Power Magnetic Devices: A Multi-Objective Design Approach

Chapter 4: Force and Torque

4.1 Energy Storage

- Forms for flux linkage equations

$$\mathbf{i} = \mathrm{i}(\boldsymbol{\lambda}, x)$$

$$\boldsymbol{\lambda} = \lambda(\mathbf{i}, x)$$

4.1 Energy Storage

- The energy transferred to the coupling field may be expressed

$$W_f = \sum_{j=1}^{J} \int_{\lambda_{j,0}}^{\lambda_{j,f}} i_j d\lambda_j - \int_{x_0}^{x_f} f_e dx$$

4.2 Calculation of field energy

- We may calculate field energy as

$$\mathrm{W}_f(\boldsymbol{\lambda}, x) = \sum_{j=1}^{J} \int_{\lambda_{j,0}}^{\lambda_{j,f}} \mathrm{i}_j(\boldsymbol{\lambda}, x)\, d\lambda_j$$

4.2 Calculation of Field Energy

- Example 4.2A

$$i = (5 + 2x)\lambda^2$$

4.2 Calculation of Field Energy

- Example 4.2B

$$i_1 = 5x\lambda_1 + (10 + 2x)e^{2\lambda_1 + 2\lambda_2}$$

$$i_2 = 7\lambda_2 + (10 + 2x)e^{2\lambda_1 + 2\lambda_2}$$

4.2 Calculation of Field Energy

4.3 Force From Field Energy

- We can show

$$f_e = -\frac{\partial W_f(\boldsymbol{\lambda}, x)}{\partial x}$$

4.3 Force From Field Energy

4.3 Force From Field Energy

4.3 Force From Field Energy

- Example 4.3A. Suppose

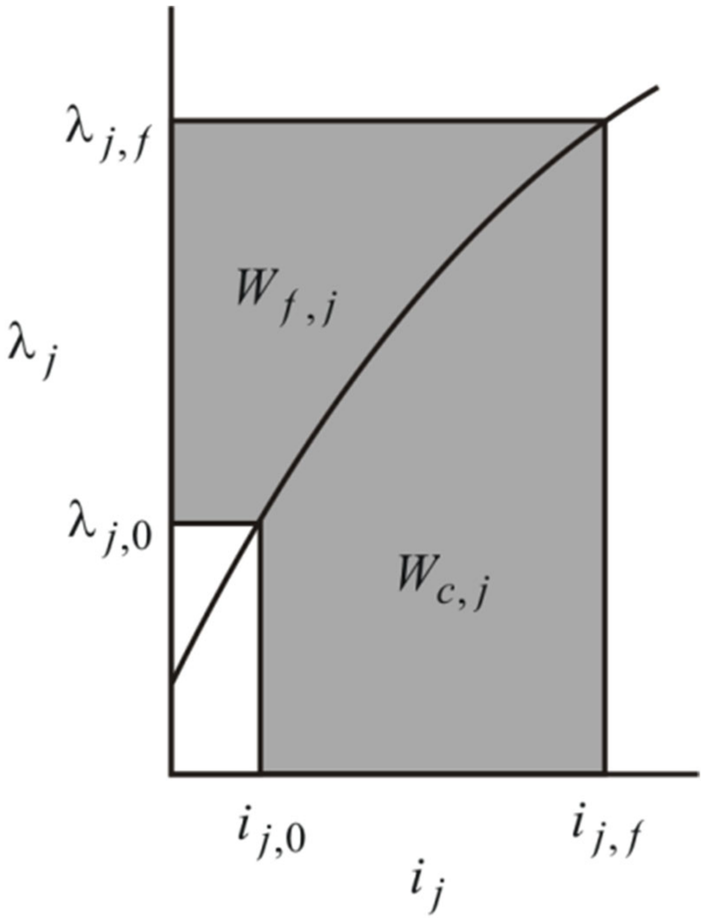
$$W_f = \frac{1}{3}(5 + 2x)\lambda^3$$

- Example 4.3B. Suppose

$$W_f = \frac{5}{2}x\lambda_1^2 + \frac{1}{2}(10 + 2x)\left(e^{2\lambda_1 + 2\lambda_2} - 1\right) + \frac{7}{2}\lambda_2^2$$

4.4 Co-Energy

- Meaning of co-energy



$$W_c + W_f = \sum_{j=1}^{J} \left(\lambda_j i_j - \lambda_{j,0} i_{j,0} \right)$$

$$\mathrm{W}_c(\mathbf{i}, x) = \sum_{j=1}^{J} \int_{i_{j,0}}^{i_{j,f}} \lambda_j(\mathbf{i}, x)\, di_j$$

4.4 Co-Energy

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- Example 4.4A

$$\lambda_1 = 2i_1 + \frac{1}{2+x}(i_1 + i_2)^{0.5}$$

$$\lambda_2 = 5i_2^{0.4} + \frac{1}{2+x}(i_1 + i_2)^{0.5}$$

4.4 Co-Energy

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- Example 4.4B

4.4 Co-Energy

4.5 Force From Co-Energy

- We can show

$$f_e = \frac{\partial W_c(\mathbf{i}, x)}{\partial x}$$

4.5 Force From Co-Energy

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- Example 4.5A. Find the force if

$$W_c = i_1^2 + \frac{2}{3}\frac{1}{2+x}(i_1+i_2)^{1.5} + \frac{5}{1.4}i_2^{1.4}$$

4.6 Conditions for Conservative Fields

- For a magnetic field to be conservative, we must have

$$\frac{\partial \mathrm{i}_j(\boldsymbol{\lambda}, x)}{\partial \lambda_k} = \frac{\partial \mathrm{i}_k(\boldsymbol{\lambda}, x)}{\partial \lambda_j}$$

$$\frac{\partial \lambda_j(\mathbf{i}, x)}{\partial i_k} = \frac{\partial \lambda_k(\mathbf{i}, x)}{\partial i_j}$$

4.6 Conditions for Conservative Fields

- Example 4.6A. Is the following system conservative?

$$\lambda_1 = 2xi_1^2 + 4i_1 i_2$$

$$\lambda_2 = \frac{7i_2}{3+x} + 5i_1$$

4.7 Magnetically Linear Systems

- A magnetically linear system may be expressed as

$$\boldsymbol{\lambda} = \mathbf{L}\mathbf{i}$$

where L is not a function of current or flux linkage.

- If a system is magnetically linear

$$W_c = \frac{1}{2}\mathbf{i}^T\mathbf{L}\mathbf{i} \qquad W_f = \frac{1}{2}\boldsymbol{\lambda}^T\mathbf{L}^{-1}\boldsymbol{\lambda} \qquad W_c = W_f$$

4.7 Magnetically Linear Systems

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4.8 Torque

- For a rotational system, the mechanical energy transferred to the coupling field may be expressed

$$W_m = -\int_{\theta_{rm,0}}^{\theta_{rm,f}} T_e \, d\theta_{rm}$$

Conclusions:

4.9 Calculating Force Using MECs

- Assuming that all branches with mechanical motion are magnetically linear, we may write

$$f_e = \frac{1}{2} \sum_{b \in B_D} F_b^2 \frac{\partial \mathrm{P}_b(x)}{\partial x}$$

4.9 Calculating Force Using MECs

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