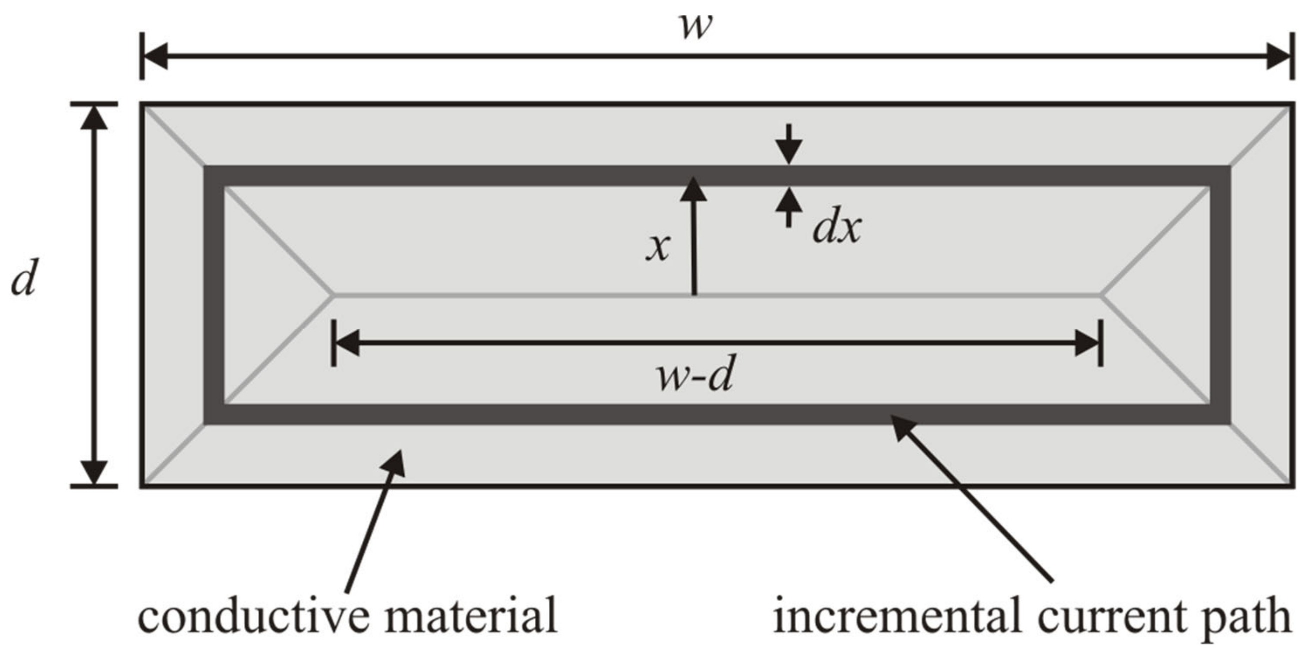
Power Magnetic Devices: A Multi-Objective Design Approach

Chapter 6: Magnetic Core Loss

6.1 Eddy Current Losses

- Eddy current lost is a resistive power loss associated with induced currents
- Let’s consider a rectangular conductor

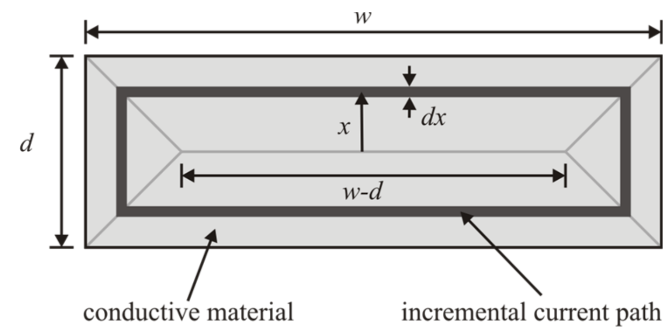


6.1 Eddy Current Losses

- Defining

$$k_1 = \min(w, d)$$

$$k_2 = |w - d|$$

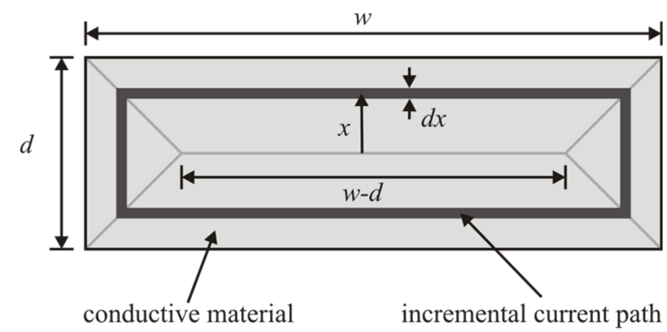


- We can show

$$P = \frac{\sigma l}{128}\left(4k_1^4 + 8k_2k_1^3 + 2k_2^2k_1^2 - 2k_2^3k_1 + k_2^4 \ln\left(1 + \frac{2k_1}{k_2}\right)\right)\frac{1}{T}\int_0^T \left(\frac{dB}{dt}\right)^2 dt$$

6.1 Eddy Current Losses

- Derivation



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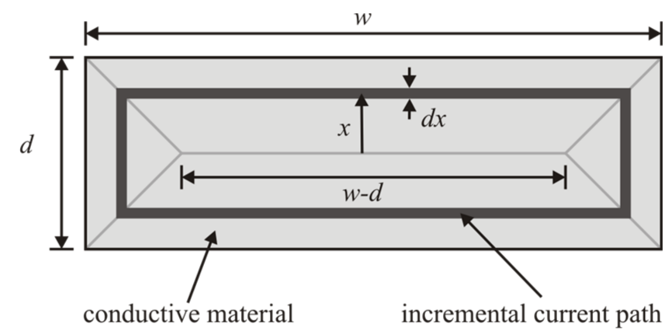

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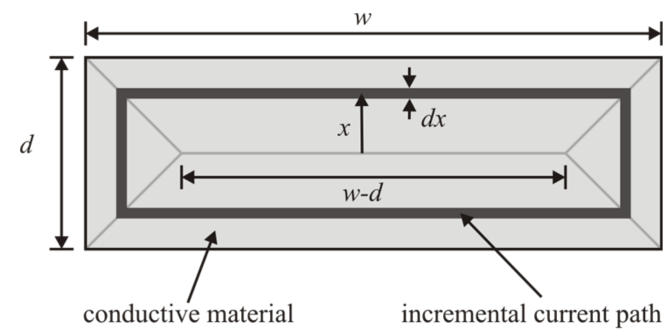

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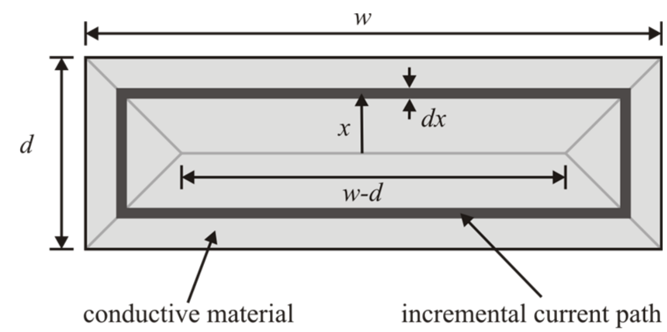

6.1 Eddy Current Losses

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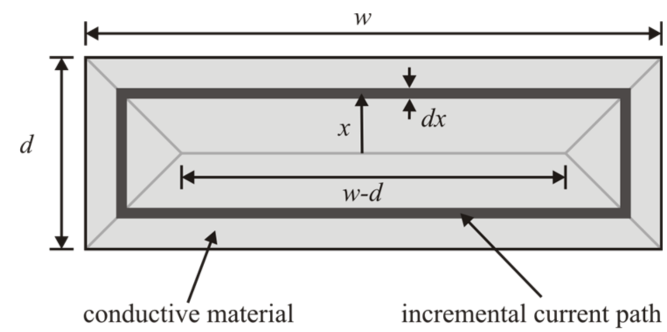


6.1 Eddy Current Losses

- Example 6.1A. Eddy current loss in a toroid

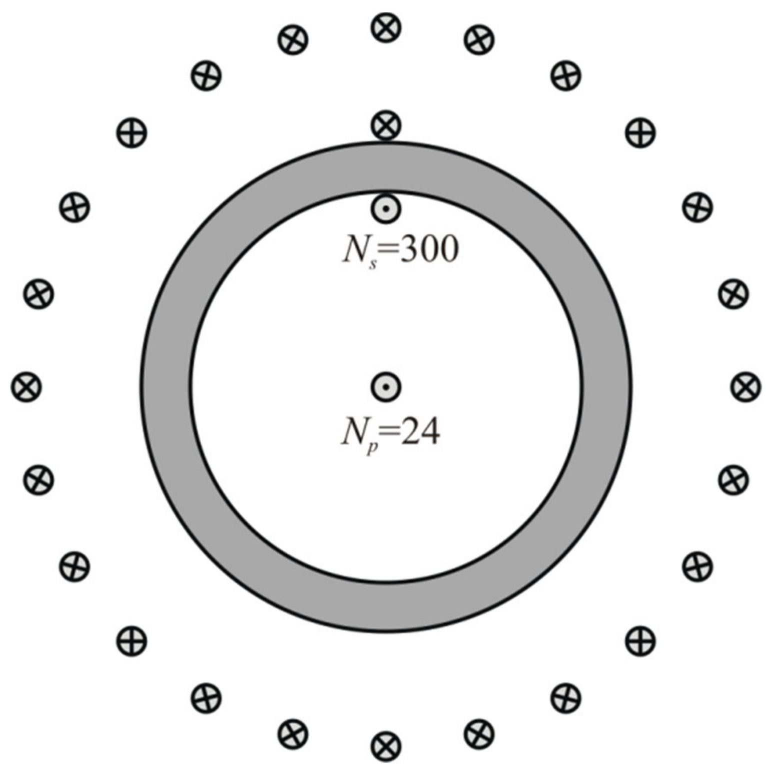


Table 6.1A-1 Aluminum alloy test samples.

Material	σ (MS/m)	μ_r	r_i (mm)	r_o (mm)	w (mm)	d (mm)
6061T6	25.3	1.23	140	150	10	12.7
6013	23.3	1.20	140	150	10	27.2

6.1 Eddy Current Losses

- Example 6.1A. Normalized power loss

$$p_n = \frac{P}{V \frac{1}{T}\int_0^T \left(\frac{dB}{dt}\right)^2 dt}$$

- Thus, with our expression for eddy current loss, we have

$$p_n = \frac{\sigma\left(4k_1^4 + 8k_2k_1^3 + 2k_2^2k_1^2 - 2k_2^3k_1 + k_2^4 \ln\left(1 + \frac{2k_1}{k_2}\right)\right)}{128wd}$$

6.1 Eddy Current Losses

- Example 6.1A. Measured power loss. We can show

$$P = \frac{N_p}{N_s}\frac{1}{T}\int_0^T v_s i_p dt$$

6.1 Eddy Current Losses

- Example 6.1A. Derivation of average power.

6.1 Eddy Current Losses

- Example 6.1A. Derivation of average power.

6.1 Eddy Current Losses

- Example 6.1A. Now we measured power, we can find the measured normalized power loss density

$$p_n = \frac{P}{V \dfrac{1}{T} \displaystyle\int_0^T \left(\frac{dB}{dt} \right)^2 dt}$$

- Clearly – we need to know B. There are two approaches.

6.1 Eddy Current Losses

- Example 6.1A. Approach 1 to finding B (predicted)
- We use

$$B = \frac{\mu_0 \mu_r i_p}{2\pi r}$$

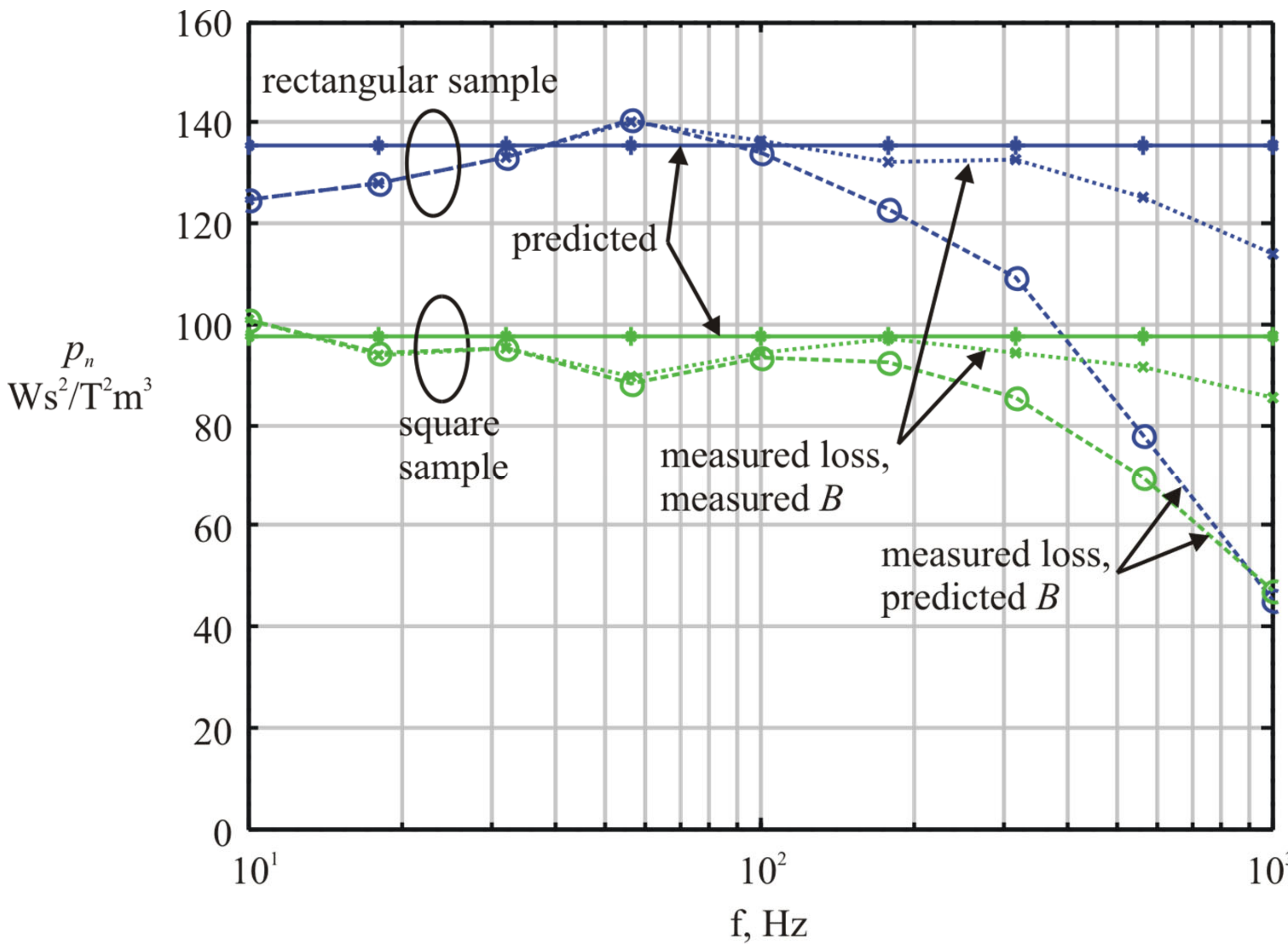
6.1 Eddy Current Losses

- Example 6.1A. Approach 1 to finding B (measured)
- We use

$$\frac{dB}{dt} = \frac{v_s}{wdN_s}$$

6.1 Eddy Current Losses

- Example 6.1A. Results:



6.1 Eddy Current Losses

- Sinusoidal excitation

$$B = B_{pk}\cos\left(\omega_e t\right) \qquad \frac{1}{T}\int_0^T \left(\frac{dB}{dt}\right)^2 dt = \frac{1}{2}\omega_e^2 B_{pk}^2$$

- Periodic excitation

$$B = \sum_{k=1}^{K} B_{a,k}\cos(k\omega_e t) + B_{b,k}\sin(k\omega_e t)$$

$$\frac{1}{T}\int_0^T \left(\frac{dB}{dt}\right)^2 dt = \frac{1}{2}\omega_e^2 \sum_{k=1}^{K} k^2 \left(B_{a,k}^2 + B_{b,k}^2\right)$$

6.1 Eddy Current Losses

- Thin laminations. Suppose $k_2 >> k_1$
- Then we can show

$$p_n = \frac{\sigma\left(4k_1^4 + 8k_2k_1^3 + 2k_2^2k_1^2 - 2k_2^3k_1 + k_2^4 \ln\left(1 + \frac{2k_1}{k_2}\right)\right)}{128wd}$$

- Can be approximated as

$$P = \frac{\sigma l}{12} k_2 k_1^3 \frac{1}{T} \int_0^T \left(\frac{dB}{dt}\right)^2 dt$$

6.1 Eddy Current Losses

- To show this

6.1 Eddy Current Losses

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- It follows that

$$p = \frac{\sigma w^2}{12} \frac{1}{T} \int_0^T \left(\frac{dB}{dt} \right)^2 dt$$

- To show this, we start with

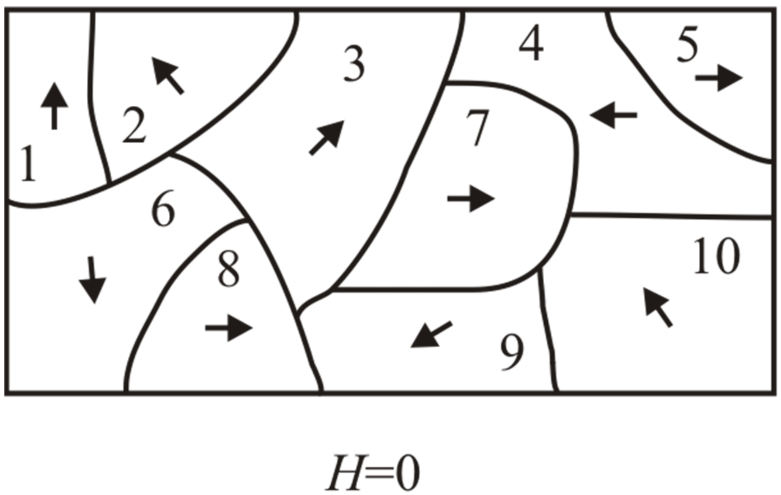
$$P = \frac{\sigma l}{12} k_2 k_1^3 \frac{1}{T} \int_0^T \left(\frac{dB}{dt} \right)^2 dt$$

6.1 Eddy Current Losses

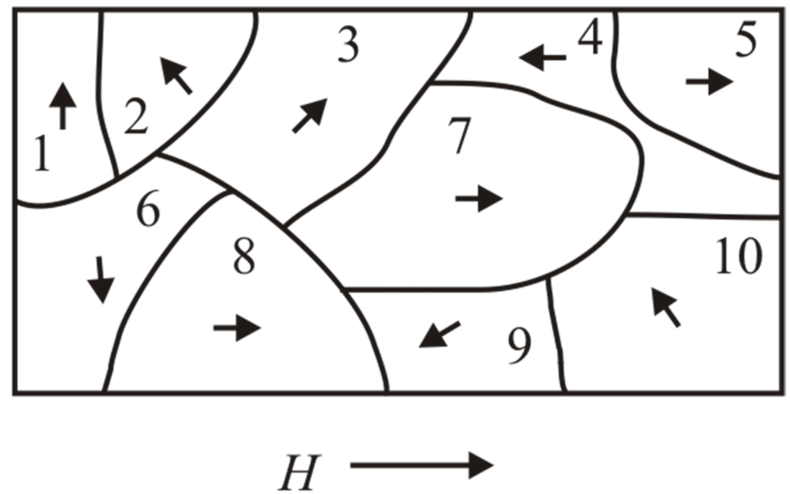
- Continuing …

6.2 Hysteresis Loss and the B-H Loop

- Domain wall motion



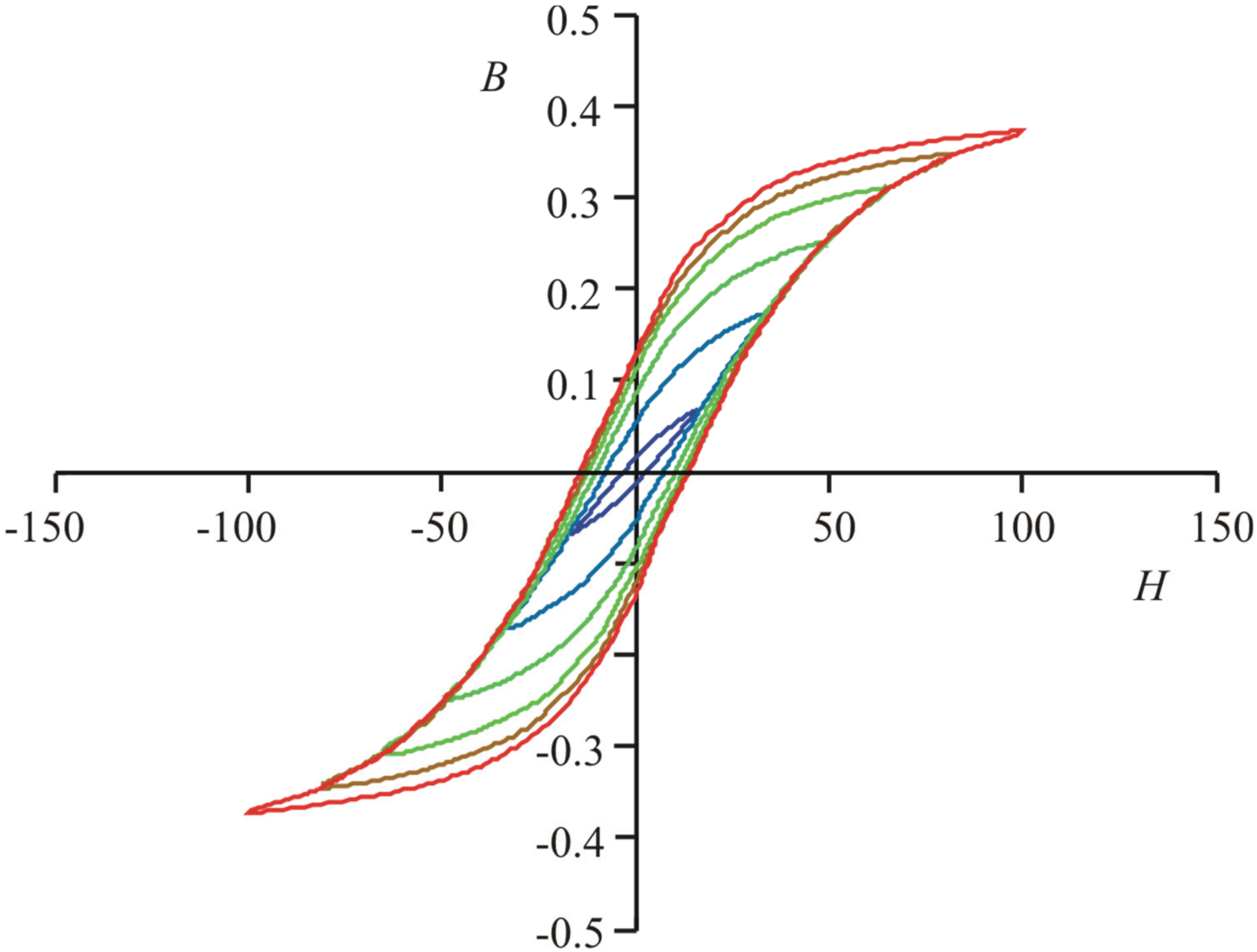
(a) domains without applied field



(b) domains with applied field

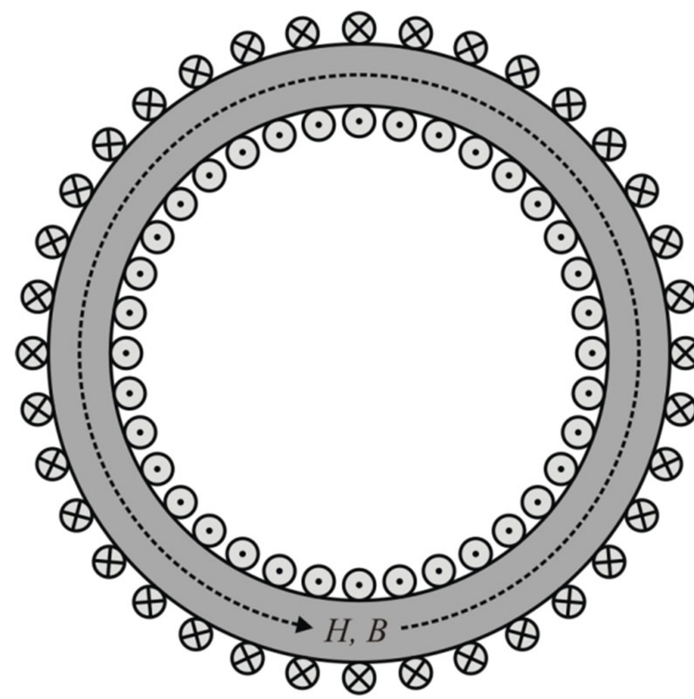
6.2 Hysteresis Loss and the B-H Loop

- Sample B-H characteristics



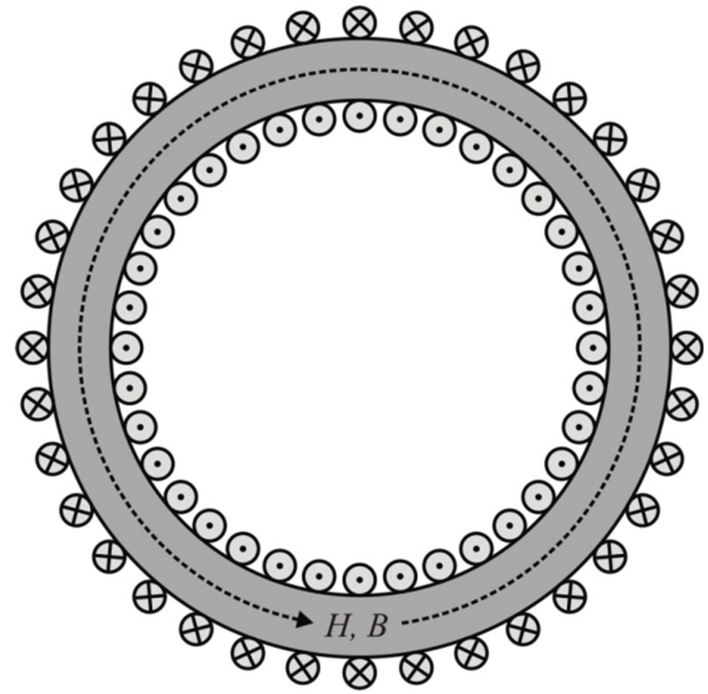
6.2 Hysteresis Loss and the B-H Loop

- The area of a B-H trajectory represent energy loss.
- The first step to do this is to show $e = \int_{B_0}^{B_T} H dB$
- To do end, consider the toroid



6.2 Hysteresis Loss and the B-H Loop

- Proceeding …

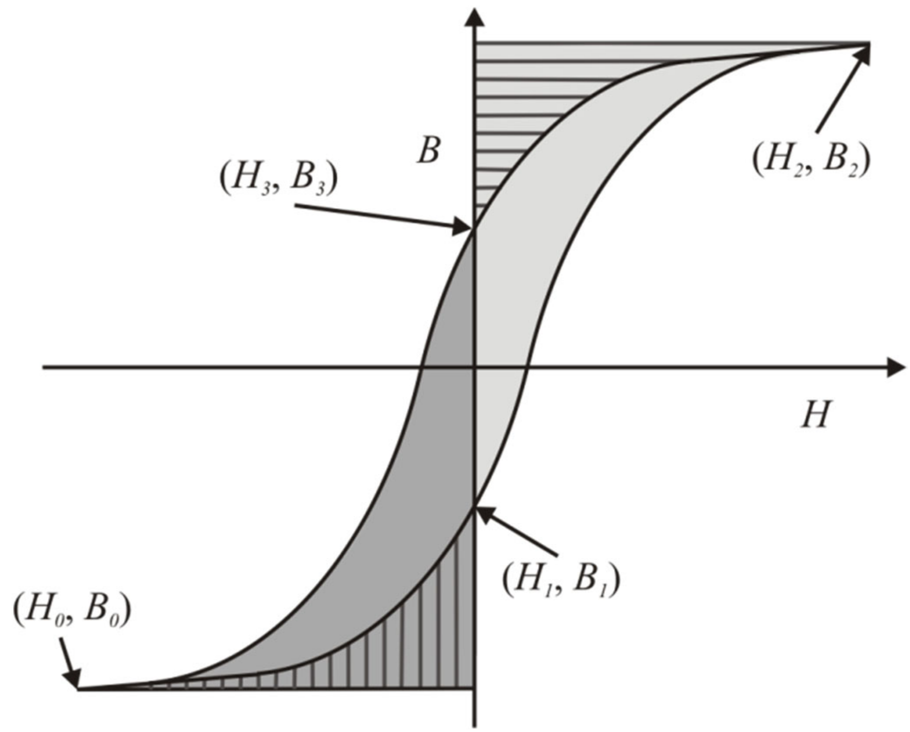


6.2 Hysteresis Loss and the B-H Loop

- Now let us consider a closed path

$$e = \underbrace{\int_{B_0}^{B_1} HdB}_{\text{Term 1}} + \underbrace{\int_{B_1}^{B_2} HdB}_{\text{Term 2}}$$

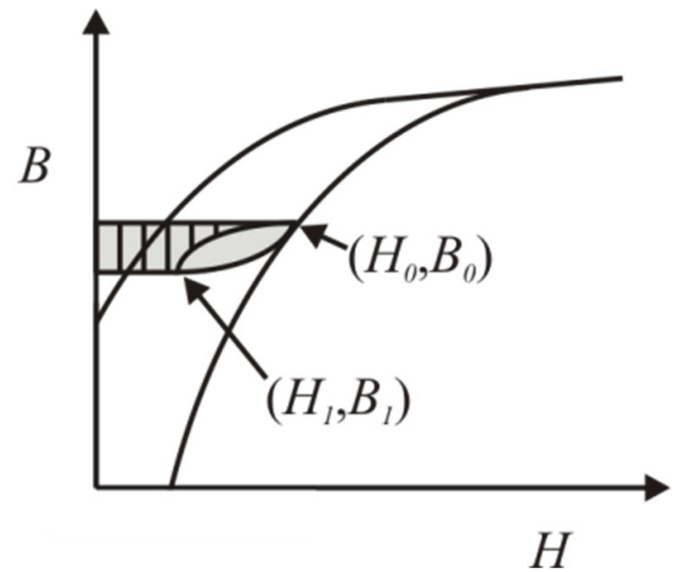
$$+ \underbrace{\int_{B_2}^{B_3} HdB}_{\text{Term 3}} + \underbrace{\int_{B_3}^{B_0} HdB}_{\text{Term 4}}$$



6.2 Hysteresis Loss and the B-H Loop

- Minor loop loss

$$e = \underbrace{\int_{B_0}^{B_1} HdB}_{\text{Term 1}} + \underbrace{\int_{B_1}^{B_0} HdB}_{\text{Term 2}}$$



6.2 Hysteresis Loss and the B-H Loop

- Now that we have established the energy loss per cycle, the average power loss may be expressed

$$p_h = f e_{BH}$$

6.3 Empirical Modeling of Core Loss

- The Steinmetz Equation

$$e_{BH} = \frac{k_h}{f_b}\left(\frac{f}{f_b}\right)^{\alpha-1}\left(\frac{B_{pk}}{B_b}\right)^{\beta}$$

$$p_h = k_h\left(\frac{f}{f_b}\right)^{\alpha}\left(\frac{B_{pk}}{B_b}\right)^{\beta}$$

6.3 Empirical Modeling of Core Loss

- The Modified Steinmetz Equation (MSE)
  - Motivated by observation that localized eddy current due to domain movement being tied rate of change
- MSE equivalent frequency

$$\Delta B = B_{mx} - B_{mn}$$

$$\dot{B} = \frac{1}{\Delta B} \int_0^T \left(\frac{dB}{dt} \right)^2 dt$$

$$f_{eq} = \frac{2}{\Delta B^2 \pi^2} \int_0^T \left(\frac{dB}{dt} \right)^2 dt$$

6.3 Empirical Modeling of Core Loss

- MSE loss equation

$$e_{BH} = \frac{k_h}{f_b}\left(\frac{f_{eq}}{f_b}\right)^{\alpha-1}\left(\frac{\Delta \mathrm{B}}{2B_b}\right)^{\beta}$$

$$p_h = k_h\left(\frac{f_{eq}}{f_b}\right)^{\alpha-1}\left(\frac{\Delta \mathrm{B}}{2B_b}\right)^{\beta}\frac{f}{f_b}$$

6.3 Empirical Modeling of Core Loss

- Note

6.3 Empirical Modeling of Core Loss

- MSE with sinusoidal flux density

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- MSE with sinusoidal flux density

6.3 Empirical Modeling of Core Loss

- Example 6.3A. MSE with triangular excitation

6.3 Empirical Modeling of Core Loss

- Example 6.3A. MSE with triangular excitation

6.3 Empirical Modeling of Core Loss

- Generalized Steinmetz Equation (GSE)
  - Assumes instantaneous loss is a function of rate of change and flux density value

$$p_i = f\left(\frac{dB}{dt}, B\right)$$

  - A suggested choice is

$$p_i = k\left|\frac{1}{f_b B_b}\frac{dB}{dt}\right|^{\alpha}\left|\frac{B}{B_b}\right|^{\beta-\alpha}$$

6.3 Empirical Modeling of Core Loss

- The GSE then becomes

$$p_h = \frac{1}{T} k \int_0^T \left| \frac{1}{f_b B_b} \frac{dB}{dt} \right|^{\alpha} \left| \frac{B}{B_b} \right|^{\beta-\alpha} dt$$

where we choose

$$k = \frac{k_h}{(2\pi)^{\alpha-1} \int_0^{2\pi} |\cos\theta|^{\alpha} |\sin\theta|^{\beta-\alpha} \, d\theta}$$

6.3 Empirical Modeling of Core Loss

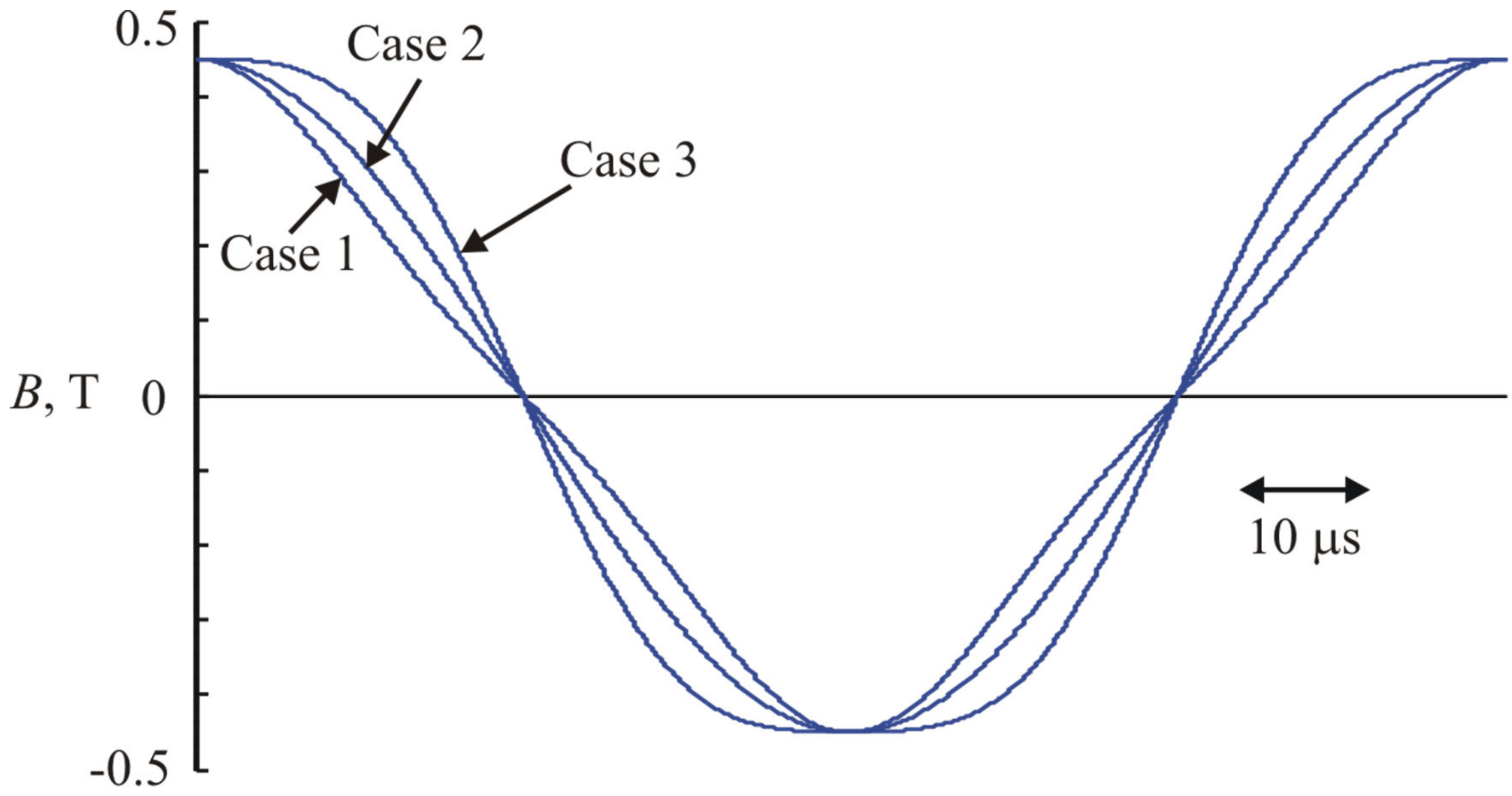
- Example 6.3B. Consider MN60LL ferrite with α=1.034, β=2.312, k_h = 40.8 W/m^3
- Let us compare MSE and GSE models for

$$B = B_1 \cos(2\pi ft) + B_3 \cos(6\pi ft)$$

- Cases
  - Case 1: B_1=0.5 T, B_3=-0.05T
  - Case 2: B_1= 0.45T, B_3=0 T
  - Case 3: B_1= 0.409 T, B_3=0.0409 T

6.3 Empirical Modeling of Core Loss

- Example 6.3 flux density waveforms



6.3 Empirical Modeling of Core Loss

- Results
  - MSE model: 87.4, 86.5, 86.2 kW/m^3
  - GSE model: 86.9, 86.5, 86.8 kW/m^3
- Comments on MSE and GSE models

6.3 Empirical Modeling of Core Loss

- Combined loss modeling
  - Eddy current loss

$$p_e = k_e \frac{1}{T}\int_0^T \left(\frac{dB}{dt}\right)^2 dt$$

  - Hysteresis loss (MSE)

$$p_h = k_h \left(\frac{f_{eq}}{f_b}\right)^{\alpha-1} \left(\frac{\Delta \mathrm{B}}{2B_b}\right)^{\beta} \frac{f}{f_b}$$

  - Combined loss model

$$p_t = k_h \left(\frac{f_{eq}}{f_b}\right)^{\alpha-1} \left(\frac{\Delta \mathrm{B}}{2B_b}\right)^{\beta} \frac{f}{f_b} + k_e \frac{1}{T}\int_0^T \left(\frac{dB}{dt}\right)^2 dt$$

6.3 Empirical Modeling of Core Loss

- Example 6.3C. Lets look at losses in M19 steel with k_h=50.7 W/m^3, α=1.34, β=1.82, and k_e=27.5·10^{-3} Am/V. We will assume

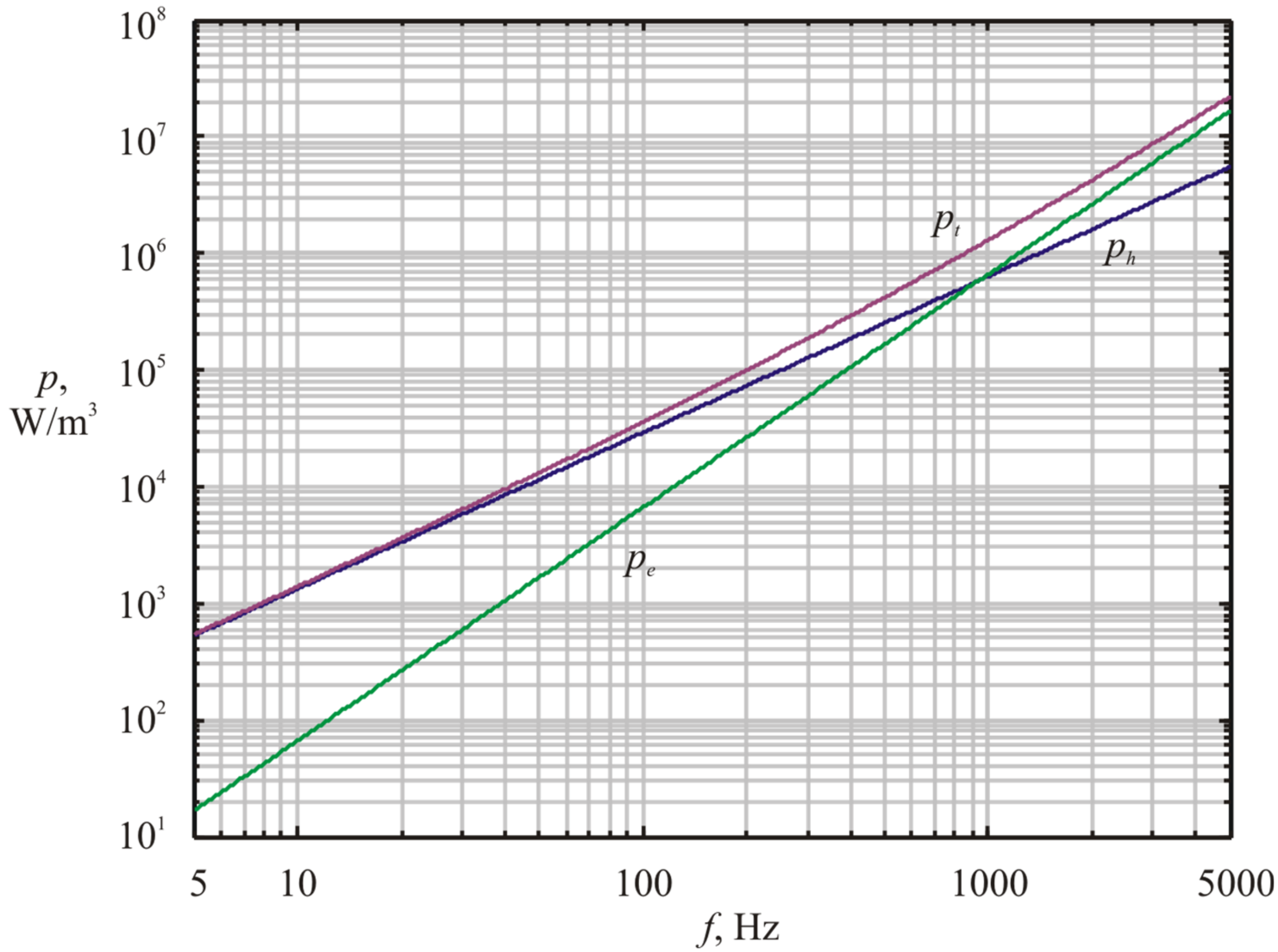
$$B = B_{pk}\cos(2\pi ft)$$

- Since the flux density is sinusoidal

$$p_e = 2\pi^2 k_e B_{pk}^2 f^2 \qquad p_h = k_h\left(\frac{f}{f_b}\right)^{\alpha}\left(\frac{B_{pk}}{B_b}\right)^{\beta}$$

6.3 Empirical Modeling of Core Loss

- Example 6.3C results



6.4 Time Domain Modeling of Core Loss

- These models can capture effects such as
  - waveforms with dc offset
  - aperiodic excitation
  - waveforms with minor loops

- Two approaches
  - Jiles-Atherton
  - Praisach

6.4 Time Domain Modeling of Core Loss

- Jiles-Atherton model

$$B = \mu_0 (H + M)$$

$$M = M_{irr} + M_{rev}$$

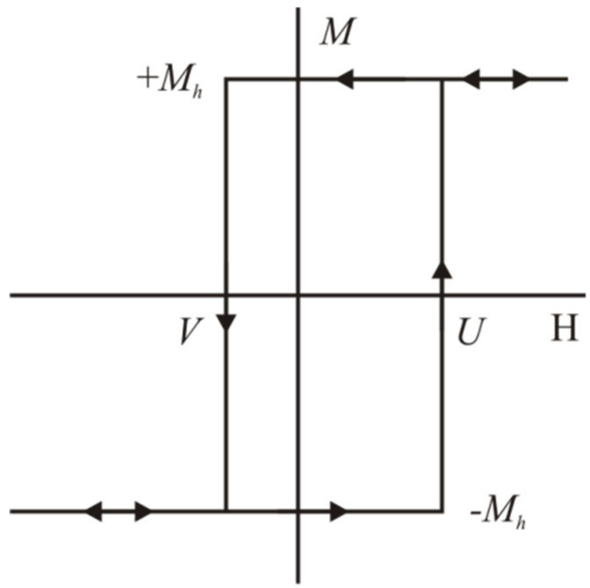
$$\frac{dM_{irr}}{dt} = \left(\frac{\mathrm{M}_{an}(H) - M_{irr}}{k\delta - \alpha(\mathrm{M}_{an}(H) - M_{irr})} \right) \frac{dH}{dt}$$

$$M_{rev} = c(\mathrm{M}_{an}(H) - M_{irr})$$

$$\delta = \begin{cases} 1 & \frac{dH}{dt} > 0 \\ 0 & \frac{dH}{dt} = 0 \\ -1 & \frac{dH}{dt} < 0 \end{cases}$$

6.4 Time Domain Modeling of Core Loss

- Preisach model is based on behavior of a hysteron



- Total magnetization based on

$$M = \int_{-\infty}^{\infty} \int_{-\infty}^{U} \mathrm{Q}(U,V)\,\mathrm{P}(U,V)\,dV\,dU$$

6.4 Time Domain Modeling of Core Loss

- Saturated magnetization

$$M_s = \int_{-\infty}^{\infty} \int_{-\infty}^{U} \mathrm{P}(U,V)\,dV\,dU$$

- Normalized magnetization

$$m = \frac{M}{M_s}$$

- Normalized hysteron density

$$\mathrm{p}(U,V) = \frac{1}{M_s}\mathrm{P}(U,V)$$

6.4 Time Domain Modeling of Core Loss

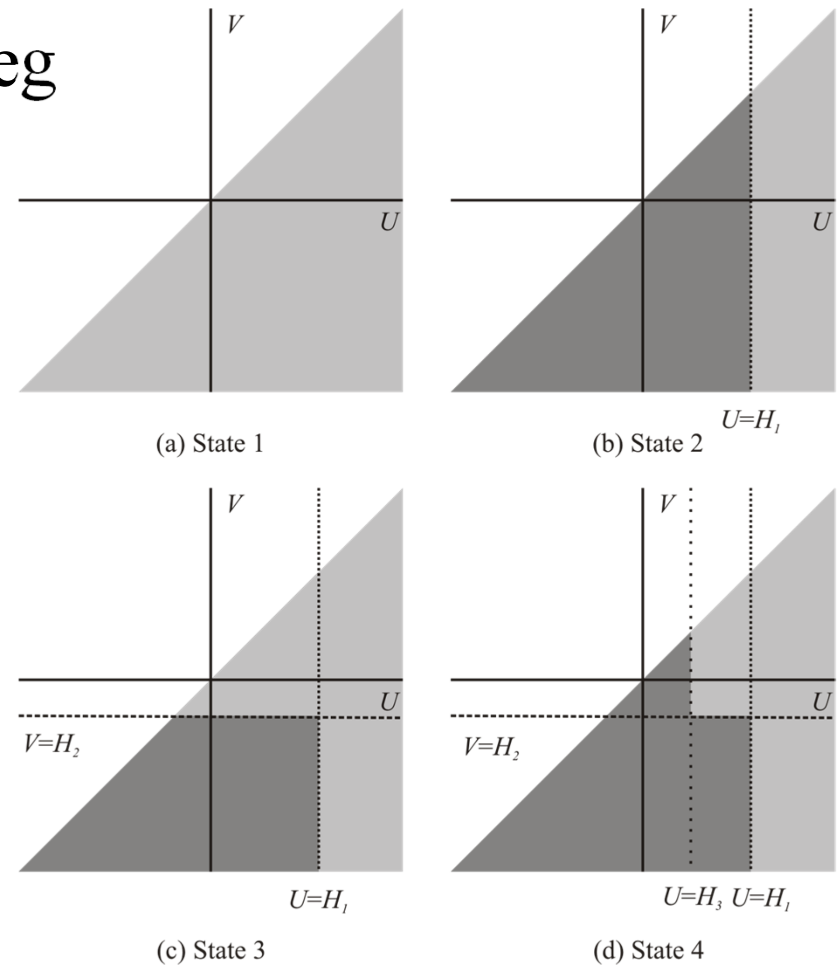
6.4 Time Domain Modeling of Core Loss

- Lets consider behavior of model
- Start in State 1 – all hysterons neg
- State 1 to State 2

$$m_{s2} = m_{s1} + 2\int_{-\infty}^{H_1}\int_{-\infty}^{U} \mathrm{p}(U,V)\,dV\,dU$$

- State 2 to State 3

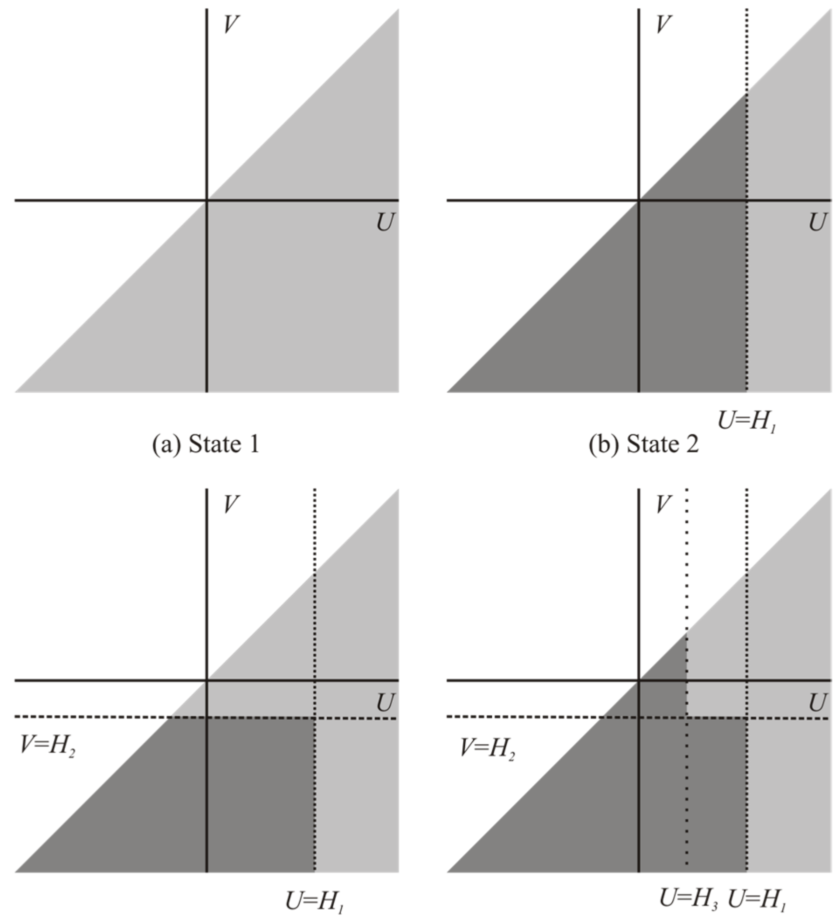
$$m_{s3} = m_{s2} - 2\int_{H_2}^{H_1}\int_{V}^{H_1} \mathrm{p}(U,V)\,dU\,dV$$



6.4 Time Domain Modeling of Core Loss

- State 3 to State 4

$$m_{s4} = m_{s3} + 2\int_{H_2}^{H_3}\int_{H_2}^{U} \mathrm{p}(U,V)\,dV\,dU$$



(a) State 1 (b) State 2 (c) State 3 (d) State 4

6.4 Time Domain Modeling of Core Loss

- Reconsider State 3 to State 4

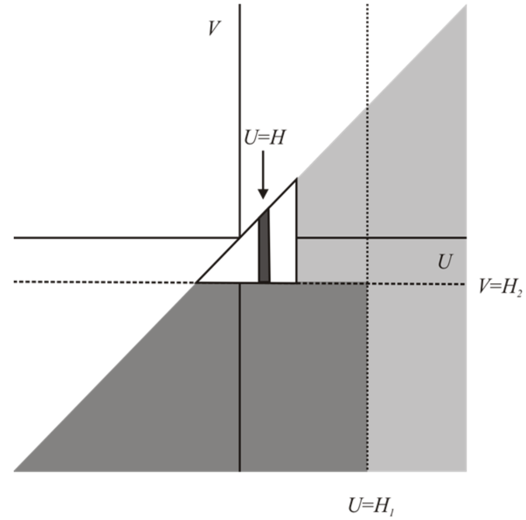
$$m = m_{s3} + 2\int_{H_2}^{H}\int_{H_2}^{U} \mathrm{p}(U,V)\,dV\,dU$$

$$\frac{dm}{dH} = 2\int_{H_2}^{H} \mathrm{p}(H,V)\,dV$$

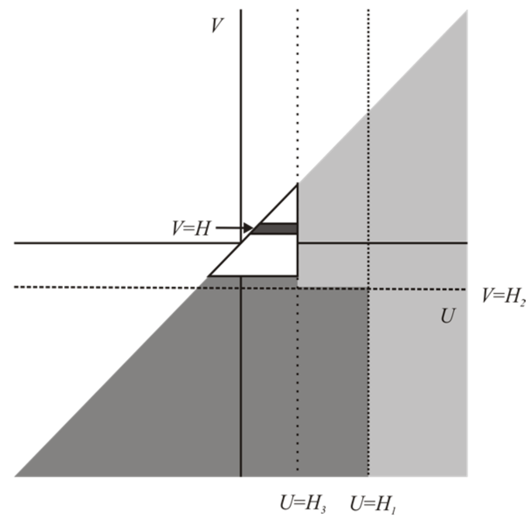
- Suppose field decreases after State 4

$$m = m_{s4} - 2\int_{H}^{H_3}\int_{V}^{H_3} \mathrm{p}(U,V)\,dU\,dV$$

$$\frac{dm}{dH} = 2\int_{H}^{H_3} \mathrm{p}(U,H)\,dU$$



(a) transition from State 3 to State 4



(b) decrease in *H* after State 4

6.4 Time Domain Modeling of Core Loss

- Completing the model

$$\frac{dm}{dt} = \frac{dm}{dH}\frac{dH}{dt}$$

$$M = M_s m$$

$$B = \mu_0 H + M$$

6.4 Time Domain Modeling of Core Loss

- Relative advantages and disadvantages of approaches