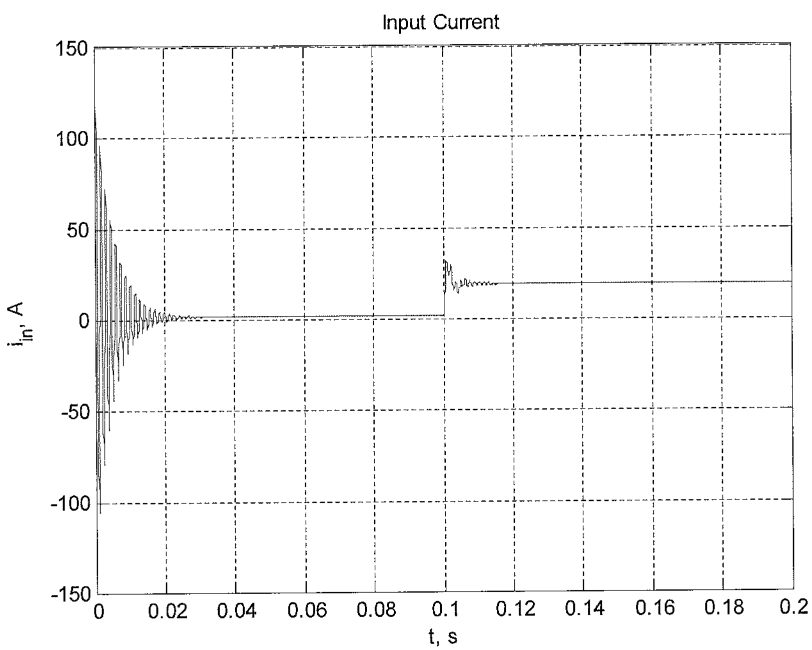


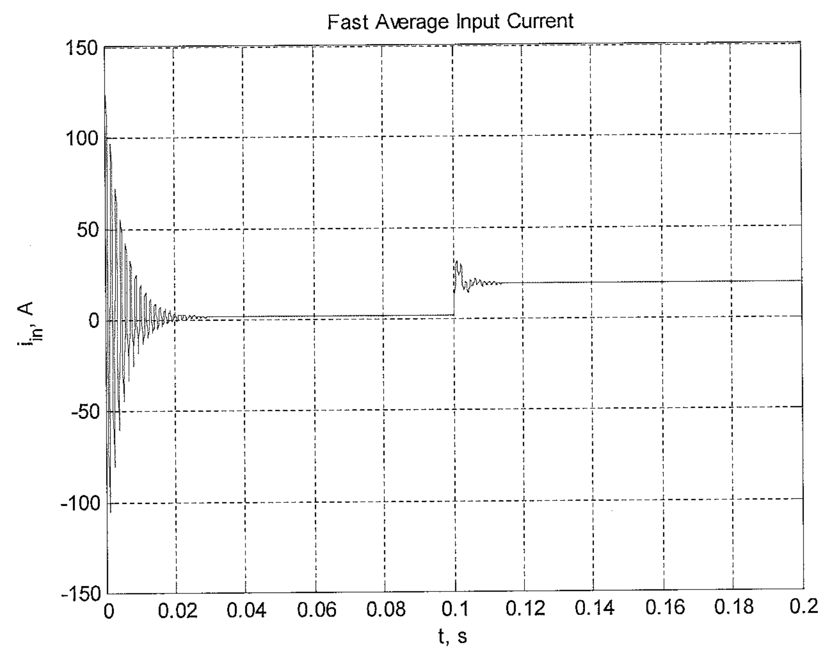
Sample Study





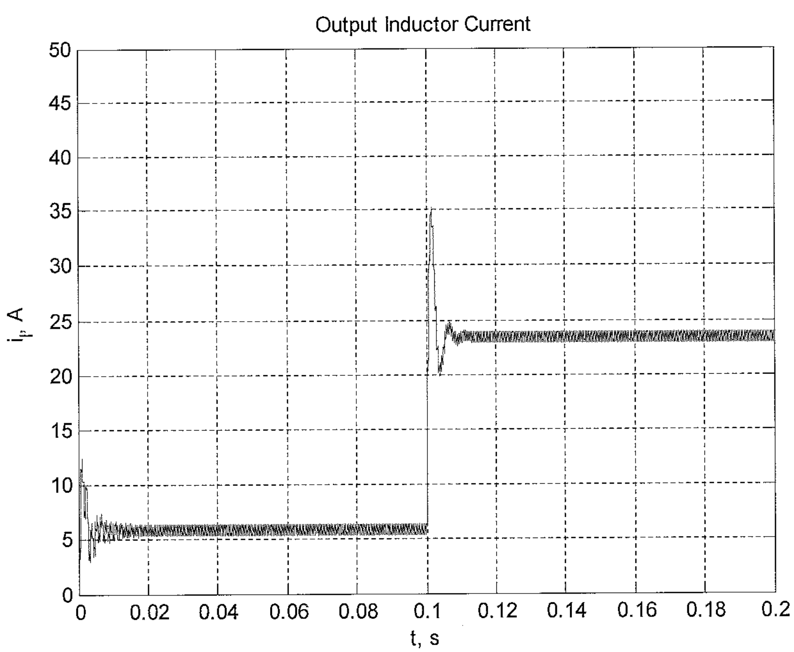
Sample Study

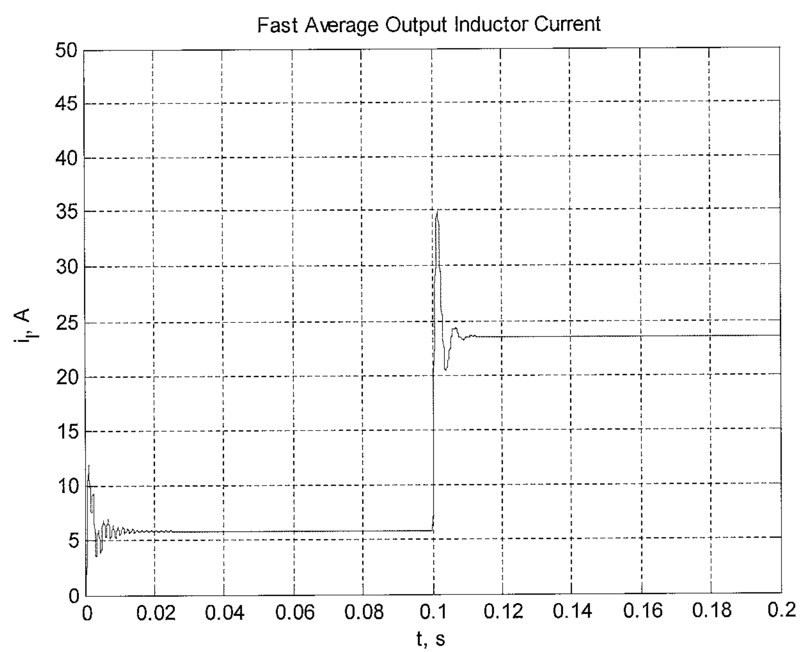




Sample Study

"switch averaging"
$\hat{i}_s$ $\hat{v}_s$





State Space Averaging (SSA)

- Basic Idea:

<u>Switch on</u>

$$pX = A_1 X + B_1 U$$
$$Y = C_1 X + D_1 U$$

<u>Switch off</u>

$$pX = A_2 X + B_2 U$$
$$Y = C_2 X + D_2 U$$

$\vdots$

Switching state N

$$pX = A_n X + B_n U$$
$$Y = C_n X + D_n U$$

State Space Averaging (SSA)

Assume Two Switching states

as approximation

$$p\hat{x} = \frac{1}{T_{sw}} \int_{t-T_{sw}}^{t} px \, d\tau$$

$$= \frac{1}{T_{sw}} \left[\int_{t \in \text{state 1}} px \, d\tau + \int_{t \in \text{state 2}} px \, d\tau \right]$$

$$= \frac{1}{T_{sw}} \left[\int_{t \in \text{state}} A_1 \hat{x} + B_1 \hat{u} \, d\tau + \int_{t \in \text{state 2}} A_2 \hat{x} + B_2 \hat{u} \, d\tau \right]$$

$$= \cancel{\frac{1}{T_{sw}}} \left[(A_1 \hat{x} + B_1 \hat{u}) d \cancel{T_{sw}} + (A_2 \hat{x} + B_2 \hat{u})(1-d) \cancel{T_{sw}} \right]$$

$$p\hat{x} = (A_1 d + A_2 (1-d)) \hat{x} + (B_1 d + B_2 (1-d)) \hat{u}$$

$$\hat{y} = (C_1 d + C_2 (1-d)) \hat{x} + (D_1 d + D_2 (1-d)) \hat{u}$$