Eigenvalues

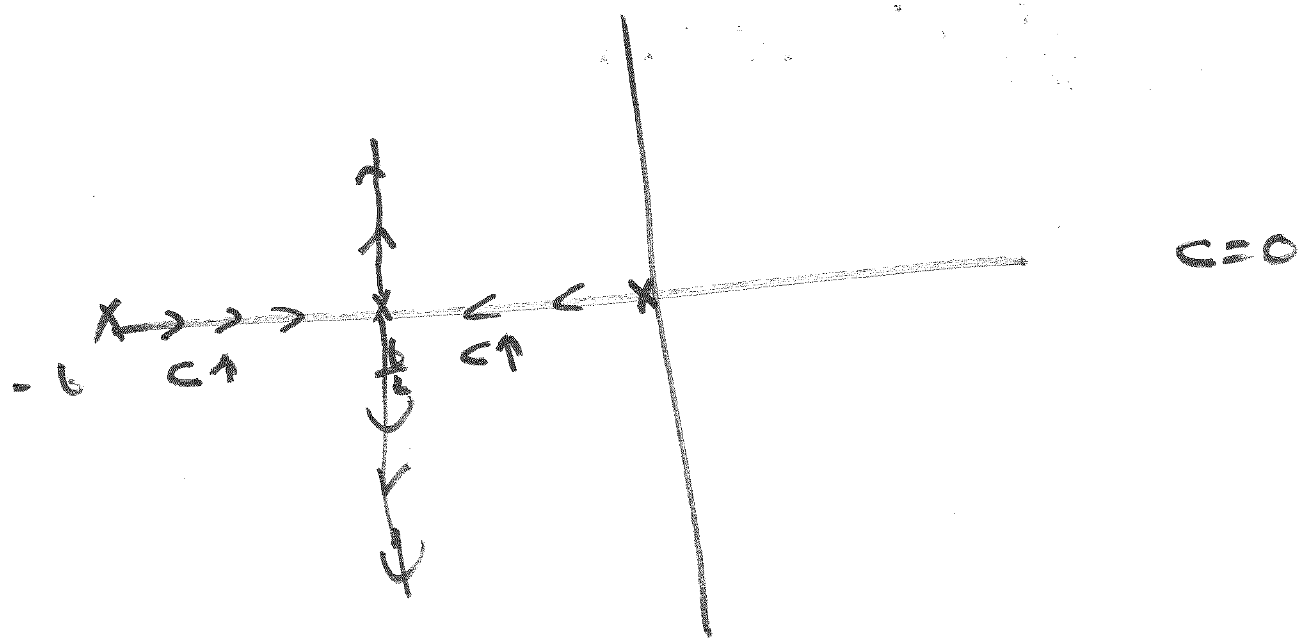
$$\det(\lambda I - A) = \det\left(\begin{bmatrix} \lambda + \frac{f}{L} & \frac{1}{L} \\ -\frac{1}{C} & \lambda - \frac{p}{C V_{c0}^2} \end{bmatrix}\right)$$

$$= \left(\lambda + \frac{f}{L}\right)\left(\lambda - \frac{p}{C V_{c0}^2}\right) + \frac{1}{LC}$$

$$= \lambda^2 + \underbrace{\left(\frac{f}{L} - \frac{p}{C V_{c0}^2}\right)}_{b} \lambda + \underbrace{\left(\frac{1}{LC} - \frac{f}{L}\frac{p}{C V_{c0}^2}\right)}_{c}$$

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2}$$

Conditions for Stability



Conditions for Stability

$b > 0$

$$\frac{r}{L} > \frac{p}{C V_{co}^2}$$

$$p < \frac{r C V_{co}^2}{L}$$

$r \quad C \quad V_{co}^2$ are good

L is bad

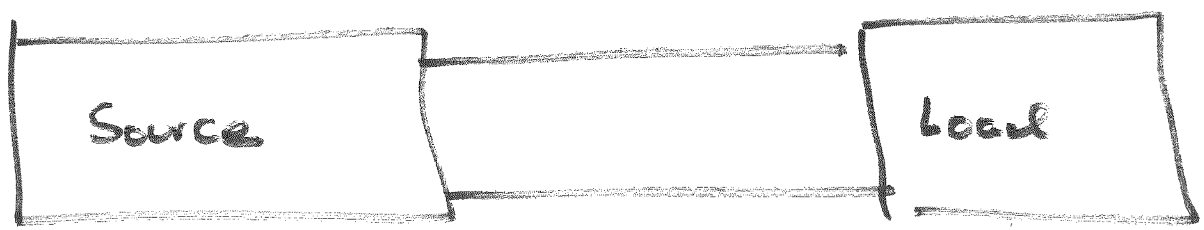
$c > 0$

$$\frac{1}{L\cancel{C}} > \frac{r}{L} \frac{p}{\cancel{C} V_{co}^2}$$

$$p < \frac{V_{co}^2}{r}$$

not relevant

Writing Specifications to Address Stability



Immittance from Linearized Models

- Going to the frequency domain

$$I(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D}$$

Immittance
(Impedance
Admittance)

$$pX = AX + BU$$

$$(sI - A)X = BU$$

$$X = (sI - A)^{-1} B U$$

$$Y = CX + DU$$

$$Y = [C(sI - A)^{-1} B + D] U$$

If U is i, $Y = v$, then Z
If U is v, $Y = i$, then Y

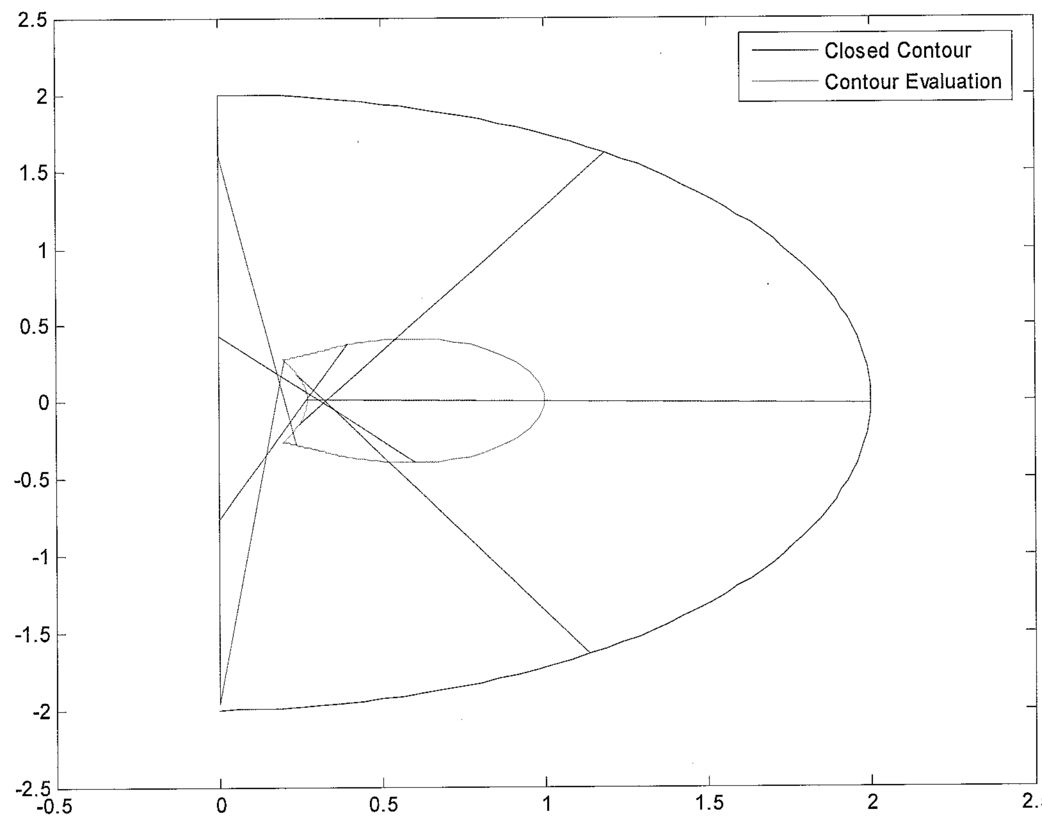
Immittance from Linearized Models

Contour Evaluations of Complex Functions

- Consider the transfer function

$$H(s) = \frac{s+1}{s^2+3s+1}$$

- A contour evaluation appears to the right

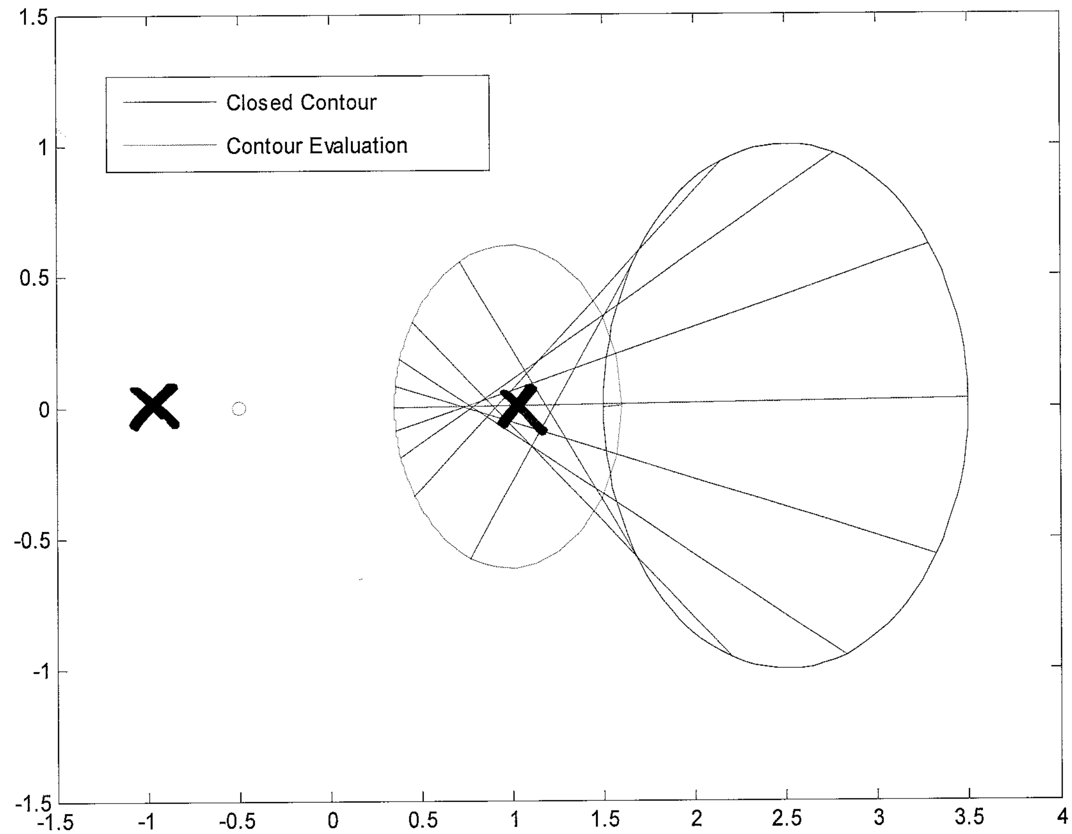


The Cauchy Principal

- A contour evaluation of a complex function will only encircle the origin if the contour contains a singularity of that function

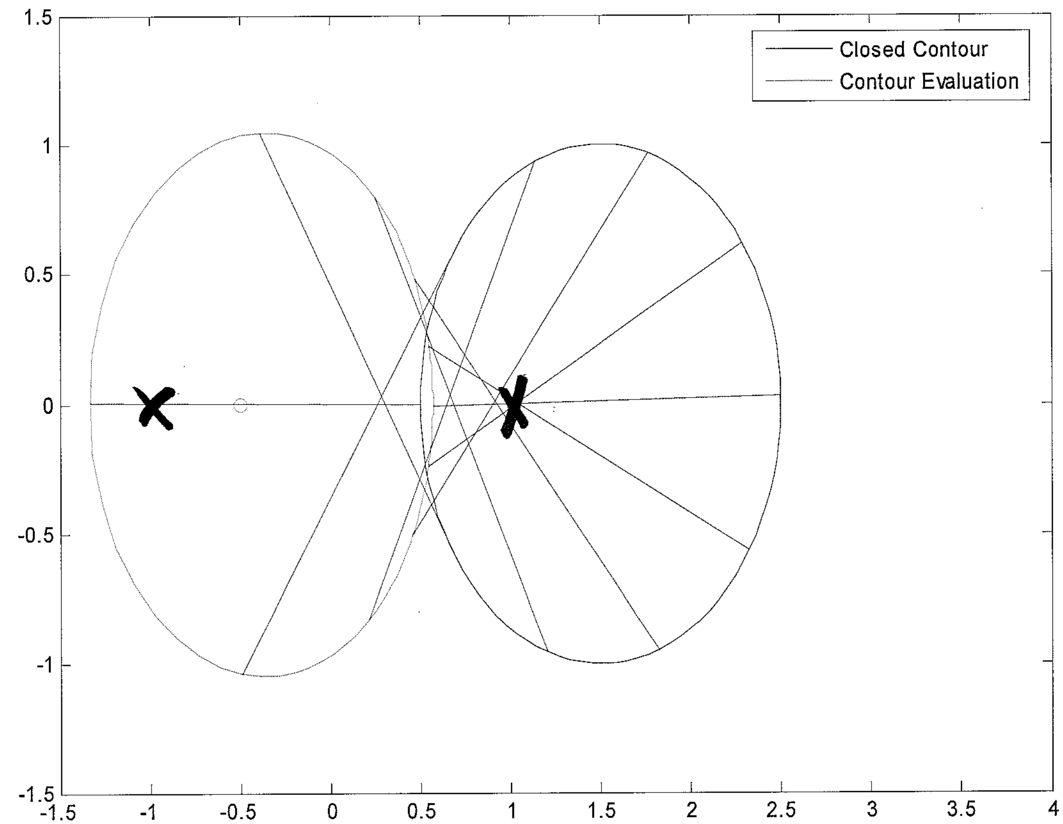
Cauchy Principal: Example 1

$$H(s)=\frac{s+0.5}{(s-1)(s+1)}$$



Cauchy Principal: Example 2

$$H(s) = \frac{s+0.5}{(s-1)(s+1)}$$

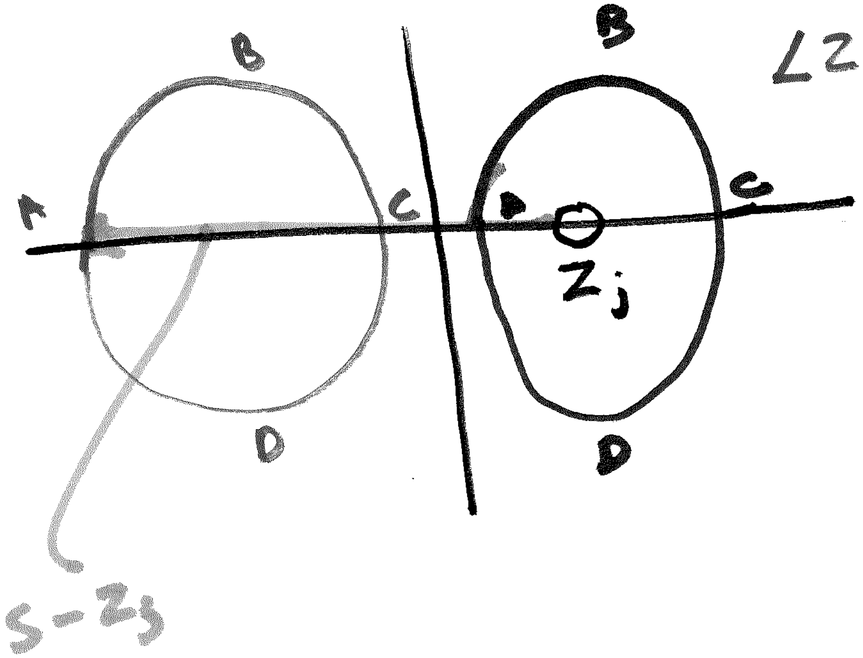


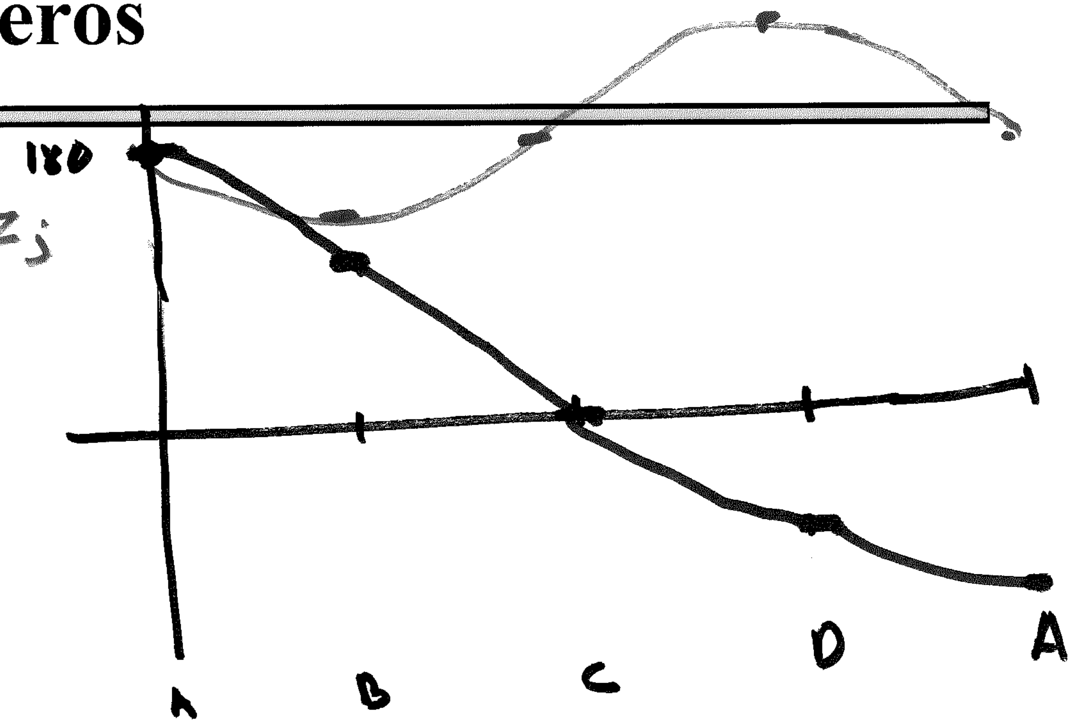
The Cauchy Principal Explained

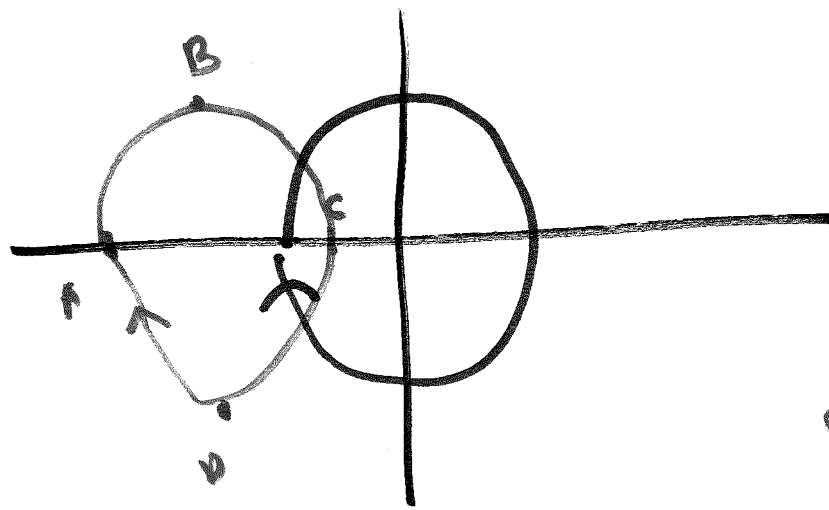
$$H(s) = a\frac{\prod_{j=1}^{n_z}(s - z_j)}{\prod_{k=1}^{n_p}(s - p_k)}$$

$$\angle H(s) = \sum_{j=1}^{n_z}\angle(s - z_j) - \sum_{j=1}^{n_p}\angle(s - p_j)$$

The Cauchy Principal: Zeros







Contour Evolution

The Cauchy Principal: Poles

For a CW contour

→ Every zero I enclose will cause a CW encirclement

→ Every pole I enclose will cause a CCW encirclement

The Cauchy Principal: Conclusions

$$N_{ccwe} = N_p - N_z$$

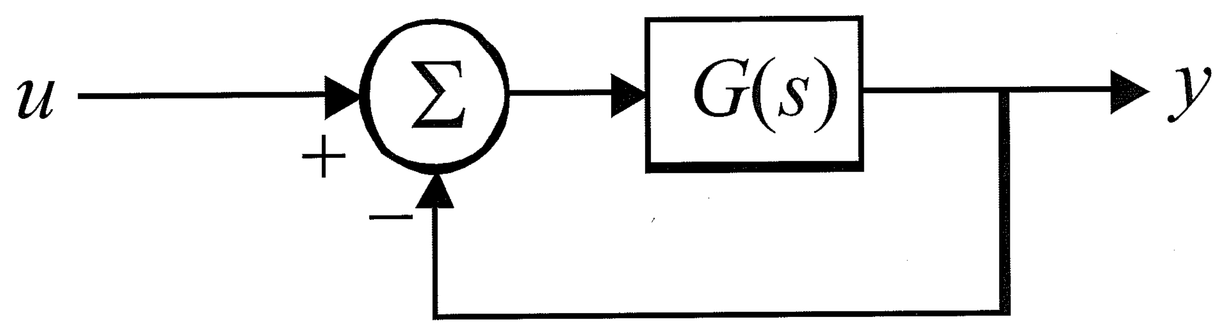
$$N_{cwe} = N_z - N_p$$

poles

zeros

The Nyquist Stability Criterion

- The Nyquist stability criterion is useful because it allows us to gain insight on the stability of a closed loop plant based on properties of an open loop plant
- Consider the following:



Nyquist Stability Criterion (Cont)

$$y = G(u - y)$$

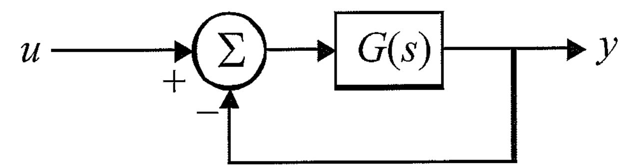
$$y(1 + G) = Gu$$

$$y = \frac{G}{1+G} u$$

$$\text{if } G = \frac{N}{D}$$

$$y = \frac{\frac{N}{D}}{1 + \frac{N}{D}} u$$

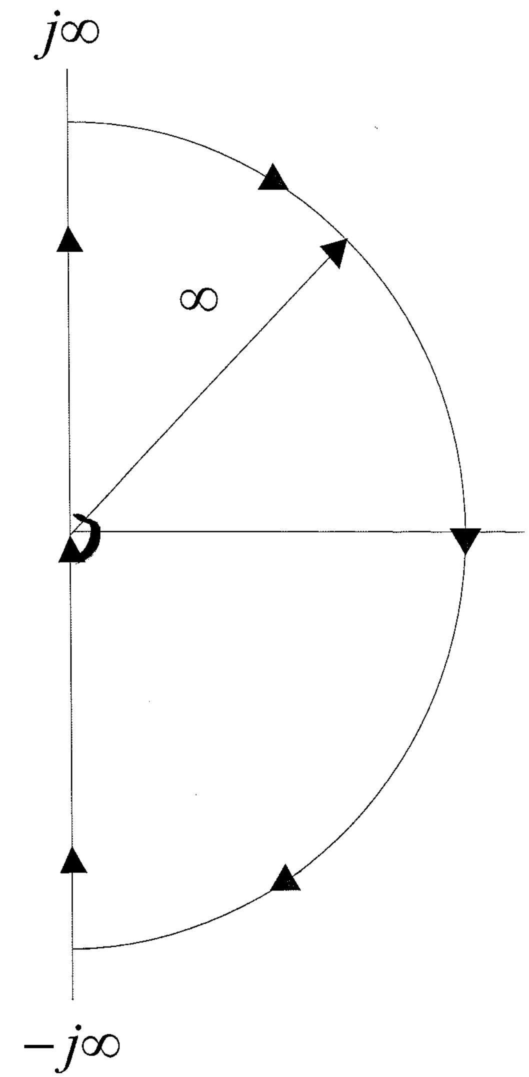
$$y = \frac{N}{D+N} u$$



we are worried about D+N

Nyquist Stability Criterion (Cont)

The Nyquist Contour



Nyquist Stability Criterion

- Consider a Nyquist evaluation of $1+G(s)$
- We have

$$1 + G = 1 + \frac{N}{D} = \frac{D+N}{D}$$

$$N_{cwe} = N_z - N_p$$

$$N_z = N_{uclp} = \#\text{ unstable closed loop poles}$$

$$N_p = N_{uolp} = \#\text{ unstable open loop poles}$$

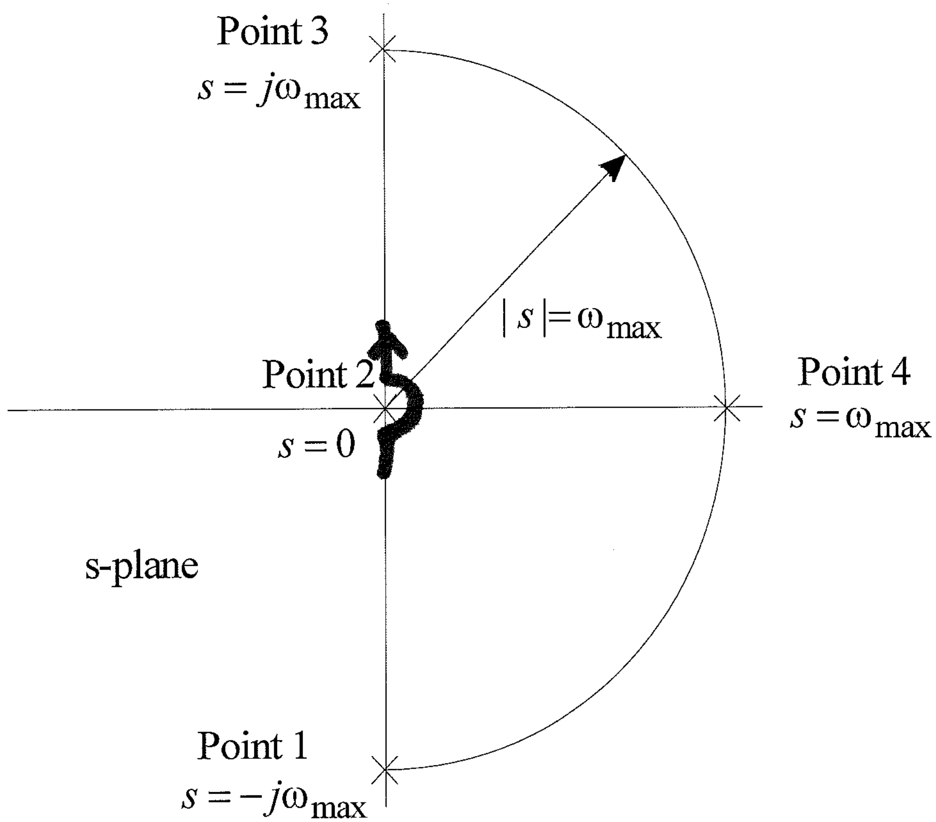
$$N_{uclp} = N_{cwe} + N_{uolp}$$

- Thus we have …

Nyquist Stability Criterion

Nyquist Stability Criterion: The number of unstable closed loop poles is equal to the number of unstable open loop poles plus the number of clockwise encirclements of -1 by the contour evaluation of the open loop plant (i.e. $G(s)$) over the Nyquist contour.

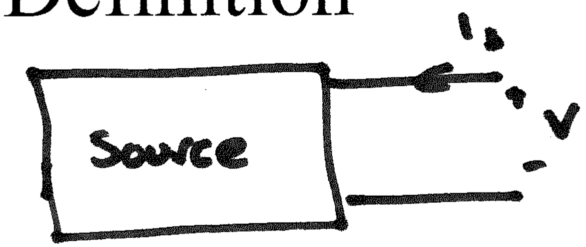
Comments on Path



G(s) to be proper
strictly proper
as $s \to \infty$ $G \to 0$

Consider a Source

- Definition



$$\frac{\Delta v}{\Delta i_s} = Z_s$$

$$(v - v_0) = Z_s(i_s - i_{s0})$$

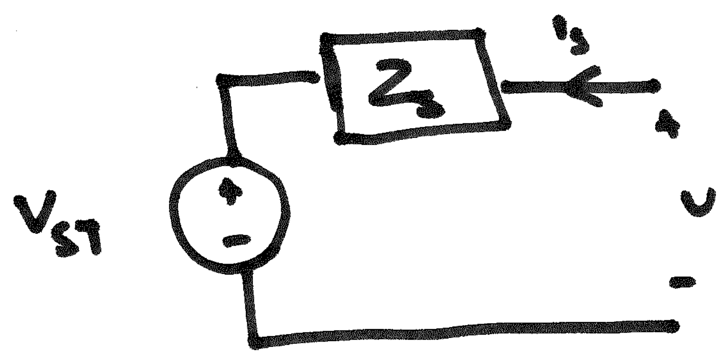
$$v = Z_s i_s + \underbrace{\left[v_0 + Z_s(0) i_{s0}\right]}_{v_{sT}}$$

- Model

$$v = Z_S(s)i_S + v_{sT}$$

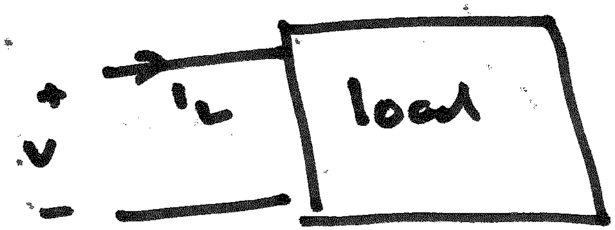
$$v_{sT} = v_0 - Z_S(0)i_{S0}$$

$$Z_s = \frac{N_s}{D_s}$$



Consider a Load

- Definition



$$\Delta i_L = Y_L \Delta v$$

$$i_L - i_{L0} = Y_L (v - v_0)$$

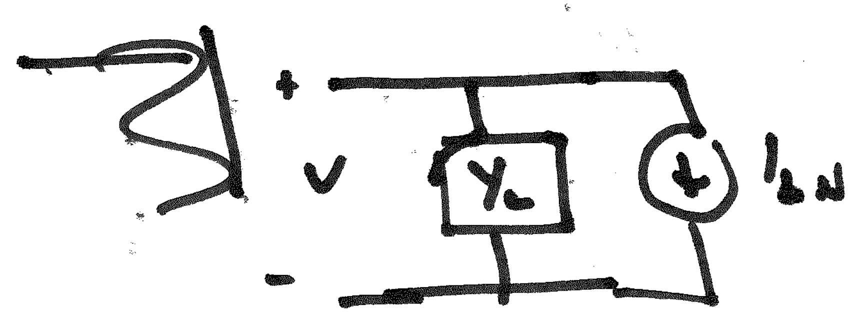
$$i_L = Y_L v + \underbrace{[\, i_{L0} + Y_L(0) v_0 \,]}_{i_{LN}}$$

- Model

$$i_l = Y_l(s)v + i_{lN}$$

$$i_{lN} = i_{l0} - Y_l(0)v_0$$

$$Y_L = \frac{N_L}{D_L}$$



Properties of Source and Load

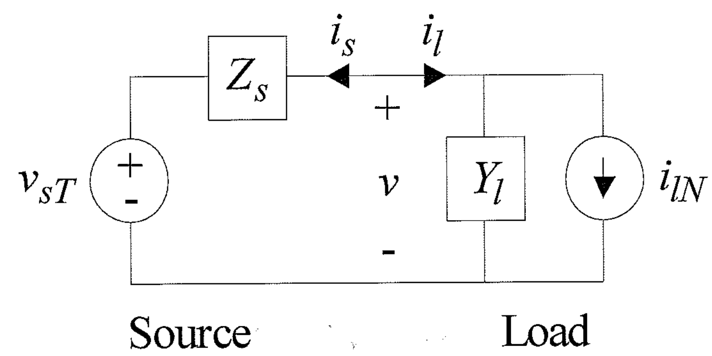
$$Z_s(s) = \frac{N_s(s)}{D_s(s)}$$

$$Y_l(s) = \frac{N_l(s)}{D_l(s)}$$

open loop stable

Neither D_s or D_l has a root in the RHP

Consider a Source-Load System



$$\frac{V - V_{sT}}{Z_s} + Y_l V + I_{lN} = 0$$

$$\left[\frac{1}{Z_s} + Y_l\right] V = \frac{V_{sT}}{Z_s} - I_{lN}$$

$$\frac{N_s}{D_s}$$

$$V = \frac{V_{sT} - Z_s I_{lN}}{1 + Z_s Y_L} = \frac{D_s V_{sT} - N_s I_{lN}}{D_s (1 + Z_s Y_L)}$$

Nyquist Immittance Criterion

- Consider

$$v = \frac{D_s v_{sT} - N_s i_{lN}}{D_s (1 + Z_s Y_l)}$$

Worried about zeros of $1 + Z_s Y_L$

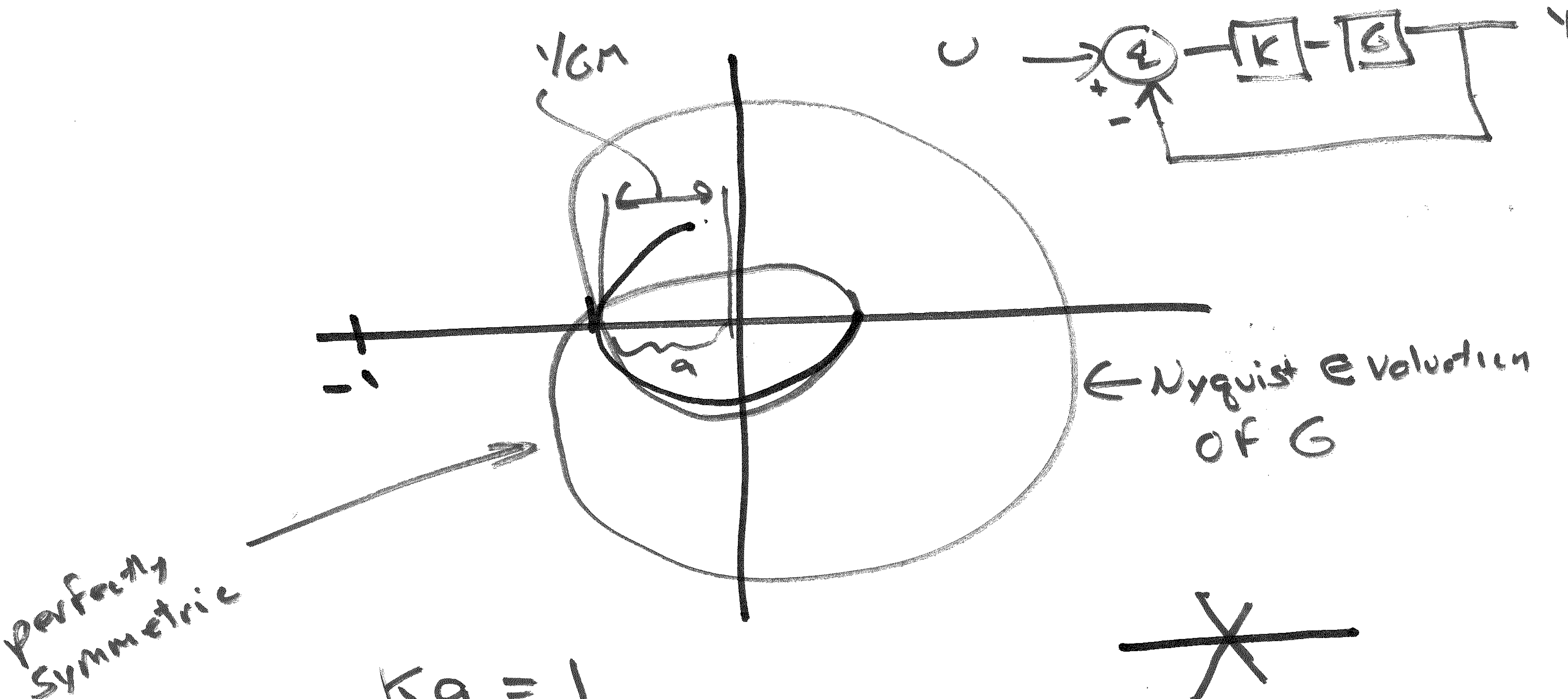
We were worried about zeros of $1 + G$

$G = Z_s Y_L$

- Thus we have the Nyquist Immittance Criterion: A source – load system is stable provided that the Nyquist evaluation of $Z_s Y_l$ does not encircle -1.

Necessary & ~~sufficient~~ sufficient

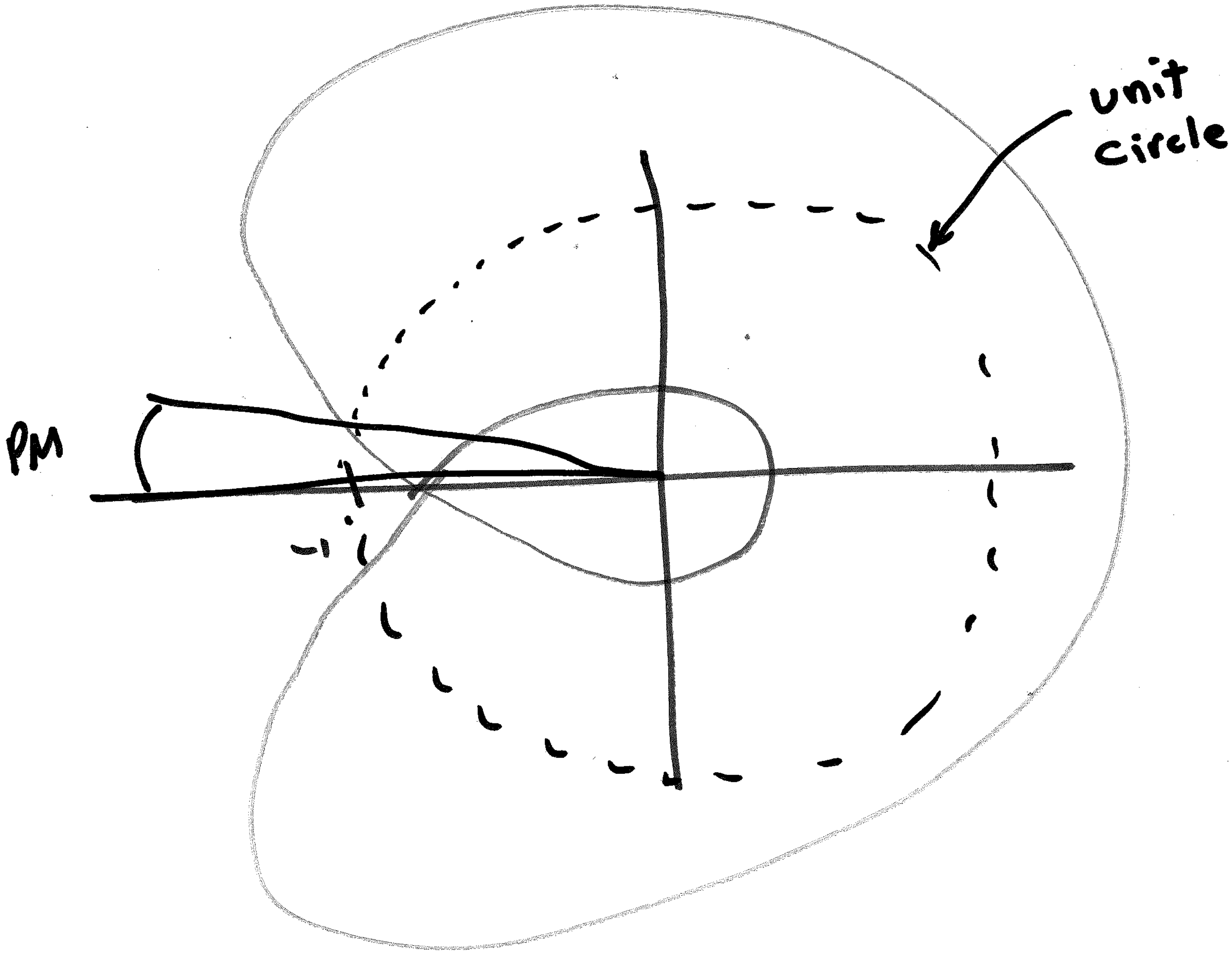
Quick Review – Gain and Phase Margin



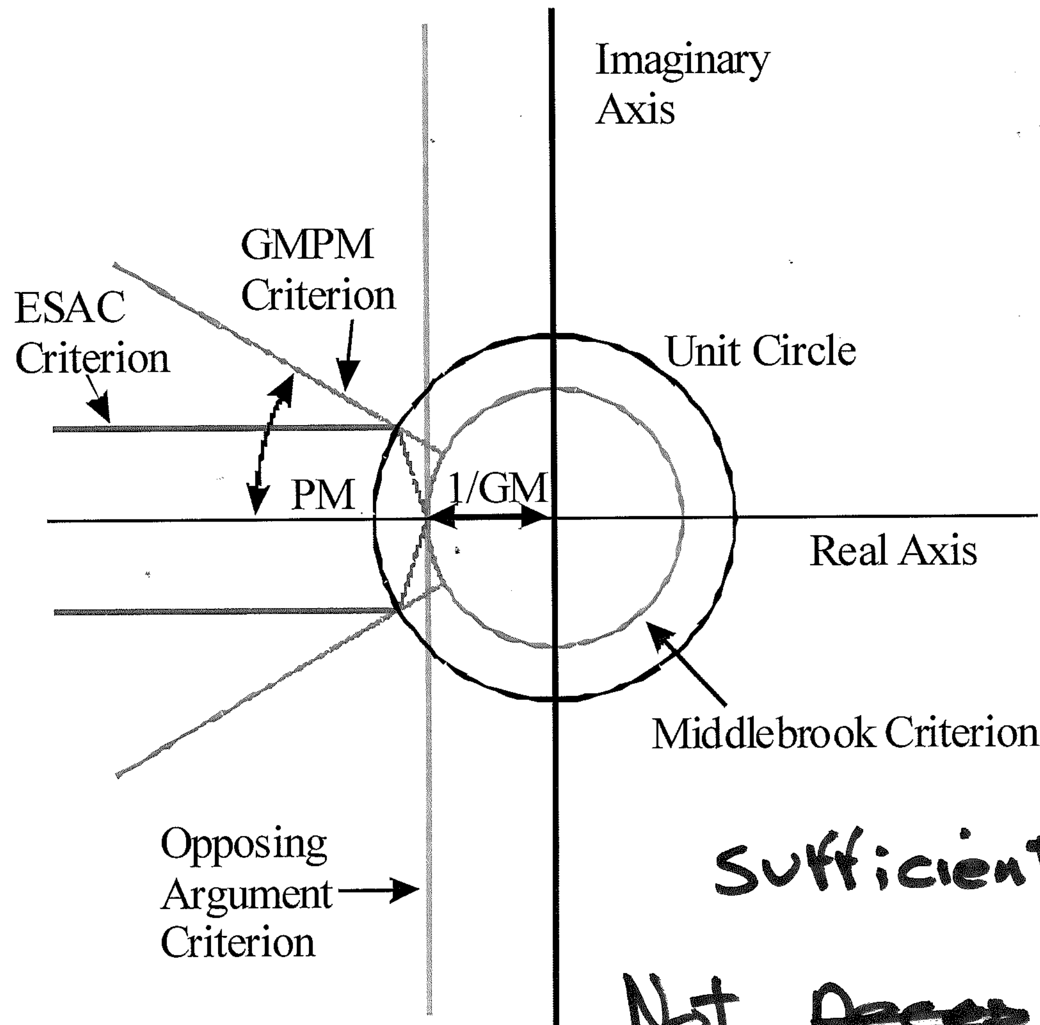
$Ka = 1$

$k = \frac{1}{a}$ Gain Margin

Quick Review – Gain and Phase Margin



Stability Criteria



Sufficient

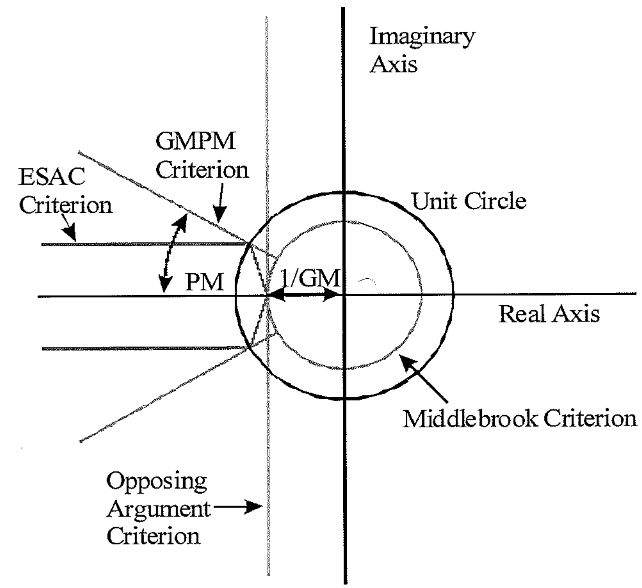
Not ~~Decess~~ neccessary

Middlebrook Criterion

- We have

$$|Z_s Y_L| < \frac{1}{GM}$$

$$|Z_s||Y_L| < \frac{1}{GM}$$

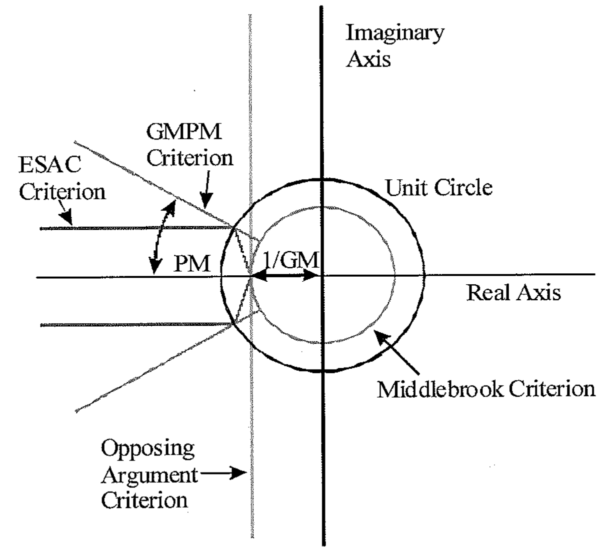


- As a design specification

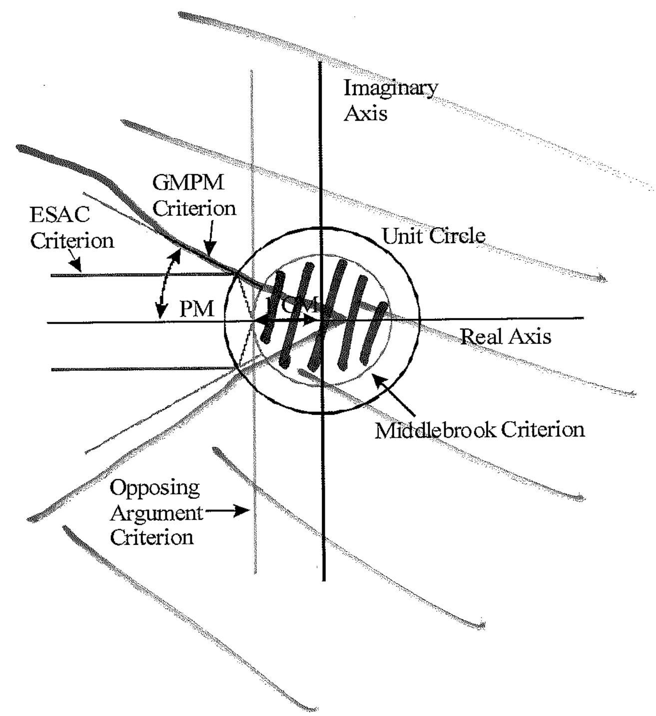
$$|Y_l| < \frac{1}{|Z_s|\,GM} \qquad |Z_s| < \frac{1}{GM|Y_l|}$$

Opposing Argument Criterion

Magnitude & Phase are
tied together



Gain and Phase Margin Criterion



$$|Y_l| < \frac{1}{|Z_s|\,GM}$$

- OR -

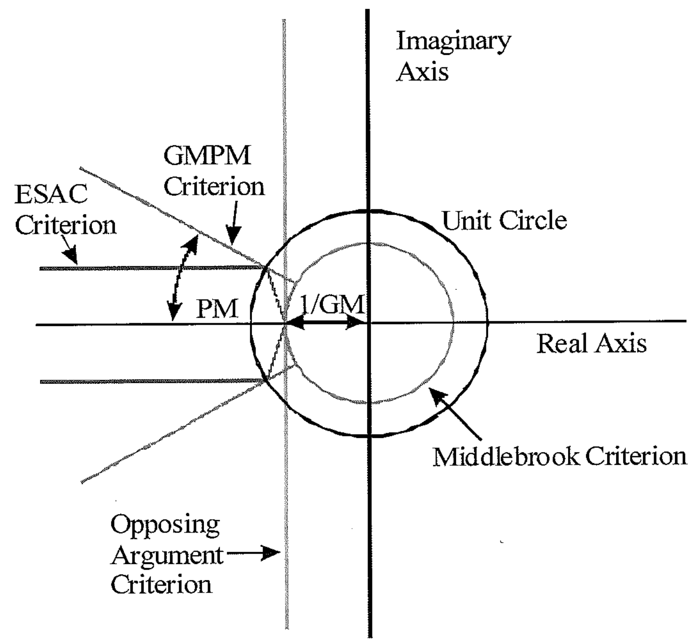
$$\angle Y_l(s) \underset{\angle}{+} \angle Z_s(s) \le \left(180^o - PM\right) \quad \text{and}$$

$$\angle Y_l(s) \underset{\angle}{+} \angle Z_s(s) \ge \left(-180^o + PM\right)$$

addition of angles mapped to -180 to 180

ESAC Criteria

Gain & Phase
tied together



Comparison of Stability Criteria

- Parameters

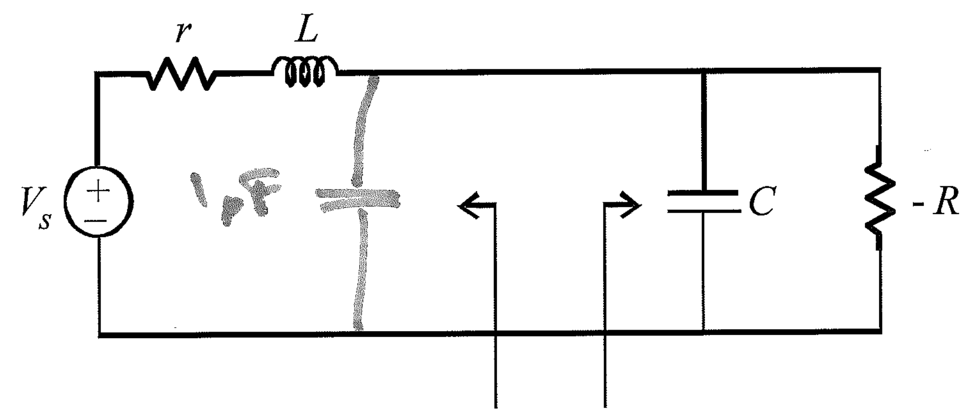
$$r = 300\ m\Omega$$

$$L = 10\ mH,$$

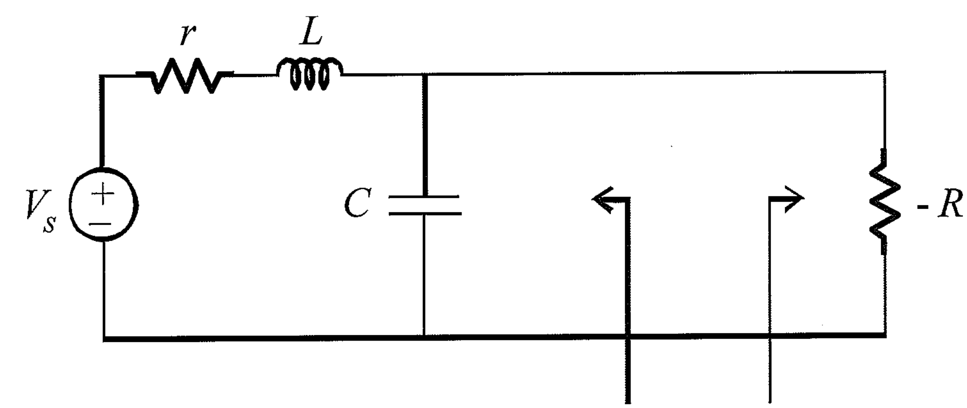
$$R = 24.3\ \Omega$$

$C = 40\ mF$ – stable

$C = 0.5\ mF$ – unstable

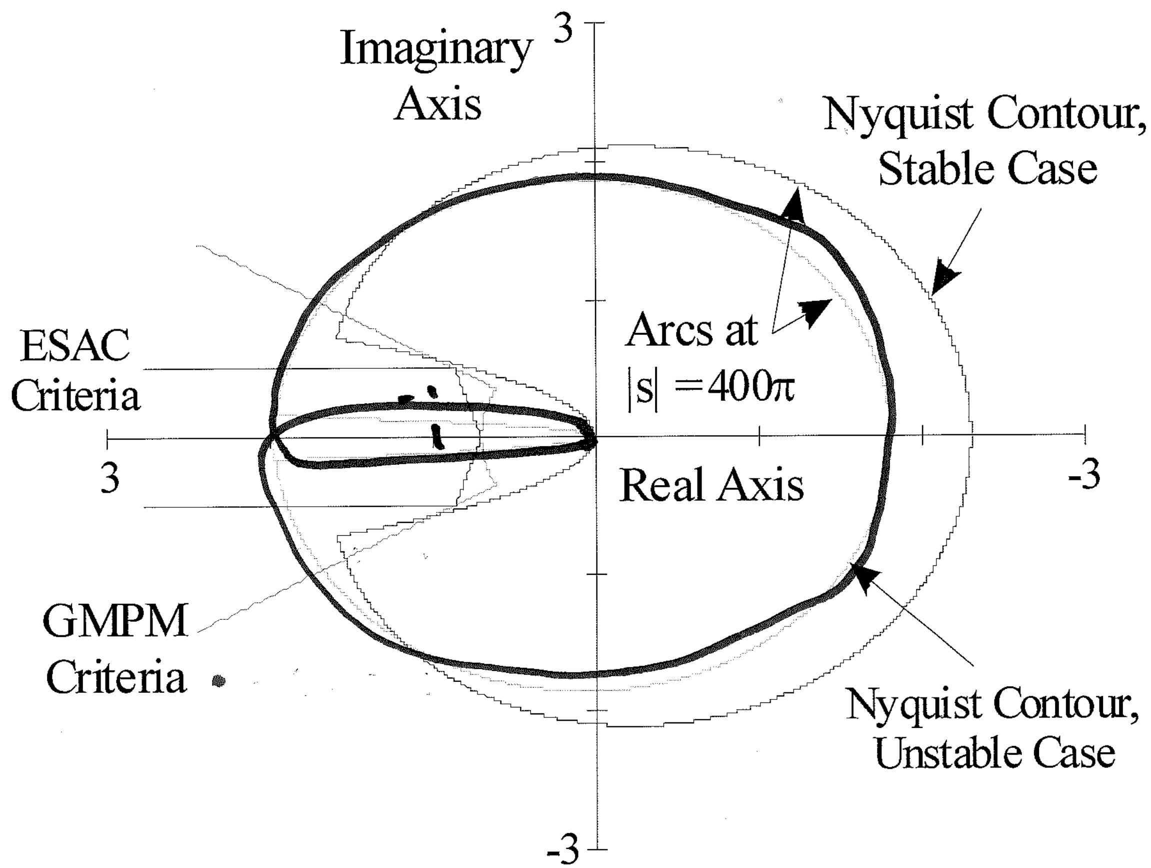


(a) component grouping 1

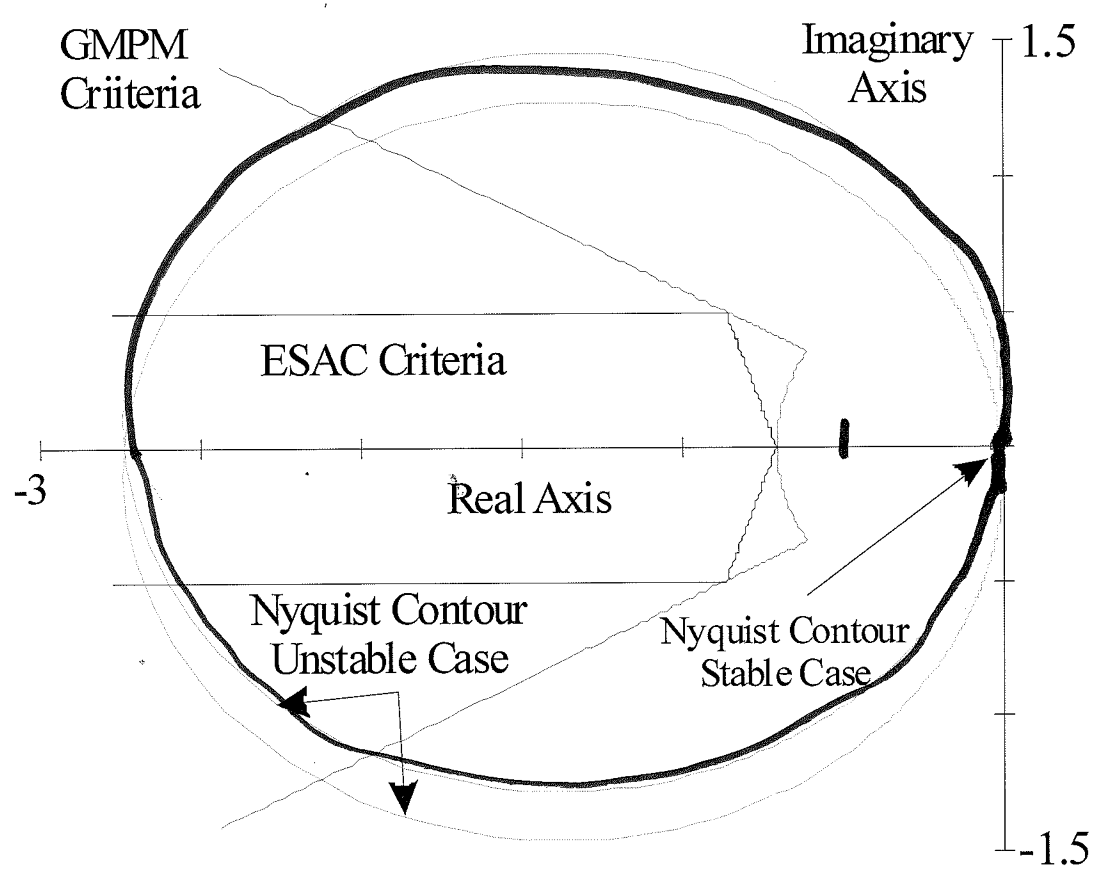


(b) component grouping 2

Component Grouping A

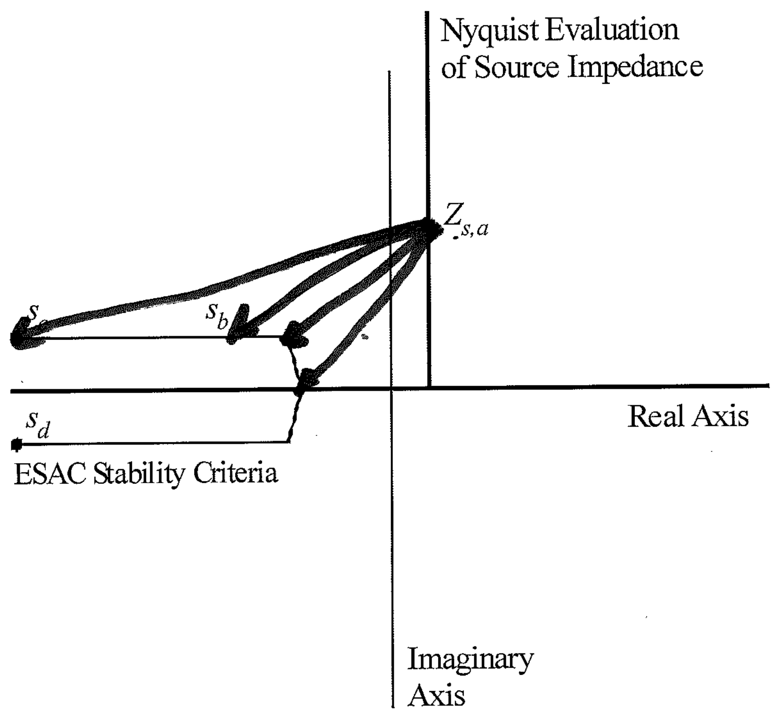


Component Grouping B



Design Specs From Arb. Stab. Crit.

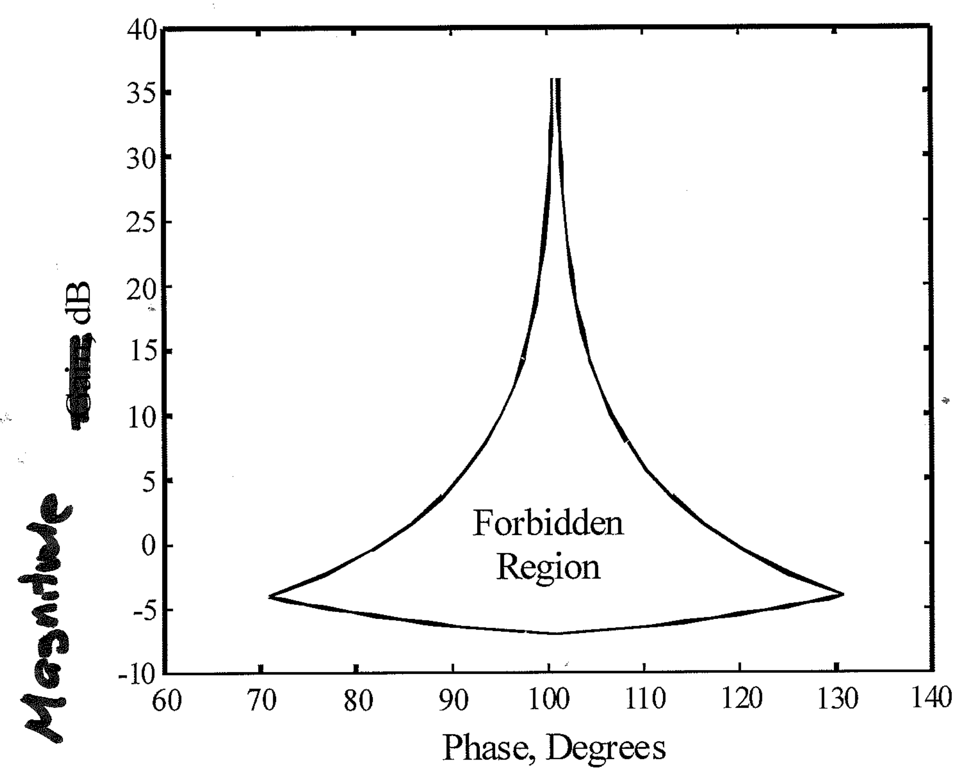
- At a frequency at a point



$$Y_{l,ab} = \frac{s_b}{Z_{s,a}}$$

Design Specs Based on Arb. Stab. Crit.

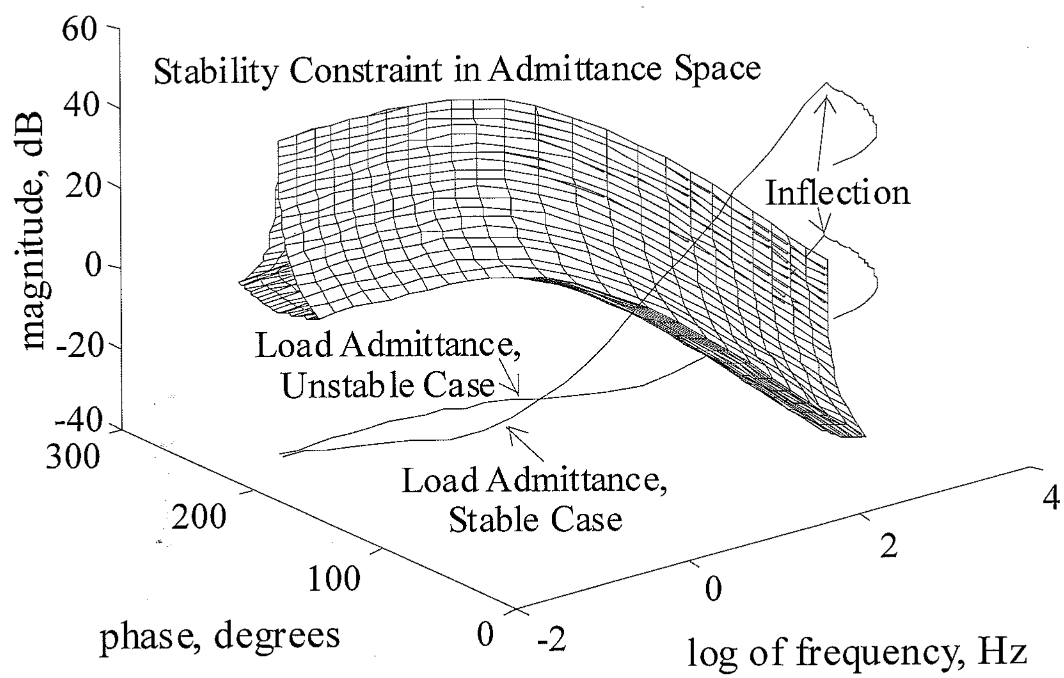
- At a frequency over all points



At some frequency

Design Specs Based on Arb. Stab. Crit.

- Over all frequencies



Artificial

Comments

- Conservativeness

Most M, OA, GMPM, ESAC Least

- Amenability to Configuration

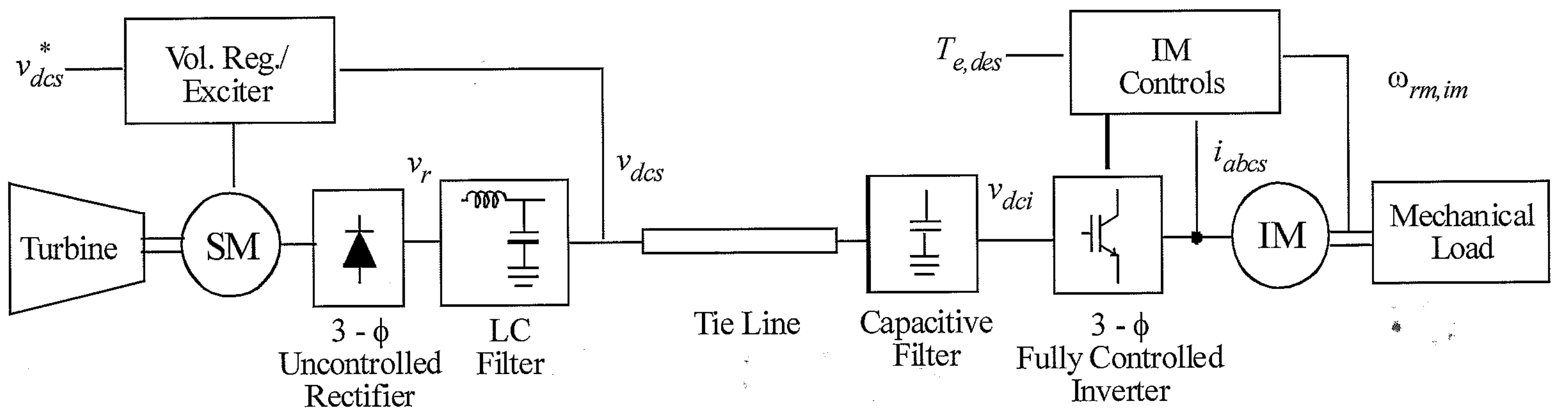
ESAC

- Simplicity of Formulating Design Specs

Easiest M, GMPM | OA ESAC Hardest

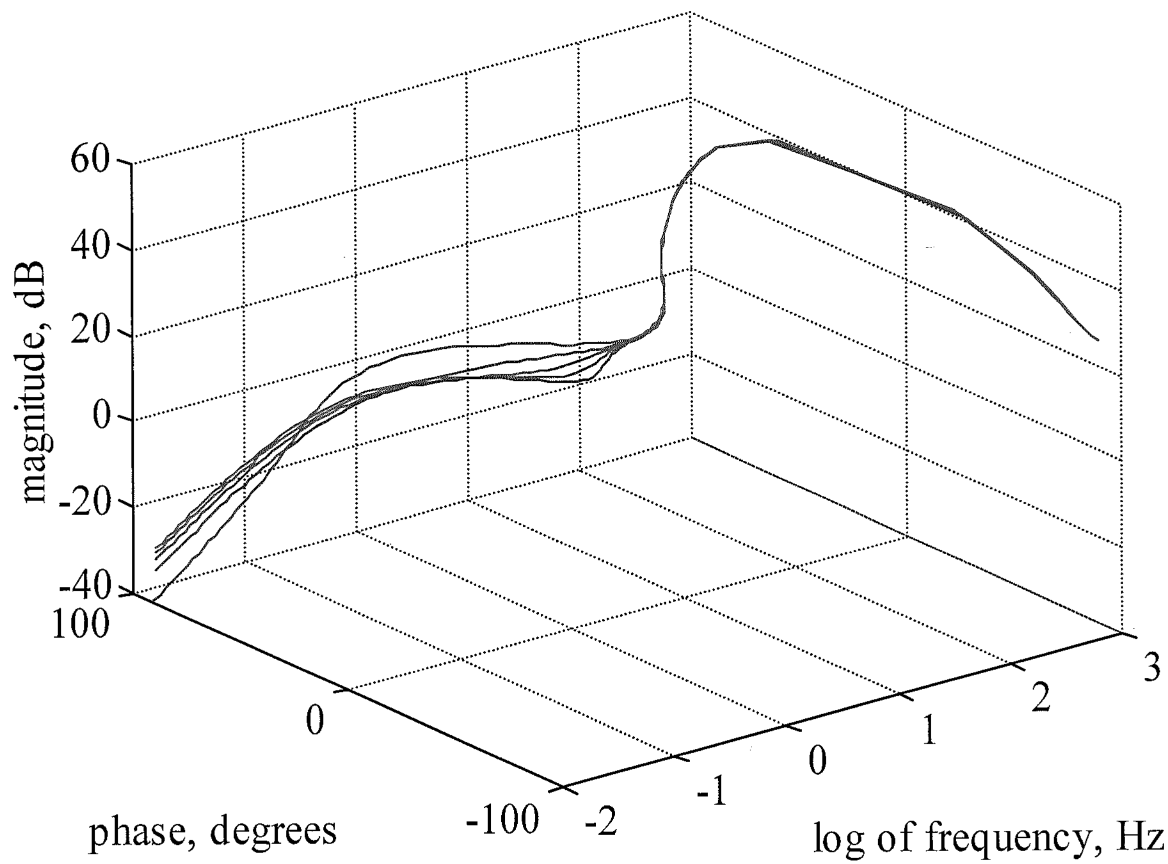
Generalized Immittance

- Consider the following system



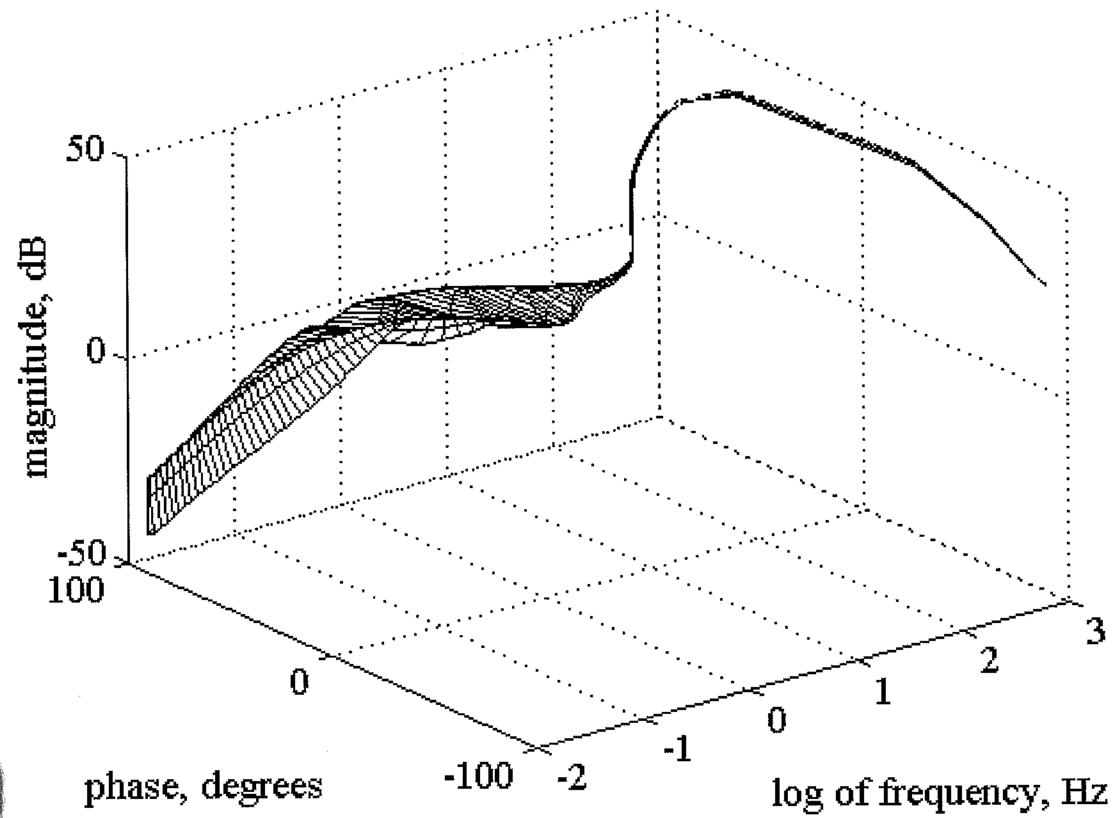
Source Impedance Vs. Operating Point

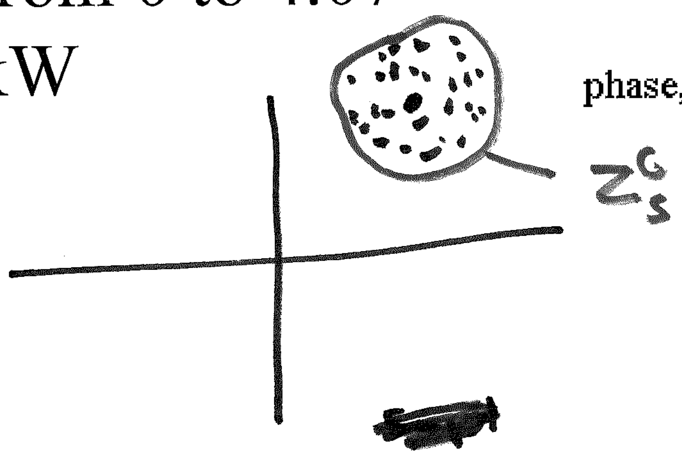
- Output Power from 0 to 1 p.u. in 5 steps



Generalized Source Impedance

- Nominal speeds from 228-279 rad/s
- Output voltage references from 380-420 V
- Output powers from 0 to 4.07 kW

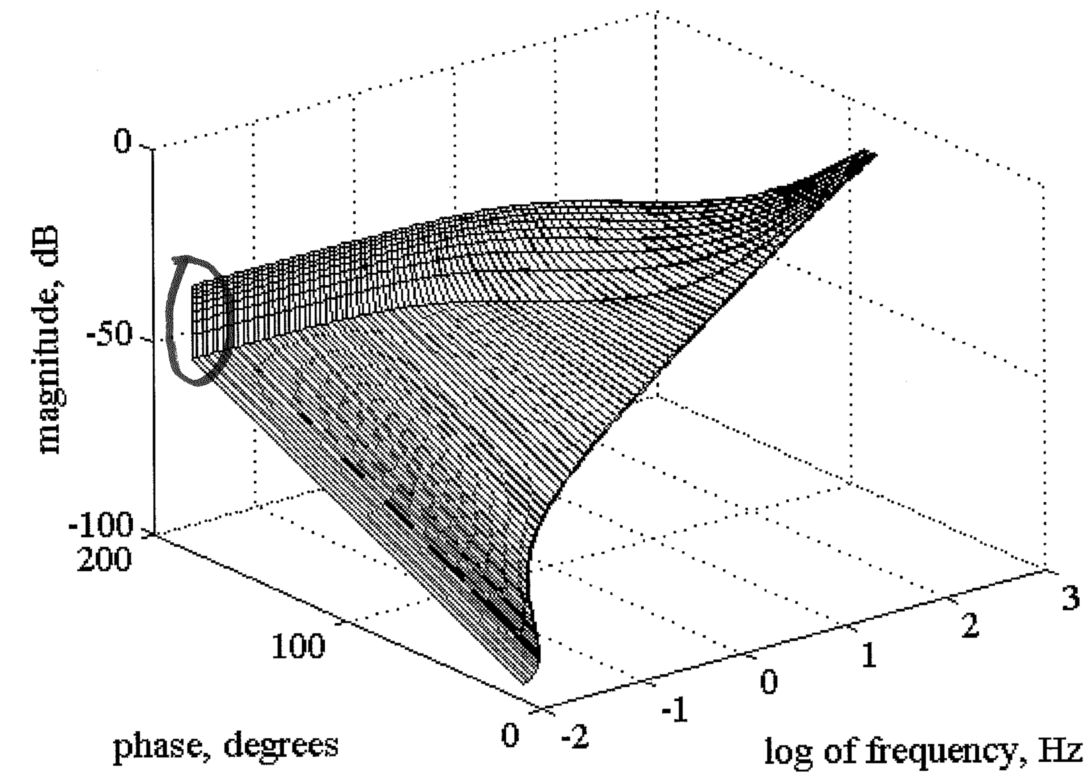




Generalized Load Admittance

- Speed from 0-200 rad/s
- Torque commands from 0-20.9 Nm
- Input voltage from 380-420 V

$i = \frac{P}{V}$



Generalized Nyquist Evaluation

- Source Impedance

$$Z(s) \Rightarrow Z^G(s)$$

- Load Admittance

$$Y(s) \Rightarrow Y^G(s)$$

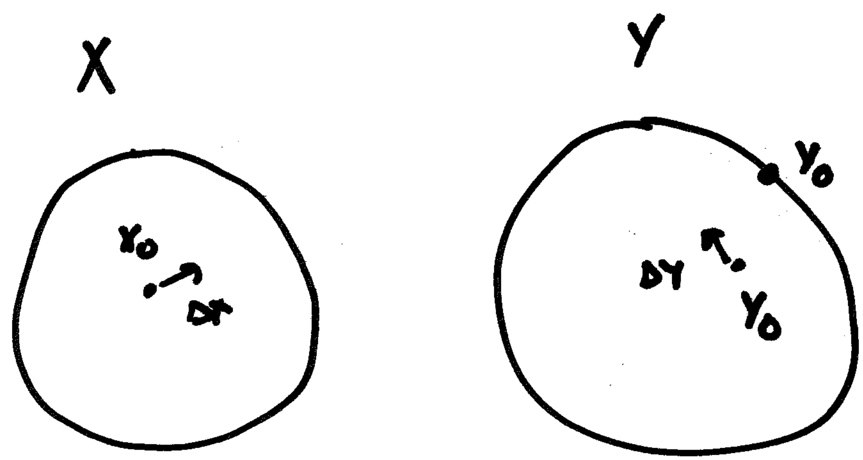
Product of Generalized Quantities

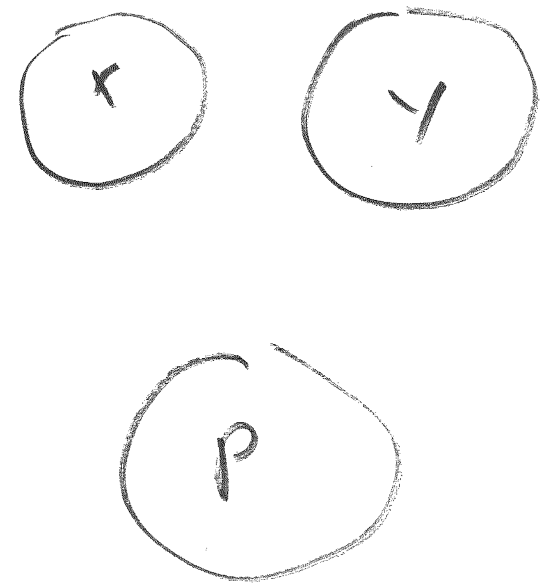
- Consider

$$x(s) \in X^G(s) \qquad y(s) \in Y^G(s)$$

$\rightarrow \quad p(s) = x(s)\,y(s)$

$p^G = X^G Y^G$





Product of Generalized Quantities

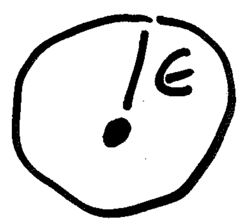
$$P = (X_0 + \Delta X)(Y_0 + \Delta Y)$$

$$= X_0 Y_0 + \underbrace{\Delta X\, Y_0 + X_0\, \Delta Y + \Delta X\, \Delta Y}_{\epsilon}$$

$$\Delta X (Y_0 + \Delta Y) + X_0\, \Delta Y = \epsilon$$

$$\Delta X = \frac{\epsilon - X_0\, \Delta Y}{Y_0 + \Delta Y}$$

$$P = X_0 Y_0 + \epsilon$$



Product of Generalized Quantities

Implications on Stability

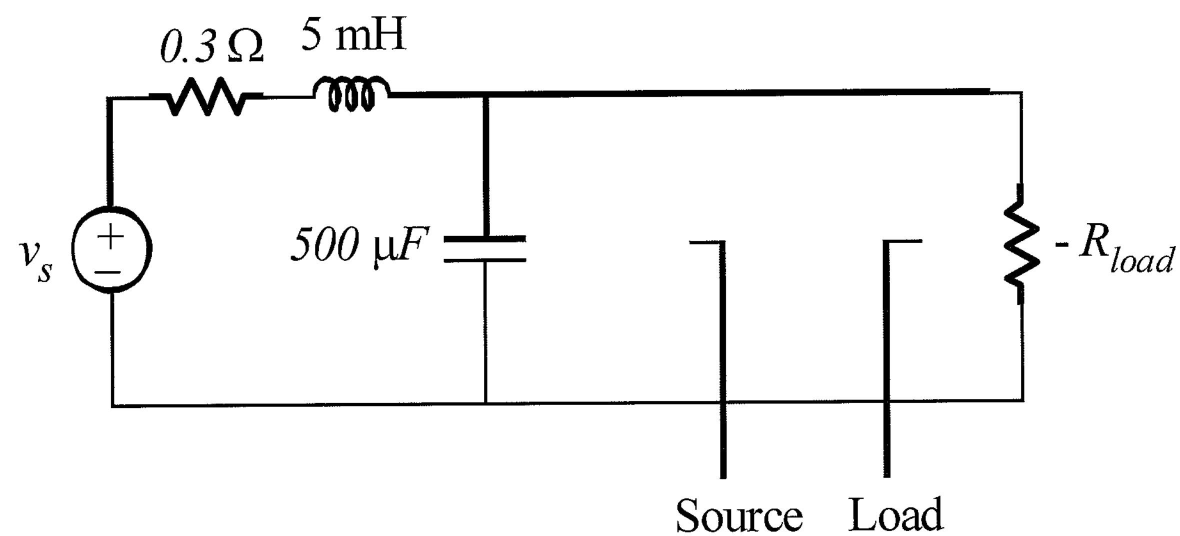
- Consider Nyquist Evaluation of $Z^G(s)Y^G(s)$

An Example

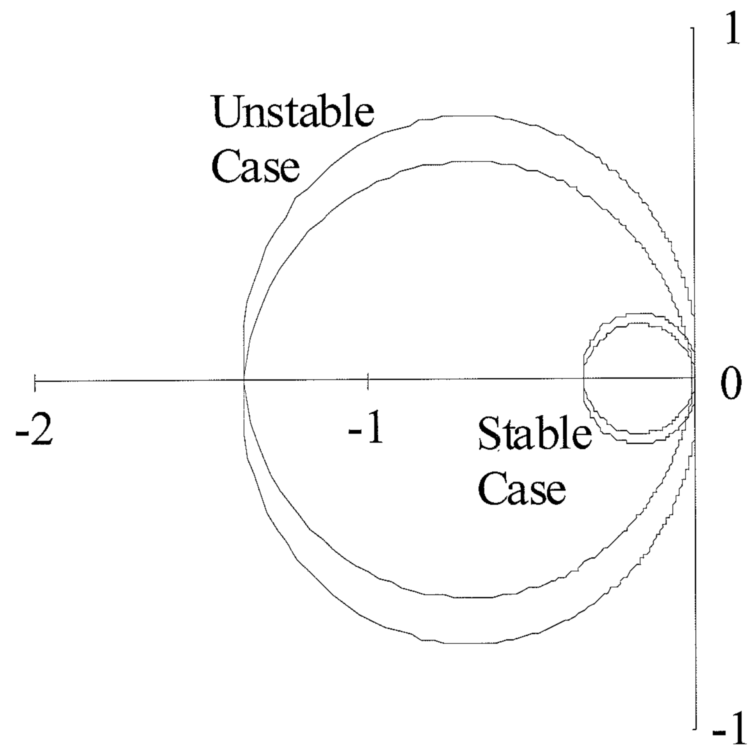
- Consider the following

$R_{load} = 97.3\ \Omega$

$R_{load} = 24.3\ \Omega$

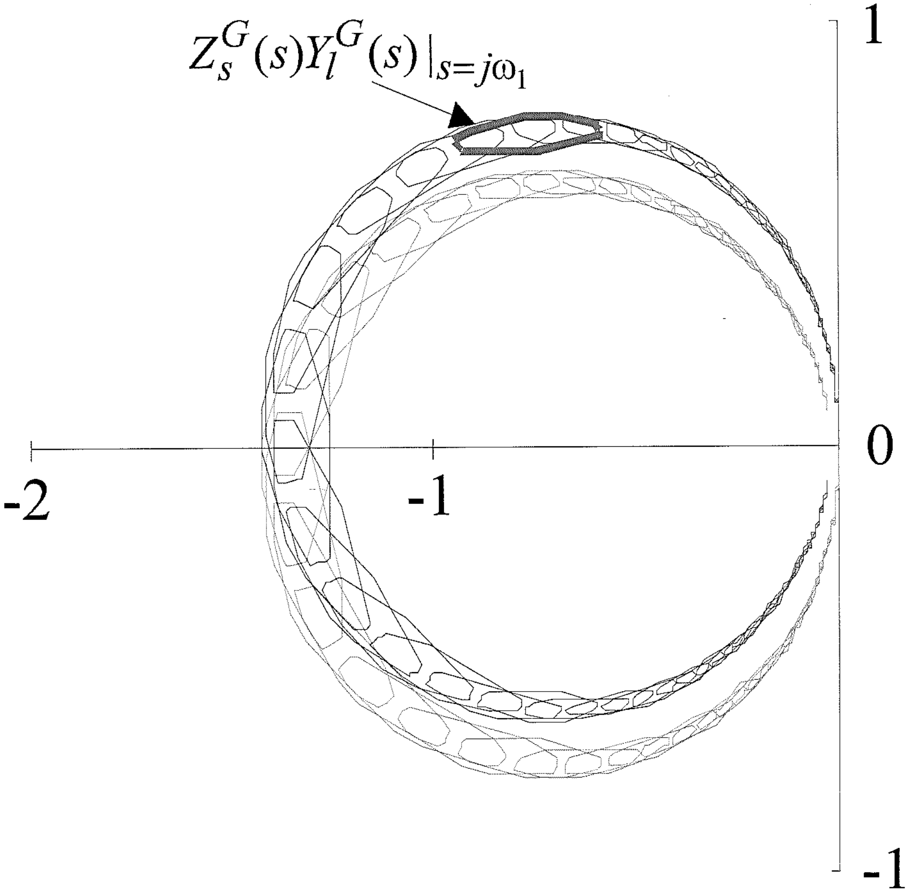


Normal Nyquist



Let's Add Variation

- Allow all parameters to vary by 1%



Generalized Middlebrook

- Standard Middlebrook

$$|Z_s(s)||Y_l(s)| < {}^{1}\!/_{GM}$$

- Generalized Middlebook

$$\|X\|_G = \max_{x \in X^G} |x| \longleftarrow \text{max at a given } \omega$$

$$\|Z_s^G(s)\|_G \|Y_l^G(s)\|_G < 1/GM$$

Generalized Middlebrook

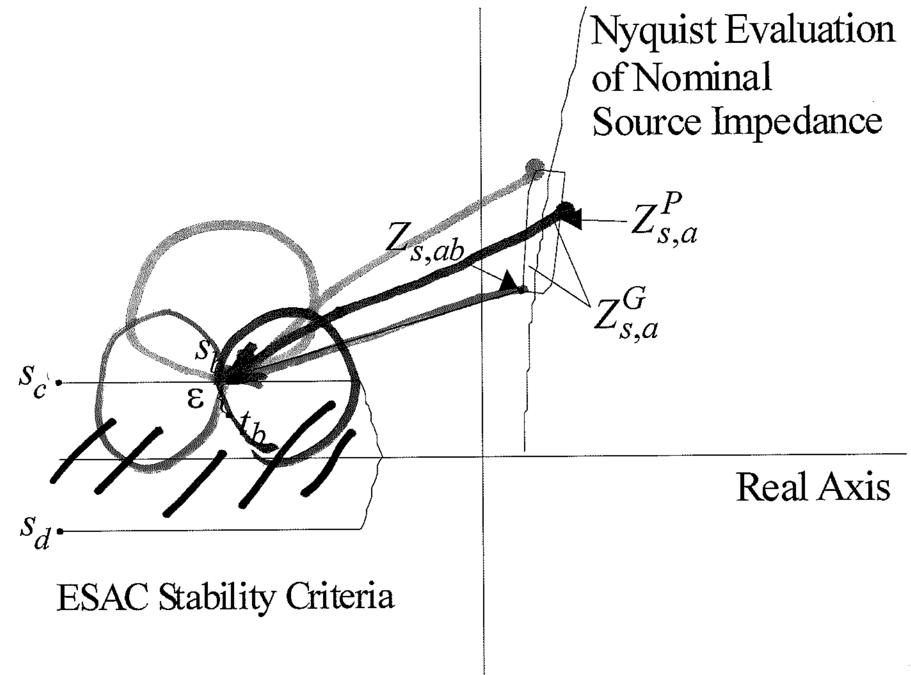
- In terms of design specs:
  - If we know source

$$\left\|Y_l^G(s)\right\|_G < \frac{1}{GM\left\|Z_s^G(s)\right\|_G}$$

  - If we know load

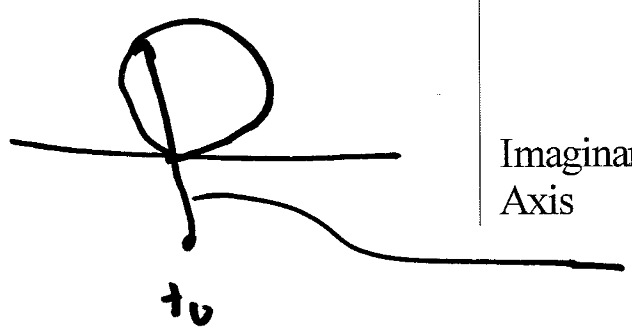
$$\left\|Z_l^G(s)\right\|_G < \frac{1}{GM\left\|Y_s^G(s)\right\|_G}$$

Generalizing Other Criteria



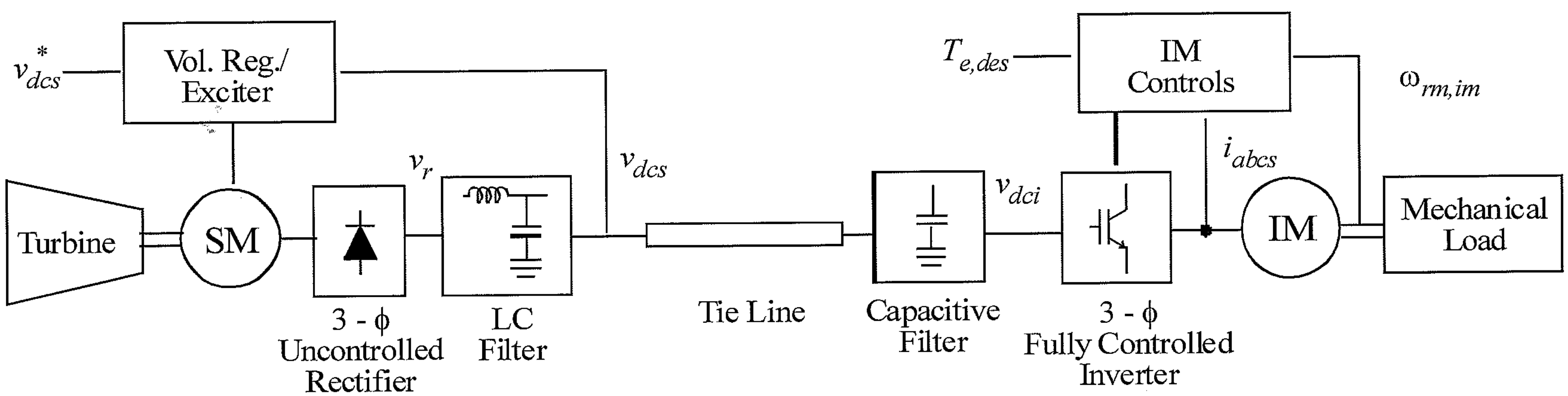
$$Y_{l,ab} = \frac{S_b}{Z_{s,ab}}$$

$$\oint_{Z^P_{s,a}} \left| t_b - \underbrace{\frac{s_b}{Z_{s,ab}}}_{Y_{l,ab}} Z_{s,a}(s) \right| ds$$

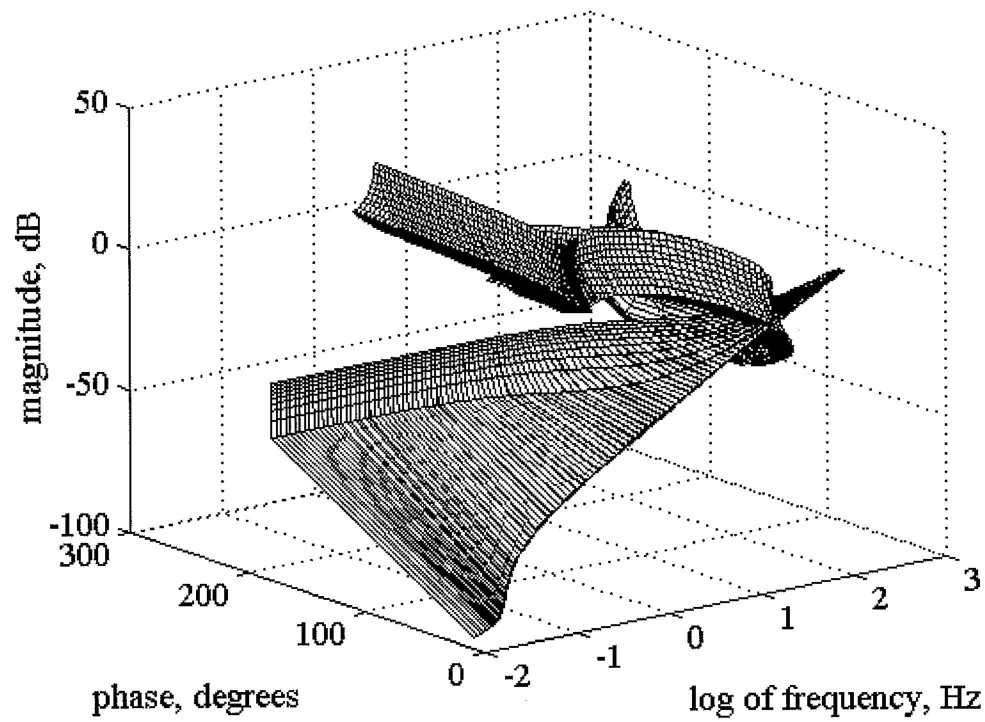


$|Y_{l,ab} Z_{s,a} - t_b|$

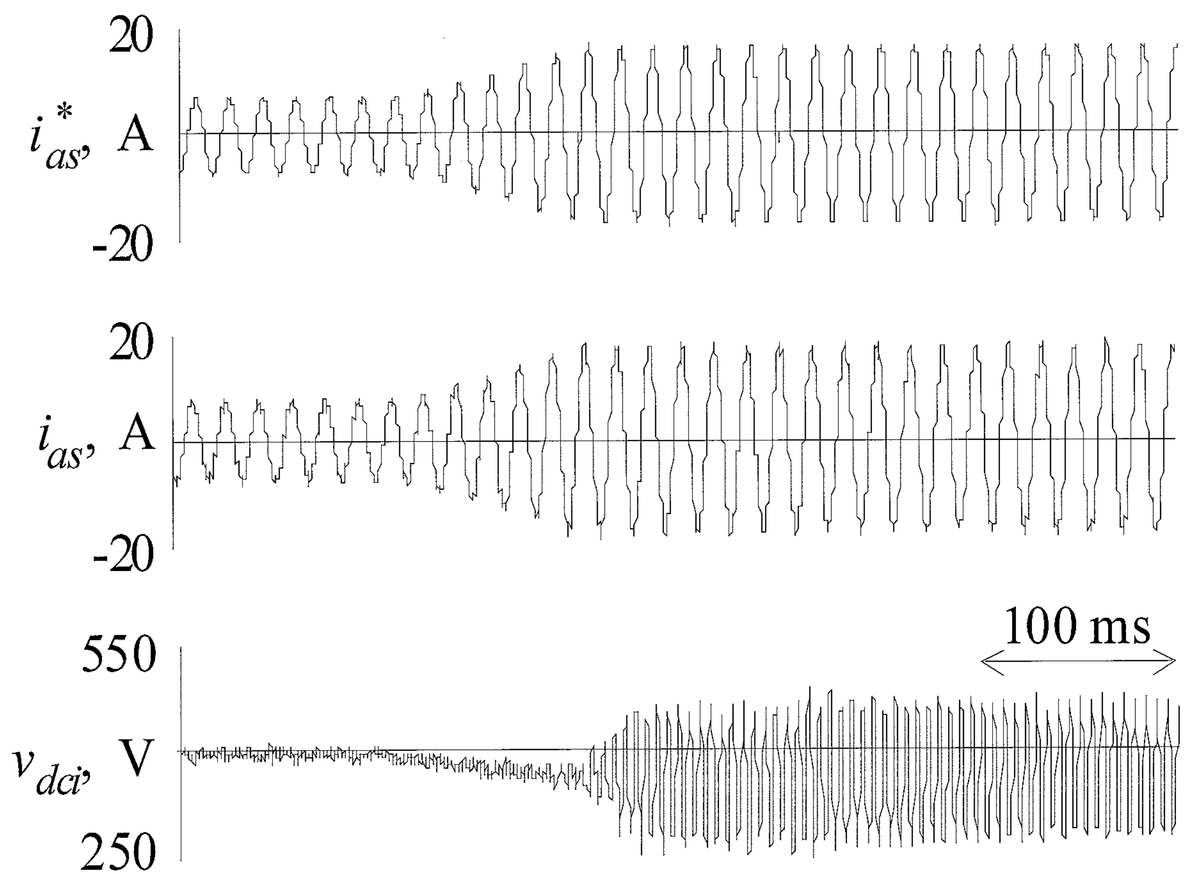
Case Study – Original System



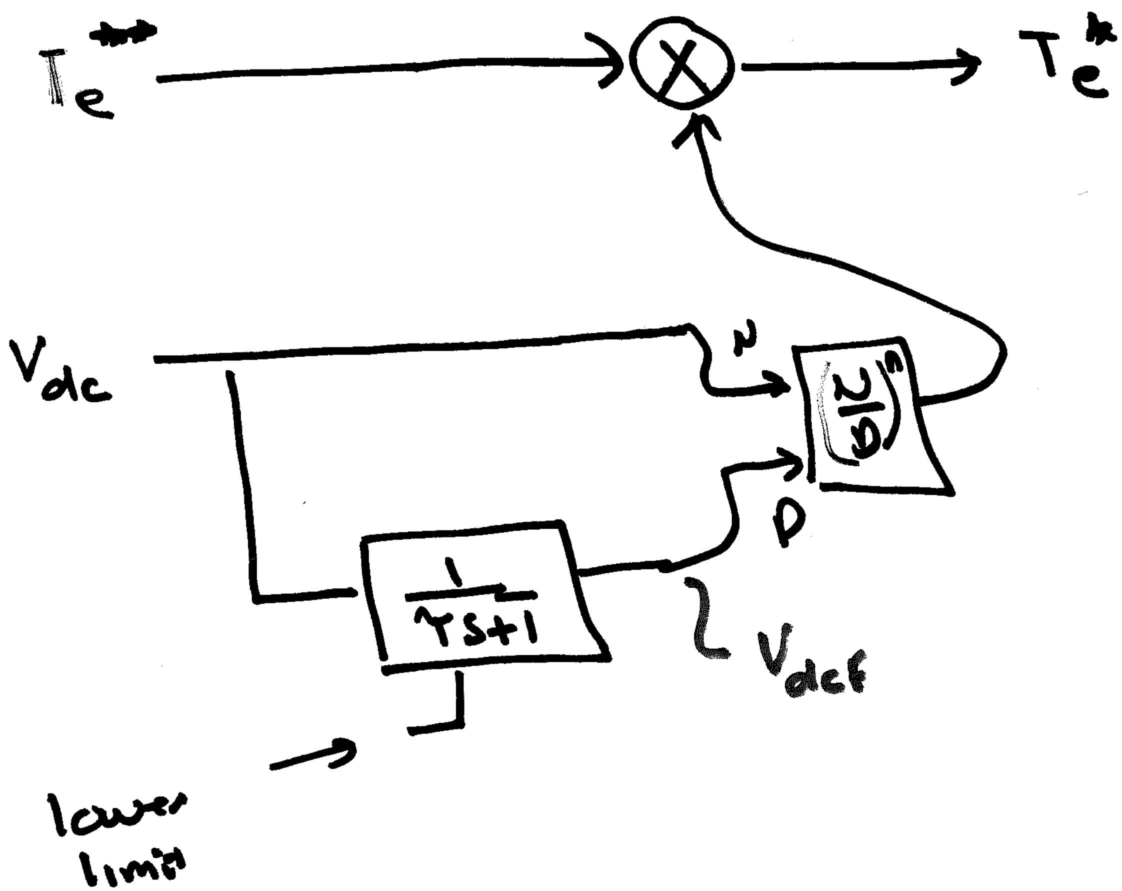
Gen. Adm. Constraint and Admittance



Original System Time Domain Perform.



Let's Modify the Control



Modified Control

$$P^* = \left(\frac{N}{D}\right)^n P^{**}$$

$$P^{**} = T_e^{**}\,\omega_{rm}$$

$$P^* = T_e^*\,\omega_{rm}$$

assume $P = P^* = \left(\frac{N}{D}\right)^n P^{**}$

$$i_{dc} = \frac{P}{V_{dc}} = \frac{1}{V_{dc}}\left(\frac{V_{dc}}{V_{dc}^*}\right)^n P^{**}$$

$$i_{dc} = \frac{V_{dc}^{n-1}}{V_{dc}^{*\,n}}\,P^{**}$$

Modified Control

Past the bandwidth of the LPF

$$V_{dcf} = V_{dc0}$$

$$I_{dc} = \frac{V_{dc}^{n-1}}{V_{dc0}^{n}} P^{**}$$

$$\Delta I_{dc} = (n-1) \frac{V_{dc0}^{n-2}}{V_{dc0}^{n}} P^{**} \Delta V_{dc}$$

$$= (n-1) \frac{P^{**}}{V_{dc0}^{2}} \Delta V_{dc}$$

$$\frac{\Delta V_{dc}}{\Delta I_{dc}} = \frac{1}{n-1} \frac{V_{dc0}^{2}}{P^{**}}$$

Modified Control

suppose $n = 0$

$$\frac{\Delta V_{dc}}{\Delta I_{dc}} = -\frac{V_{dco}^2}{p^*}$$

suppose $n = 1$

$$\frac{\Delta V_{dc}}{\Delta I_{dc}} = \infty$$ (open circuit)

suppose $n = 2$

$$\frac{\Delta V_{dc}}{\Delta I_{dc}} = \frac{V_{dco}^2}{p^*}$$ (+ resistive)

Modified Control

Including L.P.F.

$$\bar{V}_{dcf}^{-n} \qquad -n\,\bar{V}_{dcf}^{-n-1}$$

$$I_{dc} = \frac{V_{dc}^{n-1}}{\hat{V}_{dcf}^{n}}\, p^{**}$$

$$\Delta I_{dc} = (n-1)\frac{V_{dc0}^{n-2}}{\hat{V}_{dc0}^{n}}\, p^{**} \Big\}^{\Delta V_{dc}} - n\,\frac{V_{dc0}^{n-1}}{V_{dc0}^{n+1}}\, p^{**}\, \Delta V_{dcf}$$

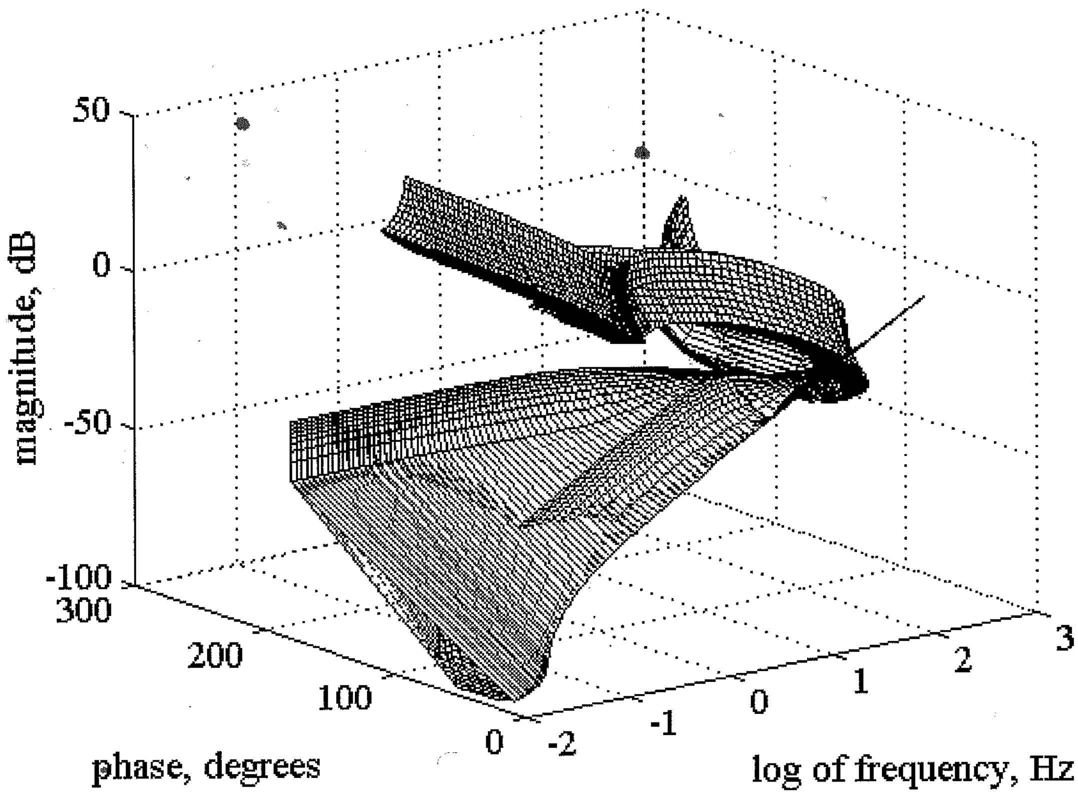
$$= \frac{p^{**}}{V_{dc0}^{2}}\left[(n-1)\Delta V_{dc} - n\,\frac{1}{\tau s+1}\,\Delta V_{dc}\right]$$

$$\frac{\Delta I_{dc}}{\Delta V_{dc}} = \frac{p^{**}}{V_{dc0}^{2}}\left[\frac{(n-1)(\tau s+1) - n}{\tau s+1}\right]$$

at $s \rightarrow 0$

$$\frac{\Delta V_{dc}}{\Delta I_{dc}} \hat{=} \frac{p^{**}}{V_{dc0}^{2}}$$

Adm. Constraint and Modified Adm



Modified System Time Domain Perform.