Power Density Comparison of Three-Phase AC Inductor Architectures

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Abstract—Three-phase inductors are often used in filtering applications in three-phase ac power distribution systems. A variety of magnetic architectures can be used to implement these inductors. In this work, two studies are performed comparing four possible architectures—UI-, EI-, Y- and delta-core. The first study focuses on low-frequency steel-core inductors; the second study focuses on a high switching frequency application. Results show that Y-core and delta-core inductors offer reduced mass for a given loss in both cases.

Keywords—3-phase inductor; multi-objective optimization, magnetic analysis

I. INTRODUCTION

Power density of all components is clearly of interest in applications such as shipboard and aerospace power systems. The focus of this paper is on the three-phase ac inductors in these systems. These may be used as part of tuned filters to eliminate particular harmonics caused by, for example, a rectifier load (this would be a low-frequency application), or in the LC or LCL filter used in conjunction with an inverter for dc/ac conversion (this would be a high-frequency application).

In the area of power filter design, much of the focus has been to determine optimal values of the filter components [1-4]. While reduction in the filter size is possible through appropriate selection of filter topology and parameter values, for a given desired parameters (i.e. inductance value) the filter size can also be reduced by selection of the inductor magnetic architecture. This paper will explore the impact of the inductor architecture on the inductor size.

Several 3-phase inductor architectures are possible. Some of them include toroidal, UI-core, and E-core geometries. E-core and EI core inductors are advantageous in that the use of a coupled magnetic path can be used to reduce overall inductor size. However, these architectures have an inherent asymmetry resulting in imbalance. For this reason, 3-phase symmetric magnetic architectures have been explored in [5]. In this paper, four architectures are compared, UI-, EI-, Y-, and delta-core.

This paper proceeds as follows. Coil geometry that is common to any inductor architecture is described in Section II. Section III comments on 3-phase inductor magnetic analysis. The description of four architectures considered in this study and their MECs are presented in Section IV. Section V describes the 3-phase inductor multi-objective optimization based design process. Two case studies comparing 3-phase inductor loss versus mass characteristics are carried out for low frequency (steel core) and high frequency (ferrite core) applications. The results of these studies are presented in Section VI followed by conclusions in Section VII.

II. COIL GEOMETRY

This section describes the details of coil windings that are common to the different architectures. Two different coil geometries are possible depending on the core post geometry, with rectangular and circular cross sections as shown in Fig. 1. In both cases, a minimum clearance, $c_{cc}$ is ensured between the core surface and the winding. In addition, the coil geometry includes a minimum conductor bend radius, $r_{cl}$. For this reason, the case of rectangular coil has rounded corners as shown in Fig. 1(a).

The winding turns are arranged in layers within a coil along the dimension $d_w$ as illustrated in Fig. 2. The variables $N_d$ and $N_l$ respectively denote the number of turns per layer and the number of layers. This results in total number of turns in the coil, $N = N_d N_l$. Each coil conductor is comprised of $N_w$ strands connected in parallel. The individual strand cross sectional area, $a_{sc}$ is given by

$$a_{sc} = a_{c} / N_w$$  \hspace{1cm} (1)

where $a_{c}$ denotes the cross-sectional area of each conductor. The parallel strands arrangement across the coil cross section is illustrated in Fig. 2.

The coil depth, $d_w$ and width, $w_w$ as depicted in Fig. 2 are obtained as

$$d_w = 2k_d N_d N_w r_{sc}$$ \hspace{1cm} (2)
$$w_w = 2k_r N_l r_{sc}$$ \hspace{1cm} (3)

where $k_d$ denotes winding build factor. The coil cross sectional area, $A_{cl}$ is expressed in terms of coil depth and width as

$$A_{cl} = d_w w_w$$ \hspace{1cm} (4)
Using the cross sectional areas, \( A_{cl} \) and \( a_c \), the coil packing factor, \( k_{pf} \) is defined as
\[
k_{pf} = \frac{A_{cl}}{(N_a)}
\]
The coil volume, \( V_{cl} \), is given by
\[
V_{cl} = A_{cs} d_w
\]
where the coil area from the top view, \( A_{cs} \) depends on the shape of the coil as
\[
A_{cs} = \begin{cases} 
\pi \left( 2w_r r_{cl} + w_{cl}^2 \right) + 2(w_{cl} + l_c)w_w & \text{rectangular} \\
\pi \left( 2w_r r_{cl} + w_{cl}^2 \right) & \text{circular}
\end{cases}
\]
Using the conductor material volumetric mass density, \( \rho_{cd} \), and volume, \( V_{cl} \), the coil mass, \( M_{cl} \) is given by
\[
M_{cl} = \rho_{cd} k_{pf} V_{cl}
\]
The coil DC resistance, \( R_{cl} \), is useful in estimating the low-frequency resistive inductor loss. This is obtained using the coil geometry and conductor material conductivity, \( \sigma_{cd} \) as
\[
R_{cl} = \frac{V_{cl} N_a^2}{\sigma_{cd} k_{pf} A_{cs}^2}
\]

General comments on the magnetic analysis of 3-phase inductors are presented in the next section.

III. Comments on Magnetic Analysis

The inductor magnetic analysis will be based on Magnetic Equivalent Circuits (MECs) which can offer highly accurate analysis of magnetic systems [6]. Based on the geometry, flux paths, and excitation currents, an MEC for each of the inductor architectures is derived, using a procedure similar to that described in [6]. Primarily, core and fringing flux paths are considered for the comparison studies. Leakage flux paths will not be considered for the sake of brevity.

A core branch permeance, \( P_{br,j} \), is defined as a function of the branch flux, \( \Phi_{br,j} \), as
\[
P_{br,j}(\Phi_{br,j}) = A_{br,j} \mu_0 (\Phi_{br,j}/A_{br,j})/l_{br,j}
\]
where \( A_{br,j} \) is the cross section of the branch, \( l_{br,j} \) is the magnetic path length and \( \mu_0 \) is the magnetic permeability defined as a function of flux density in the branch. This accounts for the magnetically non-linear nature of the core material.

The air gap flux utilizes both direct and fringing paths. The air gap permeance associated with a direct path is given by
\[
P_{gd}(A_g) = \mu_0 A_g / g
\]
where \( g \) and \( A_g \) denote the air gap length and cross sectional area, respectively.

Two possible fringing flux paths are possible depending on the geometry of the core as shown in Fig. 3. The permeance due to fringing flux is defined as function of geometrical quantities, for case (a) as
\[
P_{fa}(l, w) = \frac{\mu_0 l}{\pi} \ln\left(1 + \frac{\pi w}{g}\right)
\]
and for case (b) as,
\[
P_{fb}(l, w) = \frac{2\mu_0 l}{\pi} \ln\left(1 + \frac{\pi w}{2g}\right)
\]

In (12) and (13), the variable \( l \) denotes dimension into the page and \( w \) denotes the width of the fringing flux path.

The magnetic analysis using MEC solves for branch flux, \( \Phi_{br,j} \) using iterative solver due to the magnetically non-linear nature of the core material. The branch flux is used to calculate winding flux linkage of each phase, \( \lambda_x \), \( \forall x \in \{a', b', c'\} \) using
\[
\lambda_x = N \Phi_{br,x}
\]
The 3-phase inductor performance variables may be described using phase variables or \( qd \) variables. To transform between phase variables and \( qd \) variables, the transformation
\[
f_{qd0} = K_{x} f_{abc}
\]
is used where
\[
f_{qd0} = [f_q \ f_d \ f_0]^T
\]
\[
f_{abc} = [f_a \ f_b \ f_c]^T
\]
and the transformation matrix, \( K_x \) is
In (18), $\theta$ is the position of an arbitrary reference frame.

For the comparison studies presented in this paper, synchronous reference frame is used with the inductor current excitation in the $q$-axis. The 3-phase inductor performance is compared in terms of the incremental $q$-axis self inductance and incremental $qd$ mutual inductance as

$$L_q = \frac{\partial^2 \lambda_q}{\partial i_q^2}$$  \hspace{1cm} (19)$$

$$L_{dq} = \frac{\partial^2 \lambda_d}{\partial i_q^2}$$  \hspace{1cm} (20)

Note that ideally the $qd$ mutual inductance is zero; however asymmetry will cause a magnetic coupling between the $q$- and $d$-axis.

When operating at radian fundamental frequency $\omega_f$, the incremental inductances change with the position of the synchronous reference frame because of the inductor asymmetry due to magnetic architecture or saturation. This variation in inductance is captured by determining $q$-axis and $qd$ mutual inductances over one cycle as shown in Fig. 4.

The core branch fluxes determined using MEC analysis are also useful in estimating core loss using Modified Steinmetz Equation [7]. Flux density in core leg is obtained as

$$B_{br,i} = \Phi_{br,i} / A_{br,i}$$  \hspace{1cm} (21)

whereupon the core loss density of the core leg, $p_{br,i}$, is calculated by [7]. The branch core loss is obtained by multiplying the core loss density with branch volume as

$$P_{br,i} = p_{br,i}A_{br,i}$$  \hspace{1cm} (22)

The total core loss of the inductor is taken as summation over all core branches as

$$P_c = \sum_{i=1}^{n} P_{br,i}$$  \hspace{1cm} (23)

Because of the desire to keep this inductor comparison generic, the details of the application are not considered. Thus the inclusion of core losses is based only on the fundamental component. For some cases, the actual core loss will thus be higher than is predicted by (23).

The following section describes the geometry and MEC of each 3-phase inductor architecture considered

IV. THREE-PHASE INDUCTOR ARCHITECTURES

A. UI core

Three single-phase inductors can be used to form a 3-phase inductor. This option is simple and symmetric. A possible architecture for a single-phase inductor is a UI-core geometry, as shown in Fig. 5. This UI-core architecture is possible with both steel and ferrite as core materials.

The winding geometry takes the form of rounded rectangle as shown in Fig. 1(a).

The dimensions illustrated in Fig. 5 are related by

$$w_e = w_i + 2d_i + 2r_c$$  \hspace{1cm} (24)

$$d_s = w_w + c_d$$  \hspace{1cm} (25)

$$w_s = d_w + 2c_w$$  \hspace{1cm} (26)

The total height, $h_f$, width, $w_f$, and length, $l_f$ of the single-phase UI-core inductor are

$$h_f = w_i + g + 2d_s + w_b$$  \hspace{1cm} (27)

$$w_f = 2w_w + w_s$$  \hspace{1cm} (28)

$$l_f = l_c + 2(w_w + r_c)$$  \hspace{1cm} (29)
Using the core volumetric mass material density, \( \rho_{cr} \), the core mass, \( M_{cr} \) is given by

\[
M_{cr} = \rho_{cr} l_j (w_y + w_i)(w_y + w_x + 2d_i w_y)
\]

(30)

Using (8) to calculate the coil mass, the total mass of the 3-phase inductor is expressed

\[
M_i = 3(M_{cr} + M_{cl})
\]

(31)

The MEC of a single-phase UI-core is depicted in Fig. 6. The core permeances are defined using (10) with the magnetic path lengths and cross-sectional areas as listed in Table II. The air gap permeance, \( P_g \) includes the direct path for the flux as well as fringing flux associated with inner, outer, front, and back edges of the core end leg, and is given by

\[
P_g = P_{gd}(A_{bc}) + 2P_{fa}(w_y, \min(w_i, d_y + w_y)) + P_{fa}(l_j, \min(w_i, d_y + w_y)) + P_{fb}(l_j, \min(d_y, w_y / 2))
\]

(32)

Fig. 6. MEC of UI-core single-phase inductor

<table>
<thead>
<tr>
<th>TABLE I. UI-CORE MAGNETIC PATH LENGTHS AND AREAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b_{bc} = w_x + w_y )</td>
</tr>
<tr>
<td>( a_{bc} = w_p l_c )</td>
</tr>
</tbody>
</table>

B. EI core

An EI-core is a common 3-phase inductor architecture. As with the UI-core architecture, both steel and ferrite core materials are possible with this architecture. The cross-sections of a 3-phase EI-core inductor from the front view and side view are shown in Fig. 7(a) and Fig. 7(b) respectively.

The three coils are wrapped around the three legs of the E core. Similar to the UI-core inductor, the coils in the case of the EI-core have rounded rectangle shape. As the core end-leg width, \( w_c \), may be different from the center leg width, \( w_c \), the coil end widths wound around end-leg, \( w_{cl,e} \), and center-leg \( w_{cl,c} \), are different. From geometry,

\[
w_{cl,e} = 2w_c + w_x - 2r_{cl}
\]

(33)

\[
w_{cl,c} = 2w_c + w_x - 2r_{cl}
\]

(34)

The slots formed by the EI-core accommodate two coils resulting in slot dimensions

\[
d_s = d_w + c_w
\]

(35)

The slots formed by the EI-core accommodate two coils resulting in slot dimensions

\[
d_s = d_w + c_w
\]

(35)

The mass of the core is given by

\[
M_{cr} = \rho_{cr} l_j \left((w_y + w_i)(2w_y + 2w_x + w_c)\right)
\]

(32)

The total mass of the inductor, including the coils is given by

\[
M_i = M_{cr} + 3M_{cl}
\]

(31)

The MEC of EI core inductor is as shown in Fig. 8. The effective magnetic lengths and cross-sectional areas used to calculate the core branch permeances by using (10) are listed in Table II. The air gap permeances on the end leg, \( P_{eg} \), and center-leg, \( P_{cg} \), are given by

\[
P_{eg} = P_{gd}(A_{bc}) + 2P_{fa}(w_y, \min(w_i, d_y + w_y)) + P_{fa}(l_j, \min(w_i, d_y + w_y)) + P_{fb}(l_j, \min(d_y, w_y / 2))
\]

(32)

\[
P_{cg} = P_{gd}(A_{bc}) + 2P_{fa}(w_x, \min(w_i, d_x + w_x)) + 2P_{fa}(l_j, \min(d_x, w_y / 2))
\]

(33)

\[
P_{eg} = P_{gd}(A_{bc}) + 2P_{fa}(w_x, \min(w_i, d_x + w_x)) + 2P_{fa}(l_j, \min(d_x, w_y / 2))
\]

(34)

The total height, \( h_l \), width, \( w_l \), and length, \( l_l \) of the EI-core inductor are

\[
h_l = w_x + g + d_x + w_y
\]

(36)

\[
w_y = 2(w_y + w_x) + w_y + 2(w_x + c_w)
\]

(37)

\[
l_l = l_j + 2(w_x + r_{cl})
\]

(38)

The mass of the core is given by

\[
M_{cr} = \rho_{cr} l_j \left((w_y + w_i)(2w_y + 2w_x + w_c)\right)
\]

(39)

The total mass of the inductor, including the coils is given by

\[
M_i = M_{cr} + 3M_{cl}
\]

(40)

Fig. 7. EI-core 3-phase inductor

\[
w_y = 2w_x + 2c_{cw} + c_{cc}
\]

(41)

Fig. 8. MEC of EI-core 3-phase inductor
TABLE II. EI-CORE MAGNETIC PATH LENGTHS AND AREAS

| l_{bc} = w_e + w_c / 2 + w_c / 2 | A_{bc} = w_b l_c | l_e = d_e + w_b / 2 |
| l_{ic} = w_e + w_c / 2 + w_c / 2 | A_{ic} = w_i l_c | l_i = d_i + w_b / 2 |

C. Y core

The EI-core architecture described in the previous case has inherent asymmetry due to its geometry. This translates into variation of q- and d-axis inductances over a cycle. This may be avoided by using a physically symmetric 3-phase inductor as shown in Fig. 9. Herein, this is referred to as a Y-core architecture. This architecture can be readily implemented with ferrite material; using a laminated steel core the Y-core inductor is possible by approximating the circular post with a stepped approximation. In this case, the laminations used for the ‘Y’ part of the core are in the plane of the page in Fig. 9a; the laminations for a leg are in the plane of the page in Fig. 9b.

For the comparison studies in this paper, a circular core post is assumed for both steel and ferrite core cases as shown in Fig. 1(b). Using a bobbin of thickness, c_{uc}, the winding inner radius, r_{cl} is given as

\[ r_{cl} = r_c + c_{uc} \]  

(44)

and the winding outer radius, r_{w}, by

\[ r_w = r_0 + w_e \]  

(45)

The slot depth, d_{s} is

\[ d_s = d_w + 2c_{wc} \]  

(46)

The other geometric relations derived based on the geometry of the core in Fig. 9 are

\[ w_c = 2r_c \]  

(47)

\[ l_{cc} = \frac{2}{\sqrt{3}} \left( \frac{c_{cc}}{2} + r_w \right) \]  

(48)

\[ l_l = \frac{l_{cc} - g - \sqrt{3}/6w_c - r_c}{2} \]  

(49)

where c_{cc} is the coil-to-coil clearance.

The over all height, h_l, width, w_l, and length, l_l of the Y-core inductor are

\[ h_l = d_s + 2d_e \]  

(50)

\[ w_l = 3l_{ccr} + 2r_w \]  

(51)

\[ l_l = 3/2l_{ccr} + 2r_w \]  

(52)

Mass of the core is estimated as

\[ M_{cr} = \rho_{cr} (3V_{ccr} + 2V_{cr2}) \]  

(53)

where volumes of the vertical core and horizontal core are

\[ V_{ccr} = \pi r_c^2 (d_s + d_e) + 2(r_c + l_l) d_e w_c \]  

(54)

V_{cr2} = d_e \left( \frac{3l_{ccr} + \sqrt{3}/4w_e^2}{2} \right)  

(55)

By symmetry along the horizontal dotted line shown in Fig. 9 (b), it is sufficient to analyze only one-half of the Y-core inductor. The MEC of the Y-core inductor upper half is shown in Fig. 10. The core branch effective cross-sectional areas and lengths are listed in Table III. The air gap permeance, P_g in case of Y-core inductor is given by

\[ P_g = P_{gd}(\Phi_g) + P_{fa}(w_c, 2l_f / \pi) + 2 P_{fa}(d_e, l_l) + P_{fa}(w_c, l_l) \]  

(56)

The derivation of the permeance expression in (56) will be presented in a forth coming paper.

D. Delta Core

Another physically symmetric core geometry possible for a 3-phase inductor is shown in Fig. 10. In this case, a delta shaped horizontal cores rests on either ends of the three vertical circular posts. The air gap on each leg is split between the top and bottom ends of each post. The delta-core architecture is possible only for ferrite core material case, since stacking of laminations is not viable for the horizontal part of the core because the direction at which flux enters the delta portion of the core would cause high eddy current loss.

TABLE III. Y-CORE MAGNETIC PATHS AND AREAS

| l_p = d_s + d_e | l_{bg} = l_c + r_c | l_{cc} = l_c + \sqrt{3}/12w_c |
| A_p = \pi r_c^2 | A_{bg} = w_c d_e | A_{cc} = w_c d_e |
Concerning the coil geometry, (44)-(47) hold true for the case of delta-core geometry as well. The length between two core posts, \( l_{cc} \) is defined as

\[
l_{cc} = c_{cc} + 2\left( r_b + w_r \right)
\]  

to (57)

The overall height, \( h_I \), width, \( w_I \), and length, \( l_I \) of the delta-core inductor are

\[
h_I = d_s + 2d_c + 2g \quad \text{(58)}
\]

\[
w_I = 3l_{cc} + 2r_w \quad \text{(59)}
\]

\[
l_I = \sqrt{3} / 2l_{cc} + 2r_w \quad \text{(60)}
\]

The core mass is calculated using

\[
M_{cr} = \rho_{cr} \left( 3V_{cr1} + 6V_{cr2} \right) \quad \text{(61)}
\]

where

\[
V_{cr1} = \pi r_c^2 \left( d_s + 2d_c \right) \quad \text{(62)}
\]

\[
V_{cr2} = (l_{cc} - 2r_c) d_c w_c \quad \text{(63)}
\]

The MEC of the delta-core inductor is as shown in Fig. 12. Similar to the Y-core case, only the upper half the inductor is analyzed. The effective magnetic lengths and cross-sectional areas useful in estimating core branch permeances by using (10) are listed in Table IV. By symmetry, the fringing permeance in the region enclosed by angle \( \beta \) in Fig. 10 is of type (a), while the rest of the circular post-horizontal core has fringing flux of type (b). Taking this into account, the air gap permeance, \( g_P \) is given by

\[
g_P = P_{gd}(A_p) + P_{fb}(2 / 3\pi r_c, \min(d_c, (d_s - d_w) / 2)) + P_{fb}(4 / 3\pi r_c, \min(d_c, (d_s - d_w) / 2)) \quad \text{(64)}
\]

With the four architectures and their MECs described, the 3-phase inductor design process is presented in the next section.

V. THREE-PHASE INDUCTOR DESIGN

The comparison of 3-phase inductor architectures is carried out using a multi-objective optimization based design approach, minimizing mass and power loss in case of each architecture. Along with these two objectives, inductor design process adheres to set of design specifications as listed in Table V. These specifications include appropriate physical geometric constraints such as winding bend radius, reasonable packing factor and aspect ratio. Other constraints include current density and incremental inductance.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega )</td>
<td>Fundamental frequency (rad/s)</td>
</tr>
<tr>
<td>( i_{pk} )</td>
<td>Rated peak phase current (A)</td>
</tr>
<tr>
<td>( L_{q,mn} )</td>
<td>Minimum ( q )-axis incremental inductance required (H)</td>
</tr>
<tr>
<td>( \tau_{qL} )</td>
<td>Maximum allowed ratio of maximum ( q )-axis inductance to minimum ( q )-axis inductance</td>
</tr>
<tr>
<td>( \tau_{gqL} )</td>
<td>Maximum allowed ratio of mutual ( qd ) inductance to minimum ( q )-axis inductance</td>
</tr>
<tr>
<td>( k_{bnd} )</td>
<td>Conductor bend radius ratio</td>
</tr>
<tr>
<td>( k_{pfx} )</td>
<td>Maximum packing factor</td>
</tr>
<tr>
<td>( J_{mx} )</td>
<td>Maximum allowed current density (A rms/mm(^2))</td>
</tr>
<tr>
<td>( M_{mx} )</td>
<td>Maximum allowed mass (kg)</td>
</tr>
<tr>
<td>( P_{mx} )</td>
<td>Maximum allowed power loss (W)</td>
</tr>
<tr>
<td>( \alpha_{mx} )</td>
<td>Maximum allowed aspect ratio</td>
</tr>
</tbody>
</table>

Each constraint is implemented as comparison of design value to the specified value using the functions

\[
\text{lte}(x, x_{\text{max}}) = \begin{cases} 
1 & x \leq x_{\text{max}} \\
1 / \left( 1 + \left( x - x_{\text{max}} \right) / x_{\text{max}} \right) & x > x_{\text{max}} 
\end{cases} \quad \text{(65)}
\]

for the case where desired value is to be less than or equal to the specification and

\[
\text{gte}(x, x_{\text{max}}) = \begin{cases} 
1 / \left( 1 + \left( x - x_{\text{max}} \right) / x_{\text{max}} \right) & x < x_{\text{max}} \\
1 & x \geq x_{\text{max}} 
\end{cases} \quad \text{(66)}
\]

for the case where desired value is to greater than or equal to the specification.

First, geometric constraints are addressed followed by constraints on the inductor electrical and magnetic
performance. The conductor strands of the coil winding are ensured to have sufficient bend radius by the constraint

\[
c_i = \begin{cases} 
\text{gte}\left(w_i, 0\right) & \text{rectangular} \\
\text{gte}\left(r \cdot 2k_n, r_i\right) & \text{circular}
\end{cases} \tag{67}
\]

In the EI-core case, \( w_{cl} = \min\left(w_{cl,1}, w_{cl,2}\right) \) is used in (67) to include two coil widths \( w_{cl,1} \) and \( w_{cl,2} \).

The winding packing factor is constrained to a realizable value by using

\[
c_2 = \text{lte}\left(k_{pf}, k_{pf, m}\right) \tag{68}
\]

The third constraint is used to limit the aspect ratio of the inductor to a reasonable value by

\[
c_3 = \text{lte}\left(\alpha, \alpha_{m}\right) \tag{69}
\]

The total mass of the inductor is limited by using

\[
c_4 = \text{lte}\left(M, M_{m}\right) \tag{70}
\]

The RMS current density of the winding conductors, \( j_{rms} \), is limited to prevent overheating of the conductors by implementing the constraint as

\[
c_5 = \text{lte}\left(i_{rms}, i_{m}\right) \tag{71}
\]

where the conductor current density is \( j_{rms} = i_{pk} / \left(\sqrt{2}a_c\right) \).

The \( q \)-axis and mutual \( qd \) incremental inductances are calculated using MEC over one cycle duration using (19) and (20). Two currents excitation \( i_{q0} = \left[i_{pk} \ 0 \ 0\right] \) and \( i_{q1d} = \left[i_{pk} + \Delta i \ 0 \ 0\right] \) are used in order to evaluate the partial derivative of flux linkage with respect to current numerically. Here \( \Delta i \) denotes differential change in current. The MEC convergence is ensured by implementing constraint \( c_6 \) based on the convergence of each MEC solution.

Herein, let, \( L_{q} \) and \( L_{q0} \) denote vector representing the incremental \( q \)-axis self and \( qd \) mutual inductances over one cycle. Three constraints are placed on inductance. A minimum \( q \)-axis incremental inductance, is ensured by defining

\[
c_7 = \text{gte}\left(\min(L_{q}), L_{q, m}\right) \tag{72}
\]

The variation in \( q \)-axis inductance over one switching cycle is limited by using the constraint

\[
c_8 = \text{lte}\left(\max(L_{q}), r_{q, c} \ \min(L_{q})\right) \tag{73}
\]

The magnetic coupling between the \( q \)- and \( d \)-axis is controlled using

\[
c_9 = \text{lte}\left(\max(abs(L_{q0})), r_{q, c} \ \min(L_{q})\right) \tag{74}
\]

Power loss in each design case is comprised of the core loss and conductor DC loss. The core branch fluxes estimated during the magnetic analysis using MEC are used to calculate core flux densities and hence calculate core loss using the MSE. The last constraint is used to limit the total power loss in the inductor by defining

\[
c_{10} = \text{lte}(P, P_{max}) \tag{75}
\]

After all constraints are evaluated, \( c_{avg} \) is computed as the numerical average of \( c_i \) through \( c_{10} \). Based on \( c_{avg} \), the total inductor mass, \( M \), and the inductor loss, \( P \), the fitness function [8–9], given by

\[
f = \begin{cases} 
\left[1/M \ 1/P\right]^T & \text{if } c_{avg} = 1 \\
\epsilon \left(c_{avg} - 1\right)\left[1 \ 1\right]^T & \text{if } c_{avg} < 1
\end{cases} \tag{76}
\]

is evaluated. In (76), \( \epsilon \) is a small positive number. Inspection of (76) reveals that if any constraint is not met, both elements of \( f \) are negative; if all constraints are met the fitness function has positive elements. Maximizing \( f \) yields the Pareto-optimal front between mass and loss subject to all constraints being met.

The design space considered for the UI core architecture is

\[
\theta = \left[g \ l_c \ w_c \ w_i \ w_b \ N_d^* \ N_i^* \ N_{sc}^* a_{sc}^* \right] \tag{77}
\]

The design space used in the case of 3-phase EI-core inductor is

\[
\theta = \left[g \ l_c \ w_c \ w_i \ w_b \ N_d^* \ N_i^* \ N_{sc}^* a_{sc}^* \right] \tag{78}
\]

The design space of Y-core and delta-core architectures is

\[
\theta = \left[g \ r_c \ d_c \ N_d^* \ N_i^* N_{sc}^* a_{sc}^* \right] \tag{79}
\]

where \( N_d^* \), \( N_i^* \), \( N_{sc}^* \), and \( a_{sc}^* \) denote the desired values of the number of turns per layer, number of layers, number of parallel strands and strand cross-sectional area respectively. During optimization, the desired coil parameters are assigned real numbers and varied continuously in a given region. The actual values for the number of turns per layer, number of layers, number of parallel strands and strand cross-section are obtained using

\[
N_{x} = \text{round}(N_{x}^*) \tag{80}
\]

\[
a_{sc} = \text{awg}_\text{round}(a_{sc}^*) \tag{81}
\]

where the functions 'round' denotes integer rounding and 'awg_round' denotes rounding to closest possible American Wire Gauge (AWG) size .

VI. RESULTS AND DISCUSSION

The design specifications for the 3-phase inductor are listed in Table VI. For this study, 10JNEX900 high silicon content steel was assumed, which supports modest switching frequencies. Pareto-optimal designs satisfying the geometrical and inductance constraints are shown in Fig. 13. The three Pareto-optimal fronts correspond to three topologies considered. As can be seen, for a given power loss, the UI-
core is inferior to the other architectures. The Y-core architecture modestly outperforms the EI-core architecture. Although not included in this study, the Y-core arrangement also yields reduced proximity effect losses.

To consider the situation at higher switching frequencies, the same study is repeated with a MN60LL ferrite core material and a reduced $L_{q_{mn}}$ requirement of 80 $\mu$H. As can be seen in Fig. 14, similar observations can be made. Here, the delta-core architecture is also included and seen to have similar performance to that of the Y-core inductor.

VII. CONCLUSIONS

From the studies performed, it is evident that for a given power loss, the Y-core and delta-core inductors are mildly superior to the EI-core arrangement, while independent UI-core inductors are significantly inferior to the other arrangements considered.

| TABLE VI. 3-PHASE INDUCTOR SPECIFICATIONS |
| Symbol | Value | Symbol | Value |
| $\omega$ | 120$\pi$ rad/s | $i_{pk}$ | 50 A |
| $\Delta q$ | 0.1 A | $L_{q_{mn}}$ | 200 $\mu$H |
| $f_{ql}$ | 1.02 | $r_{dql}$ | 0.02 |
| $k_{bud}$ | 4 | $k_{w}$ | 1.02 |
| $e_{cc}$ | 2.5 mm | $e_{cc}$ | 1 mm |
| $J_{r_{max}}$ | 7.6 MA/m | $M_{n}$ | 3 kg |
| $P_{max}$ | 2 kW | $\alpha_{max}$ | 100 |

Fig. 13. Architecture comparison: Case study I – steel core for 200 $\mu$H

Fig. 14. Architecture comparison: Case study II – ferrite core for 80 $\mu$H

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