Subtransient Machine Model

$$\left.\begin{aligned}\lambda_{qs} &= L''_q\, i_{qs} + \lambda''_q \\ \lambda_{ds} &= L''_d\, i_{ds} + \lambda''_d\end{aligned}\right)\ \text{exact}$$

$$v_{qs} = \cancel{r_s i_{qs}} + \omega_r\left(L''_d\, i_{ds} + \lambda''_d\right) + p\left(L''_q\, i_{qs} + \cancel{\lambda''_q}\right)$$

$$v_{ds} = \cancel{r_s i_{ds}} - \omega_r\left(L''_q\, i_{qs} + \lambda''_q\right) + p\left(L''_d\, i_{ds} + \cancel{\lambda''_d}\right)$$

$$v_{qs} = \omega_r L''_d\, i_{ds} + L''_q\, p\, i_{qs} + \overbrace{\omega_r \lambda''_d}^{e''_q}$$

$$v_{ds} = -\omega_r L''_q\, i_{qs} + L''_d\, p\, i_{ds} \underbrace{- \omega_r \lambda''_q}_{+e''_d}$$

$$0 = ri + p\lambda$$

$$v_{fd} = r_{fd} i_{fd} + p\lambda_{fd}$$

$$v_{qs} = \omega_r \lambda_{ds} + p\lambda_{qs}$$

$$v_{ds} = -\omega_r \lambda_{qs} + p\lambda_{ds}$$

Subtransient Machine Model

Summary of Subtransient Model

$$\lambda_{qs}^{r} = L_q'' i_{qs}^{r} + \lambda_q''$$

$$e_q'' = \omega_r \lambda_d''$$

$$\lambda_{ds}^{r} = L_d'' i_{ds}^{r} + \lambda_d''$$

$$e_d'' = -\omega_r \lambda_q''$$

$$L_q'' = L_{ls} + \left(\frac{1}{L_{mq}} + \sum_{i=1}^{m} \frac{1}{L_{ldi}} \right)^{-1}$$

$$\lambda_q'' = (L_q'' - L_{ls}) \sum_{i=1}^{m} \frac{\lambda_{qi}}{L_{lqi}}$$

$$L_d'' = L_{ls} + \left(\frac{1}{L_{md}} + \frac{1}{L_{lfd}} + \sum_{i=1}^{n} \frac{1}{L_{ldi}} \right)^{-1}$$

$$\lambda_d'' = (L_d'' - L_{ls}) \left(\frac{\lambda_{fd}}{L_{lfd}} + \sum_{i=1}^{n} \frac{\lambda_{di}}{L_{ldi}} \right)$$

Derivation of Average-Value Voltage

- We have:

$$v_c = \frac{3}{\pi}\int_{\pi/3+\beta}^{2\pi/3+\beta}(v_{bs} - v_{cs})\,d\theta_r$$

$$v_{bs} = p\lambda_{bs} = \frac{d\lambda_{bs}}{dt} = \frac{d\lambda_{bs}}{d\theta_r}\frac{d\theta_r}{dt} = \frac{d\lambda_{bs}}{d\theta_r}\omega_r$$

$$v_{cs} = p\lambda_{cs} = \frac{d\lambda_{cs}}{d\theta_r}\omega_r$$

$$\overline{v}_c = \frac{3}{\pi}\omega_r(\lambda_{bs} - \lambda_{cs})\Big|_{\pi/3+\beta}^{2\pi/3+\beta}$$

Derivation of Average-Value Model

- We also have

$$i_{abcs}|_{\theta_r=\beta+\pi/3} = \begin{bmatrix} -i_{dc} & 0 & i_{dc} \end{bmatrix}^T$$

$$i_{abcs}|_{\theta_r=\beta+2\pi/3} = \begin{bmatrix} 0 & -(i_{dc}+\Delta i_{dc}) & (i_{dc}+\Delta i_{dc}) \end{bmatrix}^T$$

① Given the abc currents at each endpoint we transform to qd currents

② Given qd currents at each point we can use the subtransient model to λ_{qds} at each end point.

(in terms of i_{dc}, Δi_{dc}, β, L''_q, L''_d, λ''_q, λ''_d)

Derivation of Average-Value Model

(3) Given λ_{qds} at each end point, and again knowledge of θ_r at each end point, we can find λ_{abcs} at each enpoint

(4) Given λ_{abcs} at each ~~[illegible]~~ endpoint, we can find $\lambda_{bs} - \lambda_{cs}$ at each end point and evaluate our expression

Derivation of Average-Value Model

Output Voltage Equation

- Thus we obtain

$$\overline{v}_c = \frac{3\sqrt{3}}{\pi}\omega_r\left(-\lambda_q'' \sin\beta + \lambda_d'' \cos\beta\right) - \frac{3}{\pi}\omega_r L_c(\beta) i_{dc} - \frac{3}{\pi}\omega_r L_t(\beta)\Delta i_{dc}$$

where

$$L_c(\beta) = \frac{1}{2}(L_q'' + L_d'') + (L_d'' - L_q'')\sin(2\beta + \frac{\pi}{6})$$

commutating inductance

$$L_t(\beta) = L_q'' + L_d'' + (L_d'' - L_q'')\sin(2\beta - \frac{\pi}{6})$$

transient commutating inductance

Output Voltage Equation

Let $T = \frac{3}{\pi}\omega_r L_t(\beta)\, \Delta i_{dc}$

$$\Delta i_{dc} = p\, i_{dc} \frac{\pi/3}{\omega_r}$$

$$T = \cancel{\frac{3}{\pi}}\, \cancel{\omega_r}\, L_t(\beta) \frac{\cancel{\pi/3}}{\cancel{\omega_r}}\, p\, i_{dc}$$

$$= L_t(\beta)\, p\, i_{dc}$$

Output Voltage Equation

$$T = \frac{3\sqrt{3}}{\pi}\omega_r\left(-\lambda_q'' \sin\beta + \lambda_d'' \cos\beta\right)$$

$$= \frac{3\sqrt{3}}{\pi}\left(e_d'' \sin\beta + e_q'' \cos\beta\right)$$

$$V_{q,oc} = e_q''$$

$$V_{d,oc} = e_d''$$

$$V_{a,oc} = e_q'' \cos\theta_r + e_d'' \sin\theta_r = \underline{E_{pk} \cos(\overbrace{\theta_r + \phi}^{\theta_g})}$$

$$e_q'' \cos\theta_r + e_d'' \sin\theta_r = E_{pk} \cos\theta_r \cos\phi - E_{pk} \sin\theta_r \sin\phi$$

Output Voltage Equation

$$e_q'' = E_{pk}\cos\phi$$

$$-e_d'' = +E_{pk}\sin\phi$$

$$e_q'' - je_d'' = E_{pk}e^{j\phi}$$

$$\phi = \text{angle}(e_q'' - je_d'')$$

$$E_{pk} = \sqrt{(e_q'')^2 + (e_d'')^2}$$

$$T = \frac{3\sqrt{3}}{\pi}\left(-E_{pk}\sin\phi\sin\beta + E_{pk}\cos\beta\cos\phi\right)$$

$$T = \frac{3\sqrt{3}}{\pi}E_{pk}\cos(\beta+\phi)$$

Output Voltage Equation

Valve 3 fires when

$$\Theta_s = \frac{\pi}{3} + \alpha$$

$$\Theta_r + \phi = \frac{\pi}{3} + \alpha$$

$$\Theta_r = \frac{\pi}{3} + \alpha - \phi$$

Valve 3 fires when

$$\Theta_r = \frac{\pi}{3} + \beta$$

$$\therefore \cancel{\frac{\pi}{3}} + \beta = \cancel{\frac{\pi}{3}} + \alpha - \phi$$

$$\alpha = \beta + \phi$$

$$T = \frac{3\sqrt{3}}{\pi} E_{pk} \cos(\alpha)$$

Output Voltage Equation

- Finally we arrive at

$$\overline{v}_c = \frac{3\sqrt{3}}{\pi} E_{pk} \cos\alpha - \frac{3}{\pi}\omega_r L_c(\beta) i_{dc} - \text{[redacted]} L_t(\beta) \text{[redacted]}$$

$p\,i_{dc}$

where $E_{pk} = \sqrt{(e_q'')^2 + (e_d'')^2}$

Calculation of β

- Thus

$$\alpha = \beta + \phi$$

$$\phi = \mathrm{angle}(e_q'' - j e_d'')$$

Commutating Inductance Vs Firing Angle

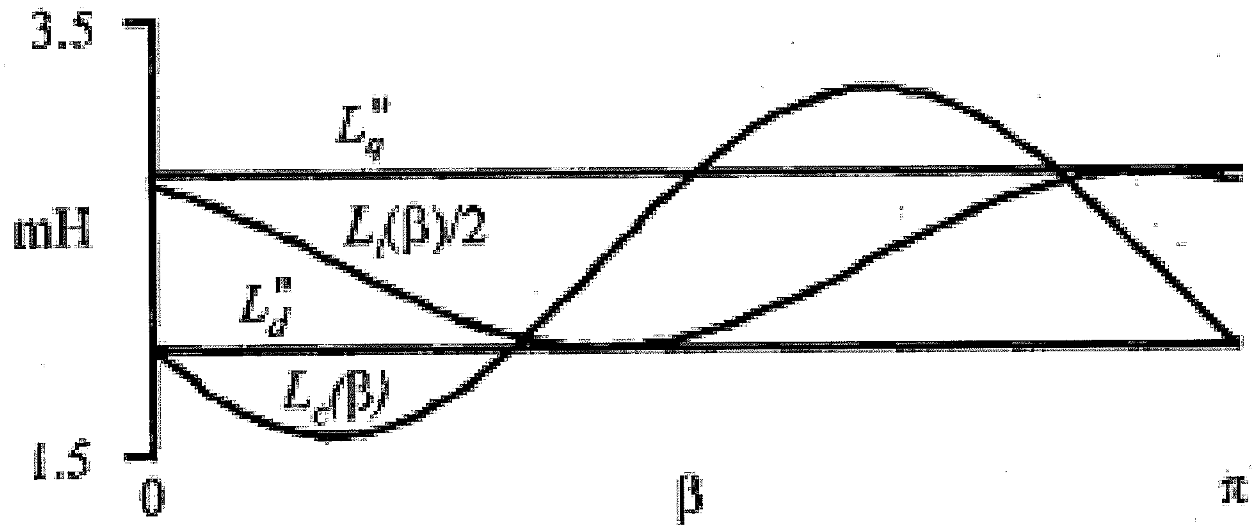


Fig. 5. Commutating and transient inductance versus firing angle.

DC Link Dynamics

- We have:

$$p\bar{i}_{dc} = \frac{\frac{3\sqrt{3}}{\pi} E_{pk} \cos\alpha - \bar{v}_{dc} - (r_l + \frac{3}{\pi}\omega_r L_c(\beta))\bar{i}_{dc}}{L_l + L_t(\beta)}$$

A-Phase Current During Commutation

- During commutation

$$v_{as} - v_{bs} = 0 \qquad p\lambda_{as} - p\lambda_{bs} = 0$$

- Thus

$$\lambda_{as} - \lambda_{bs} = K$$

- Finding K

when value 3 fires

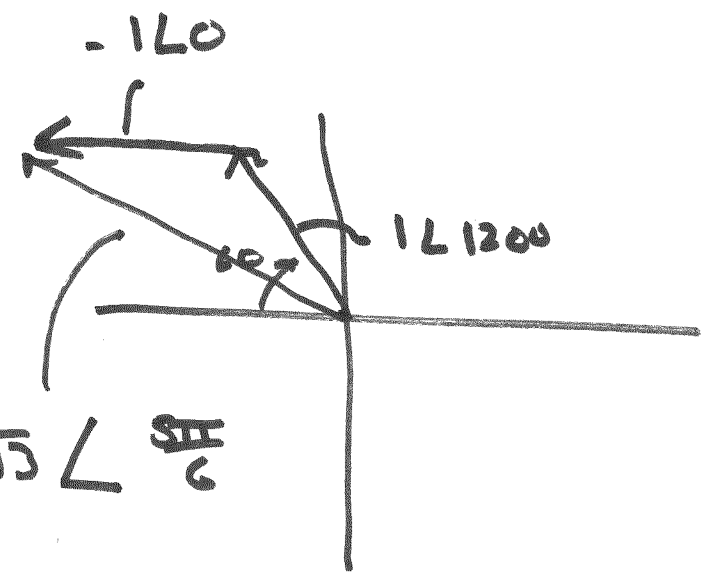
$$i_{abc} = \begin{bmatrix} -i_{dc} \\ 0 \\ i_{dc} \end{bmatrix} \qquad \theta_r = \frac{\pi}{3} + \beta$$

A-Phase Current During Commutation

$$i_{qd} = \frac{2}{3}\begin{bmatrix} c & c^- & c^+ \\ s & s^- & s^+ \end{bmatrix}\begin{bmatrix} -i_{dc} \\ 0 \\ i_{dc} \end{bmatrix}$$

$$= \frac{2}{3} i_{dc}\begin{bmatrix} c^+ - c \\ s^+ - s \end{bmatrix}$$

$$i_{qds} = \frac{2\sqrt{3}}{3} i_{dc}\begin{bmatrix} \cos(\theta_r + 5\pi/6) \\ \sin(\theta_r + 5\pi/6) \end{bmatrix}$$

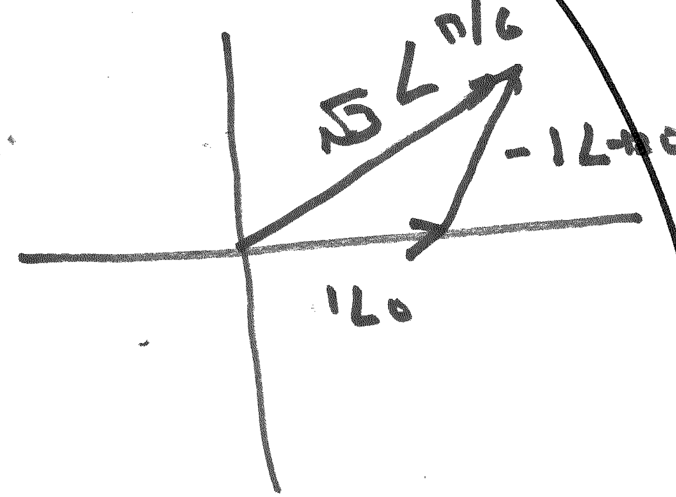


A-Phase Current During Commutation

$$\lambda_{qs} = \frac{2\sqrt{3}}{3} I_{dc} L_q'' \cos(\theta_r + 5\pi/6) + \lambda_q''$$

$$\lambda_{ds} = \frac{2\sqrt{3}}{3} I_{dc} L_d'' \sin(\theta_r + 5\pi/6) + \lambda_d''$$

$$\lambda_{abcs} = \begin{bmatrix} c & s \\ c^- & s^- \\ c^+ & s^+ \end{bmatrix} \left[\lambda_{qds}\right]$$



$$\lambda_{as} - \lambda_{bs} = \begin{bmatrix} c - c^- & s - s^- \end{bmatrix} \lambda_{qds}$$

$$= \sqrt{3} \begin{bmatrix} \cos(\theta_r + \frac{\pi}{6}) & \sin(\theta_r + \frac{\pi}{6}) \end{bmatrix} \lambda_{qds}$$

A-Phase Current During Commutation

$$\lambda_{as} - \lambda_{bs} = 2 I_{dc}\left[L_q'' \cos(\theta_r + 5\pi/6)\cos(\theta_r + \pi/6) + L_d'' \sin(\theta_r + 5\pi/6)\sin(\theta_r + \pi/6)\right]$$

$$+ \sqrt{3}\left[\lambda_q'' \cos\left(\theta_r + \frac{\pi}{6}\right) + \lambda_d'' \sin(\theta_r + \pi/6)\right]$$

$$\lambda_{as} - \lambda_{bs} = K$$

$$= I_{dc}\left[L_q''\left[-\cos 2\theta_r + \cancel{\cos\left(\frac{4\pi}{3}\right)}^{-\frac{1}{2}}\right] + L_d''\left[\cos 2\theta_r + \cancel{\cos\left(\frac{2\pi}{3}\right)}_{-\frac{1}{2}}\right]\right]$$

$$+ \sqrt{3}\left[\lambda_q'' \cos\left(\theta_r + \frac{\pi}{6}\right) + \lambda_d'' \sin(\theta_r + \pi/6)\right]$$

A-Phase Current During Commutation

$$\lambda_{as} - \lambda_{bs} = \sqrt{3}\left[\lambda_q'' \cos\left(\theta_r + \frac{\pi}{6}\right) + \lambda_d'' \sin\left(\theta_r + \pi/6\right)\right]$$
$$+ i_{dc}\left[-\frac{1}{2}\left(L_q'' + L_d''\right) + \left(L_d'' - L_q''\right)\cos 2\theta_r\right]$$

$$\theta_r = \frac{\pi}{3} + \beta$$

$$\lambda_{as} - \lambda_{bs} = \sqrt{3}\left[-\lambda_q'' \sin \beta + \lambda_d'' \cos\beta\right]$$
$$+ i_{dc}\left[-\frac{1}{2}\left(L_q'' + L_d''\right) + \left(L_d'' - L_q''\right)\cos\left(2\beta + \frac{2\pi}{3}\right)\right]$$

- Thus

$$K = \sqrt{3}(-\lambda_q'' \sin\beta + \lambda_d'' \cos\beta)$$
$$+\left[(L_d'' - L_q'')\cos(2\beta + \frac{2\pi}{3}) - \frac{1}{2}(L_q'' + L_d'')\right]\bar{i}_{dc}$$

A-Phase Current During Commutation

- Proceeding Onward

During commutation $\lambda_{as} - \lambda_{bs} = k$

$$i_{abcs} = \begin{bmatrix} i_{as} \\ -i_{as} - i_{dc} \\ i_{dc} \end{bmatrix} \qquad i_{abcs} = \begin{bmatrix} -i_{dc} \\ 0 \\ i_{dc} \end{bmatrix} \; \& \; \theta_r = \frac{\pi}{3} + \beta$$

$\downarrow$

i_{qds}

$\downarrow$

λ_{qs} & λ_{ds} (subtransient model)

$\downarrow$

$\lambda_{as} - \lambda_{bs} \rightarrow$ in terms of i_{as}, i_{dc}, λ_q'', λ_d'', L_q'', L_d'', θ_r

A-Phase Current During Commutation

- Finally we have

$$\tilde{\Theta}_r = \Theta_r - \left(\frac{\pi}{3} + \beta\right)$$

$$i_{as}(\theta_r) = \frac{K + \sqrt{3}\left(\lambda_q'' \sin(\tilde{\theta}_r + \beta) - \lambda_d'' \cos(\tilde{\theta}_r + \beta)\right)}{(L_q'' + L_d'') - (L_q'' - L_d'')\cos(2\tilde{\theta}_r + 2\beta)} - \frac{\left[(L_q'' - L_d'')\cos(2\tilde{\theta}_r + 2\beta - \frac{2\pi}{3}) + \frac{1}{2}(L_q'' + L_d'')\right]\bar{i}_{dc}}{(L_q'' + L_d'') - (L_q'' - L_d'')\cos(2\tilde{\theta}_r + 2\beta)}$$

N

$$= \frac{N}{D}$$

Minimum Firing Delay

minimum β for which valve 3 will turn on when I want it to

$$f_{\min}(\beta_{\min}) = \sqrt{3}\,(\lambda_q'' \cos\beta_{\min} + \lambda_d'' \sin\beta_{\min})$$
$$+2\bar{i}_{dc}(L_q'' - L_d'')\sin(2\beta_{\min} - \frac{\pi}{3})$$

$$f_{\min}(\beta_{\min}) = 0$$

$$\beta = \max(\beta_{\min}, \beta_c)$$

Minimum Firing Delay

$$i_{as} = \frac{N}{D}$$

$$\cancel{\frac{d i_{as}}{D} = i'_{as}}$$

$$\frac{d i_{as}}{d\theta_r} = i'_{as} = \frac{N'}{D} - \frac{N}{D^2} D' = 0$$

$$N' - \frac{N}{D} D' = 0$$

$$N' + i_{dc} D' = 0$$

$\hookrightarrow i_{as} = -i_{dc} = \frac{N}{D}$ at the moment I fire valve 3

Minimum Firing Delay

Looking back at slide 34

$$N' = \sqrt{3}\left(\lambda_q'' \cos(\cancelto{0}{\tilde{\theta}_r} + \beta_{min}) + \lambda_d'' \sin(\cancelto{0}{\tilde{\theta}_r} + \beta_{min})\right)$$

$$+ 2(L_q'' - L_d'')\sin(\cancelto{0}{2\tilde{\theta}_r} + 2\beta_{min} - \tfrac{4\pi}{3})\, i_{dc}$$

$$D' = 2(L_q'' - L_d'')\sin(\cancelto{0}{2\tilde{\theta}_r} + 2\beta_{min})$$

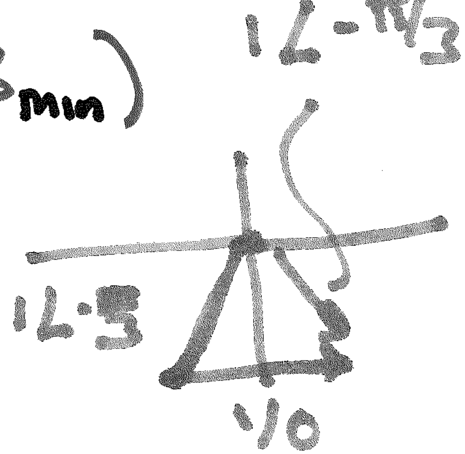
so

$$N' + i_{dc} D'$$

$$= \sqrt{3}\left(\lambda_q'' \cos\beta_{min} + \lambda_d'' \sin\beta_{min}\right)$$

$$+ 2(L_q'' - L_d'')\, i_{dc}\Big[\underbrace{\sin(2\beta_{min} - \tfrac{2\pi}{3}) + \sin(2\beta_{min})}_{\sin(2\beta_{min} - \pi/3)}\Big]$$

$$= f_{min}(\beta_{min}) = 0$$



Minimum Firing Delay

Calculation of Commutation Angle

$$i_{as} = \frac{N}{D}$$

$$\Rightarrow N = 0$$

at that point $\tilde{\theta}_r = u$

solve

$$K + \sqrt{3}\left(\lambda_q'' \sin(u+\beta) - \lambda_d'' \cos(u+\beta)\right)$$
$$- \left[\left(L_q'' - L_d''\right)\cos\left(2u + 2\beta - \tfrac{2\pi}{3}\right) + \tfrac{1}{2}\left(L_q'' + L_d''\right)\right] i_{dc} = 0$$

for u

Calculation of Commutation Angle

- Thus commutation angle found by solving

$$0 = K - \sqrt{3}\left[-\lambda_q'' \sin(\beta + u) + \lambda_d'' \cos(\beta + u)\right] -$$

$$\left[(L_q'' - L_d'')\cos(2\beta + 2u - \frac{2\pi}{3}) + \frac{1}{2}(L_q'' + L_d'')\right]\overline{i}_{dc}$$

Calculation of Inst. Q- and D-Axis Currents

$$i_{abcs} = \begin{bmatrix} i_{as} \\ -i_{as} - i_{dc} \\ +i_{dc} \end{bmatrix} \qquad i_{abcs} = \begin{bmatrix} 0 \\ -i_{dc} \\ i_{dc} \end{bmatrix}$$

during commutation during conduction

K_s^r

- Thus

$$i_{qs}^r(\tilde{\theta}_r) = \frac{2\sqrt{3}}{3}\left[-i_{as}(\tilde{\theta}_r)\sin(\tilde{\theta}_r + \beta) - i_{dc}\sin(\tilde{\theta}_r + \beta + \frac{\pi}{3})\right] \qquad i_{ds}^r(\tilde{\theta}_r) = \frac{2\sqrt{3}}{3}\left[i_{as}(\tilde{\theta}_r)\cos(\tilde{\theta}_r + \beta) + \bar{i}_{dc}\cos(\tilde{\theta}_r + \beta + \frac{\pi}{3})\right]$$

paper is incorrect

Calculation of Inst. Q- and D-Axis Currents

I_θ

Calculation of Avg. Q- and D-Axis Currents

- Approach

$$\bar{i}_{qs} = \frac{1}{\pi/3}\int_0^{\pi/3} i_{qs}\, d\tilde{\theta}_r$$

$$= \underbrace{\frac{3}{\pi}\int_0^{u} i_{qs}\, d\tilde{\theta}_r}_{i_{qs,com}} + \underbrace{\frac{3}{\pi}\int_u^{\pi/3} i_{qs}\, d\tilde{\theta}_r}_{i_{qs,cond}}$$

- Thus

$$\bar{i}^{\,r}_{qs} = \bar{i}^{\,r}_{qs,com} + \bar{i}^{\,r}_{qs,cond}$$

$$\bar{i}^{\,r}_{ds} = \bar{i}^{\,r}_{ds,com} + \bar{i}^{\,r}_{ds,cond}$$

Calculation of Avg. Q- and D-Axis Currents

- Conduction components

$$\bar{i}^{\,r}_{qs,cond} = \frac{2\sqrt{3}}{\pi}\bar{i}_{dc}\left[\cos(\beta+\frac{2\pi}{3}) - \cos(\beta+u+\frac{\pi}{3})\right]$$

$$\bar{i}^{\,r}_{ds,cond} = \frac{2\sqrt{3}}{\pi}i_{dc}\left[\sin(\beta+\frac{2\pi}{3}) - \sin(\beta+u+\frac{\pi}{3})\right]$$

- Commutation components (change pi to 4)

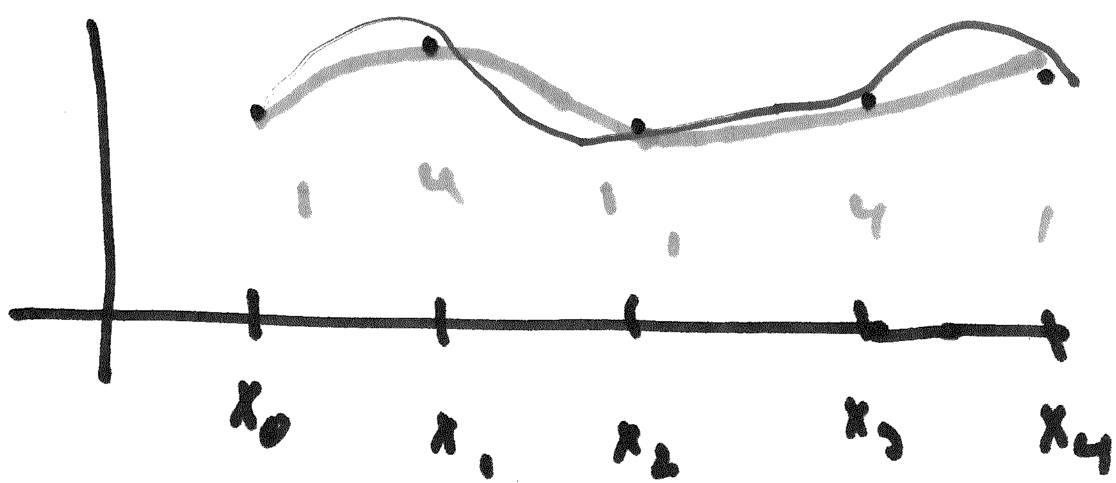
$$\bar{i}^{\,r}_{qs,com} = \frac{u}{4\pi}\left[i^r_{qs}(0) + 4i^r_{qs}(\frac{u}{4}) + 2i^r_{qs}(\frac{u}{2}) + 4i^r_{qs}(\frac{3u}{4}) + i^r_{qs}(u)\right]$$

$$\bar{i}^{\,r}_{ds,com} = \frac{u}{4\pi}\left[i^r_{ds}(0) + 4i^r_{ds}(\frac{u}{4}) + 2i^r_{ds}(\frac{u}{2}) + 4i^r_{ds}(\frac{3u}{4}) + i^r_{ds}(u)\right]$$

Simpson's Rule

$$\int_a^b F(x)\,dx = \frac{b-a}{3n}\Big[F(x_0) + 4F(x_1) + 2F(x_2) + 4f(x_3) + 2F(x_4) \cdots F(x_n)\Big]$$

$\frac{1}{\frac{\pi}{3}}$



$$= \frac{u}{12}\Big[F(x) \qquad \Big]$$

$$\frac{u}{12}\,\frac{3}{\pi} = \frac{u}{4\pi}$$

Example of Transient Performance

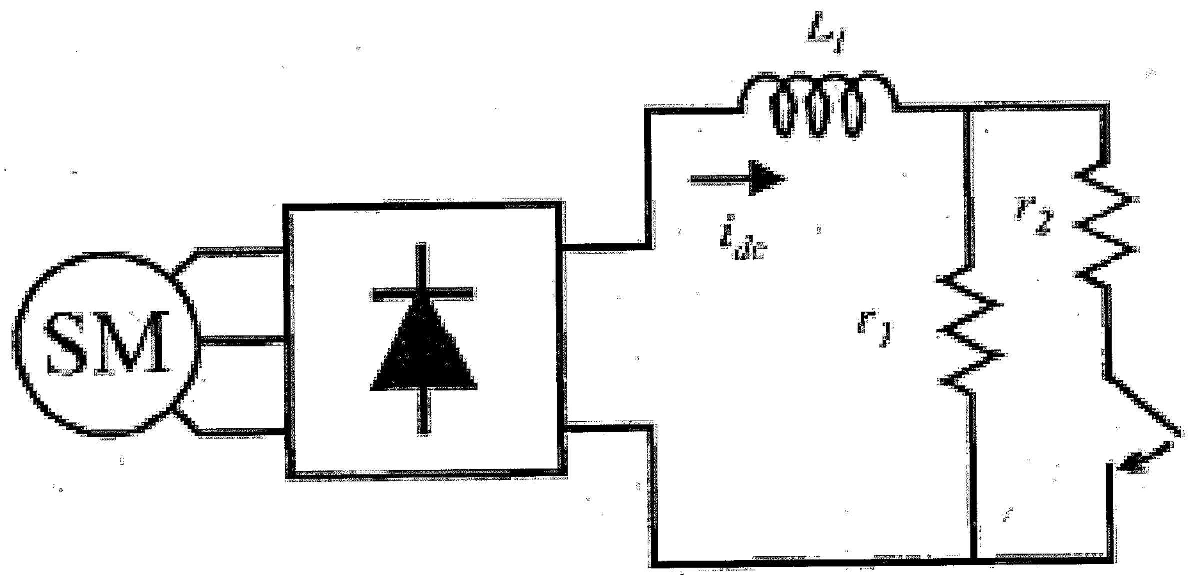


Fig. 6. System configuration for time-domain test.

Example of Transient Performance

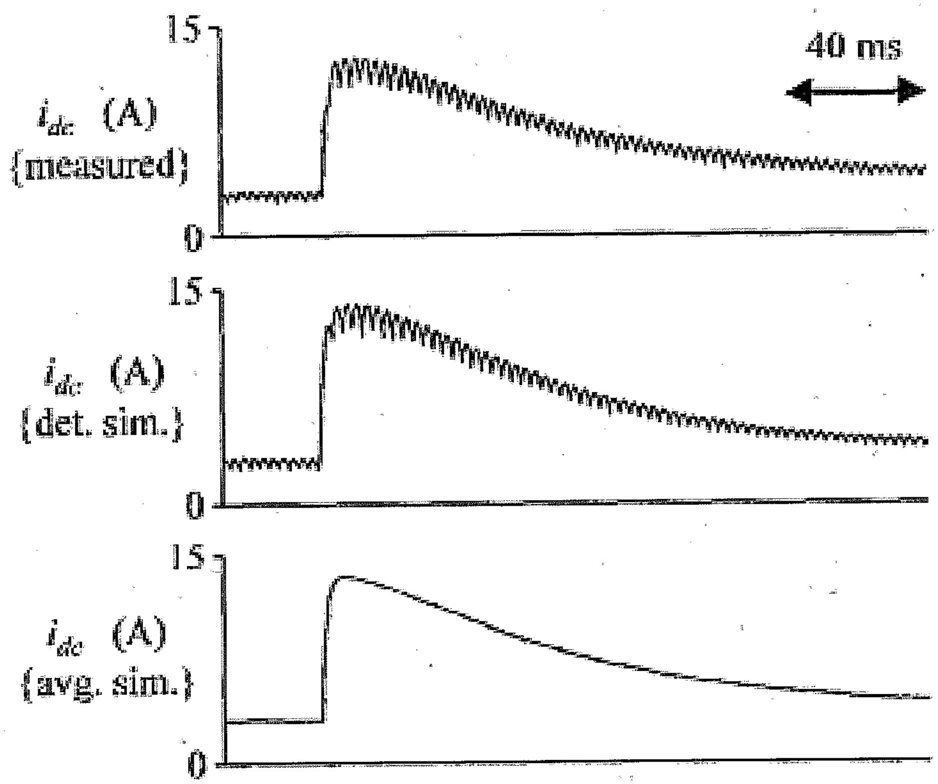


Fig. 7. DC link current during step increase in load as measured, as predicted by detailed simulation, and as predicted by average-value simulation.

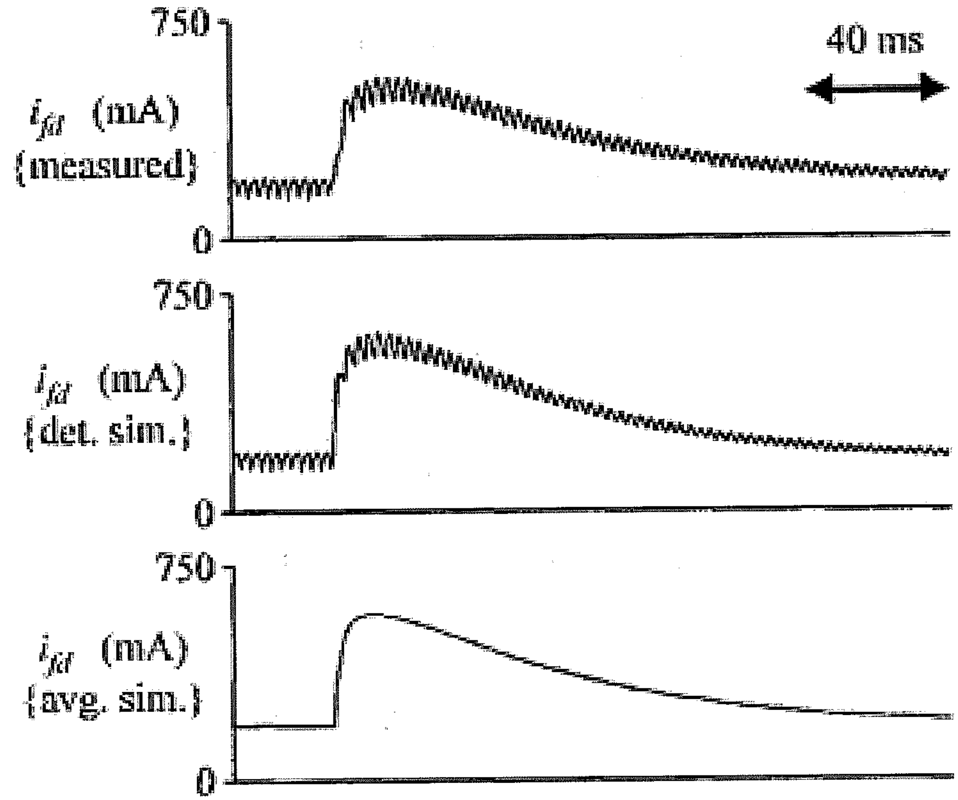


Fig. 8. Field current during step increase in load as measured, as predicted by detailed simulation, and as predicted by average-value simulation.

Frequency Domain Predictions

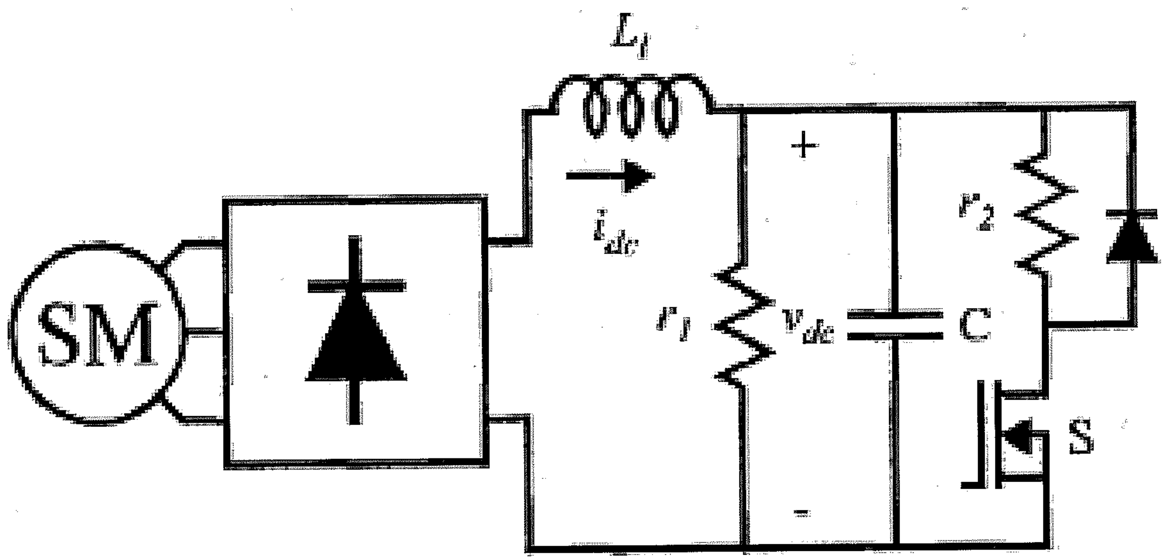
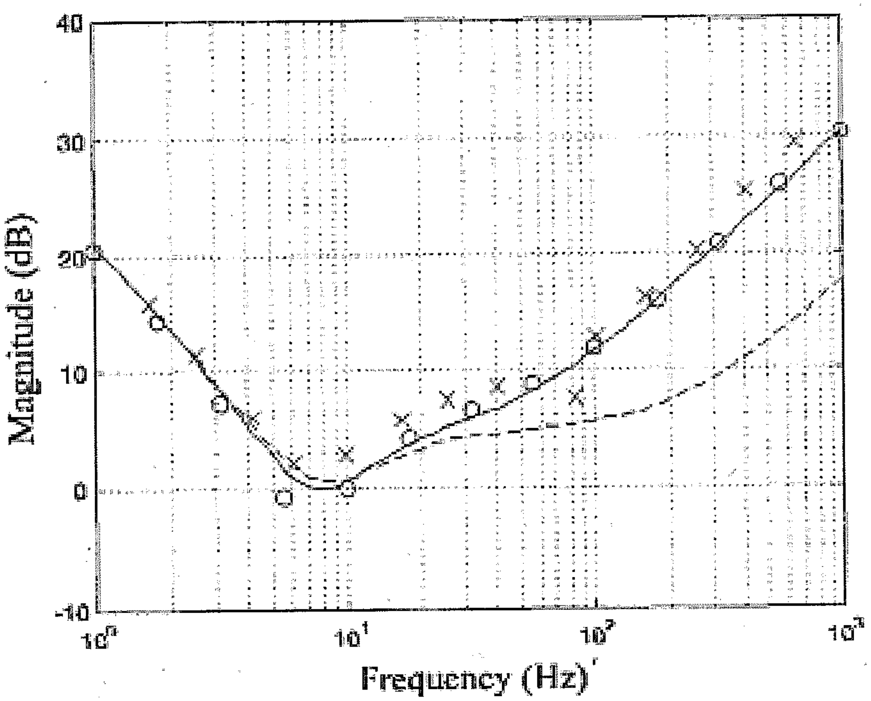


Fig. 9. System configuration for frequency-domain test.

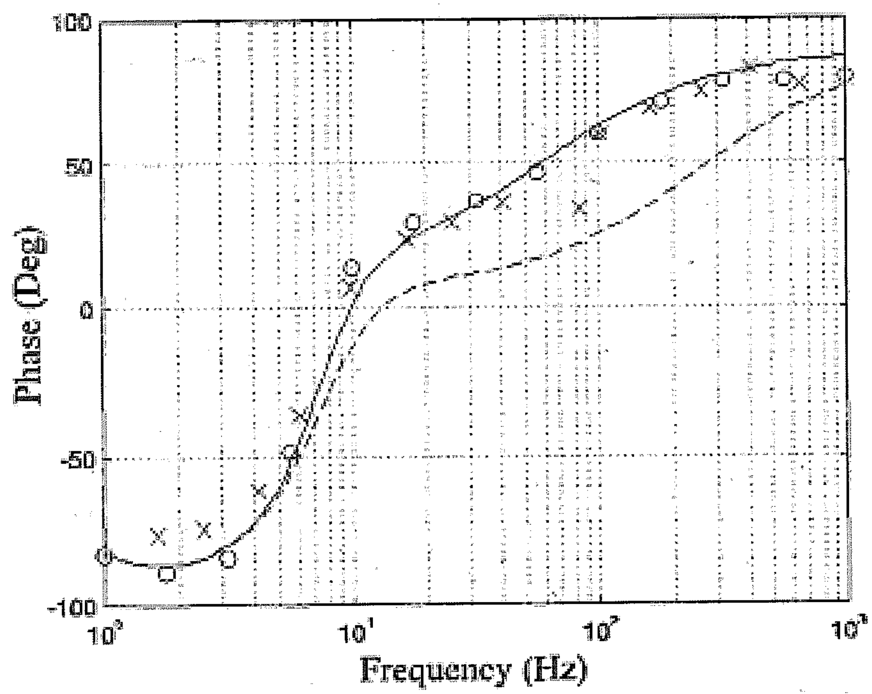
Frequency Domain Predictions



× Measured ——— Average-Value Model

○ Detailed Simulation - - - - Reduced-Order Model

Figure 10. Impedance characteristic of synchronous machine fed load-commutated converter.



× Measured ——— Average-Value Model

○ Detailed Simulation - - - - Reduced-Order Model

Figure 11. Impedance characteristic of synchronous machine fed load-commutated converter.

ECE61016 Power Electronics Converters and Systems

Stability of Power Electronic Systems
Lecture Set 9A

S.D. Sudhoff

Fall 2016

Some Definitions (Zak, 2003)

- System description (non-autonomous)

$$p\mathbf{x}(t) = \mathrm{f}(t, \mathbf{x}(t))$$

$$\mathbf{x}(t_0) = \mathbf{x}_0$$ initial condition

- System description (autonomous)

$$p\mathbf{x}(t) = \mathrm{f}(\mathbf{x}(t))$$

$$\mathbf{x}(t_0) = \mathbf{x}_0$$

Some Definitions (Zak, 2003)

- Conditions for solution

  F continuous

  all the particals are continuous

- Solution is denoted

$$\mathbf{x}(t) = \mathbf{x}(t; t_0, \mathbf{x}_0)$$

Some Definitions (Zak, 2003)

- Equilibrium point

$$\mathbf{f}(t, \mathbf{x}_e) = 0 \quad \forall t$$

- Translation of equilibrium point

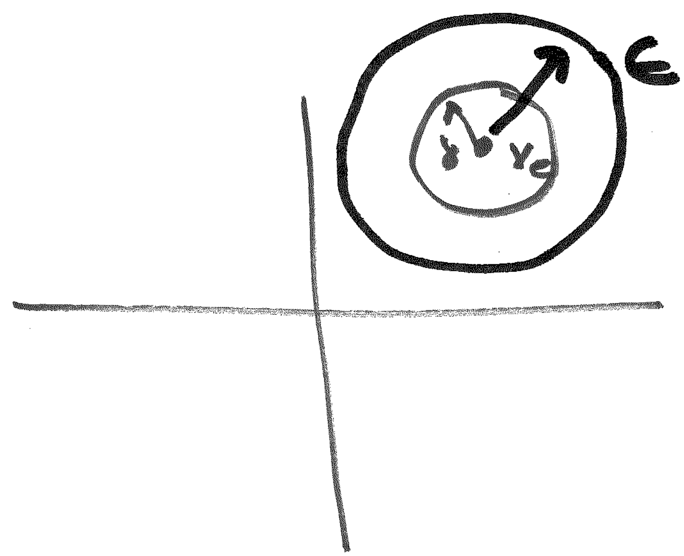
$$\tilde{\mathbf{x}} = \mathbf{x} - \mathbf{x}_e$$

$$\Delta x = x - x_0$$

Some Definitions (Zak, 2003)

- An equilibrium state $\boldsymbol{x}_e$ is said to be *stable* if for any given t_0 and any given positive scalar ε their exists a scalar $\delta(t_0,\varepsilon)$ such that

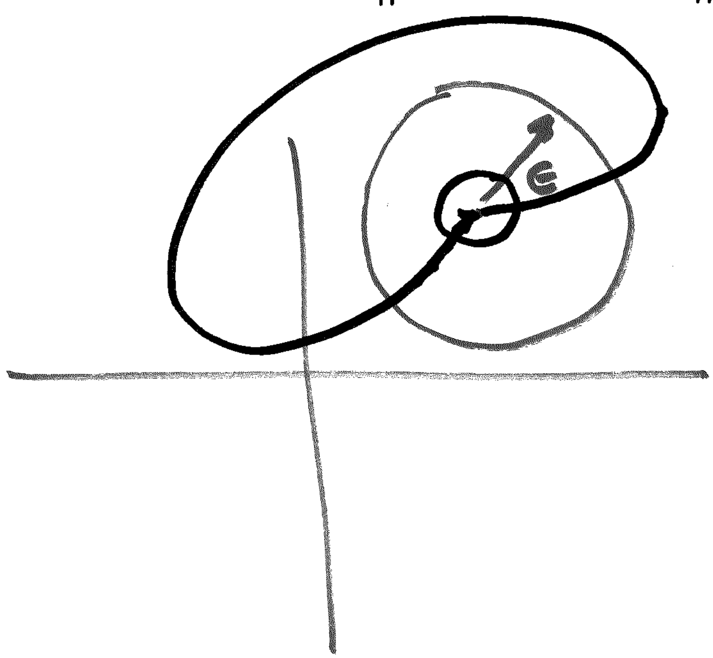
$$\|\mathbf{x}(t_0)-\mathbf{x}_e\| < \delta_0(t_0,\varepsilon) \Rightarrow \|\mathbf{x}(t;t_0,\mathbf{x}_0)-\mathbf{x}_e\| < \varepsilon \quad \forall t > t_0$$

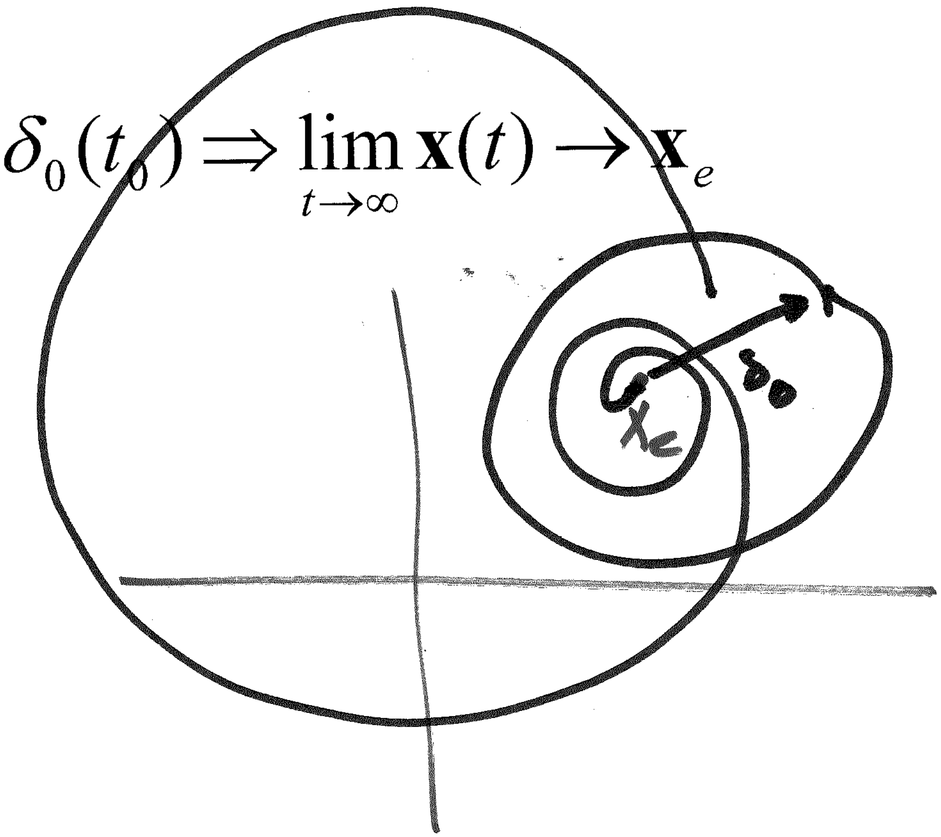


Some Definitions (Zak, 2003)

- An equilibrium state is said to be <u>*convergent*</u> or <u>*attractive*</u> if for any given t_0 there is a positive scalar $\delta_0(t_0)$

$$\left\|\mathbf{x}(t_0)-\mathbf{x}_e\right\| < \delta_0(t_0) \Rightarrow \lim_{t\to\infty}\mathbf{x}(t) \to \mathbf{x}_e$$





Some Definitions (Zak, 2003)

- An equilibrium state is *asymptotically stable* if stable and convergent
- An equilibrium state is *uniformly stable* if $\delta(t_0,\varepsilon) = \delta(\varepsilon)$
- An equilibrium state is *uniformly convergent* if $\cancel{d}_1(t_0) = \cancel{d}_1$ $\delta_1 \; \delta$.
- An equilibrium state is *uniformly asymptotically stable* if is uniformly stable and uniformly convergent

Some Definitions (Zak, 2003)

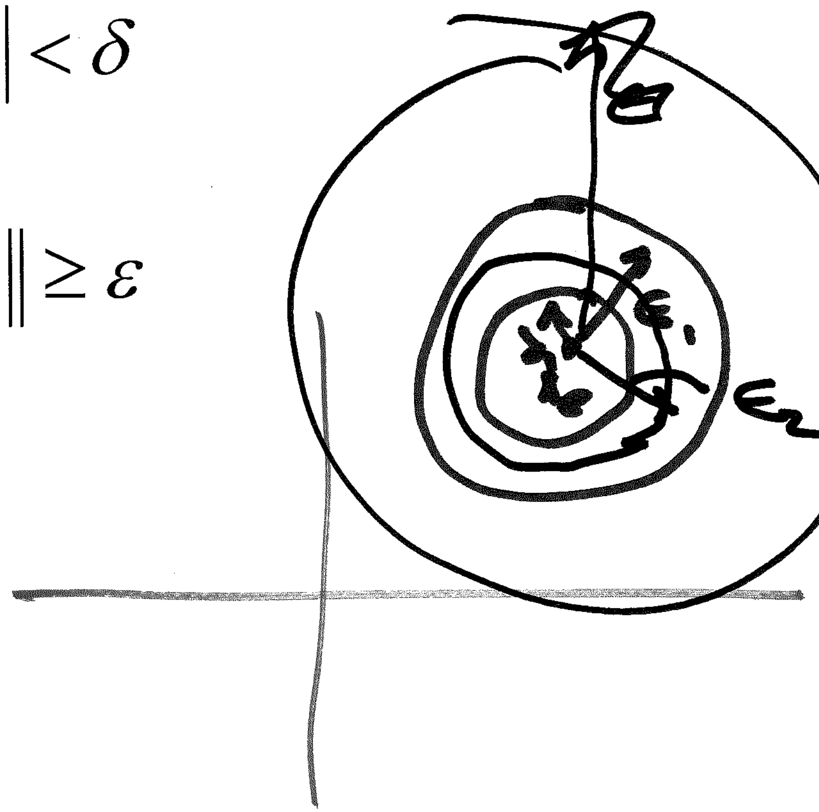
- An equilibrium state $\mathbf{x}_e$ is said to be unstable if there is an $\varepsilon > 0$ so that for any $\delta > 0$ there exists $\mathbf{x}(t_0)$ such that if

$$\|\mathbf{x}(t_0) - \mathbf{x}_e\| < \delta$$

then

$$\|\mathbf{x}(t_0) - \mathbf{x}_e\| \geq \varepsilon$$

for some $t_1 > t_0$



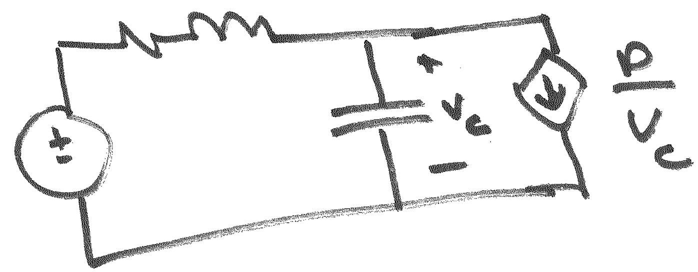
Some Definitions (Zak, 2003)

Some Definitions (Zak, 2003)

- Lyapunov's Indirect Method
- Let the origin $x=0$ be an equilibrium state of the nonlinear system $p\mathbf{x}=\mathbf{f}(\mathbf{x})$. Then the origin is an asymptotically stable equilibrium of the nonlinear system if $\mathbf{A}$, the Jacobian matrix of $\mathbf{f}$, evaluated at the origin, has all its eigenvalues in the open left-half plane.
- Take ECE675 for proof

Stability of Power Electronic Systems

- Reasons for concerns



- Subtleties

Physical system vs. Model

$px = f(x,u)$ $\quad\quad$ $\bar{x} = \frac{1}{T}\int_0^T x \, d\tau$

Physical Definition of Bus Stability

- A bus would be defined to be locally dynamically stable at a given operating condition provided that the bus voltage is constant at that operating point, and that if the bus voltage were perturbed it would return to its original value

- Bus voltage is either dc, or q- and d-axis voltage in a synchronous reference frame

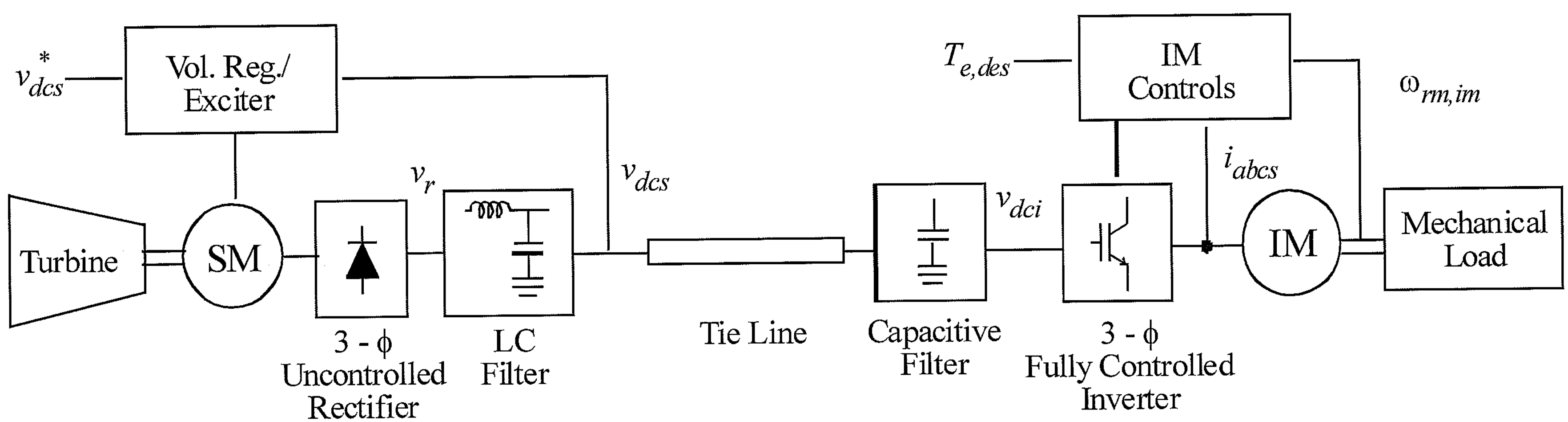
Practical Physical Def. of Bus Stability

- A bus is defined to be locally dynamically stable at a given operating point provided that the bus voltage variation is restricted to frequencies corresponding to the switching frequencies (and harmonics thereof) of the power converters in the system
- A bus is said to be regionally dynamically stable over a set of operating points if it satisfies the definition of being locally dynamically stable for all operating points in the set (region).

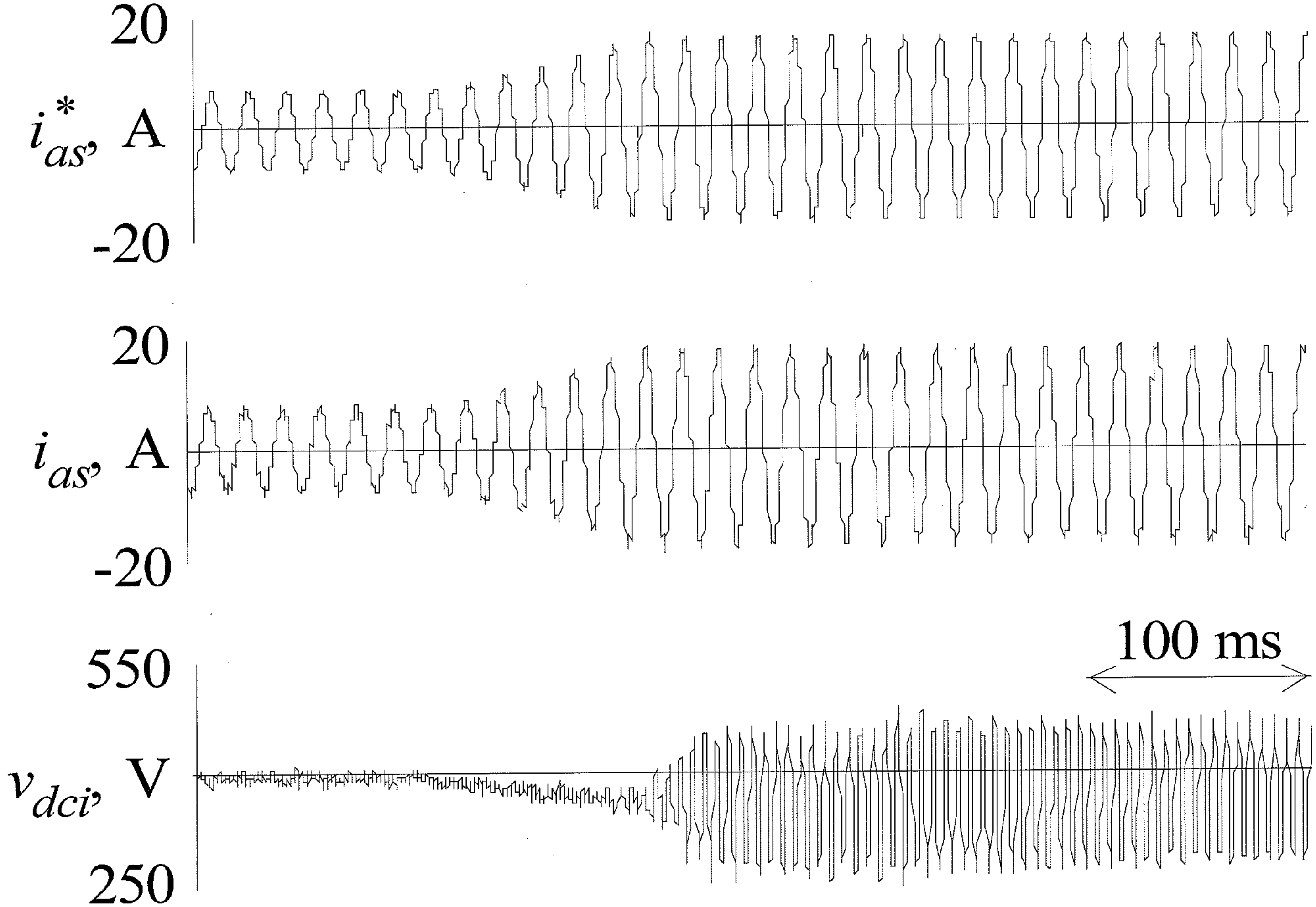
Physical Definition of System Stability

- System stability is an amalgamation of bus stability; in particular, a system is said to be locally dynamically stable at a given operating condition if all busses in the system are dynamically stable at that operating point
- A system is regionally dynamically stable if all busses in the system are regionally dynamically stable at all relevant operating conditions.

Consider a System



System Performance



General Modeling Requirements

- We assume that all components can be represented in the following form

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}, \mathbf{u})$$

$$\hat{x} = \frac{1}{T} \int_{t-T}^{t} x(\tau) d\tau$$

$$\mathbf{y} = \mathbf{g}(\mathbf{x}, \mathbf{u})$$

Model Linearization

- We will be using linear analysis techniques
- At an operating point

$$0 = \mathbf{f}(\mathbf{x}_0, \mathbf{u}_0) \qquad \mathbf{y}_0 = g(\mathbf{x}_0, \mathbf{u}_0)$$

- Perturbation from operating point

$$\Delta\mathbf{u} = \mathbf{u} - \mathbf{u}_0 \qquad \Delta\mathbf{x} = \mathbf{x} - \mathbf{x}_0 \qquad \Delta\mathbf{y} = \mathbf{y} - \mathbf{y}_0$$

Model Linearization

- Linearized System Representation

$$\frac{d\Delta\mathbf{x}}{dt} = \mathbf{A}\Delta\mathbf{x} + \mathbf{B}\Delta\mathbf{u} \qquad \Delta\mathbf{y} = \mathbf{C}\Delta\mathbf{x} + \mathbf{D}\Delta\mathbf{u}$$

- Matrix Definitions

$$\mathbf{A} = \left.\frac{\partial\mathbf{f}(\mathbf{x},\mathbf{u})}{\partial\mathbf{x}}\right|_{\mathbf{x}_0,\mathbf{u}_0} \qquad \mathbf{B} = \left.\frac{\partial\mathbf{f}(\mathbf{x},\mathbf{u})}{\partial\mathbf{u}}\right|_{\mathbf{x}_0,\mathbf{u}_0}$$

$$\mathbf{C} = \left.\frac{\partial\mathbf{g}(\mathbf{x},\mathbf{u})}{\partial\mathbf{x}}\right|_{\mathbf{x}_0,\mathbf{u}_0} \qquad \mathbf{D} = \left.\frac{\partial\mathbf{g}(\mathbf{x},\mathbf{u})}{\partial\mathbf{u}}\right|_{\mathbf{x}_0,\mathbf{u}_0}$$

Model Linearization

$$\frac{d}{dt}\overbrace{(x - x_0)}^{\Delta x} = \frac{dx}{dt} = f(x,u)$$

$$f(x,u) = \underbrace{f(x_0, u_0)}_{0} + \underbrace{\left.\frac{\partial f}{\partial x}\right|_{x_0, u_0}}_{A} \overbrace{(x - x_0)}^{\Delta x} + \underbrace{\left.\frac{\partial f}{\partial u}\right|_{x_0, u_0}}_{B} \overbrace{(u - u_0)}^{\Delta u}$$

$$\frac{d\Delta x}{dt} = A\Delta x + B\Delta u$$

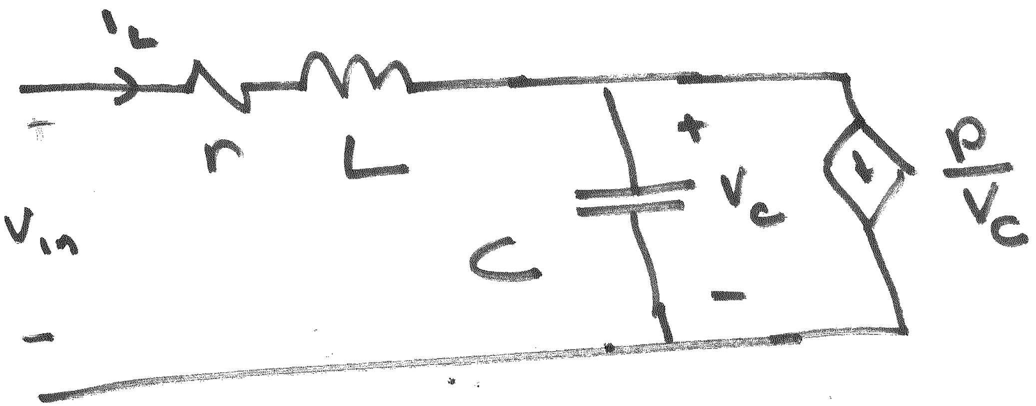
Model Linearization

$$Y = g(X, U)$$

$$Y \approx \underbrace{g(X_0, U_0)}_{Y_0} + \underbrace{\frac{\partial g}{\partial X}\Big|_{X_0, U_0}}_{C} \overbrace{(X - X_0)}^{\Delta X} + \underbrace{\frac{\partial g}{\partial U}\Big|_{X_0, U_0}}_{D} \overbrace{(Y - Y_0)}^{\Delta U}$$

$$Y - Y_0 = \Delta Y = C\,\Delta X + D\,\Delta U$$

A Simple System



$$pi_L = \frac{v_{in} - ri_L - v_c}{L}$$

$$pv_c = \frac{i_L - P/v_c}{C}$$

$$x = \begin{bmatrix} i_L \\ v_c \end{bmatrix}$$

$$y = i_L$$

A Simple System

$$F(x,u) = \begin{array}{cc} \begin{bmatrix} \dfrac{V_{in} - r I_L - V_C}{L} \\ \dfrac{I_L - P/V_C}{C} \end{bmatrix} & \begin{array}{c} f_1 \\ \\ f_2 \end{array} \end{array}$$

$$g(x,u) = I_L$$

Operating Point

$$V_c = V_{in} - r I_L = V_{in} - r\frac{P}{V_c}$$

$$V_c^2 = V_{in} V_c - rP$$

$$V_c^2 - V_{in} V_c + rP = 0$$

$$V_c = \frac{V_{in} \pm \sqrt{V_{in}^2 - 4rP}}{2}$$

Linearization

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial i_L} & \frac{\partial f_1}{\partial v_C} \\ \frac{\partial f_2}{\partial i_L} & \frac{\partial f_2}{\partial v_C} \end{bmatrix} = \begin{bmatrix} -\frac{r}{L} & -\frac{1}{L} \\ \frac{1}{C} & +\frac{P}{C V_{C0}^2} \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{\partial f_1}{\partial v_{in}} \\ \frac{\partial f_2}{\partial v_{in}} \end{bmatrix} = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}$$

Linearization

$$C = \begin{bmatrix} \frac{\partial g}{\partial i_L} & \frac{\partial g}{\partial v_C} \end{bmatrix} = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} \frac{\partial g}{\partial v_{in}} \end{bmatrix} = 0$$