Formulation of Fitness Functions

- Aggregation (or averaging) of constraints

$$\overline{c} = \frac{1}{C}\sum_{i=1}^{C} c_i$$

- We can then define a fitness function as

$$f_i = \begin{cases} \varepsilon(\overline{c}-1) & \overline{c} < 1 \\ m_i & \overline{c} = 1 \end{cases} \quad \text{(maximize metric)}$$

$$f_i = \begin{cases} \varepsilon(\overline{c}-1) & \overline{c} < 1 \\ \dfrac{1}{m_i} & \overline{c} = 1 \end{cases} \quad \text{(minimize metric)}$$

Formulation of Fitness Functions

- An approach to minimize computational effort

calculate constraint c_i

$C_I = C_I + 1$

$C_S = C_S + c_i$

if ($C_S < C_I$)

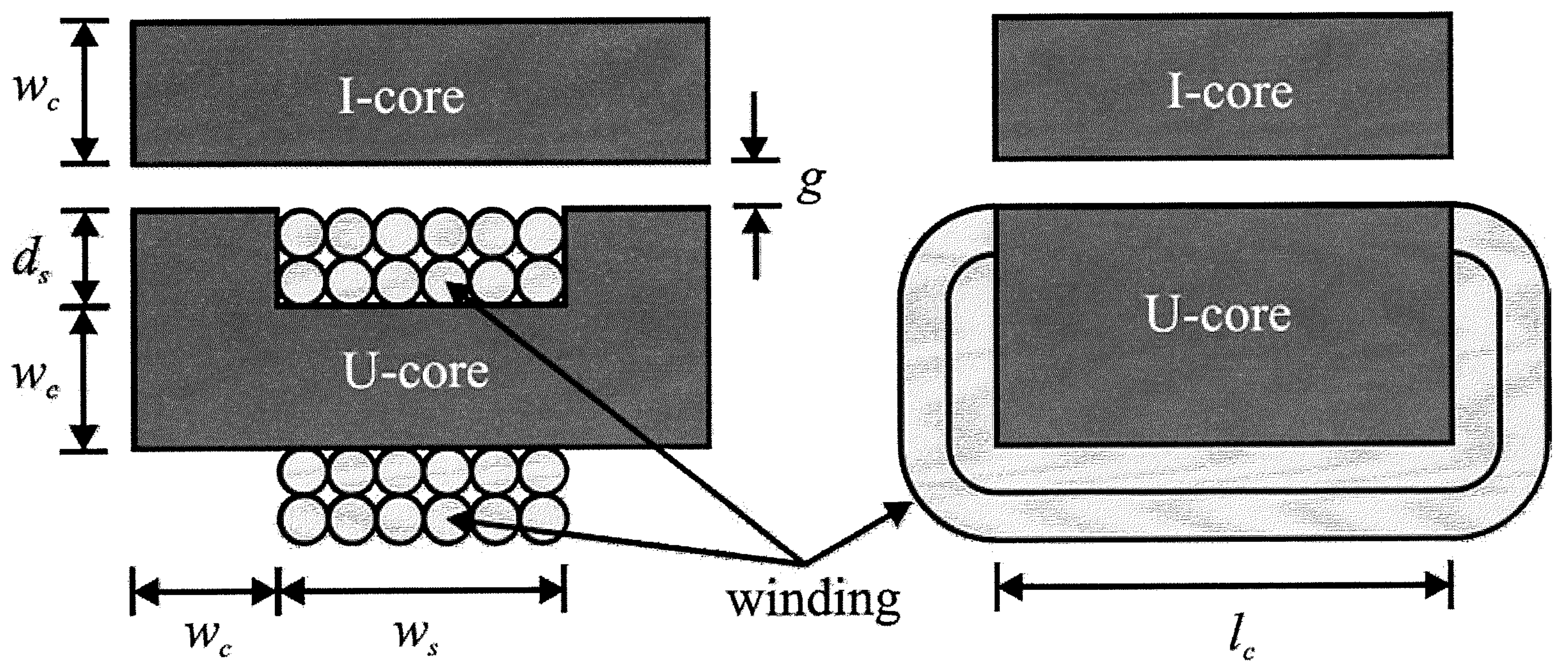
$$\mathbf{f} = \varepsilon \left(\frac{C_S - C}{C} \right) [1 \quad 1 \quad \cdots \quad 1]^T$$

return

end

A Design Example

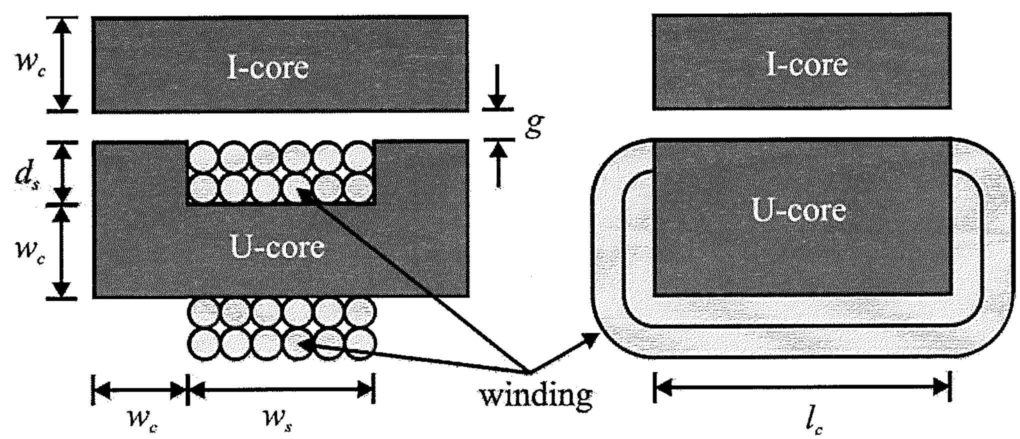
- Consider a UI core inductor



(a) cross section (b) side view

Free Parameters

$$\mathbf{x} = \begin{bmatrix} N^* & d_s & w_s & w_c & l_c & g \end{bmatrix}^T$$



(a) cross section (b) side view

Analysis

$$N = \mathrm{round}(N^*)$$

$$M = 2(2w_c + w_s + d_s)l_c w_c \rho_{mc} + (2l_c + 2w_c + \pi d_s)d_s w_s k_{pf} \rho_{wc}$$

$$P_{rt} = \frac{(2l_c + 2w_c + \pi d_s)N^2 i_{rt}^2}{d_s w_s k_{pf} \sigma_{wc}}$$

$$L = \frac{\mu_0 l_c w_c N^2}{2g}$$

$$B_{rt} = \frac{\mu_0 N i_{rt}}{2g}$$

$$J_{rt} = \frac{Ni}{w_s d_s k_{pf}}$$

Constraints

$$c_1 = \mathrm{gte}(L, L_{mn})$$

$$c_2 = \mathrm{lte}(B_{rt}, B_{mx})$$

$$c_3 = \mathrm{lte}(J_{rt}, J_{mx})$$

$$c_4 = \mathrm{lte}(P_{rt}, P_{mx})$$

$$c_5 = \mathrm{lte}(M, M_{mx})$$

$$\overline{c} = \frac{1}{5}\left(c_1 + c_2 + c_3 + c_4 + c_5\right)$$

Fitness

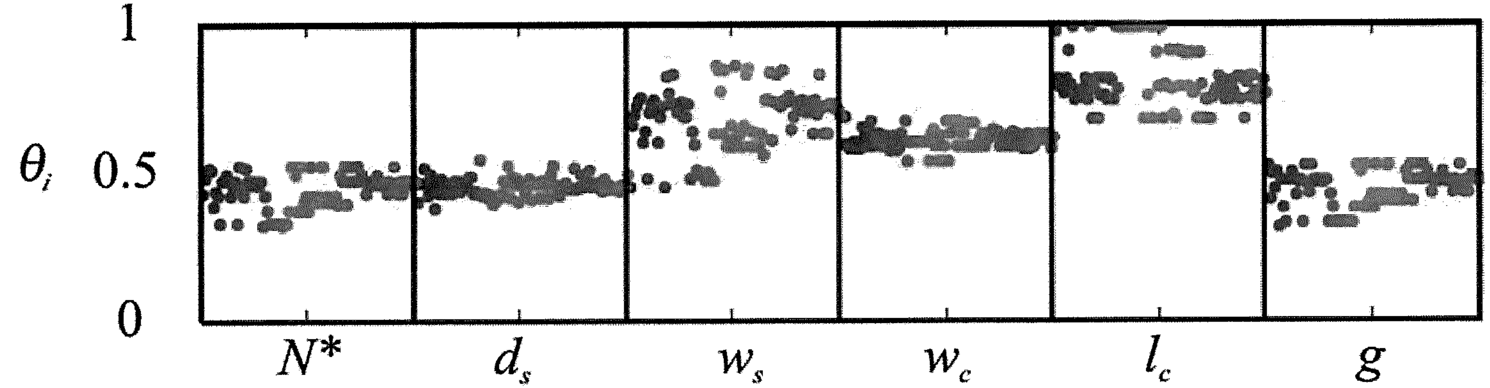
$$f = \begin{cases} \varepsilon(\overline{c} - 1) & \overline{c} < 1 \\ \dfrac{1}{M} & \overline{c} = 1 \end{cases}$$

$$f = \begin{cases} \varepsilon(\overline{c} - 1)\begin{bmatrix} 1 & 1 \end{bmatrix}^T & \overline{c} < 1 \\ \begin{bmatrix} \dfrac{1}{M} & \dfrac{1}{P_{rt}} \end{bmatrix}^T & \overline{c} = 1 \end{cases}$$

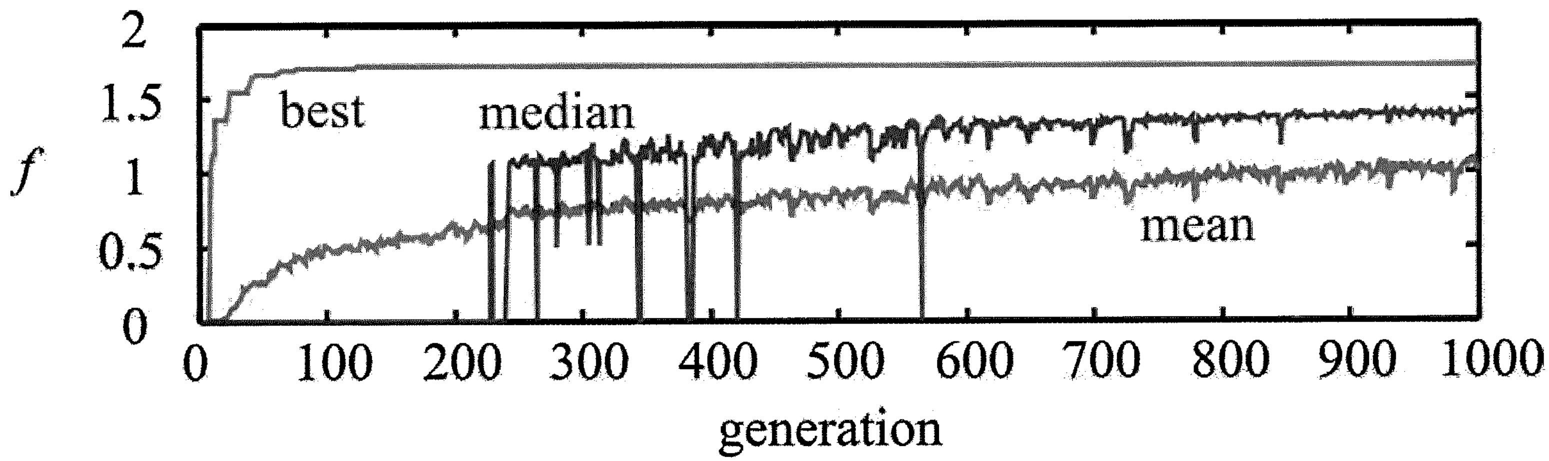
Domain of Design Parameters

parameter	N	d_s (m)	w_s (m)	w_c (m)	l_c (m)	g (m)
min. value	1	10^{-3}	10^{-3}	10^{-3}	10^{-3}	10^{-5}
max. value	10^{3}	10^{-1}	10^{-1}	10^{-1}	10^{-1}	10^{-2}
encoding	log	log	log	log	log	log
chromosome	1	1	1	1	1	1

Single Objective Optimization Study



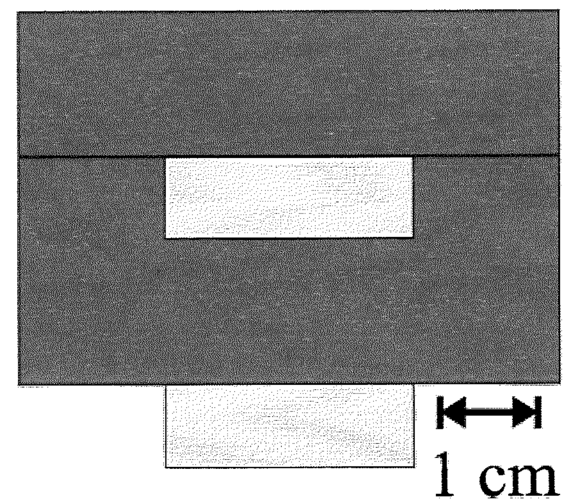
(a) gene distribution



(b) fitness evolution

UI-Core Design

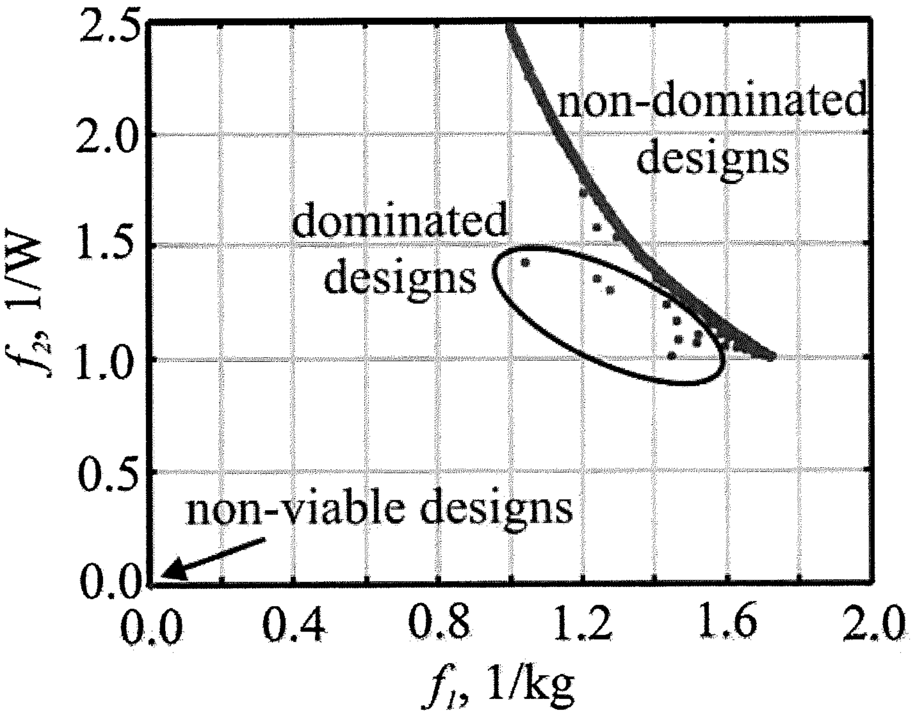
Turns $N = 25$ ($N^* = 25.3$)
Slot depth d_s (mm) = 8.39
Slot width w_s (mm) = 25.5
Core width w_c (mm) = 15.0
Core length l_c (mm) = 43.3
Air gap g (mm) = 0.255
Current density (A/mm^2) = 1.67
Flux density (T) = 0.617
Inductance (mH) = 1
Mass (kg) = 0.578



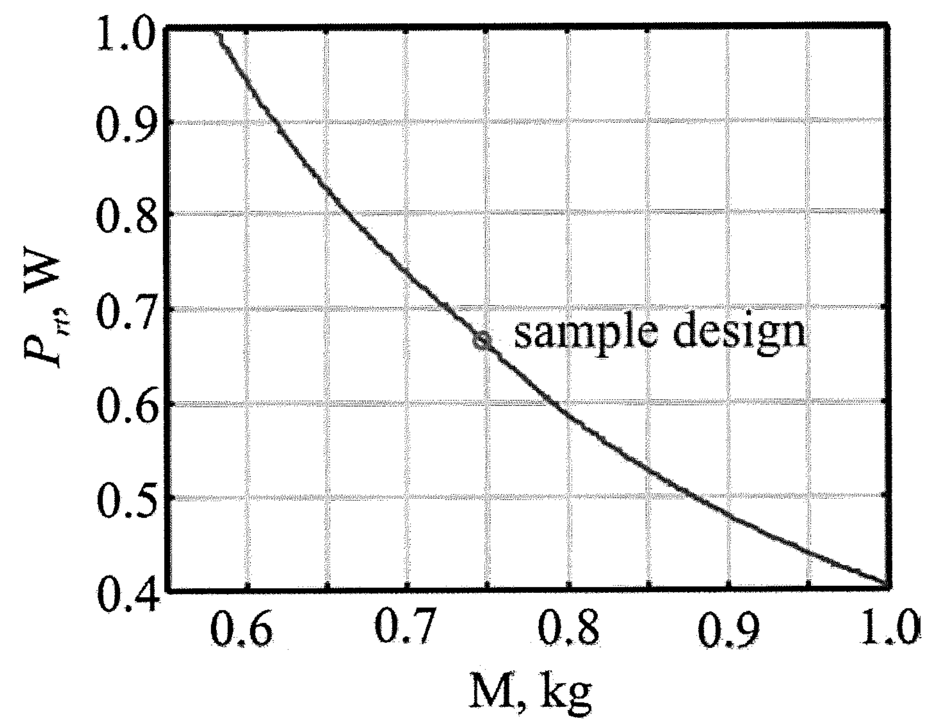
Design Repeatability

- Running 100 studies yields standard deviations (normalized to means)
  - N: 11%
  - d_s: 4.4%
  - w_s: 17%
  - w_c: 6.4%
  - l_c: 14%
  - g: 11%
  - M: 1%

Multi-Objective Optimization

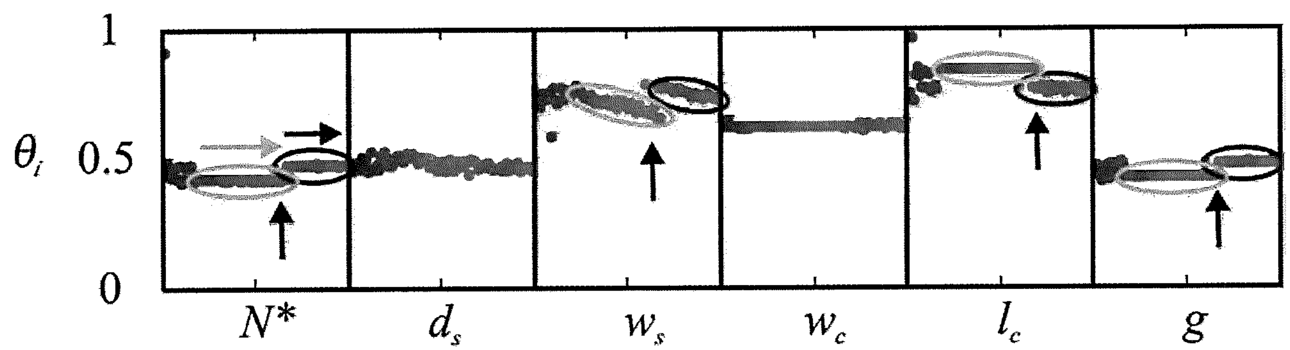


(a) objective space

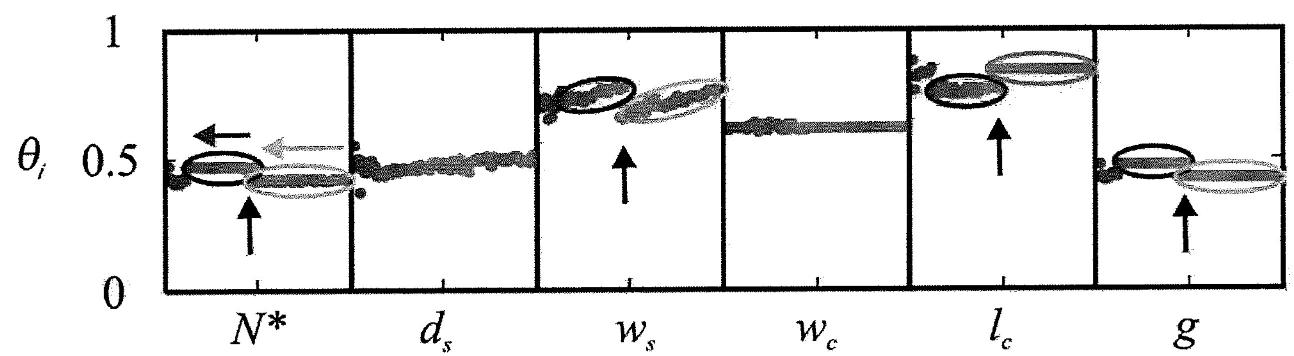


(b) Pareto-optimal front

Population Distribution



(a) gene distribution sorted by objective 1



(b) gene distribution sorted by objective 2

Optional Reading

- K. Deb, *Multi-Objective Optimization using Evolutionary Algorithms,* John Wiley & Sons, Inc., 2001
- S.D. Sudhoff, *Power Magnetic Devices: A Multi-Objective Design Approach*, Wiley/IEEE Press, 2014 (Chapter 1)

ECE61016 Power Electronics Converters and Systems

Homework 2

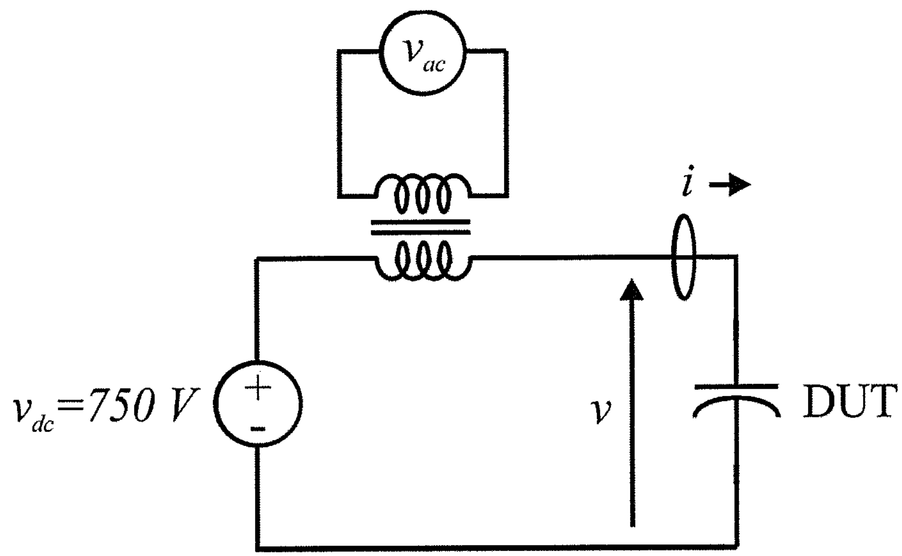
S.D. Sudhoff

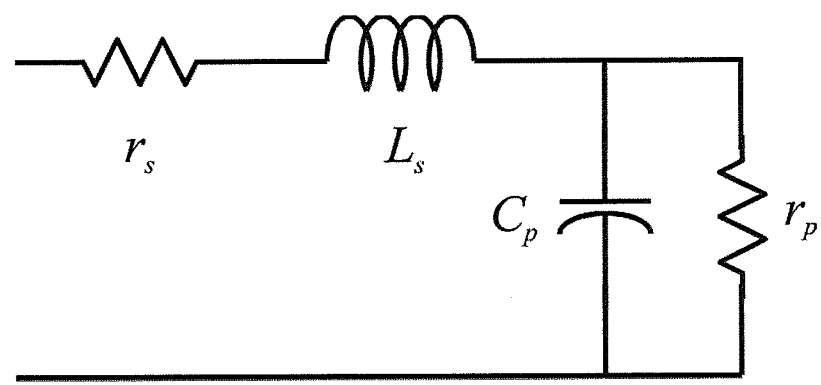
Fall 2016

Aspects of Assignment

- Problem 3: Repeat Problem 1 from Assignment 1, except use Simulink. Print relevant files/scripts. Plot waveforms on same scale that we used last time.
- Problem 3B: Suppose the inductance and capacitance are all subject to +/- 20% random and uncorrelated variations. Does the control still work? Use brute force with at least 500 simulation runs to form a conclusion.
- Problem 3C: Repeat 3B for +/- 40% variations.
- Problem 4: Identify the parameters of a capacitor, by formulating a single-objective optimization problem and using GOSET. Print relevant Matlab files, and parameters values.

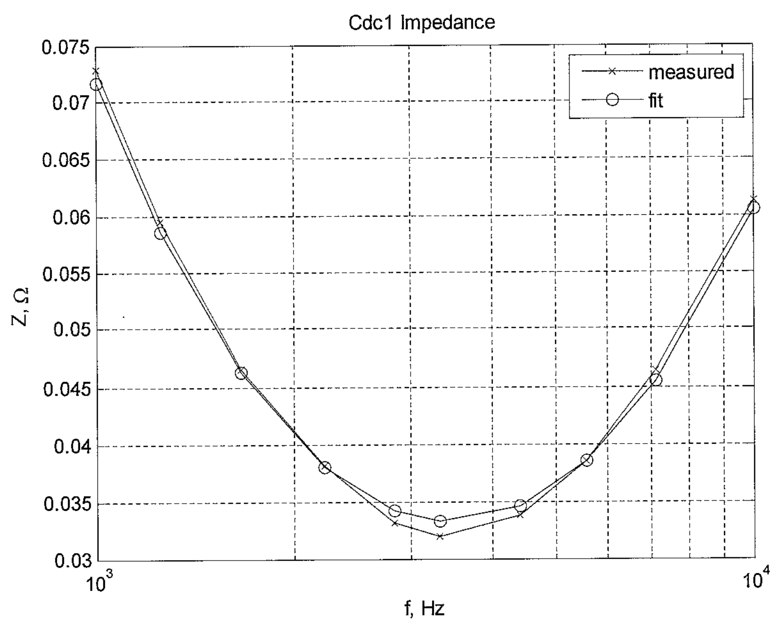
Capacitor Frequency Response

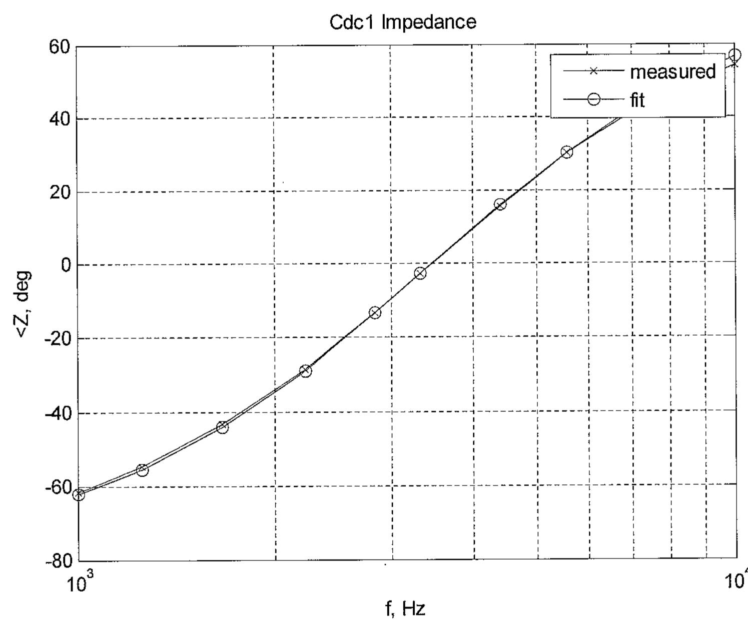




Assume $r_p = 10^5$
Find r_s, L_s, and C_p

Capacitor Frequency Response





The file Cdc1_Impedance contains structure data
data.Zm – vector magnitude of impedance (Ohms)
data.Za – vector of angle of impedance (rad)
data.f – vector of frequency (Hz)
data.w – vector of radian frequency (rad/s)

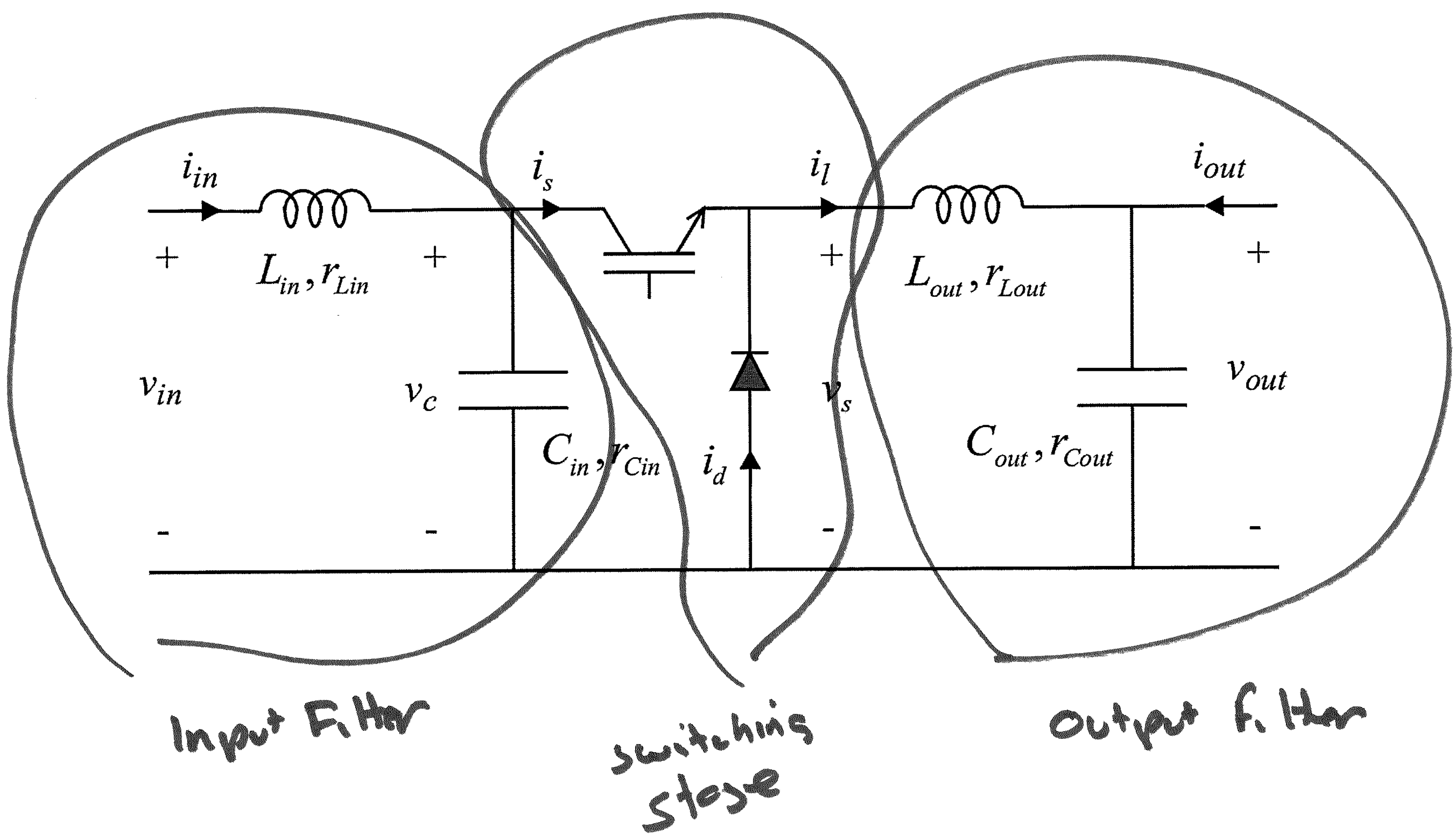
ECE61016 Power Electronics Converters and Systems

Lecture Set 3A: Steady-State Analysis of Buck Converter

S.D. Sudhoff

Fall 2016

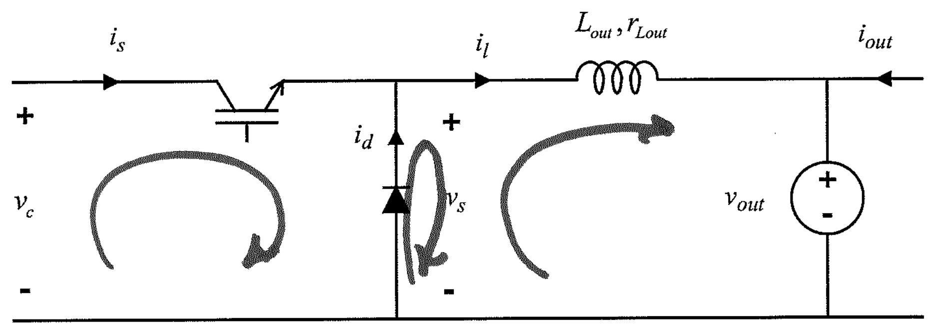
Buck Converter

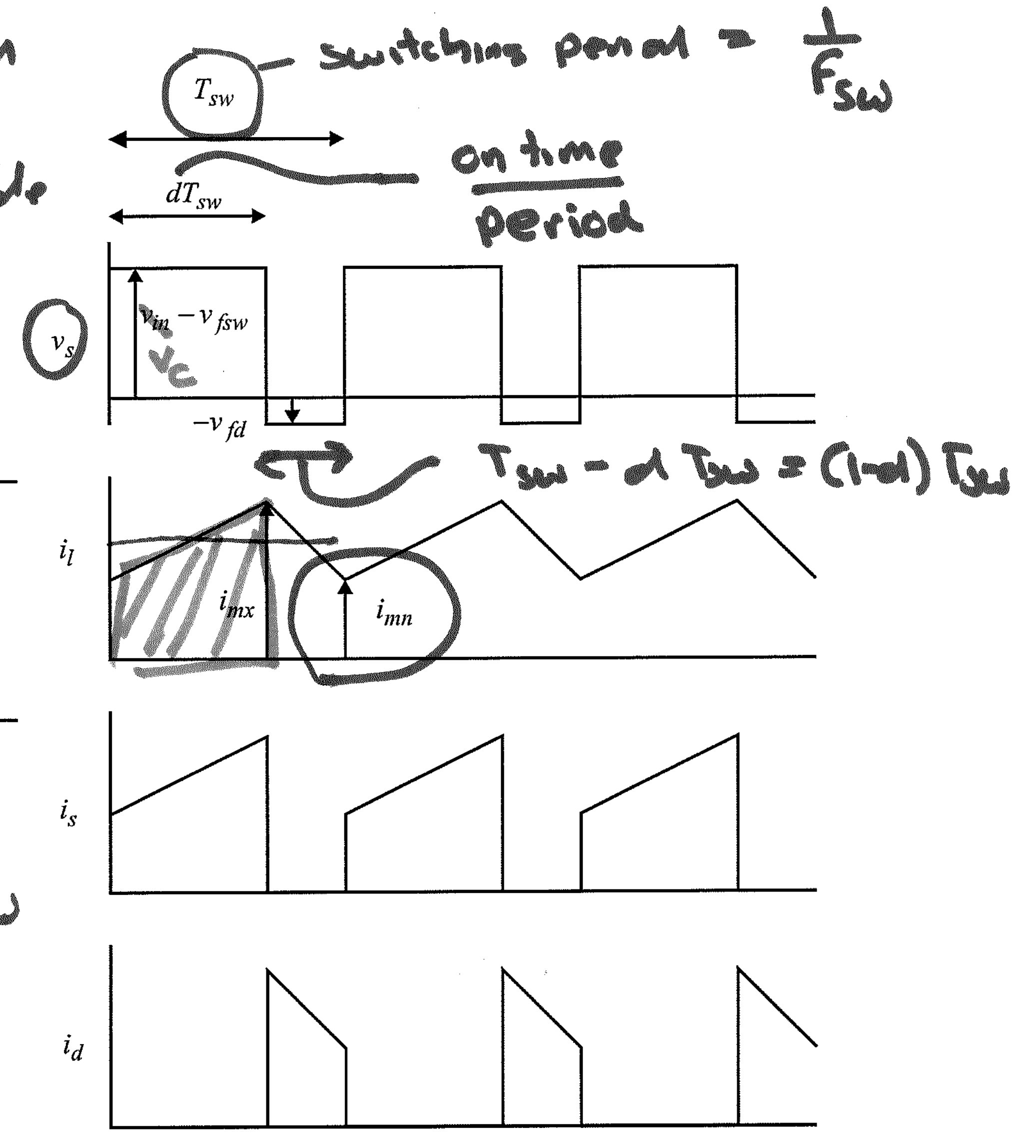


Continuous Current (CC) Buck Operation

V_{fsw} → Forward on-state drop on switch

V_{fd} → Forward on-state diode drop





Switch on

$$-V_C + V_{FSW} + V_S = 0 \qquad V_S = V_C - V_{FSW}$$

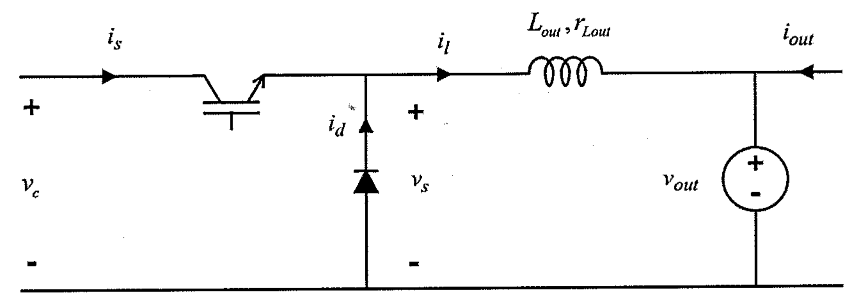
Switch off

$$-V_{Fd} + V_S = 0 \qquad V_S = -V_{Fd}$$

CC Buck Operation

- Definition of average:

$$\bar{x} = \frac{1}{T_{SW}} \int_{t_{ss}}^{t_{ss}+T_{SW}} x(t)dt$$



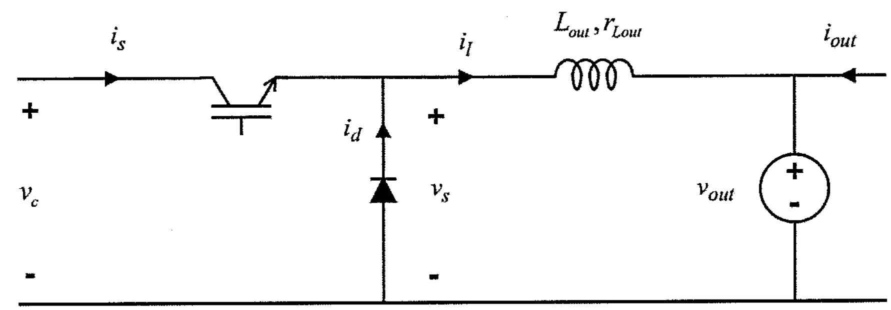
- Average switch point voltage

$$\bar{V}_s = \frac{1}{T_{sw}} \int_0^{T_{sw}} V_s \, dt = \frac{1}{\cancel{T_{sw}}} \left[(V_c - V_{Fsw}) \, d \cancel{T_{sw}} + (-V_{Fd})(1-d)\cancel{T_{sw}} \right]$$

$$= (\bar{V}_c - V_{Fsw}) d - (1-d) V_{Fd}$$

CC Buck Operation

- Average switch current



$$\overline{I_l} = \frac{1}{T_{sw}} \int_0^{T_{sw}} i_l \, dt$$

$$\boxed{\overline{I_l} = \frac{1}{\cancel{T_{sw}}} \frac{1}{2}(I_{max} + I_{min}) \cancel{T_{sw}}}$$

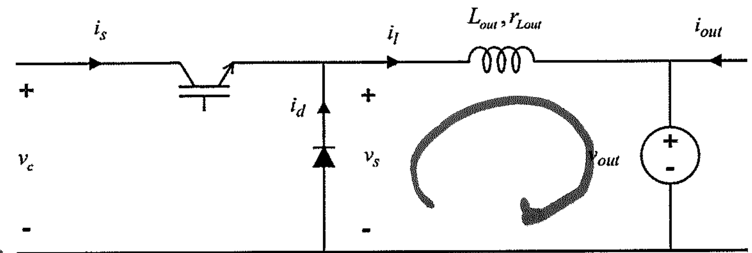
$$\overline{I_s} = \frac{1}{T_{sw}} \int_0^{dT_{sw}} i_l \, dt$$

$$= \frac{1}{\cancel{T_{sw}}} \frac{1}{2}(I_{max} + I_{min}) d\cancel{T_{sw}}$$

$$\boxed{\overline{I_s} = d\,\overline{I_l}}$$

CC Buck Operation

- Relationship of output voltage and switch point voltage



$$-V_s + L_{out}\frac{di_l}{dt} + r_{Lout} i_l + V_{out} = 0$$

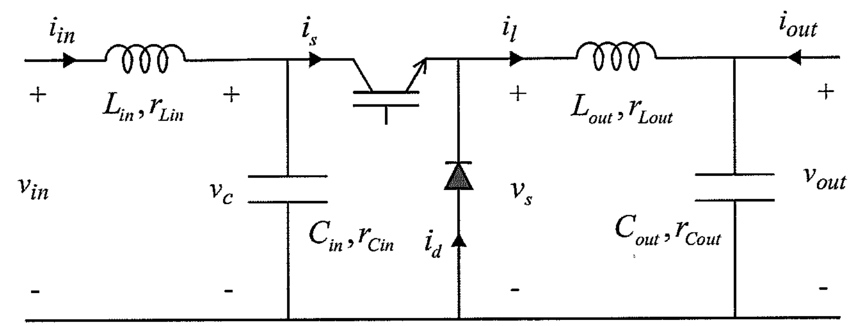
$$\overline{X} = \frac{1}{T_{sw}}\int_0^{T_{sw}} x \, dt$$

$$-\overline{V_s} + \underbrace{\cancel{\frac{1}{T_{sw}}\int_0^{T_{sw}} L_{out}\frac{di_l}{dt}\, dt}}_{0} + r_{Lout}\overline{I_l} + \overline{V_{out}} = 0$$

$$\boxed{-\overline{V_s} + r_{Lout}\overline{I_l} + \overline{V_{out}} = 0}$$

CC Buck Operation

- Relationship of input and input capacitor voltages



$$-v_{in} + L_{in}\frac{di_{in}}{dt} + r_{Lin} i_{in} + v_c = 0$$

$$\Downarrow$$

$$-\overline{V_{in}} + r_{Lin}\overline{I_{in}} + \overline{V_c} = 0$$

CC Buck Operation

- Summary of average-vale steady-state CC operation

$$\overline{i_l} = -\overline{i_{out}}$$

$$\overline{i_s} = d\overline{i_l}$$

$$\overline{v_{out}} = \overline{v_s} - r_{Lout}\overline{i_l}$$

$$\overline{v_s} = d(\overline{v_c} - v_{fsw}) - (1-d)v_{fd}$$

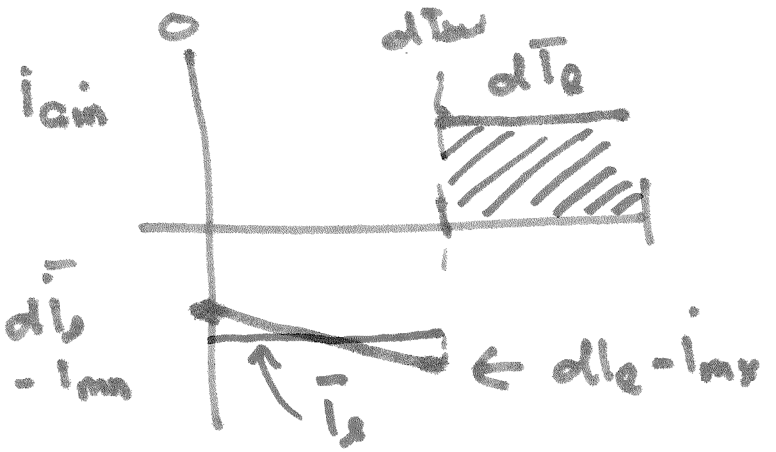
$$\overline{v_c} = \overline{v_{in}} - r_{Lin}\overline{i_s}$$

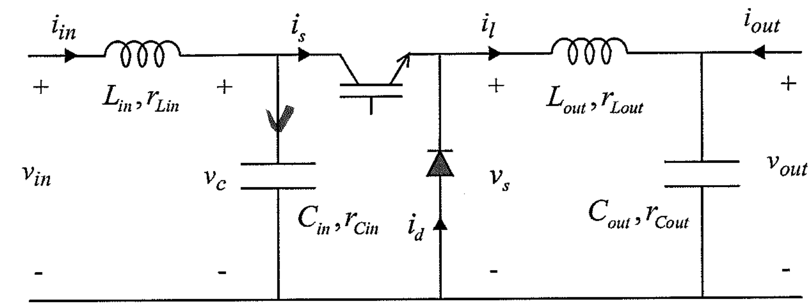
u

$\underline{I_{imn} > 0}$

CC Input Capacitor Current & Voltage Ripple

- Waveforms





$$i_{cin} = i_{in} - i_s$$
$$= d i_l - i_s$$

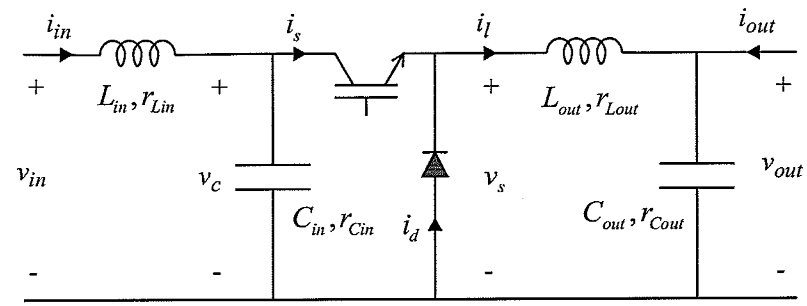
$$X_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} X^2\, dt}$$

$$T_{sw}\, I_{c,rms}^2 = \int_0^{T_{sw}} i_{cin}^2\, dt$$

$$= \int_0^{dT_{sw}} (d\bar{I}_l - \bar{I}_l)^2\, dt + \int_{dT_{sw}}^{T_{sw}} (d\bar{I}_l)^2\, dt$$

CC Input Capacitor Current & Voltage Ripple

- Analysis



$$\cancel{T_{sw}}\, I_{Cin,rms}^2 = \overline{I_\ell}^2 (d-1)^2 d \cancel{T_{sw}} + \overline{I_\ell}^2 d^2 (1-d) \cancel{T_{sw}}$$

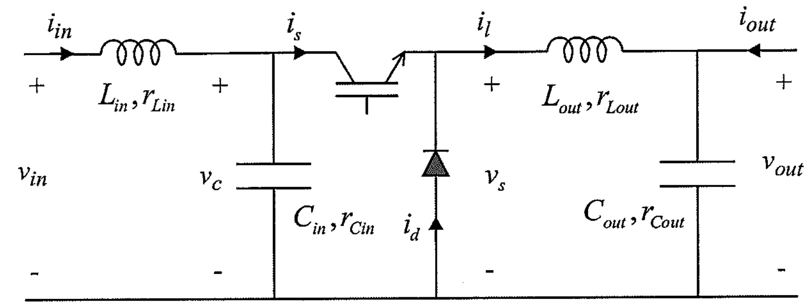
$$= \overline{I_\ell}^2 d \left[\cancel{d^2} - 2d + 1 + d - \cancel{d^2} \right]$$

$$= \overline{I_\ell}^2 d (1-d)$$

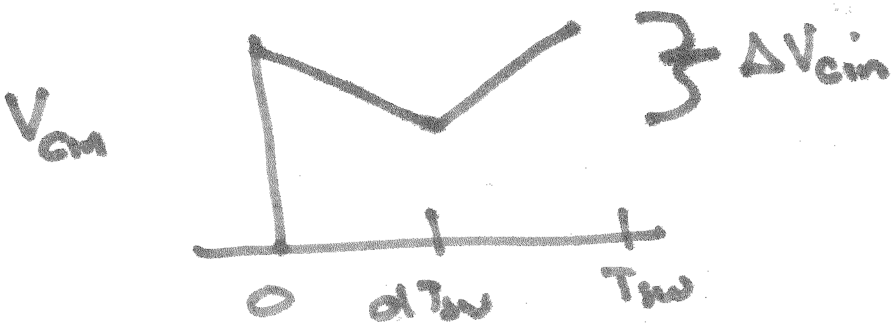
$$\boxed{I_{Cin,rms} = \overline{I_\ell} \sqrt{d(1-d)}}$$

CC Input Capacitor Current & Voltage Ripple

- Analysis



$$i_{Cin} = C_{in} \frac{dv_{Cin}}{dt}$$



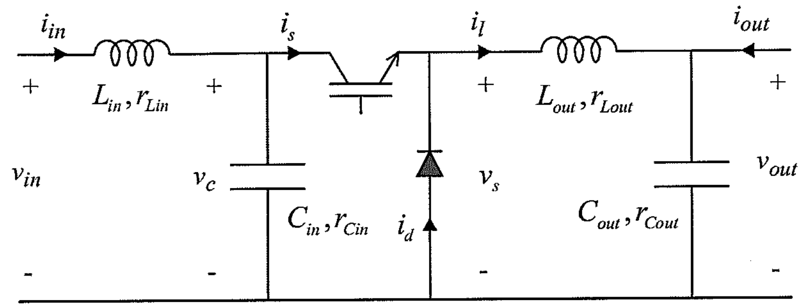
$$V_{Cin}(T_{sw}) = V_{Cin}(dT_{sw}) + \int_{dT_{sw}}^{T_{sw}} \frac{i_{Cin}}{C_{in}} dt$$

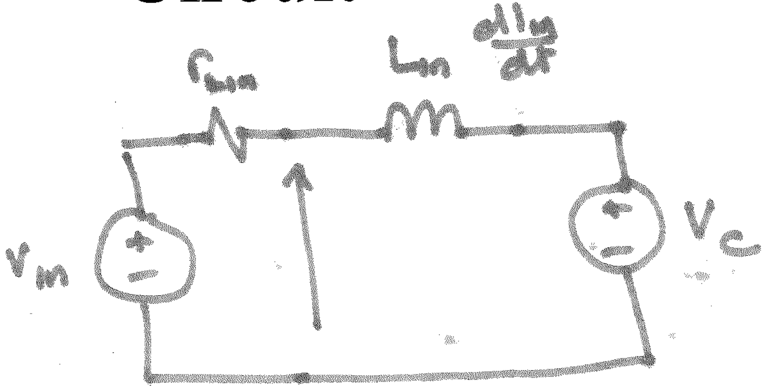
$$\Delta V_{Cin} = \frac{1}{C_{in}} \int_{dT_{sw}}^{T_{sw}} i_{Cin}\, dt = \frac{1}{C_{in}} d\bar{I}_l (1-d) T_{sw}$$

$$\boxed{\Delta V_{Cin} = \frac{1}{C_{in} f_{sw}} \bar{I}_l\, d(1-d)}$$

CC Input Inductor Current Ripple

- Circuit

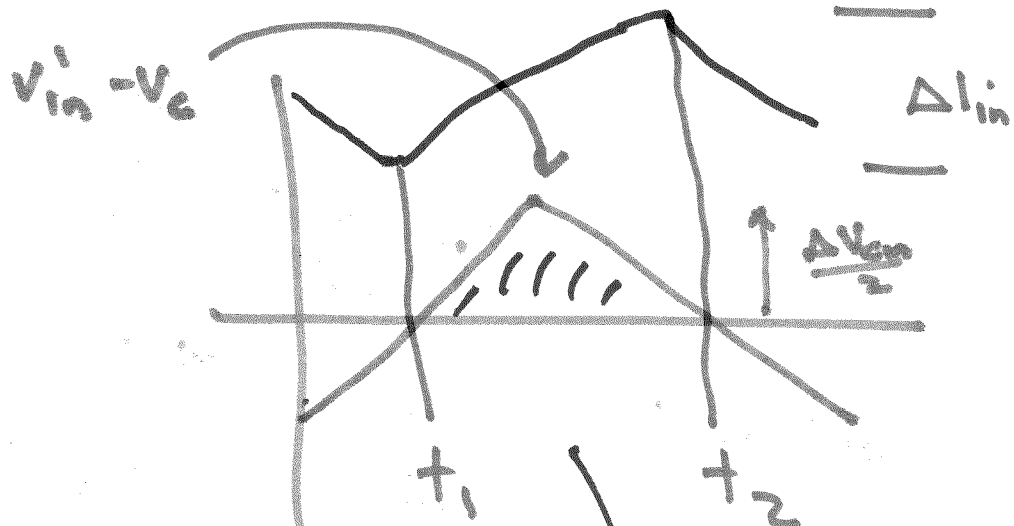




$$v_{in}' = v_{in} - r_{Lin} \overline{di_l}$$

$$-v_{in}' + L_{in}\frac{di_{in}}{dt} + v_c = 0$$

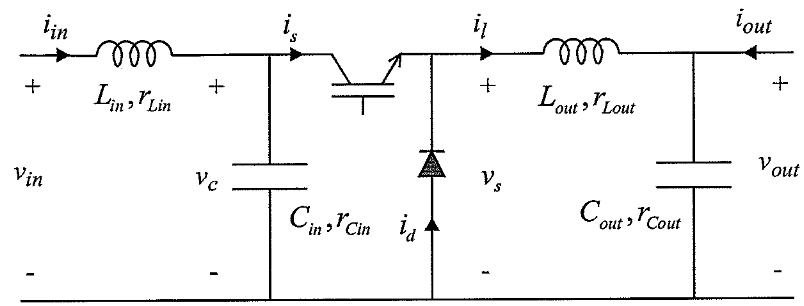
$$\frac{di_{in}}{dt} = \frac{1}{L_{in}}\left(v_{in}' - v_c\right)$$



$$i_{in}(t_2) = i_{in}(t_1) + \int_{t_1}^{t_2} \frac{1}{L_{in}}\left(v_{in}' - v_c\right) dt$$

CC Input Inductor Current Ripple

- Waveforms

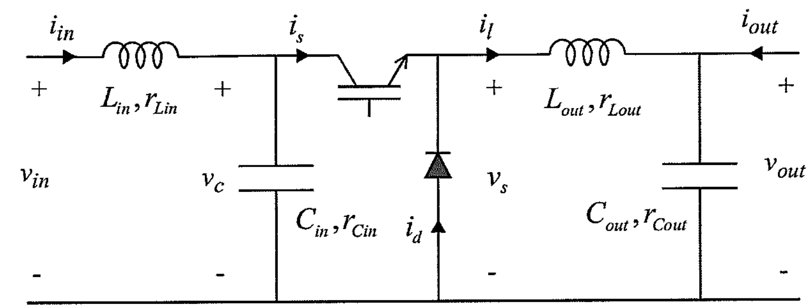


$$\Delta I_{in} = \frac{1}{L_{in}} \frac{1}{2} \frac{1}{2} T_{sw} \frac{\Delta V_{cin}}{2}$$

$$\boxed{\Delta I_{in} = \frac{1}{8} \frac{1}{L_{in} f_{sw}} \Delta V_{cin}}$$

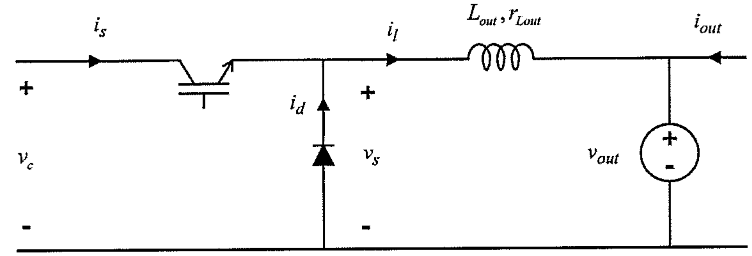
CC Input Inductor Current Ripple

- Analysis



CC Output Inductor Current Ripple

- On state



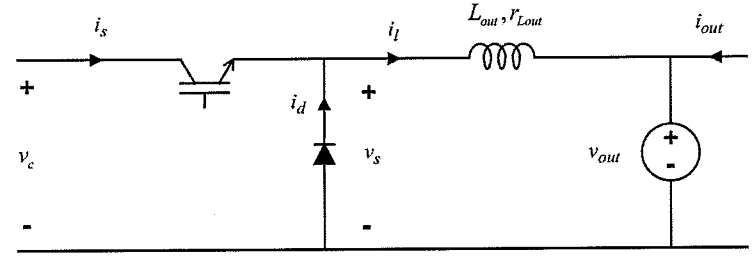
$$-V_c + V_{Fsw} + r_{Lout}\cancel{i_l} + L_{out}\frac{di_l}{dt} + V_{out} = 0 \qquad \approx \bar{I}_l$$

$$L_{out}\frac{di_l}{dt} = V_c - V_{Fsw} - r_{Lout}\bar{I}_l - V_{out}$$

$$L_{out}\frac{I_{max} - I_{min}}{dT_{sw}} = V_c - V_{Fsw} - r_{Lout}\bar{I}_l - V_{out}$$

CC Output Inductor Current Ripple

- Off state



$$-V_{fd} + L_{out}\frac{di_l}{dt} + r_{Lout}\cancel{i_l}^{\overline{I_l}} + V_{out} = 0$$

$$L_{out}\frac{di_l}{dt} = -V_{out} + V_{fd} - r_{Lout}\overline{I_l}$$

$$+L_{out}\frac{I_{max} - I_{min}}{(1-d)T_{sw}} = +V_{out} - V_{fd} + r_{Lout}\overline{I_l}$$

CC Output Inductor Current Ripple

- In conclusion:

$$\left(i_{mx} - i_{mn}\right)\frac{L_{out}}{dT_{sw}} = v_{in} - v_{fsw} - r_{Lout}\bar{i}_l - v_{out} \quad on$$

$$\left(i_{mx} - i_{mn}\right)\frac{L_{out}}{(1-d)T_{sw}} = v_{fd} + r_{Lout}\bar{i}_l + v_{out} \quad off$$

- Combining

$$\frac{L_{out}}{T_{sw}}\left(I_{mx} - I_{mn}\right)\left[\frac{1}{d} + \frac{1}{1-d}\right] = V_{in} - V_{fsw} + V_{fd}$$

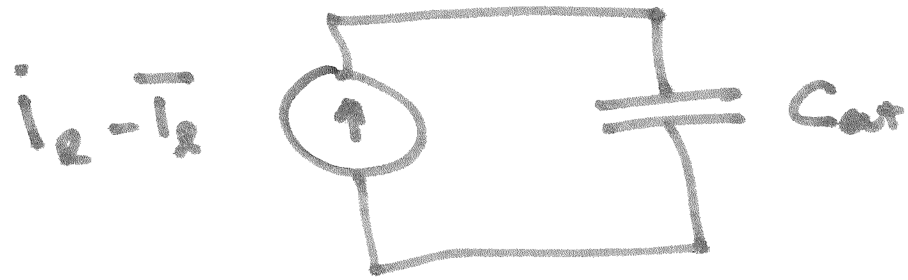
$$\frac{1}{d} + \frac{1}{1-d} = \frac{1-d+d}{d(1-d)} = \frac{1}{d(1-d)}$$

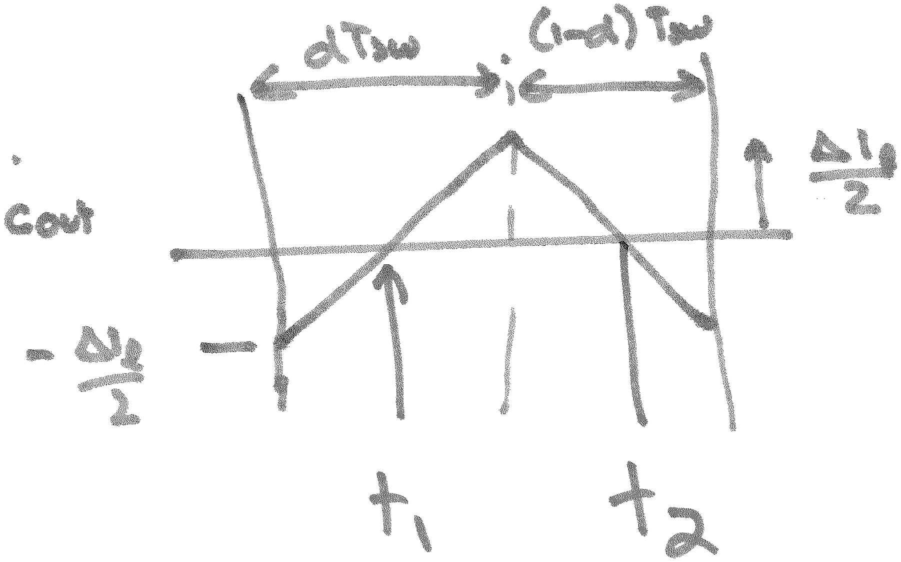
$$I_{mx} - I_{mn} = \boxed{\Delta I_{Lout} = \frac{\left(V_{in} - V_{fsw} + V_{fd}\right) d(1-d)}{f_{sw}\, L_{OUT}}}$$

CC Output Inductor Current Ripple

CC Output Capacitor Current & Voltage Ripple

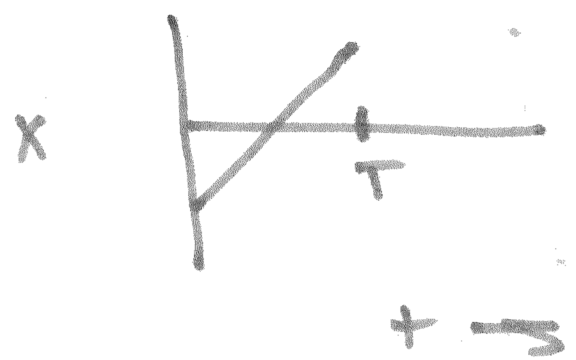
- Circuit





CC Output Capacitor Current & Voltage Ripple

- Waveforms



$$x = \alpha + \beta t$$

$$\int_0^T x^2 dt = \int_0^T \alpha^2 + 2\alpha\beta t + \beta^2 t^2 dt$$

$$= \alpha^2 T + \alpha\beta T^2 + \frac{1}{3}\beta^2 T^3$$

$$T_{sw} I_{Cout,rms}^2 = \int_0^{T_{sw}} I_{Cout}^2 dt$$

$$= \int_0^{dT_{sw}} \left[-\frac{\Delta I_L}{2} + \frac{\Delta I_L}{dT_{sw}} t\right]^2 dt$$

shifted time

$$+ \int_0^{(1-d)T_{sw}} \left[\frac{\Delta I_L}{2} - \frac{\Delta I_L}{(1-d)T_{sw}} t\right]^2 dt$$

CC Output Capacitor Current & Voltage Ripple

- Analysis

$$\cancel{T_{sw}}\, I^2_{Cout,rms} = \left(\frac{\Delta I_L}{2}\right)^2 d\cancel{T_{sw}} - \frac{\Delta I_L}{2}\frac{\Delta I_L}{d\cancel{T_{sw}}} d^2 \cancel{T_{sw}^2} + \frac{1}{3}\frac{(\Delta I_L)^2}{(d\cancel{T_{sw}})^2}(d\cancel{T_{sw}})^3$$

$$+ \left(\frac{\Delta I_L}{2}\right)^2 (1-d)\cancel{T_{sw}} - \frac{\Delta I_L}{2}\frac{\Delta I_L}{(1-d)\cancel{T_{sw}}}\left((1-d)\cancel{T_{sw}}\right)^2$$

$$+ \frac{1}{3}\left(\frac{\Delta I_L}{(1-d)\cancel{T_{sw}}}\right)^2 (1-d)^3 \cancel{T_{sw}^3}$$

$$= \Delta I_L^2 \left[\frac{1}{4}d - \frac{1}{2}d + \frac{1}{3}d\right]$$

$$+ \Delta I_L^2 \left[\frac{1}{4}(1-d) - \frac{1}{2}(1-d) + \frac{1}{3}(1-d)\right]$$

$$= \frac{\Delta I_L^2}{12}\left[d + (1-d)\right] = \frac{\Delta I_L^2}{12}$$

$$\boxed{I_{Cout,rms} = \frac{\Delta I_L}{2\sqrt{3}}}$$