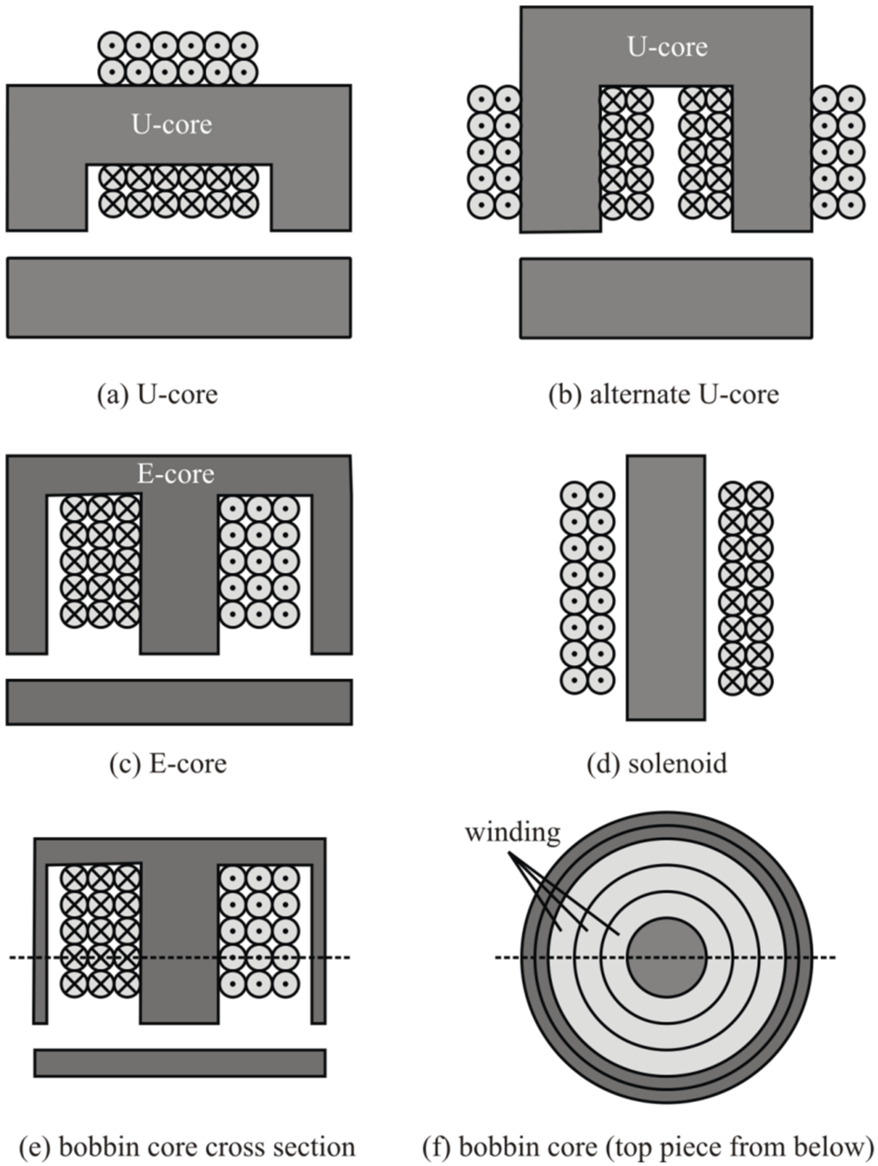
Power Magnet Devices: A Multi-Objective Design Approach

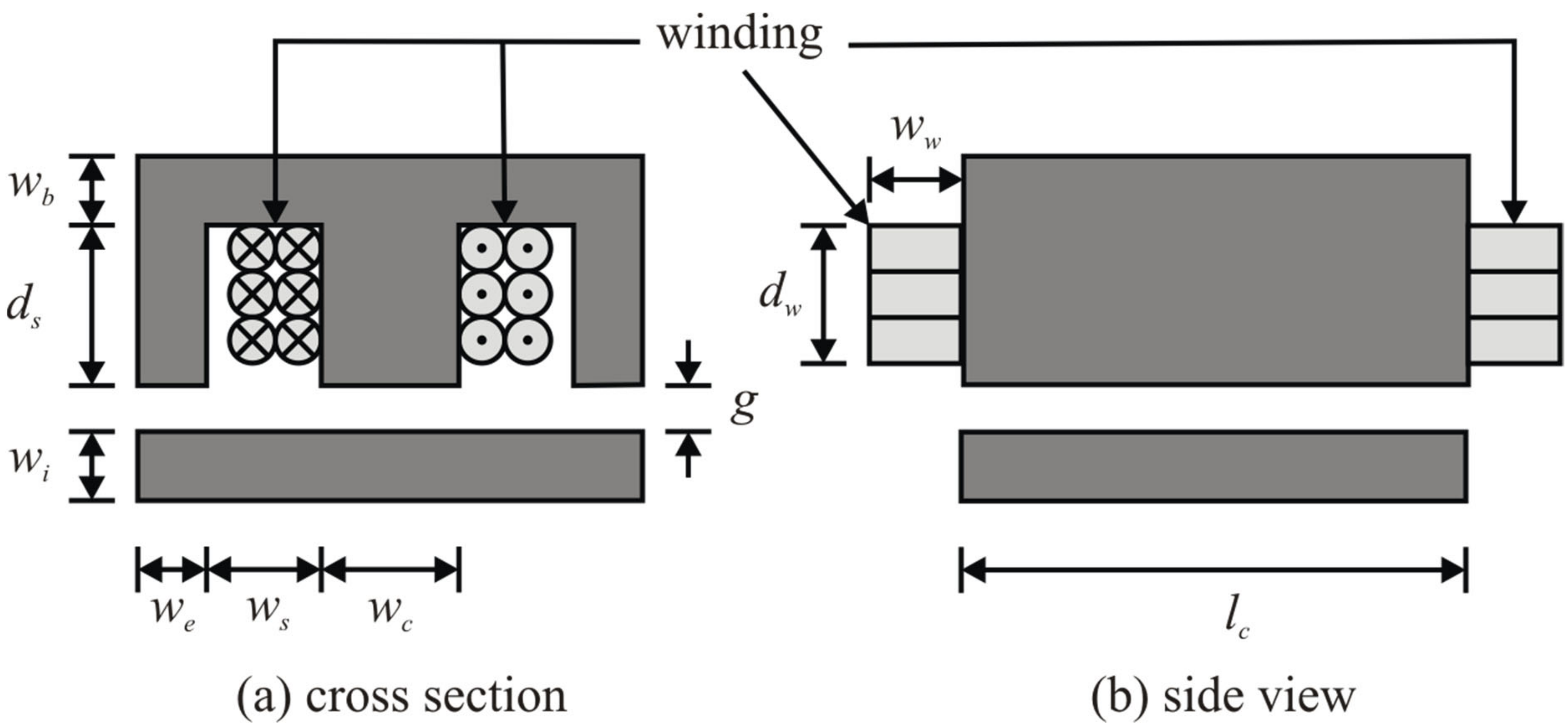
Chapter 5: Introduction to Electromagnet Design

5.1 Common Electromagnet Architectures



5.2 Analysis of an EI Core Electromagnet

- Configuration



5.2 Analysis of an EI Core Electromagnet

- Electrical analysis

$$A_{cl} = w_w d_w$$

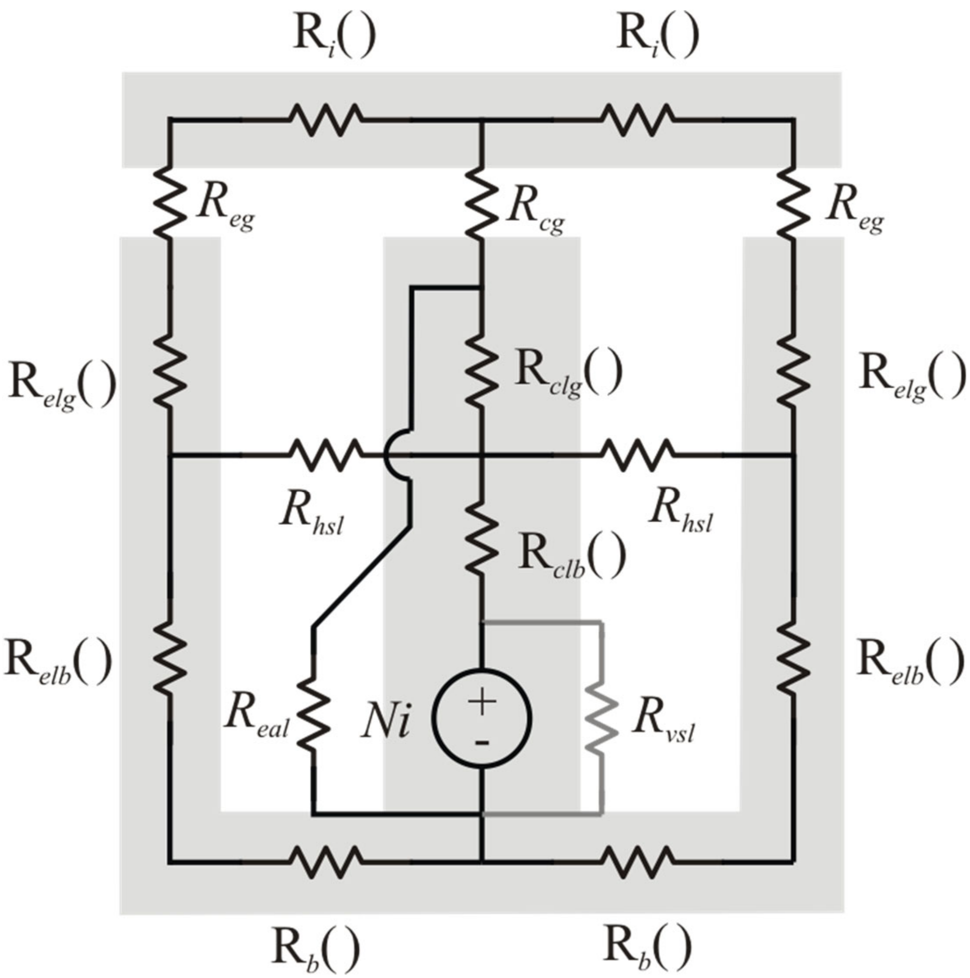
$$V_{cl} = d_w \left(\pi w_w^2 + (2l_c + 2w_c) w_w \right)$$

$$R_{cl} = \frac{V_{cl} N^2}{k_{pf} A_{cl}^2 \sigma}$$

$$i = \frac{v}{R_{cl}}$$

5.2 Analysis of an EI Core Electromagnet

- Magnetic analysis



$$\mathrm{R}_b(\Phi) = \frac{2w_s + w_c + w_e}{2w_b l_c \mu_B(\Phi/(w_b l_c))}$$

$$\mathrm{R}_i(\Phi) = \frac{2w_s + w_c + w_e}{2w_i l_c \mu_B(\Phi/(w_i l_c))}$$

$$\mathrm{R}_{elb}(\Phi) = \frac{2d_h + w_b}{2w_e l_c \mu_B(\Phi/(w_e l_c))}$$

$$\mathrm{R}_{elg}(\Phi) = \frac{d_s - d_h}{w_e l_c \mu_B(\Phi/(w_e l_c))}$$

$$\mathrm{R}_{clb}(\Phi) = \frac{2d_h + w_b}{2w_c l_c \mu_B(\Phi/(w_c l_c))}$$

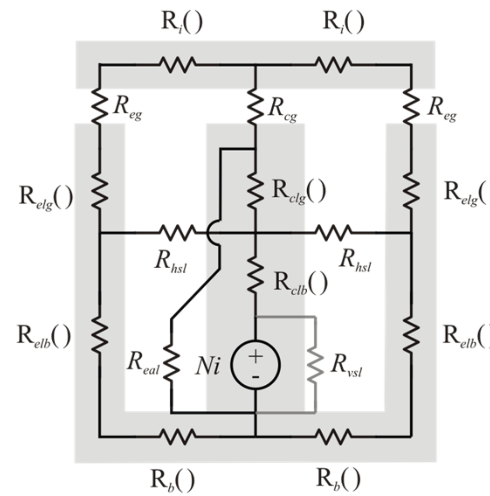
$$\mathrm{R}_{clg}(\Phi) = \frac{d_s - d_h}{w_c l_c \mu_B(\Phi/(w_c l_c))}$$

5.2 Analysis of an EI Core Electromagnet

- In expressions for magnetizing reluctances

$$d_h = \begin{cases} \sqrt{d_w(d_s - d_w/2)} & d_w \geq 2d_s/3 \\ d_s/2 + d_w/4 & d_w \leq 2d_s/3 \end{cases}$$

5.2 Analysis of an EI Core Electromagnet



- End airgap permeance

$$P_{eg} = P_{egd} + P_{efoc} + P_{efic} + 2P_{eff}$$

$$P_{egd} = \frac{\mu_0 w_e l_c}{g}$$

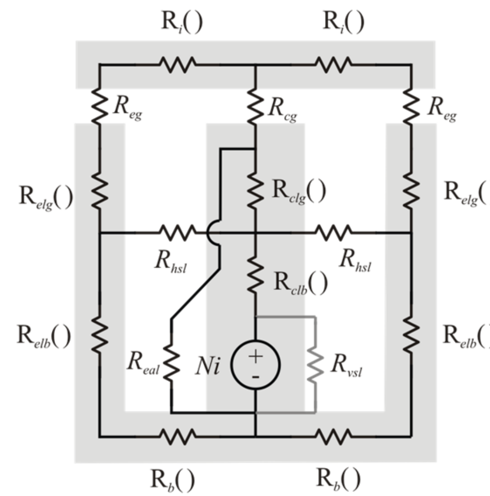
$$P_{efoc} = \frac{\mu_0 l_c}{\pi} \ln\left(1 + \frac{\pi}{g}\min(w_i, d_s + w_b)\right)$$

$$P_{efic} = \frac{2\mu_0 l_c}{\pi} \ln\left(1 + \frac{\pi}{4}\frac{\min(2d_s, w_s)}{g}\right)$$

$$P_{eff} = \frac{\mu_0 w_e}{\pi} \ln\left(1 + \pi\frac{\min(w_i, d_s + w_b)}{g}\right)$$

5.2 Analysis of an EI Core Electromagnet

- Center airgap permeance



$$P_{cg} = P_{cgd} + 2P_{efic} + 2P_{cff}$$

$$P_{cgd} = \frac{\mu_0 w_c l_c}{g}$$

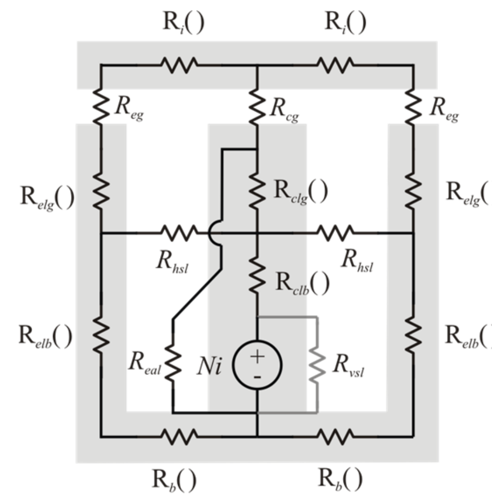
$$P_{cff} = \frac{\mu_0 w_c}{\pi} \ln\left(1 + \pi \frac{\min(w_i, d_s + w_b)}{g} \right)$$

5.2 Analysis of an EI Core Electromagnet

- Slot leakage

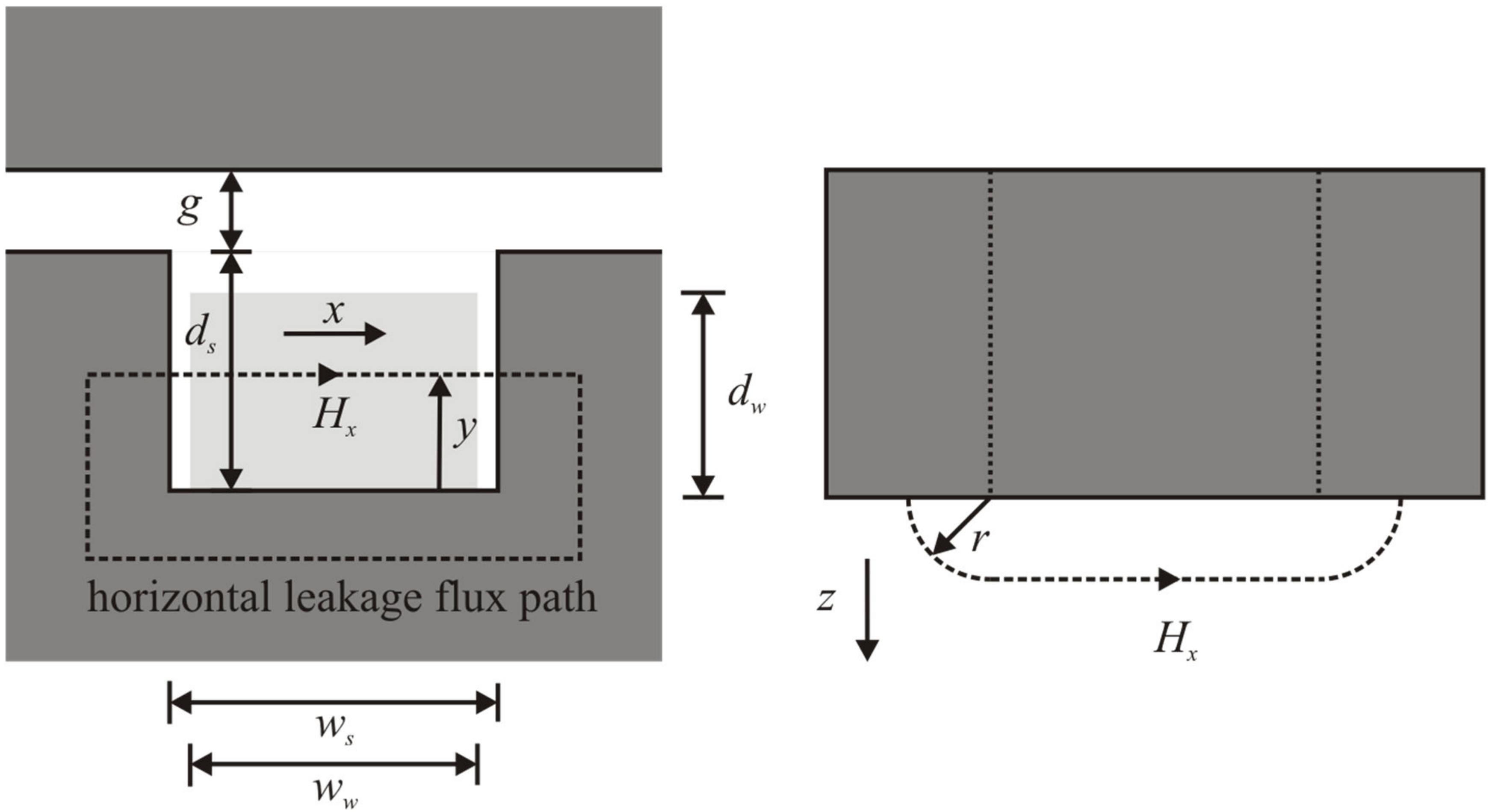
$$P_{hsl} = \frac{\mu_0 l_c \left(3d_s - 2d_w\right)}{3w_s}$$

$$P_{vsl} = \frac{\mu_0 l_c (3w_s - 2w_w)}{6(d_s + g)}$$



5.2 Analysis of an EI Core Electromagnet

- End slot leakage



(a) front view (b) top view

5.2 Analysis of an EI Core Electromagnet

- We obtain

$$P_{hsl} = \frac{\mu_0\left(3d_s - 2d_w\right)}{3}\left(\frac{l_c}{w_s} + \frac{2}{\pi}\ln\left(1 + \frac{\pi \min(2w_e, w_c)}{2w_s}\right)\right)$$

$$P_{vsl} = \frac{\mu_0(3w_s - 2w_w)}{6}\left(\frac{l_c}{d_s + g} + \frac{2}{\pi}\ln\left(1 + \frac{\pi \min(w_i, w_b)}{d_s + g}\right)\right)$$

5.2 Analysis of an EI Core Electromagnet

- Derivation of new portion of P_{hsl}

5.2 Analysis of an EI Core Electromagnet

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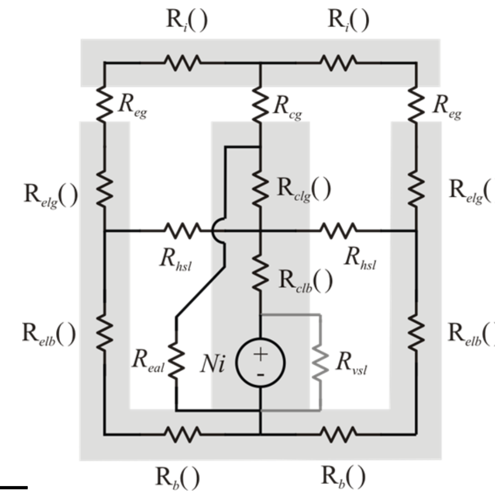
5.2 Analysis of an EI Core Electromagnet

- Front face leakage



$$P_{eal} = \frac{2}{R_{eale}} + 2P_{eali}$$

$$w_{e2} = d_s - d_w$$

$$R_{eale} = \frac{w_w(w_b + w_{e2}) + \sqrt{(2d_w + w_b + w_{e2})(w_b + w_{e2})w_b w_{e2}}}{\mu_0 w_c w_b w_{e2}}$$

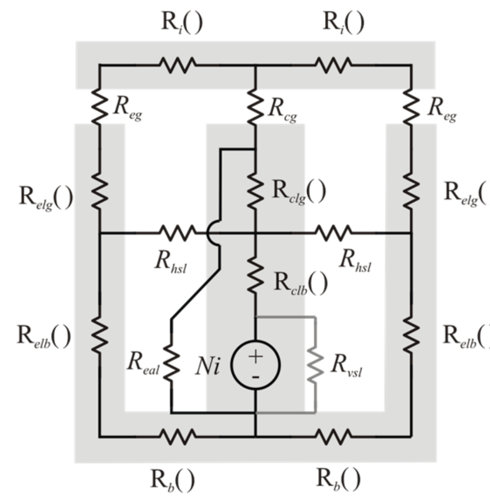
$$P_{eali} = \frac{\mu_0 w_c}{256 w_w^2 d_w^2}\begin{bmatrix} 16k_2^4 + 16\sqrt{2}k_1 k_2^3 + 4k_1^2 k_2^2 - \cdots \\ 2\sqrt{2}k_1^3 k_2 + k_1^4 \ln\left(1 + \frac{2\sqrt{2}k_2}{k_1}\right) \end{bmatrix}$$

$$k_2 = \begin{cases} \sqrt{2}w_w & d_w > 2w_w \\ d_w/\sqrt{2} & d_w \le 2w_w \end{cases} \qquad k_1 = \left|d_w - 2w_w\right|$$

5.2 Analysis of an EI Core Electromagnet

- Expression for force

$$f_e = \frac{1}{2} \sum_{b \in B_D} F_b^2 \frac{\partial \mathrm{P}_b(x)}{\partial x}$$



$$f_e = -\frac{1}{2}\left(\frac{1}{2}\frac{1}{P_{eg}^2}\frac{\partial P_{eg}}{\partial g} + \frac{1}{P_{cg}^2}\frac{\partial P_{cg}}{\partial g}\right)\Phi_1^2 - \frac{1}{2}N^2 i^2 \frac{\partial P_{vsl}}{\partial g}$$

5.2 Analysis of an EI Core Electromagnet

- We have

$$\frac{\partial P_{eg}}{\partial x} = \frac{\partial P_{egd}}{\partial x} + \frac{\partial P_{efoc}}{\partial x} + \frac{\partial P_{efic}}{\partial x} + 2\frac{\partial P_{eff}}{\partial x}$$

$$\frac{\partial P_{egd}}{\partial g} = -\frac{\mu_0 w_e l_c}{g^2}$$

$$\frac{\partial P_{efoc}}{\partial g} = -\frac{\mu_0 l_c}{g} \frac{\min(w_i, d_s + w_b)}{g + \pi \min(w_i, d_s + w_b)}$$

$$\frac{\partial P_{efic}}{\partial g} = -\frac{2\mu_0 l_c}{g} \frac{\min(2d_s, w_s)}{4g + \pi \min(2d_s, w_s)}$$

$$\frac{\partial P_{eff}}{\partial g} = -\frac{\mu_0 w_e}{g} \frac{\min(w_i, d_s + w_b)}{g + \pi \min(w_i, d_s + w_b)}$$

5.2 Analysis of an EI Core Electromagnet

- and have

$$\frac{\partial P_{cg}}{\partial g}=\frac{\partial P_{cgd}}{\partial g}+2\frac{\partial P_{efic}}{\partial g}+2\frac{\partial P_{cff}}{\partial g}$$

$$\frac{\partial P_{cdg}}{\partial g}=-\frac{\mu_0 w_c l_c}{g^2}$$

$$\frac{\partial P_{cff}}{\partial g}=-\frac{\mu_0 w_c}{g}\frac{\min(w_i,d_s+w_b)}{g+\pi\min(w_i,d_s+w_b)}$$

$$\frac{\partial P_{vsl}}{\partial g}=-\frac{\mu_0(3w_s-2w_w)}{6(d_s+g)}\left(\frac{l_c}{d_s+g}+\frac{2\min(w_i,w_b)}{d_s+g+\pi\min(w_i,w_b)}\right)$$

5.2 Analysis of an EI Core Electromagnet

- Limits as the gap goes to zero

5.2 Analysis of an EI Core Electromagnet

- Example 5.2A. Let's take a look at a EI core
- Dimensions (in mm):

$$w_s = 37 \quad w_b = 27.3 \quad w_e = 28.7 \quad w_c = 57.4$$

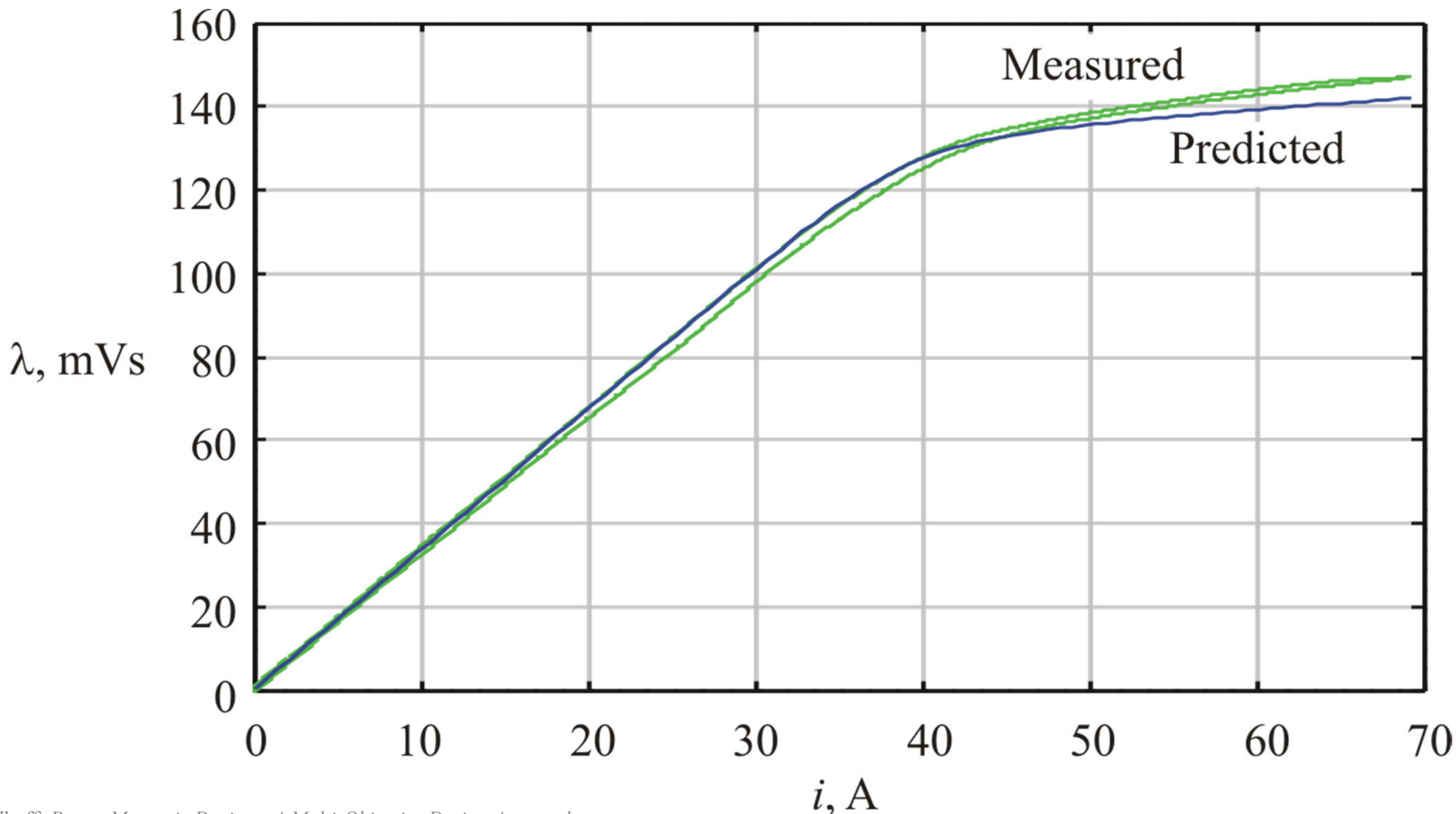
$$d_s = 48.8 \quad w_i = 30.5 \quad l_c = 119.8 \quad g = 2.96$$

$$w_w = 15.8 \quad d_w = 44.4$$

- Winding
  – N=40, a_c = 11 μm^2

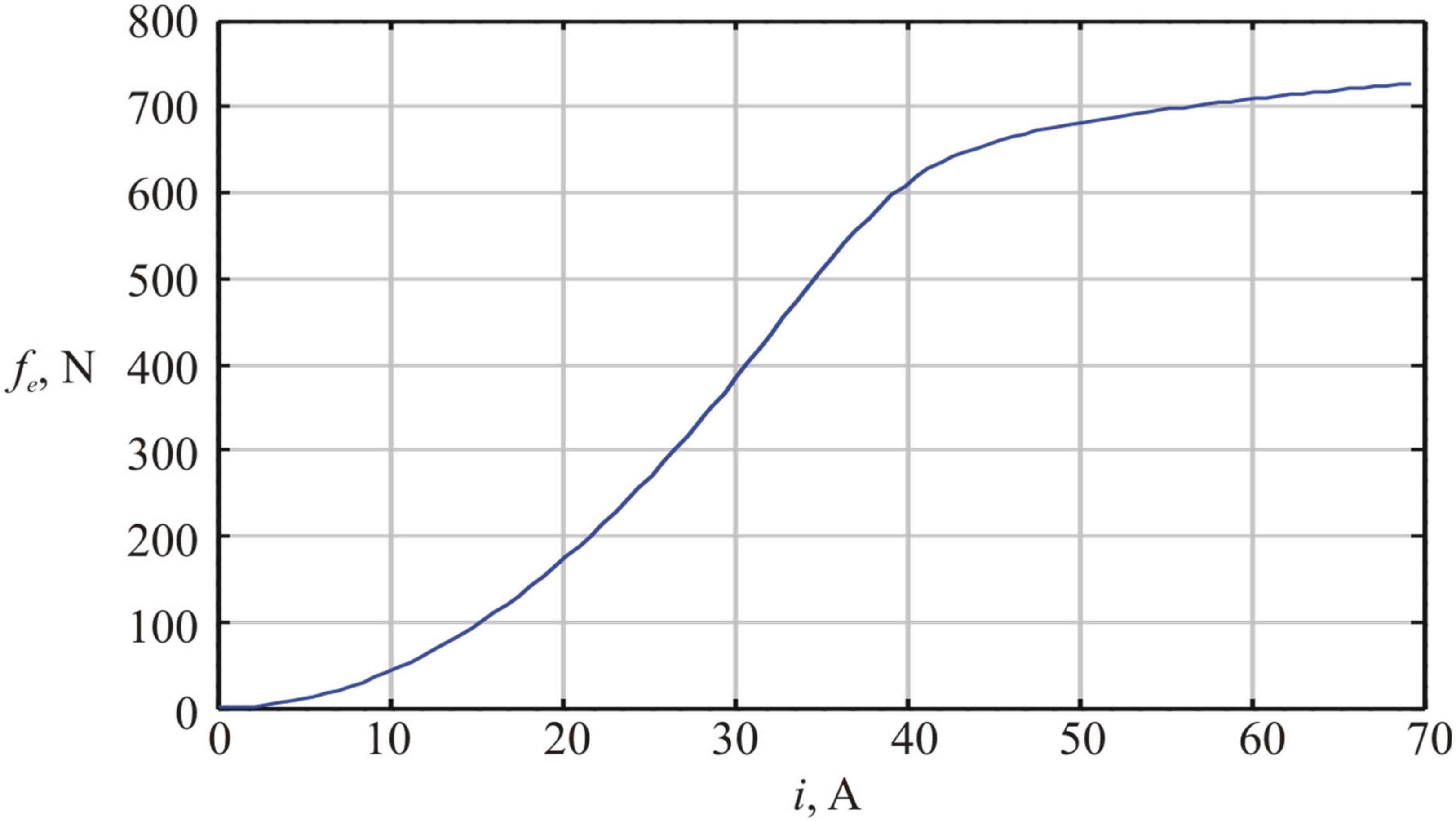
5.2 Analysis of an EI Core Electromagnet

- Predicted resistance: 24.7 mΩ
- Measured resistance: 30.5 mΩ



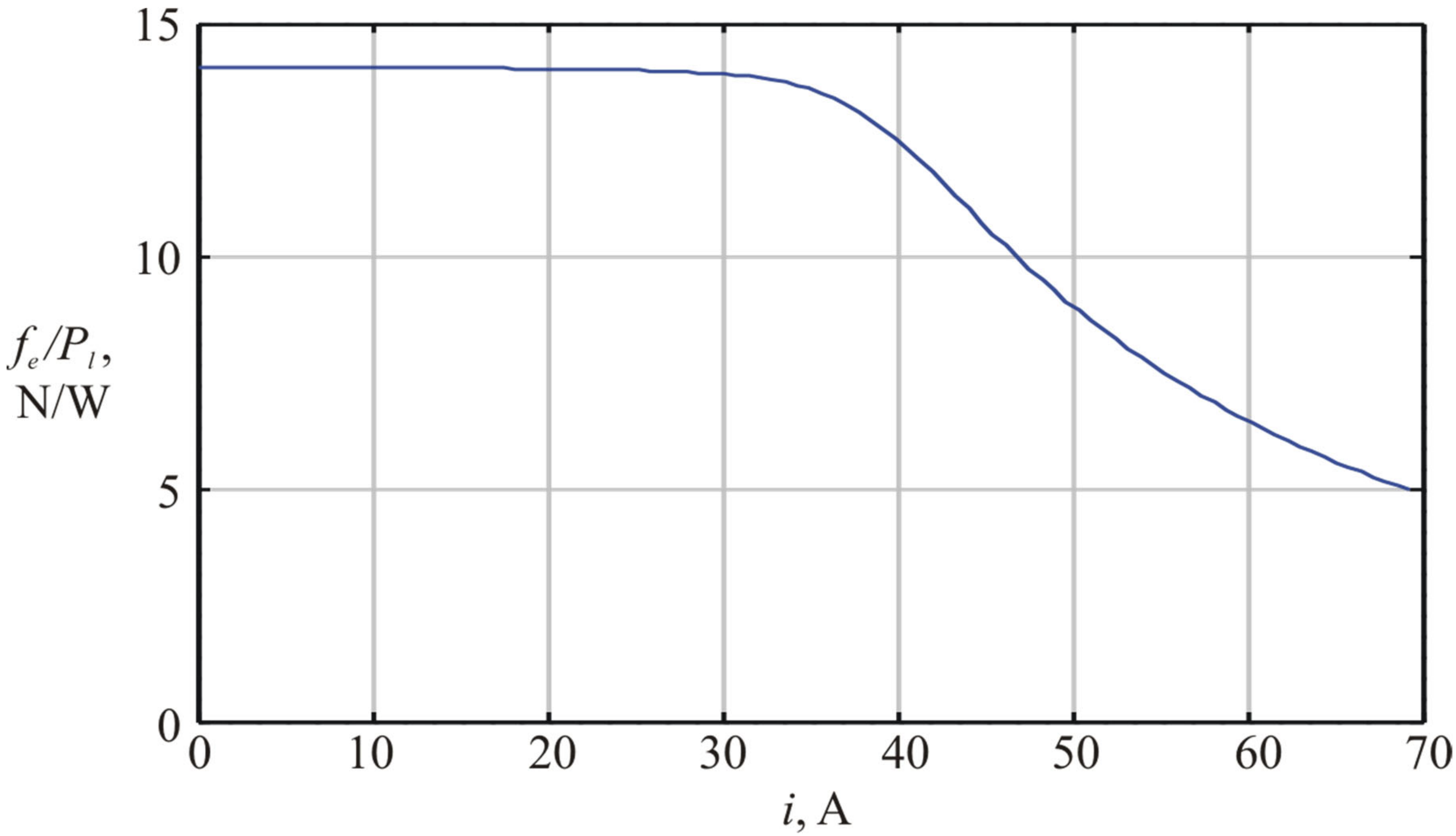
5.2 Analysis of an EI Core Electromagnet

- Force versus current (predicted)



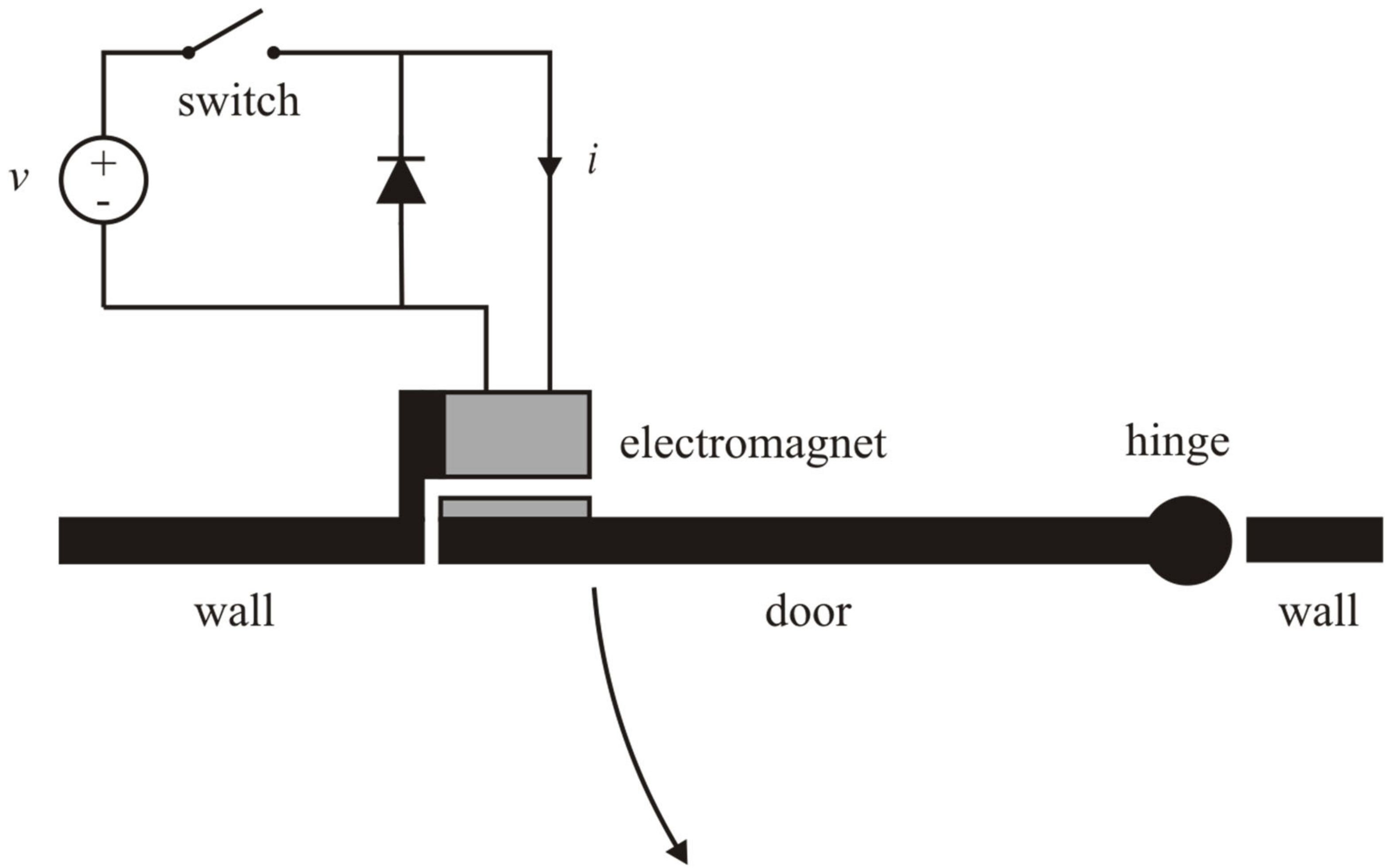
5.2 Analysis of an EI Core Electromagnet

- Force over power versus current (predicted)



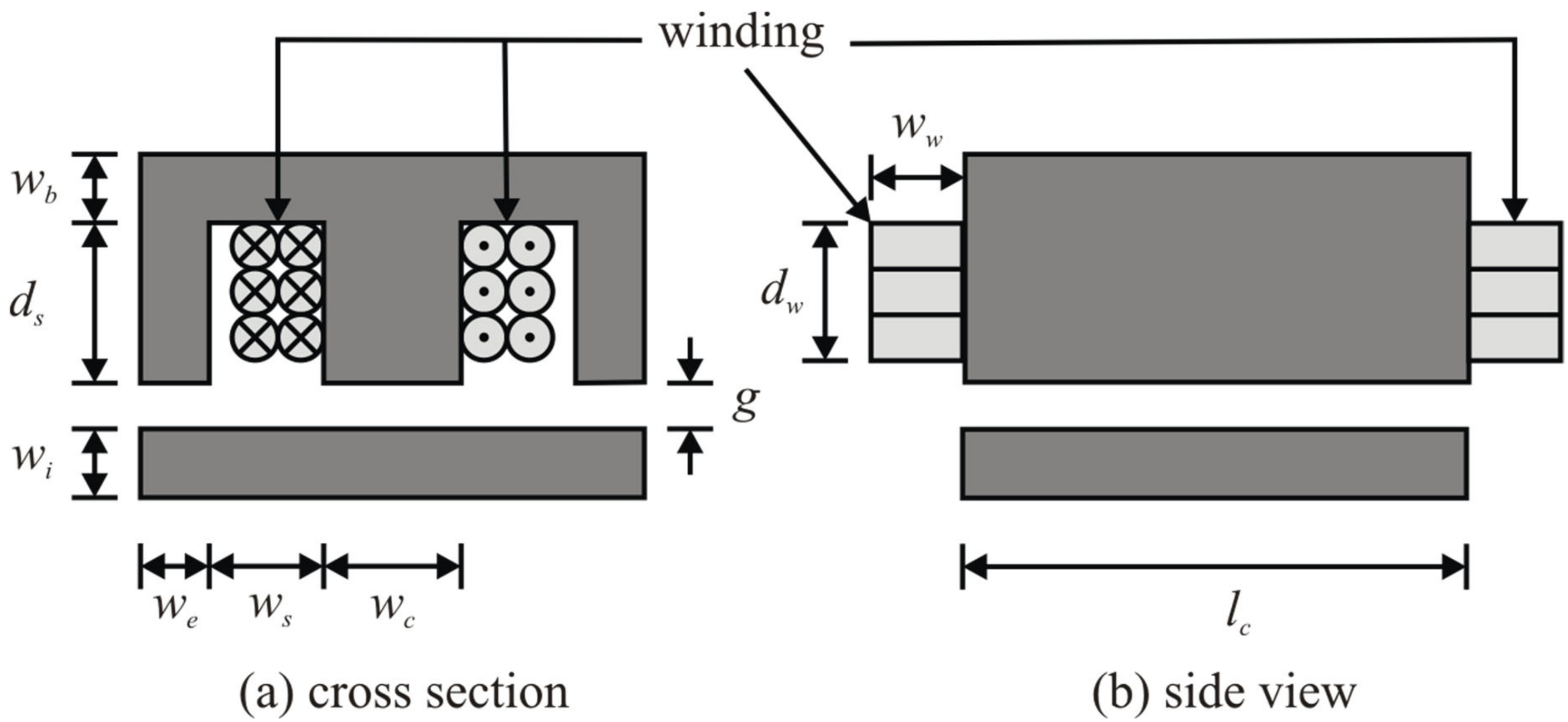
5.3 EI Core Electromagnet Design

- Configuration



5.3 EI Core Electromagnet Design

- Design space



$$\boldsymbol{\theta} = [T_{cr}\ T_{cd}\ w_c\ r_{ec}\ r_{ic}\ r_{bc}\ a_c^*\ N^*\ N_w^*\ N_d^*]^T$$

5.3 EI Core Electromagnet Design

- Widths

$$w_e = \frac{1}{2} r_{ec} w_c \qquad w_i = \frac{1}{2} r_{ic} w_c \qquad w_b = \frac{1}{2} r_{bc} w_c$$

- Conductors

$$a_c = \text{round}_{WG}(a_c^*)$$

$$r_c = \sqrt{\frac{a_c}{\pi}}$$

$$N = \text{round}(N^*)$$

5.3 EI Core Electromagnet Design

- Dimensions

$$N_w = \text{round}(N_w^*) \qquad N_d = \text{round}(N_d^*)$$

$$w_w = 2 r_c k_b N_w \qquad d_w = 2 r_c k_b N_d$$

$$w_s = w_w + c_w$$

$$d_s = d_w + c_d$$

$$l_c = 2w_s + 2w_e + w_c$$

$$h_E = w_i + g + d_s + w_b$$

$$w_E = 2w_s + 2w_e + w_c$$

$$l_E = l_c + 2w_w$$

5.3 EI Core Electromagnet Design

- Metrics

$$V_E = h_E w_E l_E$$

$$P_E = \frac{v^2}{R_{cl}}$$

5.3 EI Core Electromagnet Design

- Constraints

$$c_1 = \text{gte}(N_d N_w, N)$$

$$k_{pf} = \frac{N a_c}{w_w d_w}$$

$$c_2 = \text{lte}(k_{pf}, k_{pf,mx})$$

$$J = \frac{i}{a_c}$$

$$c_3 = \text{lte}(J, \mathrm{J}_{\text{mx}}(T_{cd}))$$

5.3 EI Core Electromagnet Design

- More constraints

$$\alpha_L = \frac{\max(h_E, w_E, l_E)}{\min(h_E, w_E, l_E)}$$

$$c_4 = \text{lte}(\alpha_L, \alpha_{mx})$$

$$c_5 = \text{lte}(V_E, V_{Emxa})$$

$$c_6 = \text{lte}(P_E, P_{Emxa})$$

$$c_7 =$$

$$c_8 = \text{gte}(f_e, f_{Emnr})$$

5.3 EI Core Electromagnet Design

- Pseudo-code for fitness function

calculate V_E , P_E

calculate constraints c_1 through c_6

$C_S = c_1 + c_2 + c_3 + c_4 + c_5 + c_6$

$C_I = 6$

if ($C_S < C_I$)

$$\mathbf{f} = \varepsilon\left(\frac{C_s - C}{C}\right)\begin{bmatrix}1\\1\end{bmatrix}$$

return

end

construct MEC and calculate c_7 and c_8

$C_s = C_s + c_7 + c_8$

$C_I = C_I + 2$

if ($C_S < C_I$)

$$\mathbf{f} = \varepsilon\left(\frac{C_s - C}{C}\right)\begin{bmatrix}1\\1\end{bmatrix}$$

return

end

$$\mathbf{f} = \begin{bmatrix}\dfrac{1}{V_E}\\[2mm]\dfrac{1}{P_E}\end{bmatrix}$$

return

5.4 Case Study

- Specifications

Table 5.4-1 Electromagnet specifications.

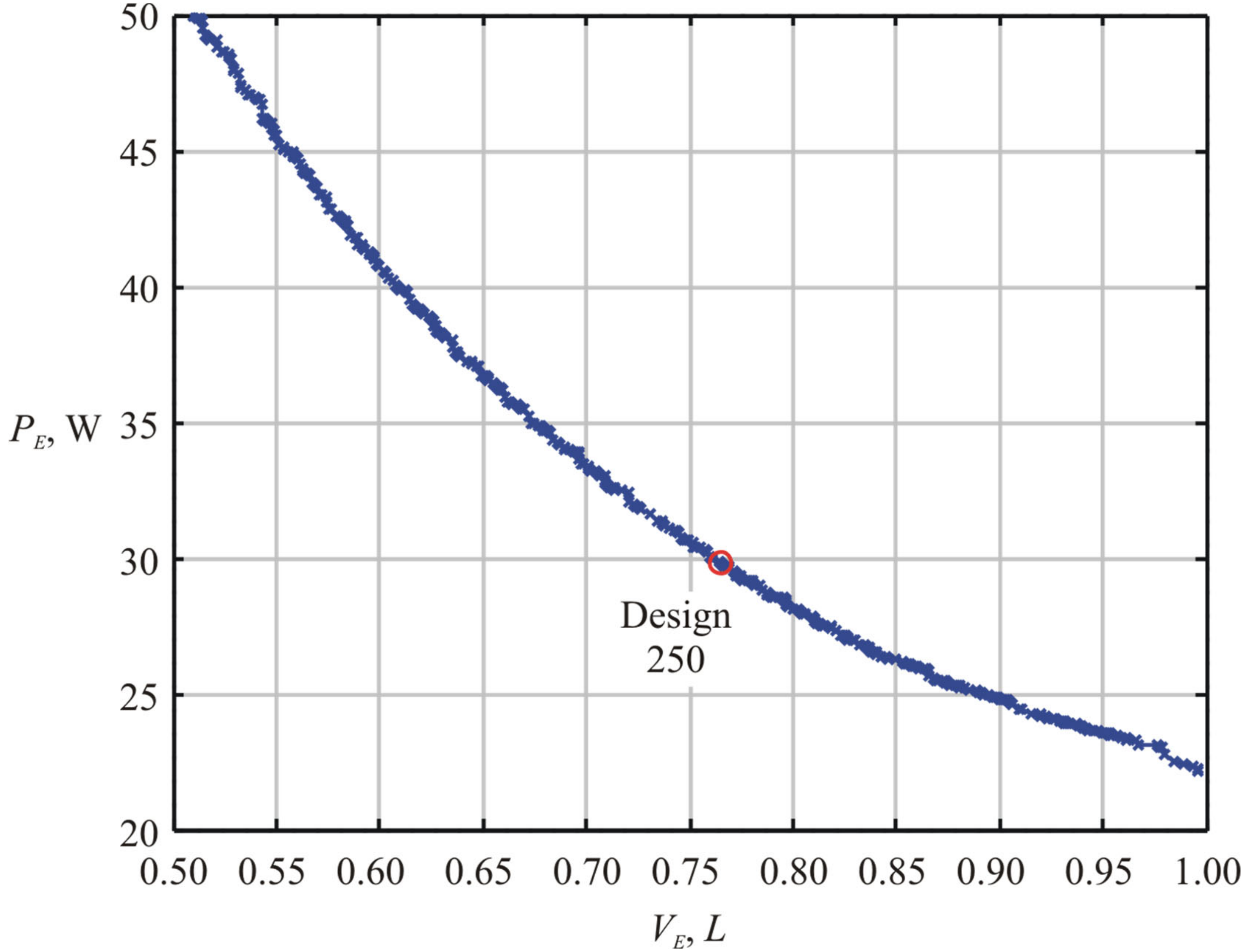
Symbol	Description	Value
v	applied voltage	12 V
g	air gap	1 mm
α_{mxa}	maximum allowed aspect ratio	3
V_{Emxa}	maximum allowed volume	1 L
P_{Emxa}	maximum allowed loss	50 W
k_{pf-mxa}	maximum allowed packing factor	0.7
k_b	winding build factor	1.05
f_{Emnr}	minimum required electromagnetic force	2500 N
c_w	winding clearance in width	2 mm
c_d	winding clearance in depth	2 mm

5.4 Case Study

- Design space

Par.	Description	Min.	Max.	Enc.	Gene
T_{cr}	Core material	1	5	int	1
T_{cd}	Conductor material	1	2	int	2
w_c	Center leg width	$2\cdot 10^{-3}$	10^{-1}	log	3
r_{ec}	Twice w_e to w_c ratio	0.5	1.5	lin	4
r_{ic}	Twice w_i to w_c ratio	0.25	1.5	lin	5
r_{bc}	Twice w_b to w_c ratio	0.25	1.5	lin	6
a_c^*	Cross sectional conductor area (m^2)	10^{-9}	10^{-4}	log	7
N^*	Desired number of turns	1	10^3	log	8
N_w^*	Desired slot width in conductors	1	10^3	log	9
N_d^*	Desired slot depth in conductors	1	10^3	log	10

5.4 Case Study



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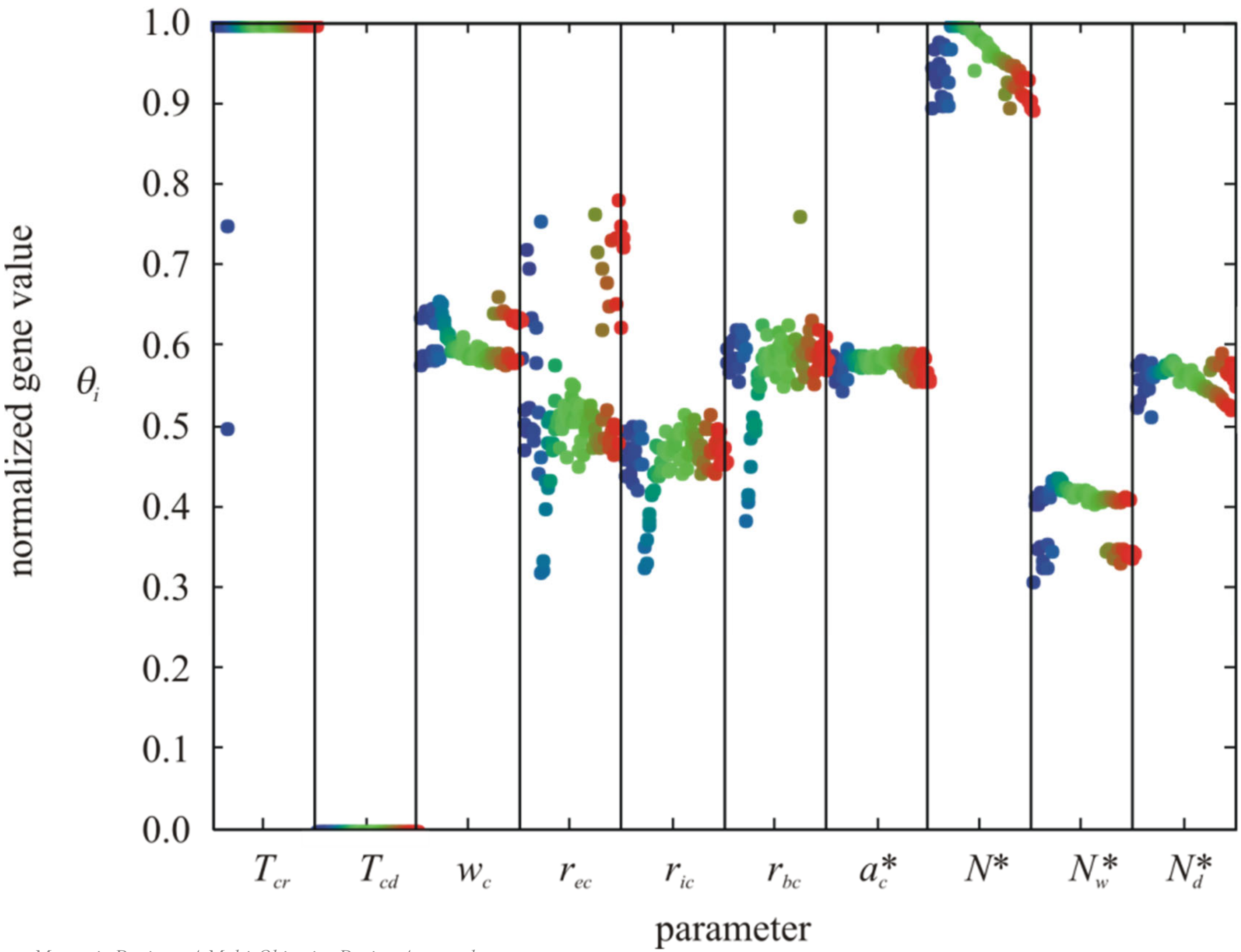

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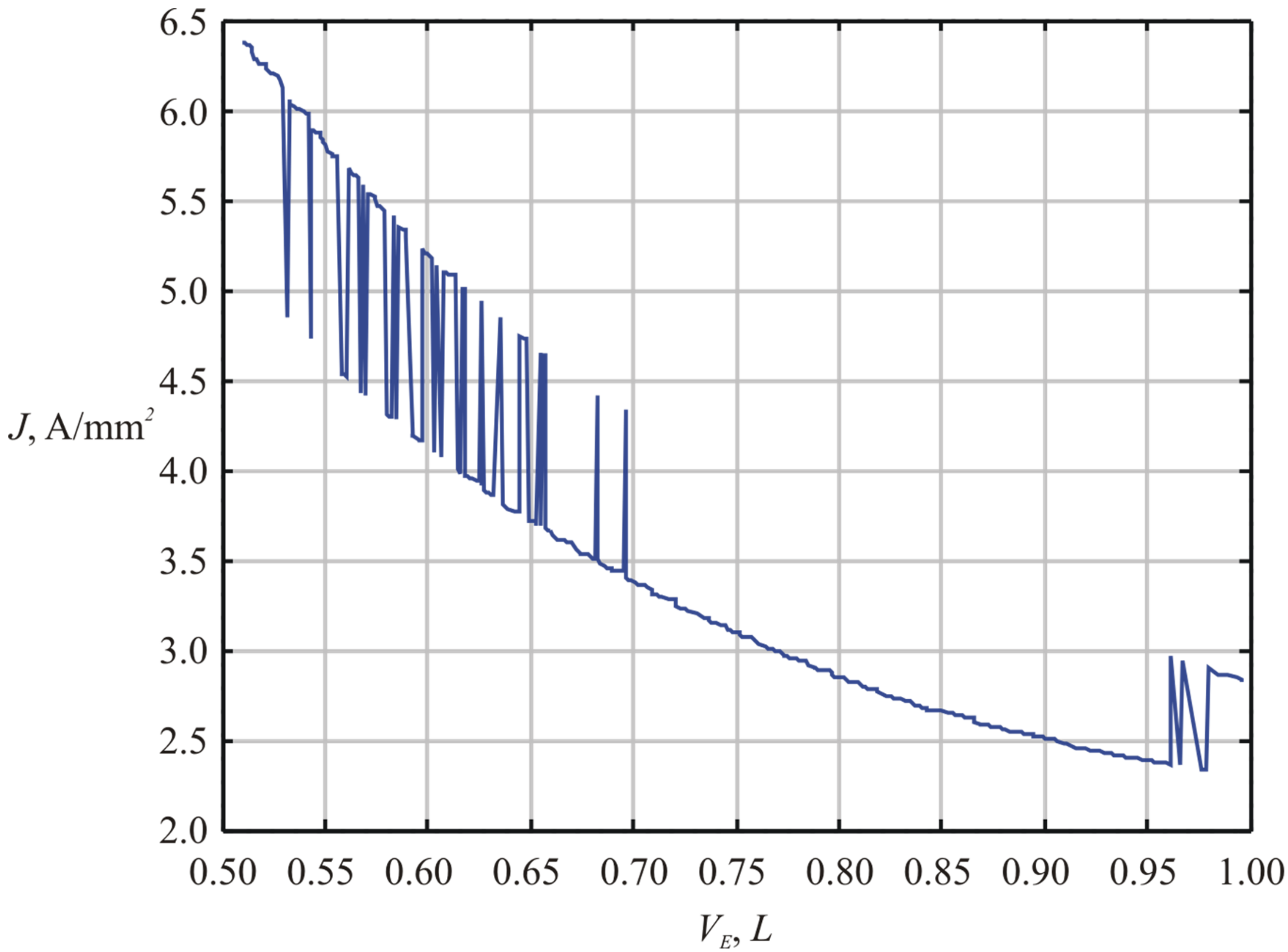

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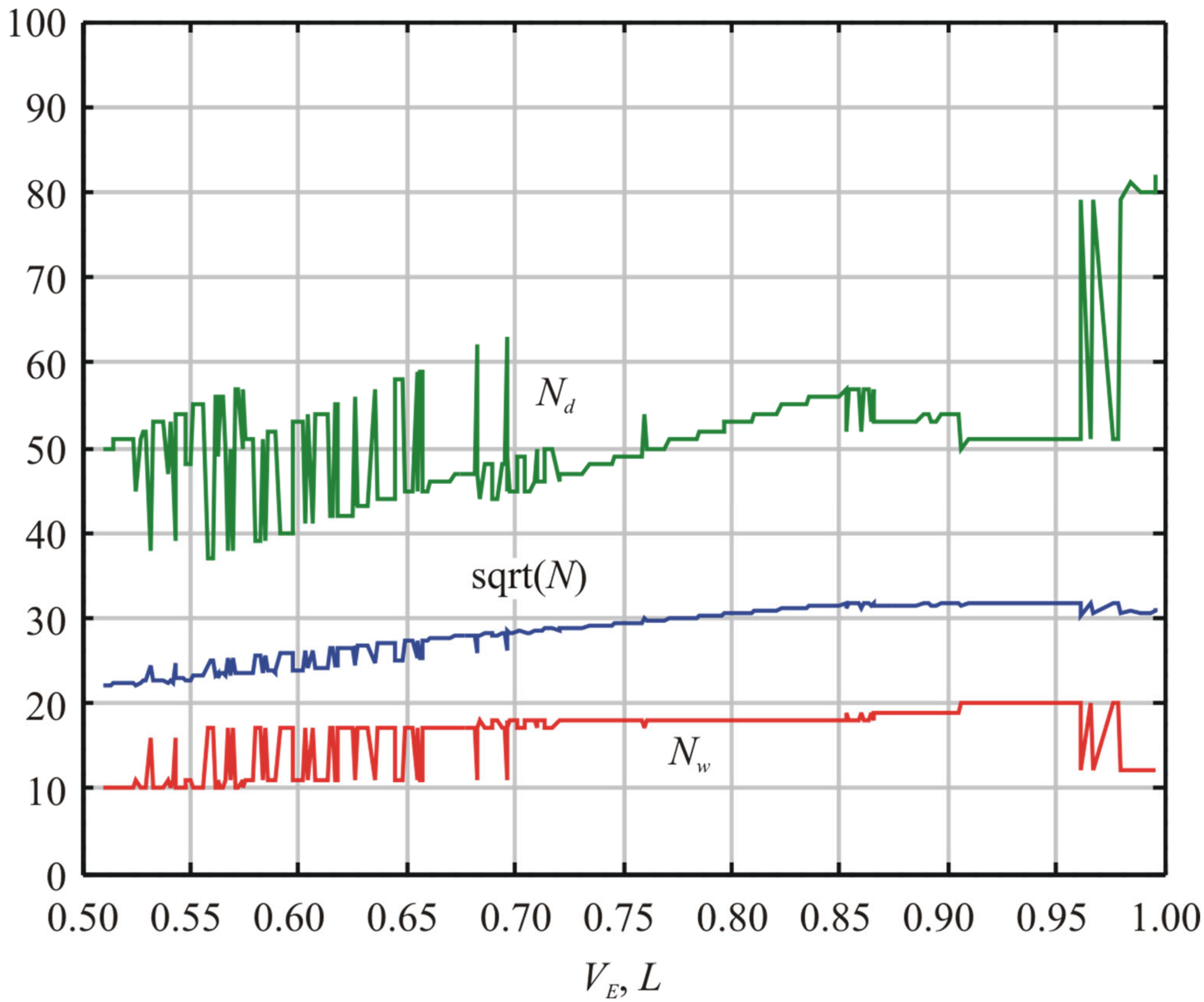

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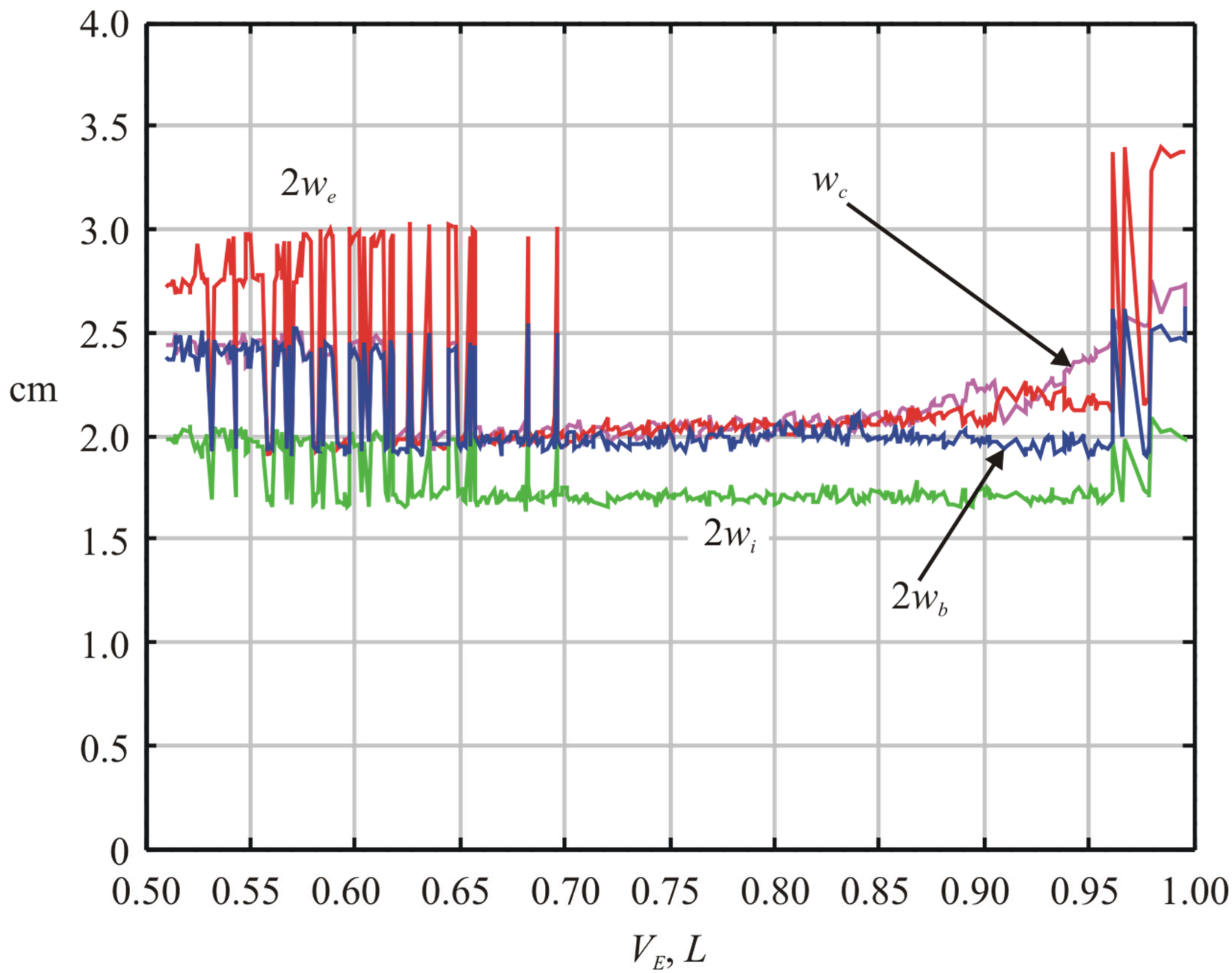

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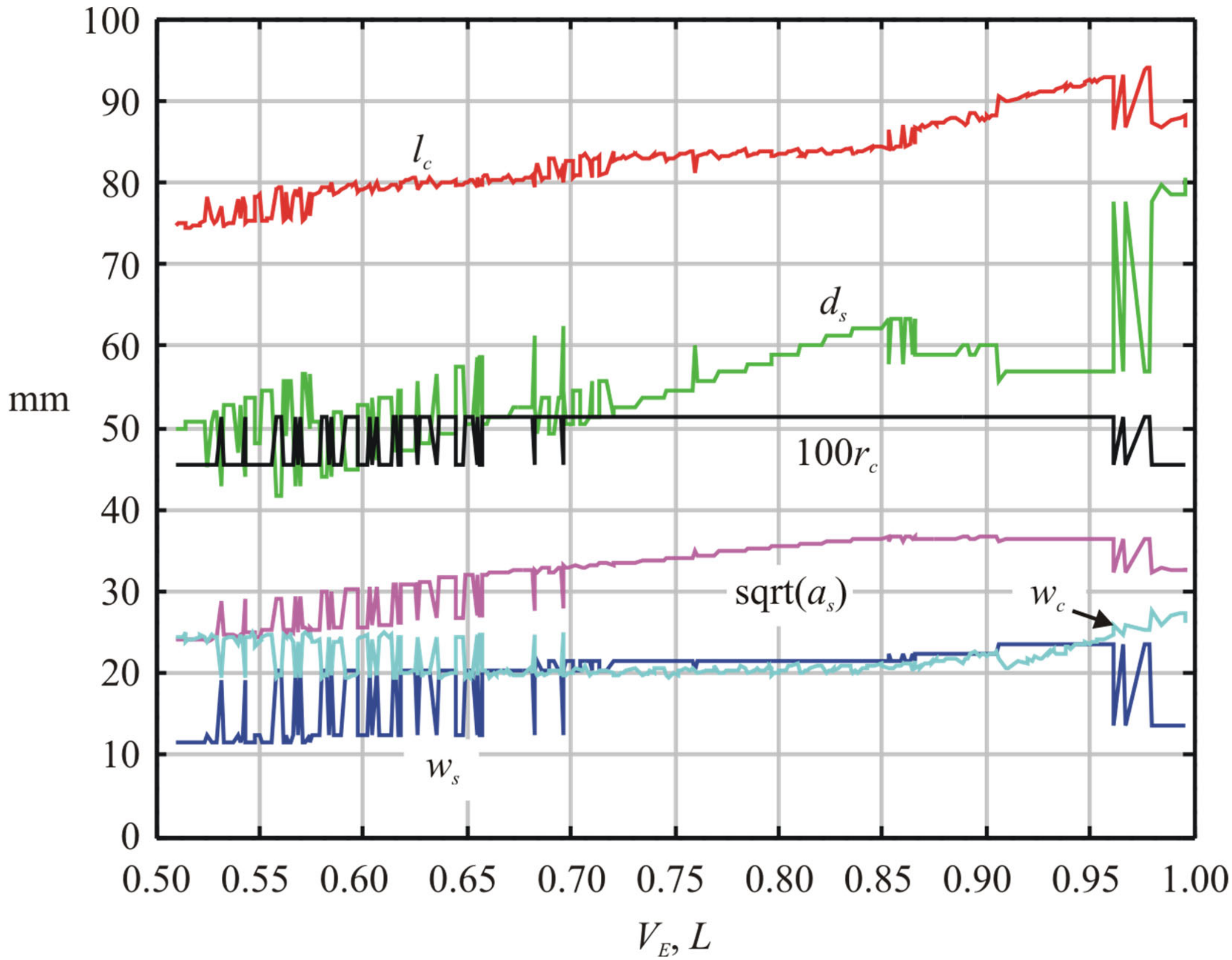
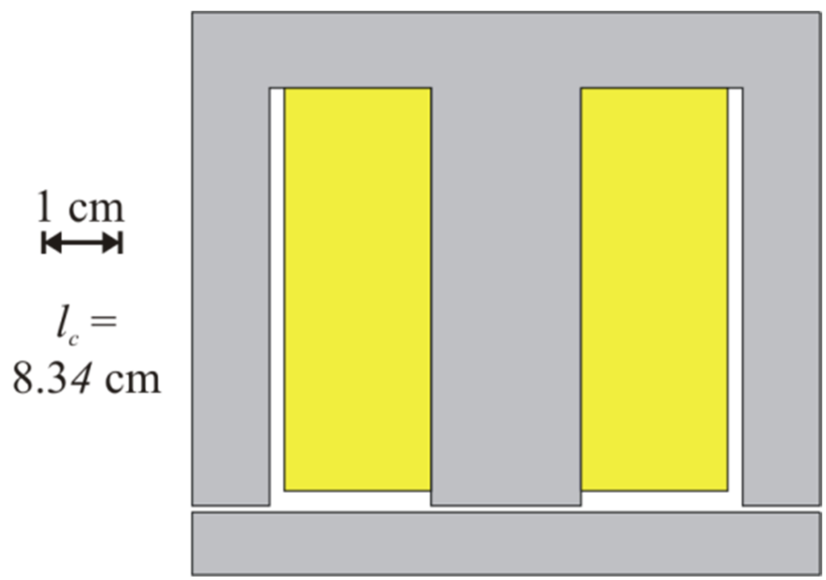


5.4 Case Study

EI Core Data	Winding Data	Metrics
Material = Hiperco50	Material = Copper	V_E = 764 mL
w_c = 2.02 cm	AWG = 18	P_E = 29.9 W
w_e = 1.03 cm	a_c = 0.823 mm^2	i_E = 2.49 A
w_i = 8.36 mm	N = 882	J_E = 3.03 A/mm^2
w_b = 9.93 mm	N_w = 18	R_E = 4.82 Ω
w_s =2.13 cm	N_d = 50	h_E = 7.50 cm
d_s = 5.57 cm	d_w = 5.37 cm	w_E = 8.34 cm
l_c = 8.34 cm	w_w = 1.93 cm	l_E = 12.2 cm



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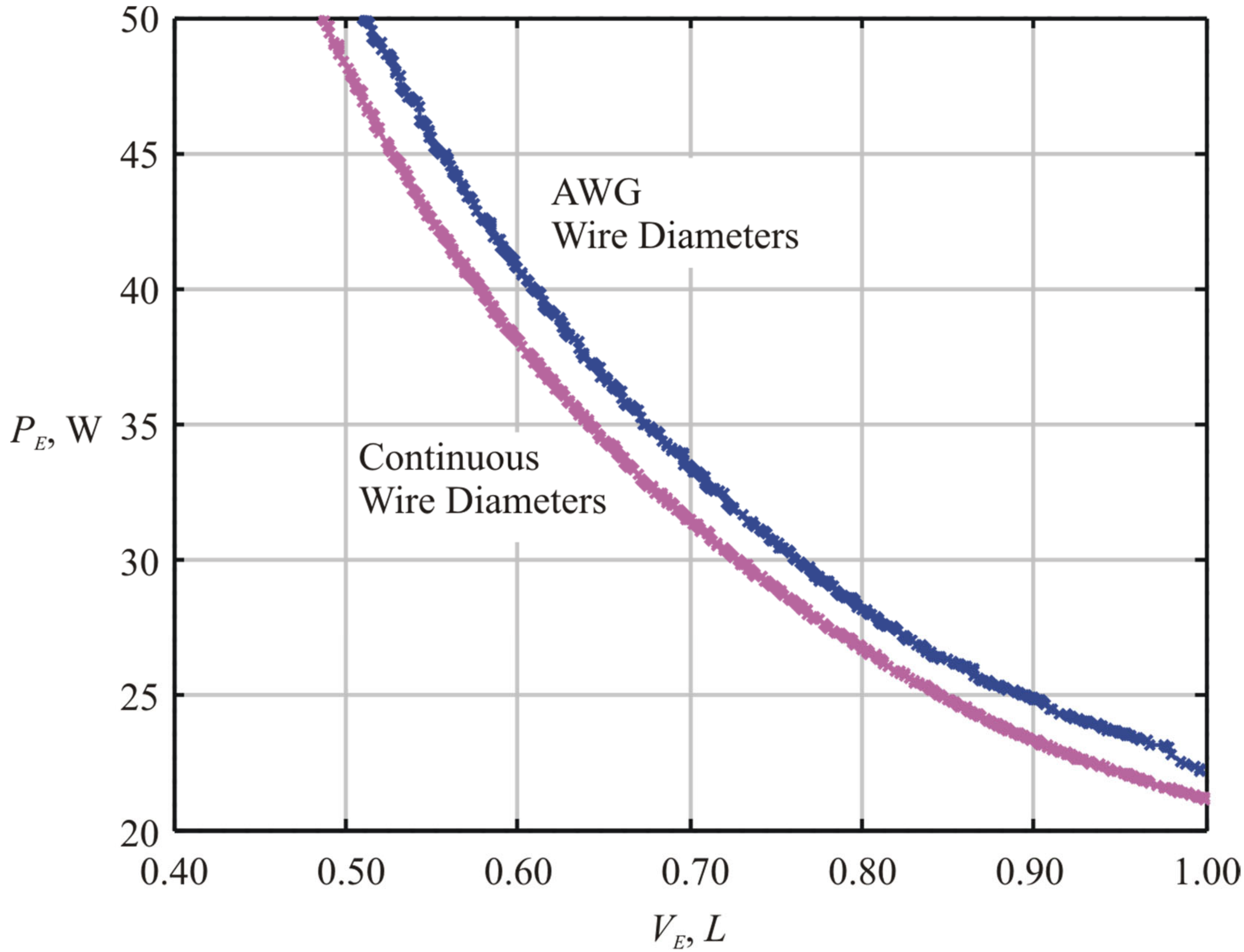

5.4 Case Study