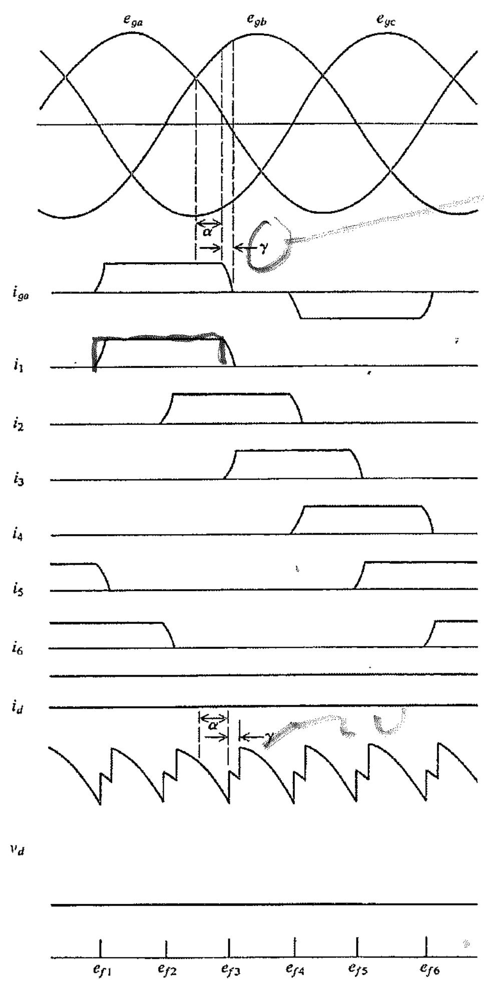
Phase Delay and Commutation



γ is the commutation angle

Phase Delay, Commutation, Changing Current

- Assumptions

$$v_{abcg} = \sqrt{2}E \begin{bmatrix} \cos(\theta_g) \\ \cos(\theta_g - 2\pi/3) \\ \cos(\theta_g + 2\pi/3) \end{bmatrix}$$

$$i_{abcg}\Big|_{\theta_g = \frac{\pi}{3}+\alpha} = \begin{bmatrix} -\hat{i}_d \\ 0 \\ \hat{i}_d \end{bmatrix}$$

value 3 turns on

$$i_{abcg}\Big|_{\theta_g = \frac{2\pi}{3}+\alpha} = \begin{bmatrix} 0 \\ -\hat{i}_d - \Delta\hat{i}_d \\ \hat{i}_d + \Delta\hat{i}_d \end{bmatrix}$$

change in $\hat{i}_d$ over the switching interval

value 4 turns on

Derivation

$$\hat{v}_d = \frac{3}{\pi} \int_{\frac{\pi}{3}+\alpha}^{\frac{2\pi}{3}+\alpha} (v_{bs} - v_{cs})\, d\theta_g$$

$$\theta_g = \omega_g t + \theta_g(0)$$

$$d\theta_g = \omega_g\, dt$$

$$\frac{1}{dt} = \omega_g \frac{1}{d\theta_g}$$

$$V_{bs} = V_{bg} + L_c\, p\, i_{bg}$$

$$V_{cs} = V_{cg} + L_c\, p\, i_{cg}$$

$$\hat{V}_d = \frac{3}{\pi} \int_{\frac{\pi}{3}+\alpha}^{2\pi/3+\alpha} V_{bg} - V_{cg}\, d\theta_g + \frac{3}{\pi} \int_{\frac{\pi}{3}+\alpha}^{\frac{2\pi}{3}+\alpha} L_c\, p\,(i_{bg} - i_{cg})\, d\theta_g$$

$$\hat{V}_d = \frac{3\sqrt{3}}{\pi} \sqrt{2} E \cos\alpha + \frac{3}{\pi} L_c \omega_g \int_{\frac{\pi}{3}+\alpha}^{\frac{2\pi}{3}+\alpha} \frac{d}{d\theta_g}(i_{bg} - i_{cg})\, d\theta_g$$

Derivation

$$\hat{V}_d = \frac{3\sqrt{6}}{\pi} E \cos\alpha + \frac{3}{\pi} L_c \omega_g \left(i_{bg} - i_{cg} \right) \Big|_{\frac{\pi}{3}+\alpha}^{\frac{2\pi}{3}+\alpha}$$

at $\Theta_s = \frac{2\pi}{3} + \alpha$

$$i_{bg} - i_{cg} = -2\hat{i}_d - 2\Delta\hat{i}_d$$

at $\Theta_s = \frac{\pi}{3} + \alpha$

$$i_{bg} - i_{cg} = -\hat{i}_d$$

so

$$\left(i_{bg} - i_{cg} \right) \Big|_{\frac{\pi}{3}+\alpha}^{\frac{2\pi}{3}+\alpha} = -\cancel{2}\hat{i}_d - 2\Delta\hat{i}_d \cancel{- \hat{i}_d}$$

Derivation

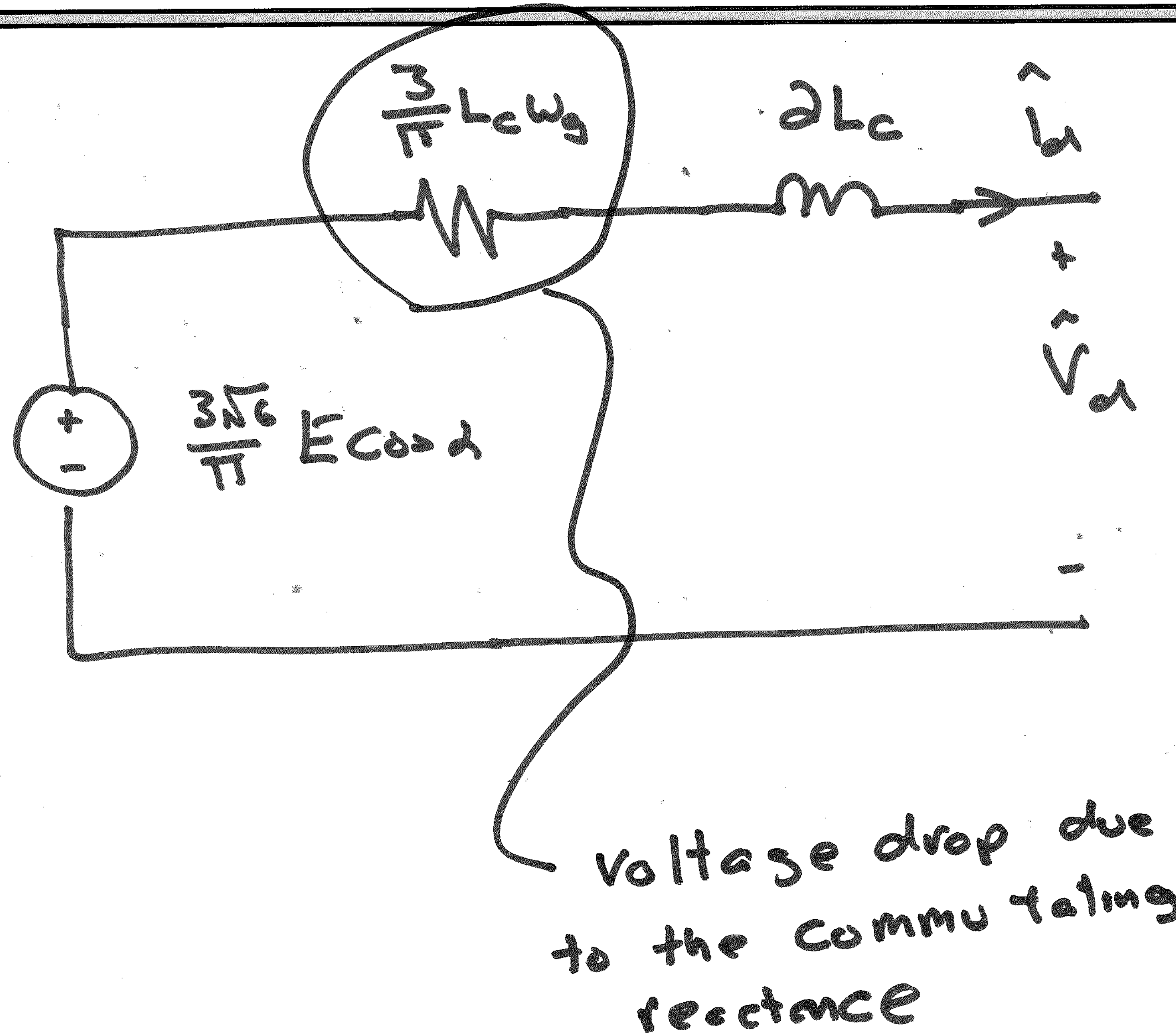
$$\hat{V}_d = \frac{3\sqrt{6}}{\pi} E \cos\alpha + \frac{3}{\pi} L_c \omega_s \left(-\hat{I}_d - 2\Delta\hat{I}_d \right)$$

$$\Delta\hat{I}_d \approx p\hat{I}_d \frac{\frac{\pi}{3}}{\omega_s}$$

$$\hat{V}_d = \frac{3\sqrt{6}}{\pi} E \cos\alpha - \frac{3}{\pi} L_c \omega_s \hat{I}_d - 2\cancel{\frac{3}{\pi}} L_c \cancel{\omega_s} \frac{\cancel{\frac{\pi}{3}}}{\cancel{\omega_s}} p\hat{I}_d$$

$$\hat{V}_d = \frac{3\sqrt{6}}{\pi} E \cos\alpha - \frac{3}{\pi} L_c \omega_s \hat{I}_d - 2L_c p\hat{I}_d$$

Derivation



$\frac{3}{\pi} L_c \omega_g$
$2L_c$
$\hat{i}_d$
+
$\hat{V}_d$
-
$\frac{3\sqrt{6}}{\pi} E \cos\alpha$
voltage drop due
to the commutating
reactance

Derivation

- Finally we obtain

$$\hat{v}_d = \frac{3\sqrt{6}}{\pi} E \cos\alpha - \frac{3}{\pi} L_c \omega_g \hat{i}_d - 2L_c p \hat{i}_d$$

Integration with DC Link Dynamics

$$-\hat{v}_d + r_{dc}\hat{i}_d + L_{dc}\,p\hat{i}_d + \hat{e}_d = 0$$

$$-\left[\frac{3\sqrt{6}}{\pi}E\cos\alpha - \frac{3}{\pi}L_c\omega_g\hat{i}_d - 2L_c\,p\hat{i}_d\right]$$

$$+ r_{dc}\hat{i}_d + L_{dc}\,p\hat{i}_d + \hat{e}_d = 0$$

$$(L_{dc} + 2L_c)\,p\hat{i}_d = \frac{3\sqrt{6}}{\pi}E\cos\alpha - \hat{e}_d - \left(r_{dc} + \frac{3}{\pi}L_c\omega_g\right)\hat{i}_d$$

$$p\hat{i}_d = \frac{\frac{3\sqrt{6}}{\pi}E\cos\alpha - \hat{e}_d - \left(r_{dc} + \overbrace{\frac{3}{\pi}L_c\omega_g}^{\text{normally dominates } r_{dc}}\right)\hat{i}_d}{L_{dc} + 2L_c}$$

Integration with DC Link Dynamics

- Thus

$$p\hat{i}_d = \frac{\frac{3\sqrt{6}}{\pi} E\cos\alpha - (r_{dc} + \frac{3}{\pi} L_c \omega_g)\hat{i}_d - e_d}{L_{dc} + 2L_c}$$

Commutation Angle

- During Commutation after valve 3 turns on

$$i_{abcg} = \begin{bmatrix} i_{ag} \\ -\hat{i}_d - i_{ag} \\ \hat{i}_d \end{bmatrix}$$

$$v_{as} - v_{bs} = 0$$

$$V_{as} = V_{ag} + L_c \, p \, i_{ag}$$

$$V_{bs} = V_{bg} + L_c \, p \, i_{bg}$$

$$i_{bg} = -\hat{i}_d - i_{ag}$$

$$p \, i_{bg} = - p \, i_{ag} \quad \text{(neglecting } p\hat{i}_d\text{)}$$

$$V_{bs} = V_{bg} - L_c \, p \, i_{ag}$$

$$V_{as} - V_{bs} = V_{ag} - V_{bg} + 2L_c \, p \, i_{ag}$$

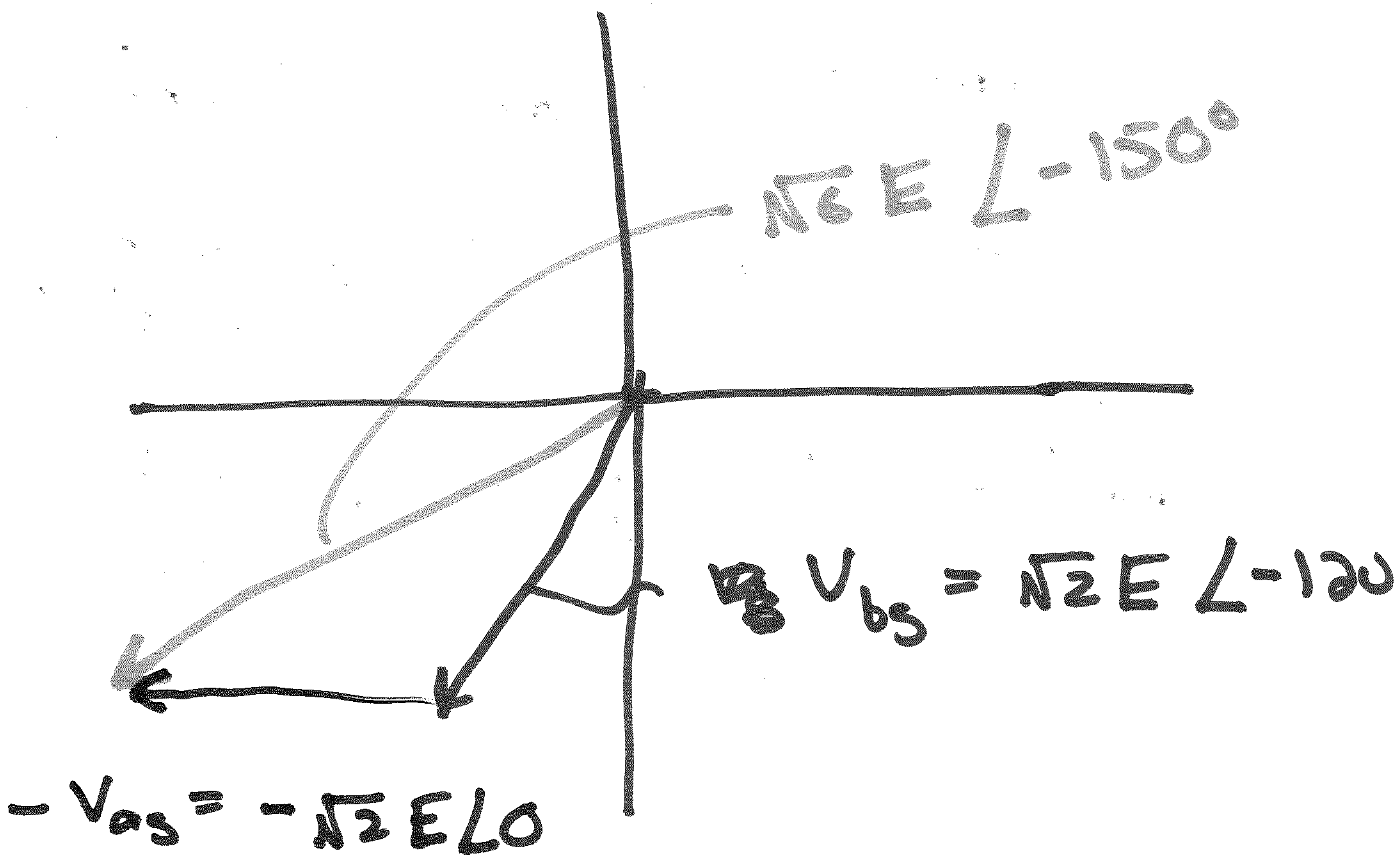
Commutation Angle

$$p\, i_{ag} = \frac{1}{2L_c}\left(v_{bg} - v_{ag}\right)$$

$$\omega_g \frac{d i_{ag}}{d\theta_g} = \frac{1}{2L_c}\left(v_{bg} - v_{ag}\right)$$

$$\frac{d i_{ag}}{d\theta_g} = \frac{1}{2L_c\omega_g}\left(v_{bg} - v_{ag}\right)$$

Commutation Angle



$$V_{bs} - V_{as} = \sqrt{6}E\cos(\theta_s - 5\pi/6)$$

Commutation Angle

$$\boxed{\frac{d i_{ag}}{d\theta_g} = \frac{\sqrt{6}E}{2L_c\omega_g}\cos(\theta_g - 5\pi/6)}$$

$$i_{ag} = \underbrace{-I_d}_{\text{I.C.}} + \int_{\frac{\pi}{3}+\alpha}^{\theta_g} \frac{d i_{ag}}{d\theta_g}\, d\theta_g$$

$c' = -s$

$s' = c$

$$i_{ag} = -I_d + \frac{\sqrt{6}E}{2L_c\omega_g}\left[\sin(\theta_g - 5\pi/6) - \underbrace{\sin\left(\frac{\pi}{3} + \alpha - \frac{5\pi}{6}\right)}\right]$$

$$\sin\left(\alpha - \frac{\pi}{2}\right) = -\cos\alpha$$

Commutation Angle

$$i_{as} = -i_d + \frac{\sqrt{6}\,E}{2 L_c \omega_s}\left[\sin\left(\theta_s - 5\pi/6\right) + \cos\alpha\right]$$

Commutation Angle

- Thus we get

$$i_{ag} = -\hat{i}_d - \frac{\sqrt{6}E}{2l_c\omega_g}\left[\sin\left(\theta_g + \frac{\pi}{6}\right) - \cos(\alpha)\right]$$

Commutation Angle

$$0 = -\hat{I}_d - \frac{\sqrt{6}E}{2L_c\omega_g}\left[\sin\left(\overbrace{\frac{\pi}{3}+\alpha+u}^{\theta_g}+\frac{\pi}{6}\right) - \cos\alpha\right]$$

$$\frac{2L_c\omega_g I_d}{\sqrt{6}E} = \cos\alpha - \underbrace{\sin\left(\alpha+u+\frac{\pi}{2}\right)}_{\cos(\alpha+u)}$$

$$\cos(\alpha+u) = \cos\alpha - \frac{2L_c\omega_g I_d}{\sqrt{6}E}$$

$$\boxed{u = \operatorname{acos}\left(\cos\alpha - \frac{2L_c\omega_g I_d}{\sqrt{6}E}\right) - \alpha}$$

Commutation Angle

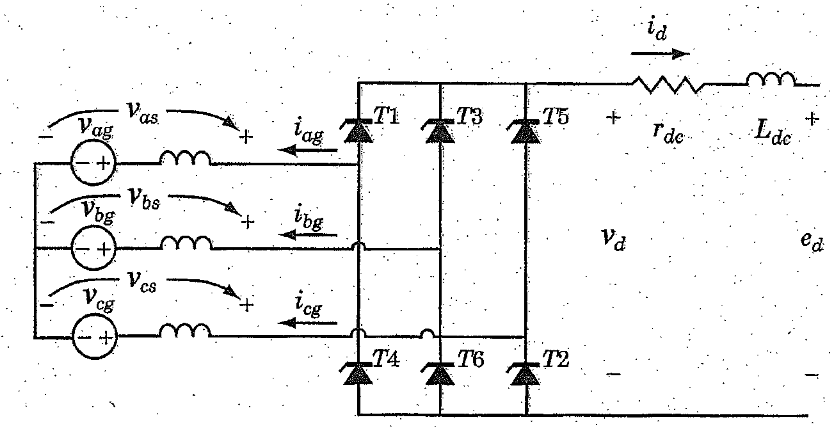
Commutation Angle

- Finally, we arrive at

$$u = -\alpha + \arccos\left(\cos(\alpha) - \frac{2 l_c \omega_g i_d}{\sqrt{6} E} \right) \qquad (11.3\text{-}43)$$

Restrictions on Firing And Commutation

- Let's think about the a-phase current



Recall

$$\frac{di_{as}}{d\theta_s} = \frac{\sqrt{6}E}{2L_c\omega_s}\cos\left(\theta_s - \frac{5\pi}{6}\right)$$

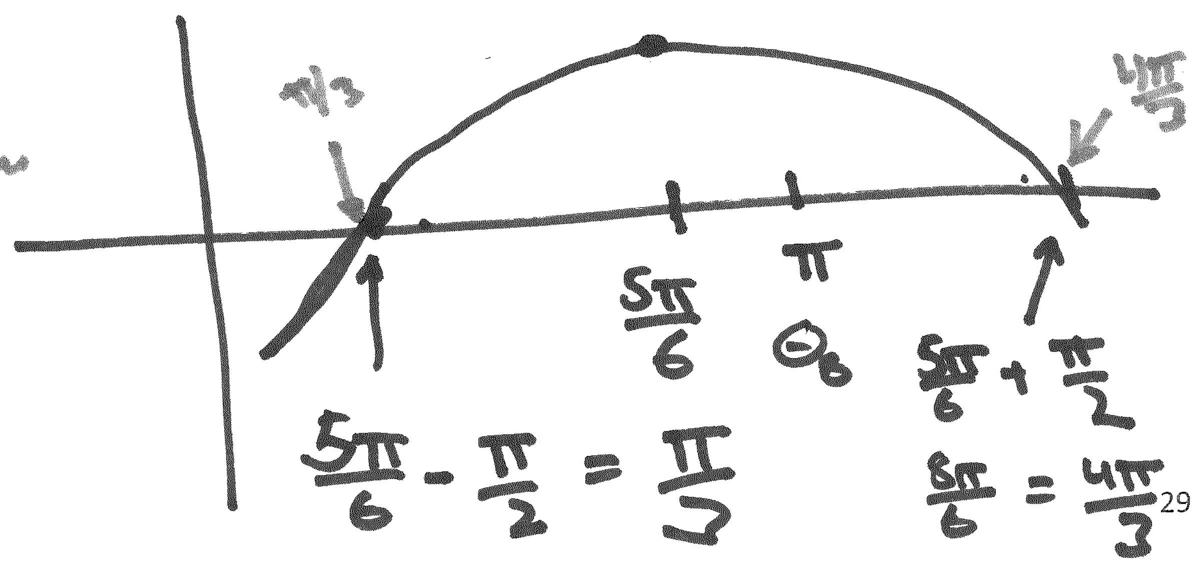
~~$\alpha > \pi$~~

$\theta_s > \frac{\pi}{3}$ when we turn valve 3 on

$\boxed{\alpha > 0}$

$\frac{\pi}{3} + \alpha + u \leq \frac{4\pi}{3}$

$\boxed{\alpha + u \leq \pi}$



Restrictions on Firing And Commutation

Restrictions on Firing and Commutation

- Thus, we must have
  - (1) $\alpha>0$
  - (2) $\alpha+u<\pi$

Modification for Mode II Operation

- We can use our work for Mode 2 as well as Mode 1 if we make some additional modifications…

$U = \frac{\pi}{3}$

Command valve 3 to turn on at $\theta_s = \frac{\pi}{3} + \alpha$

Valve 3 turns on at $\theta_s = \frac{\pi}{3} + \alpha_{eff}$

α_{eff} is whatever it takes to get $U = \frac{\pi}{3}$

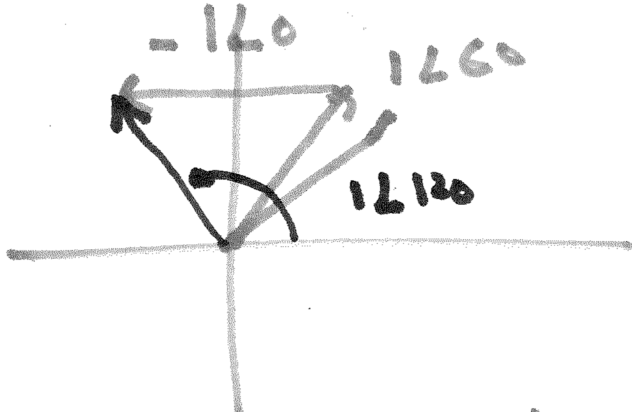
Modification for Mode II Operation

From slide 28

$$U = -\alpha + \operatorname{acos}\left(\cos(\alpha) - \frac{2L_c \omega_g I_d}{\sqrt{6}E}\right)$$

$$\cos\left(\frac{\pi}{3} + \alpha_{eff}\right) = \cos\alpha_{eff} - \frac{2L_c \omega_g I_d}{\sqrt{6}E}$$

$$\cos\left(\alpha_{eff} + \frac{\pi}{3}\right) - \cos\alpha_{eff} = -\frac{2L_c \omega_g I_d}{\sqrt{6}E}$$



$$\cos\left(\alpha_{ff} + \frac{2\pi}{3}\right) = -\frac{2L_c \omega_g I_d}{\sqrt{6}E}$$

$$\cos\left(\alpha_{eff} - \frac{\pi}{3}\right) = \frac{2L_c \omega_g I_d}{\sqrt{6}E}$$

$$\alpha_{eff} = \frac{\pi}{3} + \operatorname{acos}\left(\frac{2L_c \omega_g I_d}{\sqrt{6}E}\right)$$

Modification for Mode II Operation

- Thus, the effective firing angle must obey

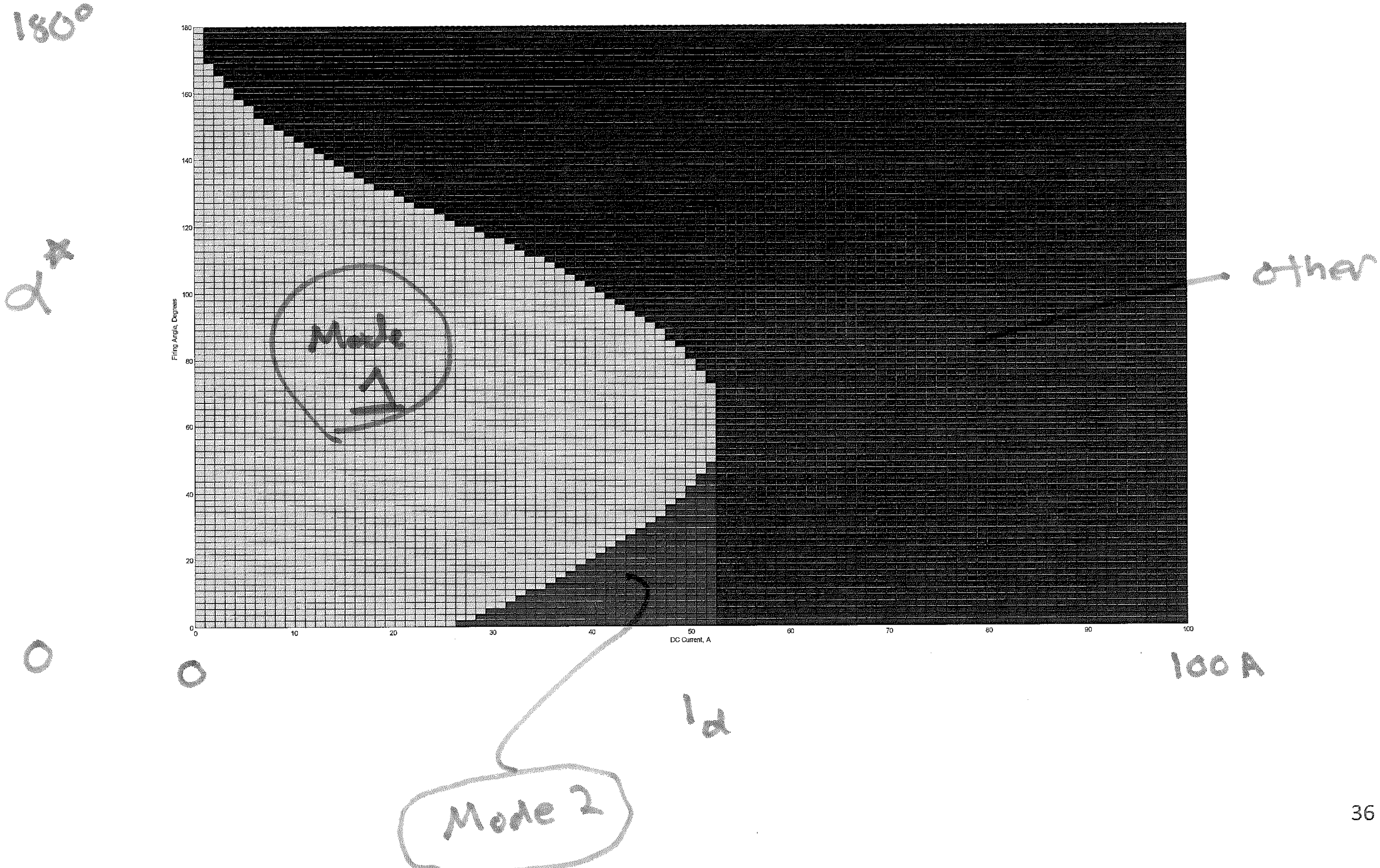
$$\cos(\alpha_{eff} - \pi / 3) = \frac{2 l_c \omega_g i_d}{\sqrt{6} E}$$

Numerical Example

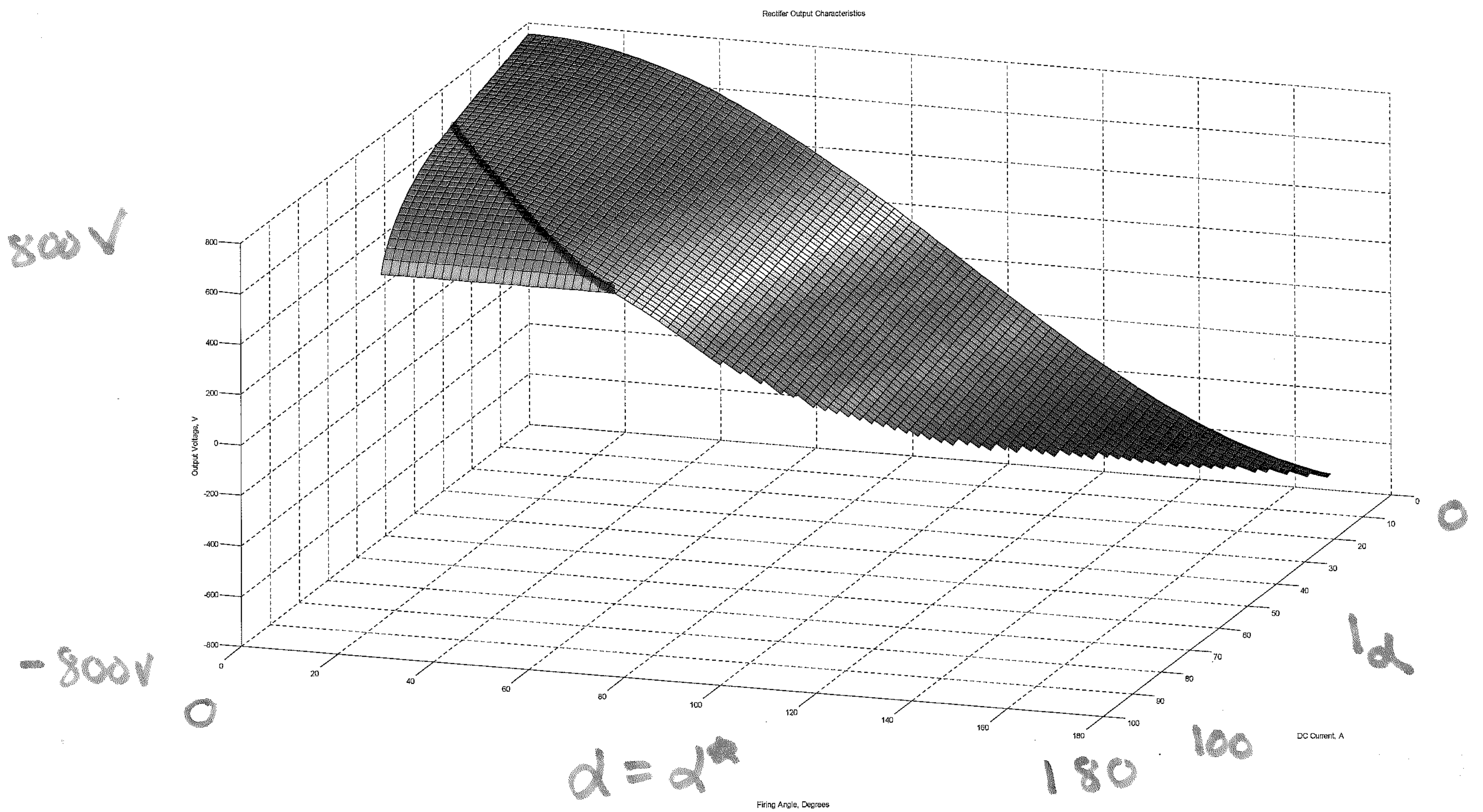
- Consider a system with a 560 V l-l rms voltage source, 20 mH commutating inductance, and operating at 60 Hz
- Let's look at the output characteristics as dc link current varied from 0 to 100 A

Operating Mode

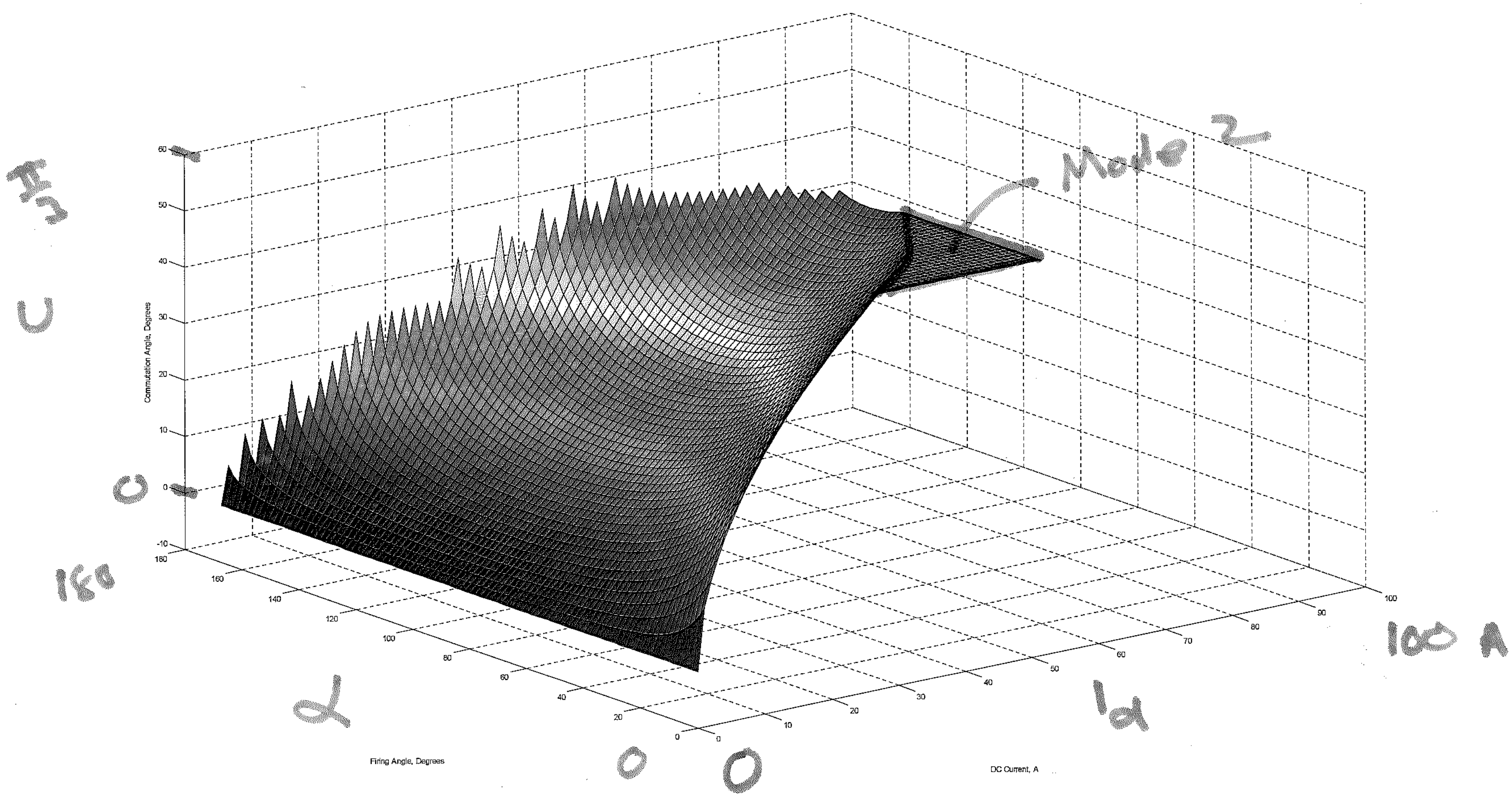
$\alpha = \alpha^*$



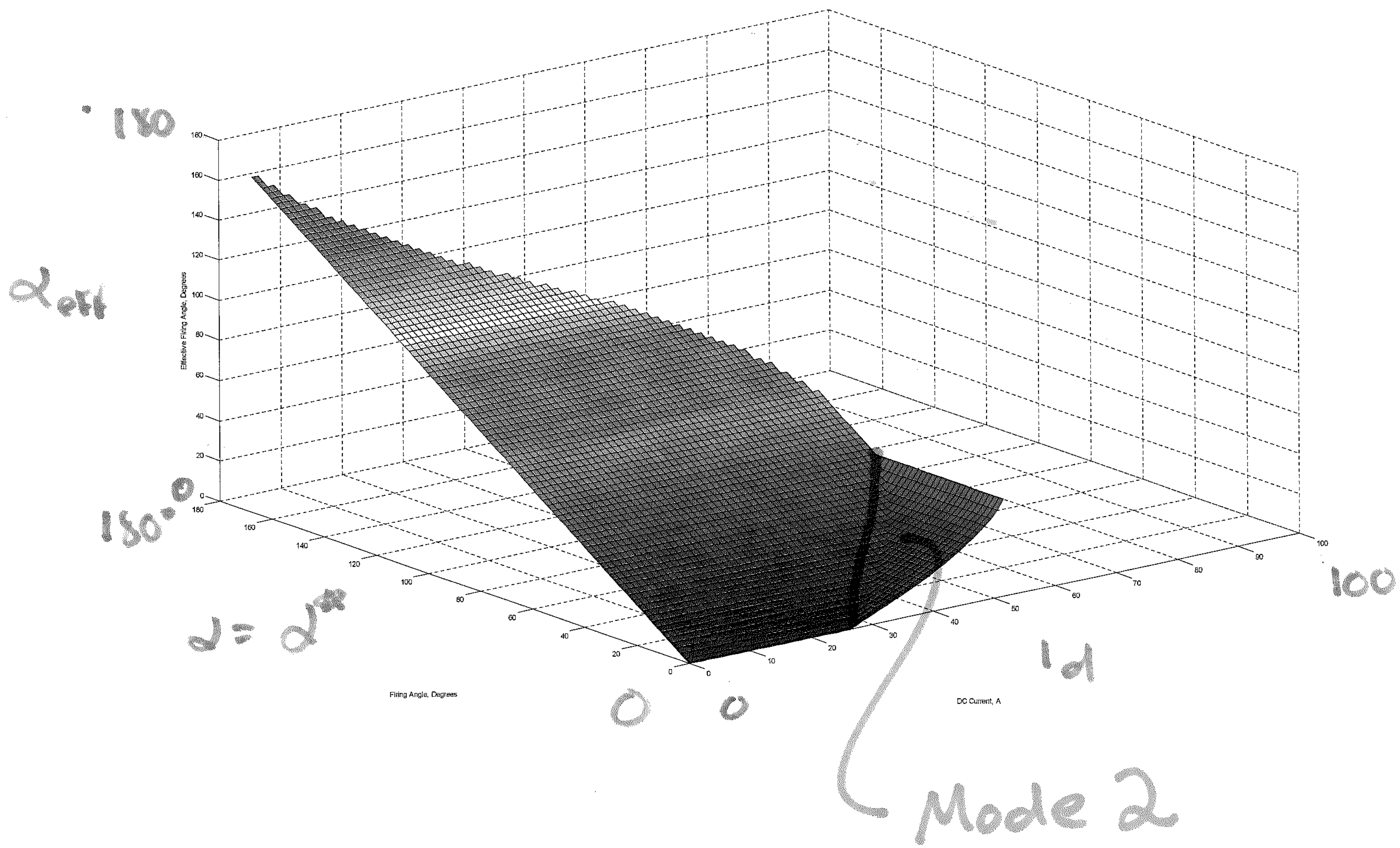
Output Voltage



Commutation Angle



Effective Firing Angle



Calculation of AC Currents

- Let's consider the current in the generation source reference frame, i.e. the reference frame wherein

$\theta = \theta_g$

$$v_{abcg} = \sqrt{2}E \begin{bmatrix} \cos(\theta_g) \\ \cos(\theta_g - 2\pi/3) \\ \cos(\theta_g + 2\pi/3) \end{bmatrix}$$

General Approach

- The average q- and d-axis current may be expressed
- DC variables, qd variables in the synchronous ref. frame, repeat every 60°

$$\bar{i}_{qg}^{g} = \frac{3}{\pi}\int_{\frac{\pi}{3}+\alpha}^{\frac{2\pi}{3}+\alpha} i_{qg}^{g}(\theta_g)d\theta_g$$

$$\bar{i}_{dg}^{g} = \frac{3}{\pi}\int_{\frac{\pi}{3}+\alpha}^{\frac{2\pi}{3}+\alpha} i_{dg}^{g}(\theta_g)d\theta_g$$

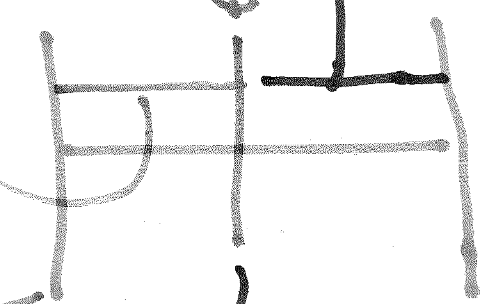
Written a mean diff

Commutation and Conduction Components

Commutation sub-interval

conduction sub-interval

$\theta_s = \frac{\pi}{3} + d_{eff} + u$



$\theta_s = \frac{\pi}{3} + d_{eff}$

$\theta_s = \frac{2\pi}{3} + d_{eff}$

- Breaking the integrals up we have

$$\bar{i}_{qg}^{g} = \bar{i}_{qg,com}^{g} + \bar{i}_{qg,cond}^{g}$$

$$\bar{i}_{dg}^{g} = \bar{i}_{dg,com}^{g} + \bar{i}_{dg,cond}^{g}$$

- where

$$\bar{i}_{qg,com}^{g} = \frac{3}{\pi} \int_{\frac{\pi}{3}+\alpha}^{\frac{\pi}{3}+\alpha+u} i_{qg,com}^{g}(\theta_g) d\theta_g \qquad \bar{i}_{qg,cond}^{g} = \frac{3}{\pi} \int_{\frac{\pi}{3}+\alpha+u}^{\frac{2\pi}{3}+\alpha} i_{qg,cond}^{g}(\theta_g) d\theta_g$$

$$\bar{i}_{dg,com}^{g} = \frac{3}{\pi} \int_{\frac{\pi}{3}+\alpha}^{\frac{\pi}{3}+\alpha+u} i_{dg,com}^{g}(\theta_g) d\theta_g \qquad \bar{i}_{dg,cond}^{g} = \frac{3}{\pi} \int_{\frac{\pi}{3}+\alpha+u}^{\frac{2\pi}{3}+\alpha} i_{dg,cond}^{g}(\theta_g) d\theta_g$$

Calculation of Commutation Component

- We start with

$$i_{abcg} = \begin{bmatrix} i_{ag} \\ -\bar{i}_d - i_{ag} \\ \bar{i}_d \end{bmatrix}$$

$$i_{ag}(\theta_g) = -\bar{i}_d + \frac{1}{l_c \omega_e} \frac{\sqrt{3}}{2} \sqrt{2} E \left[\cos(\alpha) - \cos\left(\theta_g - \frac{\pi}{3} \right) \right]$$

$$i_{gd}^{s} = K^{s} \Big|_{UTK} i_{abcg}$$

Calculation of Commutation Component

- Skipping the math, we arrive at

$$\begin{aligned}\bar{i}^{g}_{qg,com} &= \frac{2\sqrt{3}}{\pi}\bar{i}_d\left[\sin(u+\alpha-5\pi/6)-\sin(\alpha-5\pi/6)\right] \\ &+\frac{3}{\pi}\frac{\sqrt{2}E}{l_c\omega_g}\cos\alpha\left[\cos(u+\alpha)-\cos(\alpha)\right] \\ &+\frac{1}{4}\frac{3}{\pi}\frac{\sqrt{2}E}{l_c\omega_g}\left[\cos(2\alpha)-\cos(2\alpha+2u)\right]\end{aligned} \tag{11.5-53}$$

$$\begin{aligned}\bar{i}^{g}_{dg,com} &= \frac{2\sqrt{3}}{\pi}\bar{i}_d\left[-\cos(u+\alpha-5\pi/6)+\cos(\alpha-5\pi/6)\right] \\ &+\frac{3}{\pi}\frac{\sqrt{2}E}{l_c\omega_g}\cos\alpha\left[\sin(u+\alpha)-\sin(\alpha)\right] \\ &+\frac{1}{4}\frac{3}{\pi}\frac{\sqrt{2}E}{l_c\omega_g}\left[\sin(2\alpha)-\sin(2\alpha+2u)\right]-\frac{3}{\pi}\frac{\sqrt{2}E}{2l_c\omega_g}u\end{aligned} \tag{11.5-54}$$

Calculation of Conduction Component

- During the conduction interval

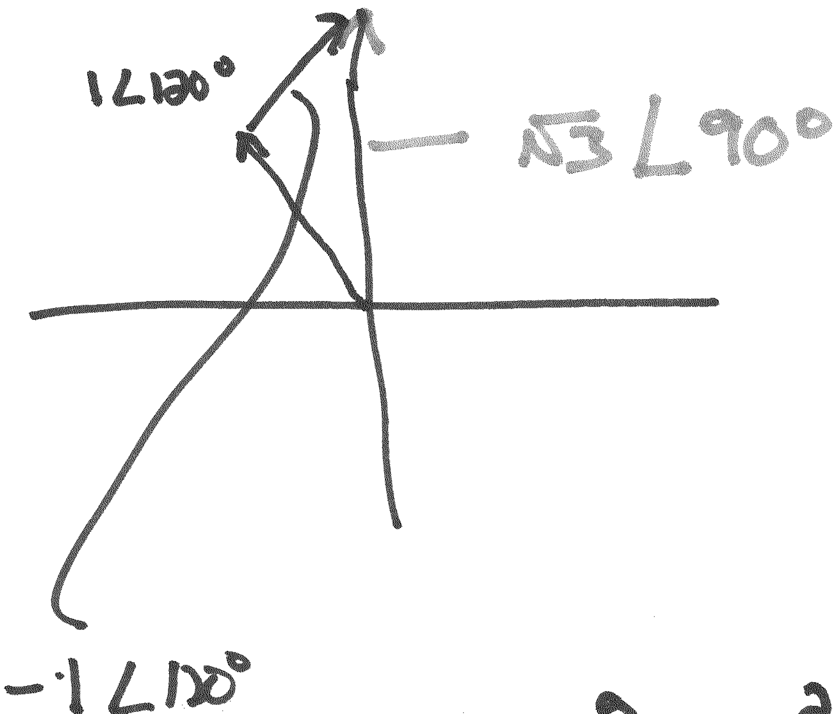
$$i_{abcg} = \begin{bmatrix} 0 \\ -\bar{i}_d \\ \bar{i}_d \end{bmatrix}$$

$c = \cos(\theta_s)$

$$i_{qd}^{s} = \frac{2}{3} \begin{bmatrix} c & c^- & c^+ \\ s & s^- & s^+ \end{bmatrix} \begin{bmatrix} 0 \\ -\bar{i}_d \\ \bar{i}_d \end{bmatrix}$$

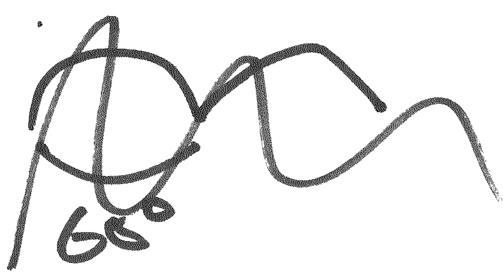
$$= \frac{2}{3} \bar{i}_d \begin{bmatrix} c^+ - c^- \\ s^+ - s^- \end{bmatrix}$$

Calculation of Conduction Component



$$i_{qd}^{s} = \frac{2\sqrt{3}}{3} i_d \begin{bmatrix} \cos(\theta_s + \frac{\pi}{2}) \\ \sin(\theta_s + \frac{\pi}{2}) \end{bmatrix}$$

$$i_{qd}^{s} = \frac{2\sqrt{3}}{3} i_d \begin{bmatrix} -\sin\theta_s \\ \cos\theta_s \end{bmatrix}$$



Calculation of Conduction Component

$$i^{a}_{q,cond} = \frac{\cancel{3}}{\pi} \int_{\frac{\pi}{3}+\alpha_{eff}+u}^{\frac{2\pi}{3}+\alpha_{eff}} \frac{2\sqrt{3}}{\cancel{2}} i_d \left(-\sin\theta_s\right) d\theta_s$$

$$C' = -S$$
$$S' = C$$

$$= \frac{2\sqrt{3}}{\pi} i_d \cos\theta_s \Big|_{\frac{\pi}{3}+\alpha_{eff}+u}^{\frac{2\pi}{3}+\alpha_{eff}}$$

$$= \frac{2\sqrt{3}}{\pi} i_d \left[\cos\left(\alpha_{eff} + \frac{2\pi}{3}\right) - \cos\left(\alpha_{eff} + u + \frac{\pi}{3}\right)\right]$$

$$+\frac{\pi}{2} \quad -\frac{\pi}{2}$$

Calculation of Conduction Component

- Thus we arrive at

α_{eff}

$$\bar{i}_{qg,cond}^{g} = \frac{2\sqrt{3}}{\pi}\bar{i}_d\left[\sin\left(\alpha + \frac{7\pi}{6}\right) - \sin\left(\alpha + u + \frac{5\pi}{6}\right)\right] \tag{11.5-55}$$

$$\bar{i}_{dg,cond}^{g} = \frac{2\sqrt{3}}{\pi}\bar{i}_d\left[-\cos\left(\alpha + \frac{7\pi}{6}\right) + \cos\left(\alpha + u + \frac{5\pi}{6}\right)\right] \tag{11.5-56}$$

Non-Generator Reference Frame Interface

- Basic Approach: The Frame-to-Frame Transformation

$$\mathbf{f}_{qd}^{y} = {}^{x}\mathbf{K}^{y}\mathbf{f}_{qd}^{x}$$

$${}^{x}\mathbf{K}^{y} = \begin{bmatrix} \cos(\theta_y - \theta_x) & -\sin(\theta_y - \theta_x) \\ \sin(\theta_y - \theta_x) & \cos(\theta_y - \theta_x) \end{bmatrix}$$

$${}^{a}K^{g} = \begin{bmatrix} \cos(\theta_x - \theta_y) & \sin(\theta_x - \theta_y) \\ -\sin(\theta_x - \theta_y) & \cos(\theta_x - \theta_y) \end{bmatrix}$$

Non-Generator Reference Frame Interface

- Now we have

$$\mathbf{v}_{qd,g}^{a} = \begin{bmatrix} v_{qg}^{a} \\ v_{dg}^{a} \end{bmatrix}$$

And we want

$$\mathbf{v}_{qd}^{g} = \begin{bmatrix} \sqrt{2}E \\ 0 \end{bmatrix}$$

- So choose

$$\theta_y = \theta_g \qquad \theta_x = \theta_a \qquad \phi_g = \cancel{\theta_x}_{a} - \cancel{\theta_y}_{g}$$

- Then

$${}^{a}\mathbf{K}^{g} = \begin{bmatrix} \cos(\phi_g) & \sin(\phi_g) \\ -\sin(\phi_g) & \cos(\phi_g) \end{bmatrix}$$

$${}^{g}K^{a} = \begin{bmatrix} c & -s \\ s & c \end{bmatrix}$$

$\cos(\phi_g)$

Non-Generator Reference Frame Interface

- Solving for E and ϕ_g

$$V^a_{qd} = {}^aK^g \, V^g_{qd}$$

$$= \begin{bmatrix} c & -s \\ s & c \end{bmatrix} \begin{bmatrix} \sqrt{2}E \\ 0 \end{bmatrix}$$

$$V^a_q = \sqrt{2}\, E \cos\phi_g$$

$$V^a_d = \sqrt{2}\, E \sin\phi_g$$

$$V^a_q + jV^a_d = \sqrt{2}E\, e^{j\phi_g}$$

+

$$\Rightarrow \sqrt{2}E = \sqrt{(V^a_q)^2 + (V^a_d)^2} \qquad \phi_g = \text{angle}\left(V^a_q + jV^a_d\right)$$

Non-Generator Reference Frame Interface

- Taking care of the currents

$$\mathbf{i}_{qd}^{g} = {}^{a}\mathbf{K}^{g}\,\mathbf{i}_{qd}^{a}$$

$$\mathbf{i}_{qd}^{a} = \left({}^{a}\mathbf{K}^{g}\right)^{-1}\mathbf{i}_{qd}^{g} = {}^{g}\mathbf{K}^{a}\,\mathbf{i}_{qd}^{g}$$

$${}^{g}\mathbf{K}^{a} = \begin{bmatrix} \cos(\phi_g) & -\sin(\phi_g) \\ \sin(\phi_g) & \cos(\phi_g) \end{bmatrix}$$

AVM Model Summary

- Step 1: Compute voltage parameters

$$E = \frac{1}{\sqrt{2}}\sqrt{\left(v_{qg}^{a}\right)^{2} + \left(v_{dg}^{a}\right)^{2}}$$

$$\phi_g = \mathrm{angle}(v_{qg}^{a} + jv_{dg}^{a})$$

- Step 2: Compute commutation and effective firing angles

$$u = -\alpha + \arccos\left(\cos(\alpha) - \frac{2l_c\omega_g i_d}{\sqrt{6}E}\right)$$

$$\alpha_{eff} = \frac{\pi}{3} - a\cos\left(\frac{2l_c\omega_g i_d}{\sqrt{6}E}\right)$$

AVM Model Summary

- Step 2: A little more detail…

Assume Mode 1

$\alpha_{eff} = \alpha$

iff u is defined, and $u < \frac{\pi}{3}$, and $u + \alpha_{eff} \leq \pi$

then done

otherwise

Assume Mode 2

iff α_{eff} is defined $u + \alpha_{eff} \leq \pi$

then done $(u = \pi/3)$

otherwise

out of luck

AVM Model Summary

- Step 3: DC Link Dynamics α_{eff}

$$p\hat{i}_d = \frac{\frac{3\sqrt{6}}{\pi} E \cos \cancel{\alpha} - (r_{dc} + \frac{3}{\pi} l_c \omega_g)\hat{i}_d - e_d}{L_{dc} + 2l_c}$$

- Step 4: AC Currents

$$\bar{i}^g_{qg,com} = \frac{2\sqrt{3}}{\pi}\bar{i}_d\left[\sin(u+\alpha-5\pi/6) - \sin(\alpha-5\pi/6)\right] + \frac{3}{\pi}\frac{\sqrt{2}E}{l_c\omega_g}\cos\alpha\left[\cos(u+\alpha) - \cos(\alpha)\right] + \frac{1}{4}\frac{3}{\pi}\frac{\sqrt{2}E}{l_c\omega_g}\left[\cos(2\alpha) - \cos(2\alpha+2u)\right]$$

$$\bar{i}^g_{dg,com} = \frac{2\sqrt{3}}{\pi}\bar{i}_d\left[-\cos(u+\alpha-5\pi/6) + \cos(\alpha-5\pi/6)\right] + \frac{3}{\pi}\frac{\sqrt{2}E}{l_c\omega_g}\cos\alpha\left[\sin(u+\alpha) - \sin(\alpha)\right] + \frac{1}{4}\frac{3}{\pi}\frac{\sqrt{2}E}{l_c\omega_g}\left[\sin(2\alpha) - \sin(2\alpha+2u)\right] - \frac{3}{\pi}\frac{\sqrt{2}E}{2l_c\omega_g}u$$

$\alpha = \alpha_{eff}$

AVM Model Summary

- Step 4: Continued …

$\alpha = \alpha_{eff}$

$$\overline{i}_{qg,cond}^{g} = \frac{2\sqrt{3}}{\pi}\overline{i}_d\left[\sin\left(\alpha + \frac{7\pi}{6}\right) - \sin\left(\alpha + u + \frac{5\pi}{6}\right)\right]$$

$$\overline{i}_{qg}^{g} = \overline{i}_{qg,com}^{g} + \overline{i}_{qg,cond}^{g}$$

$$\overline{i}_{dg,cond}^{g} = \frac{2\sqrt{3}}{\pi}\overline{i}_d\left[-\cos\left(\alpha + \frac{7\pi}{6}\right) + \cos\left(\alpha + u + \frac{5\pi}{6}\right)\right]$$

$$\overline{i}_{dg}^{g} = \overline{i}_{dg,com}^{g} + \overline{i}_{dg,cond}^{g}$$

- Step 5: Transform currents to desired reference frame

ECE61016 Power Electronics Converters and Systems

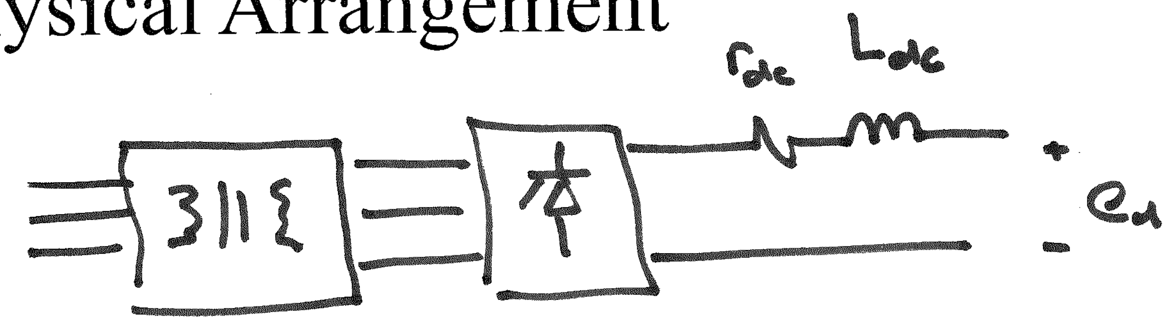
AC/DC Conversion with Passive Rectifiers Lecture Set 7B

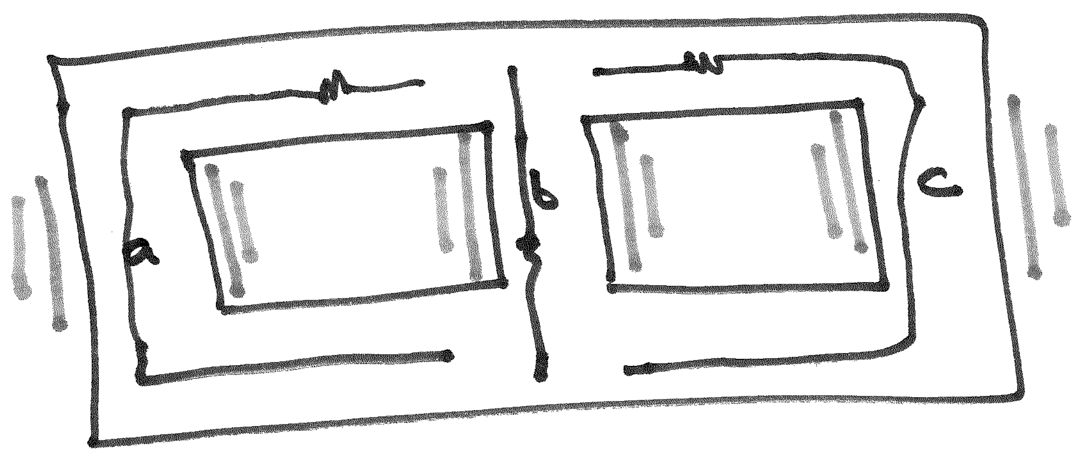
S.D. Sudhoff

Fall 2016

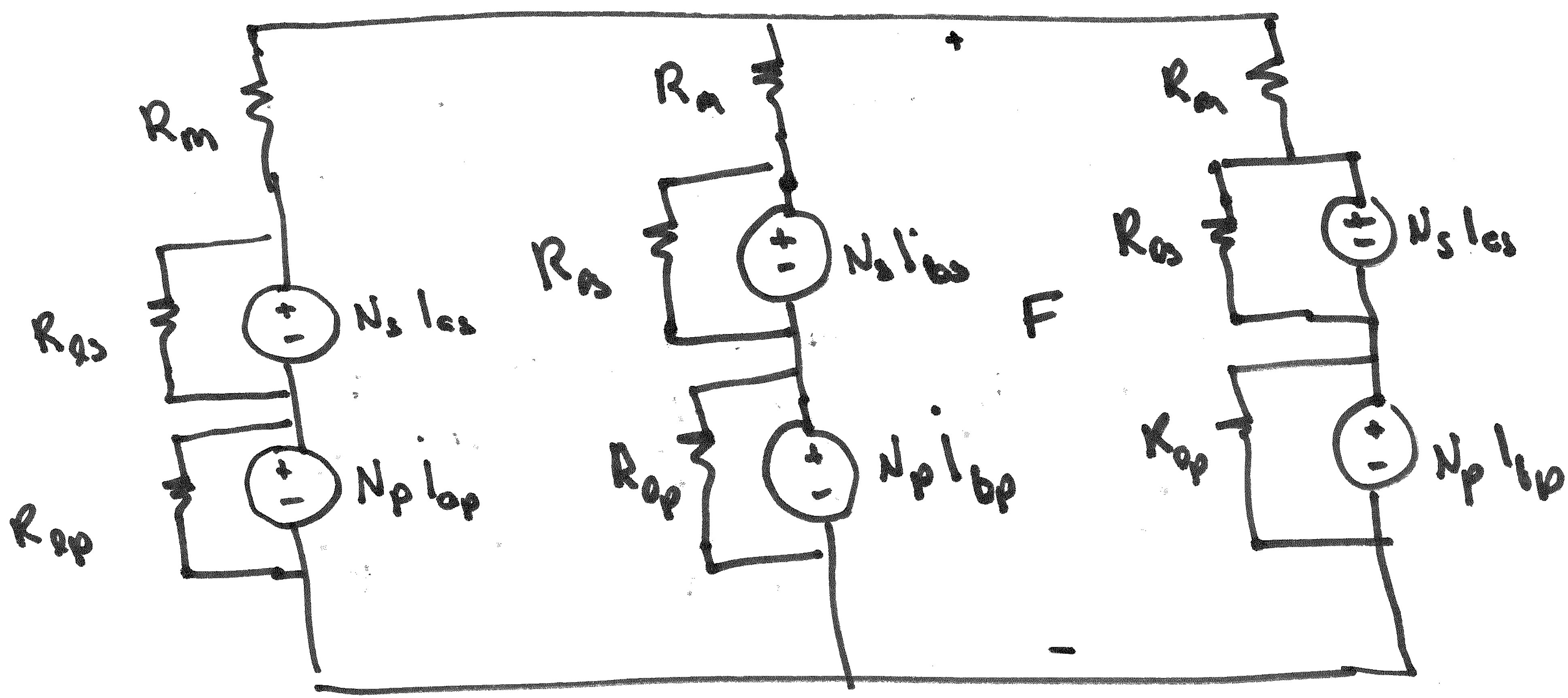
The Transformer Rectifier

- Physical Arrangement





Transformer Equiv. Circuit



Modification of Equivalent Circuit

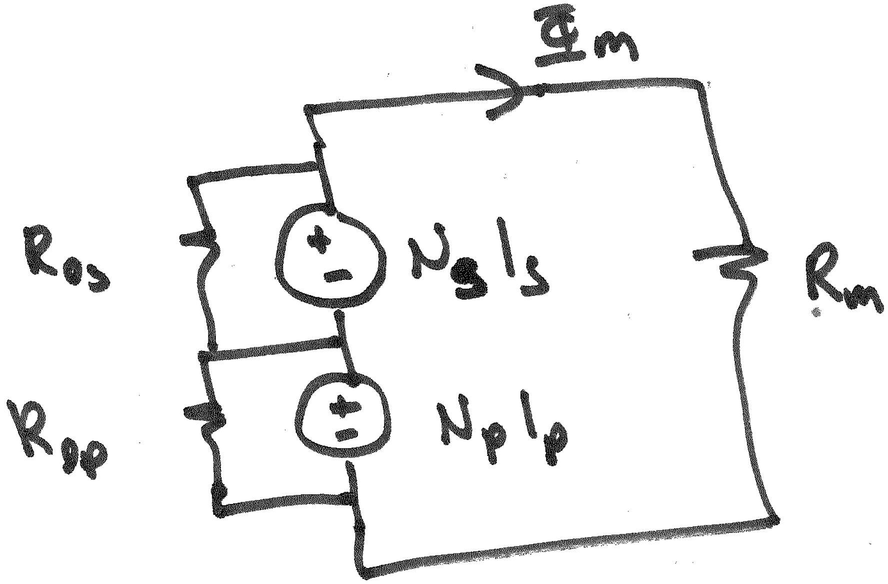
$$\frac{F - N_s i_{as} - N_p i_{ap}}{R_m} + \frac{F - N_s i_{bs} - N_p i_{bp}}{R_m} + \frac{F - N_s i_{cs} - N_p \cancel{i_{cp}}\, i_{cp}}{R_m} = 0$$

Assume ① $i_{as} + i_{bs} + i_{cs} = 0$

② $i_{ap} + i_{bp} + i_{cp} = 0$

then $F = 0$

Modification of Equivalent Circuit



$$\Phi_m = \frac{N_s I_s + N_p I_p}{R_m}$$

$$\lambda_s = N_s\left(\frac{N_s I_s}{R_{\ell s}} + \Phi_m\right) = \overbrace{\frac{N_s^2}{R_{\ell s}}}^{L_{\ell s}} I_s + N_s \Phi_m$$

$$\lambda_p = N_p\left(\frac{N_p I_p}{R_{\ell p}} + \Phi_m\right) = \underbrace{\frac{N_p^2}{R_{\ell p}}}_{L_{\ell p}} I_p + N_p \Phi_m$$

Modification of Equivalent Circuit

$$\lambda_s = L_{\ell s} i_s + \frac{N_s^2}{R_m} i_s + \frac{N_s N_p}{R_m} i_p$$

$$\lambda_p = L_{\ell p} i_p + \frac{N_p^2}{R_m} i_p + \frac{N_s N_p}{R_m} i_s$$

Let :

$$\lambda_p' = \beta \lambda_p$$

$$v_p' = \beta v_p$$

$$i_p' = \frac{1}{\beta} \cdot i_p$$

Modification of Equivalent Circuit

$$\lambda_s = L_{ls} i_s + \frac{N_s^2}{R_m} i_s + \frac{N_s N_p}{R_m} \beta\, i_p'$$

$$= \underbrace{\left[L_{ls} + \frac{N_s^2}{R_m} - \frac{N_s N_p}{R_m}\beta \right]}_{L_{ls}'} i_s + \underbrace{\left[\frac{N_s N_p}{R_m}\beta \right]}_{L_m} (i_s + i_p')$$

$$\lambda_p' = \beta \left[L_{lp} \beta\, i_p' + \frac{N_p^2}{R_m} \beta\, i_p' + \frac{N_s N_p}{R_m} i_s \right]$$

$$= \underbrace{\left[L_{lp}\beta^2 + \frac{N_p^2}{R_m}\beta^2 - \frac{N_s N_p}{R_m}\beta \right]}_{L_{lp}'} i_p' + \underbrace{\frac{N_s N_p}{R_m}\beta}_{L_m} (i_s + i_p')$$

Modification of Equivalent Circuit

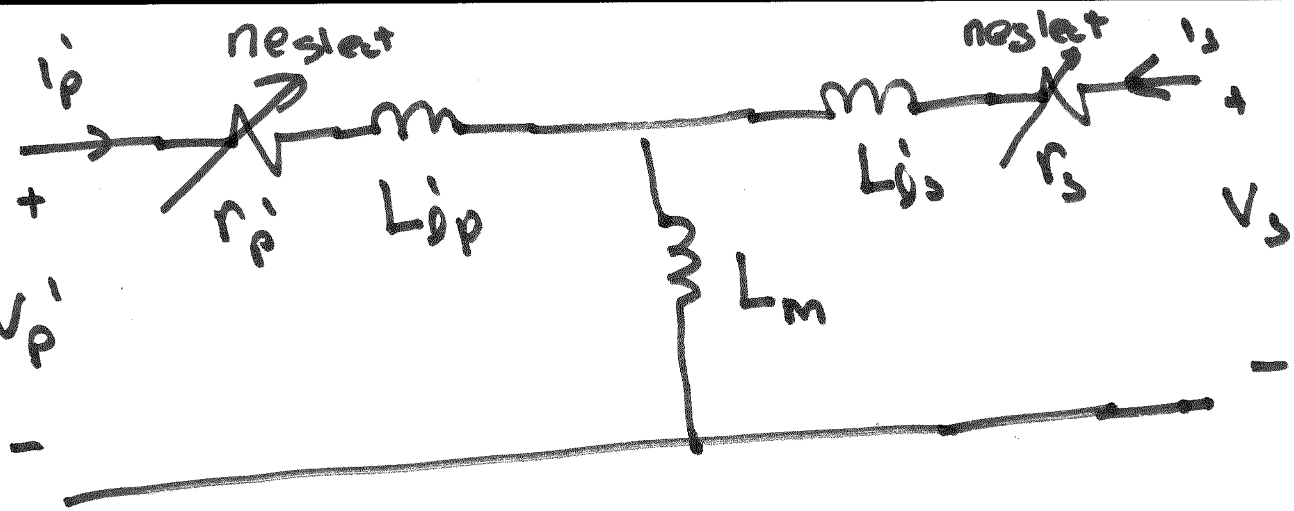
$$V_s = r_s i_s + \frac{d\lambda_s}{dt}$$

$$V_p = r_p i_p + \frac{d\lambda_p}{dt}$$

$$V_p' = \beta \left[r_p \beta i_p' + \frac{d}{dt}\left(\frac{1}{\beta} \lambda_p'\right) \right]$$

$$= \underbrace{r_p \beta^2}_{r_p'} i_p' + \frac{d\lambda_p'}{dt}$$

Modification of Equivalent Circuit



Modification of Equivalent Circuit

Choosing β

We want $L'_{\ell p} = 0$

$$L_{\ell p}\beta^{\cancel{2}} + \frac{N_p^2}{R_m}\beta^{\cancel{2}} - \frac{N_s N_p}{R_m}\cancel{\beta} = 0$$

$$\beta = \frac{\frac{N_s N_p}{R_m}}{L_{\ell p} + \frac{N_p^2}{R_m}} = \frac{N_s N_p}{R_m L_{\ell p} + N_p^2}$$

$$\beta = \frac{N_s N_p}{N_p^2}\left[\frac{1}{\frac{R_m}{N_p^2}L_{\ell p} + 1}\right] = \frac{N_s}{N_p}\left[\frac{1}{\frac{L_{\ell p}}{L_{mp}} + 1}\right]$$

$$L_m = \frac{N_s N_p}{R_m}\beta \qquad L_{mp} = \frac{N_p^2}{R_m}$$

Modification of Equivalent Circuit

$$L'_{ls} = L_{ls} + \frac{N_s^2}{R_m} - \frac{N_s \cancel{N_p}}{R_m} \frac{N_s}{\cancel{N_p}} \left[\frac{1}{\frac{L_{lp}}{L_{mp}} + 1} \right]$$

$$= L_{ls} + \frac{N_s^2}{R_m} \left[1 - \frac{L_{mp}}{L_{lp} + L_{mp}} \right]$$

$$= L_{ls} + \frac{N_s^2}{R_m} \left[\frac{L_{lp}}{L_{lp} + L_{mp}} \right]$$

$$= L_{l\cancel{s}} + \left(\frac{N_s}{N_p}\right)^2 \frac{L_{mp} L_{lp}}{L_{lp} + L_{mp}}$$

$$L'_{ls} = L_{l\cancel{s}} + \left(\frac{N_s}{N_p}\right)^2 \left[L_{lp} \parallel L_{mp} \right] = L_c$$

commutating inductance

$$\approx L_{l\cancel{s}} + \left(\frac{N_s}{N_p}\right)^2 L_{lp}$$

Modification of Equivalent Circuit