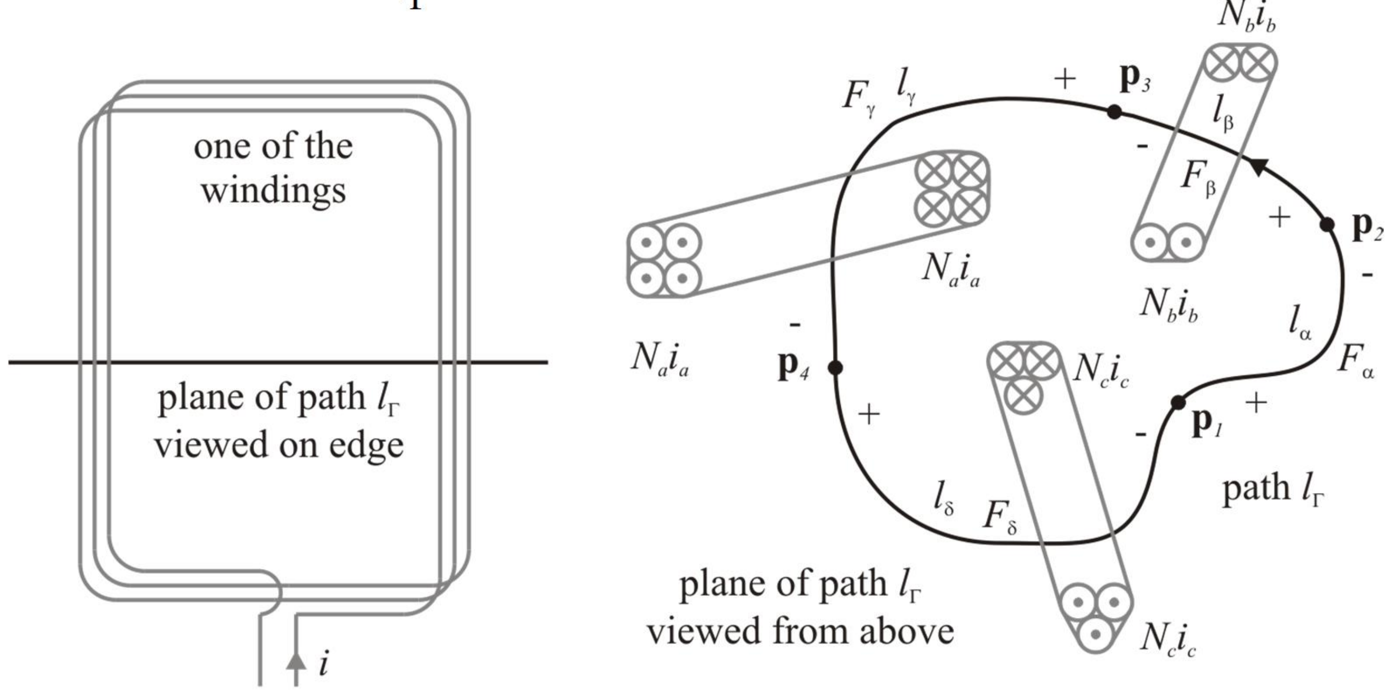
Power Magnetic Devices: A Multi-Objective Design Approach

Chapter 2 Magnetics and Magnetic Equivalent Circuits

2.1 Ampere's Law, MMF, Kirchoff's MMF Law

- Ampere's Law

$$\oint_{l_\Gamma} \mathbf{H} \cdot d\mathbf{l} = i_{enc,\Gamma}$$



(a) winding configuration

(b) path of integral

2.1 Ampere's Law, MMF, Kirchoff's MMF Law

- Enclosed current and MMF sources

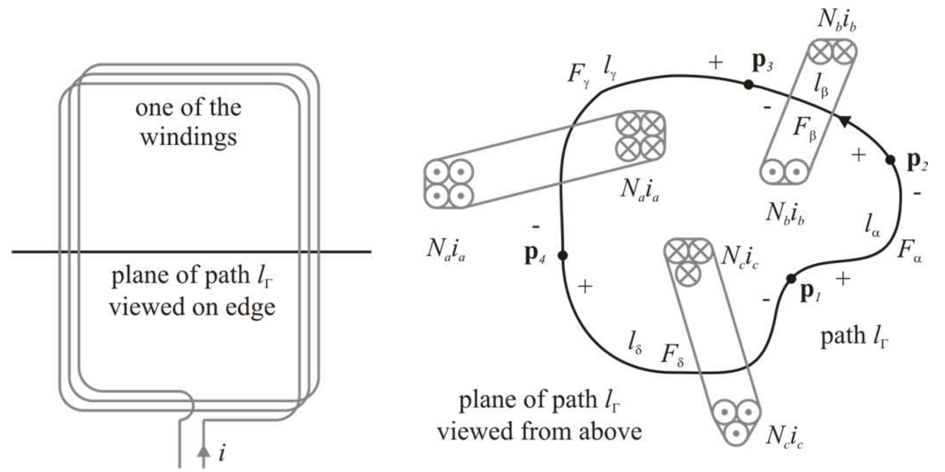
$$i_{enc,\Gamma} =$$

$$F_a =$$

$$F_b =$$

$$F_c =$$

$$i_{enc,\Gamma} = \sum_{s \in S_\Gamma} F_s$$



(a) winding configuration (b) path of integral

2.1 Ampere's Law, MMF, Kirchoff's MMF Law

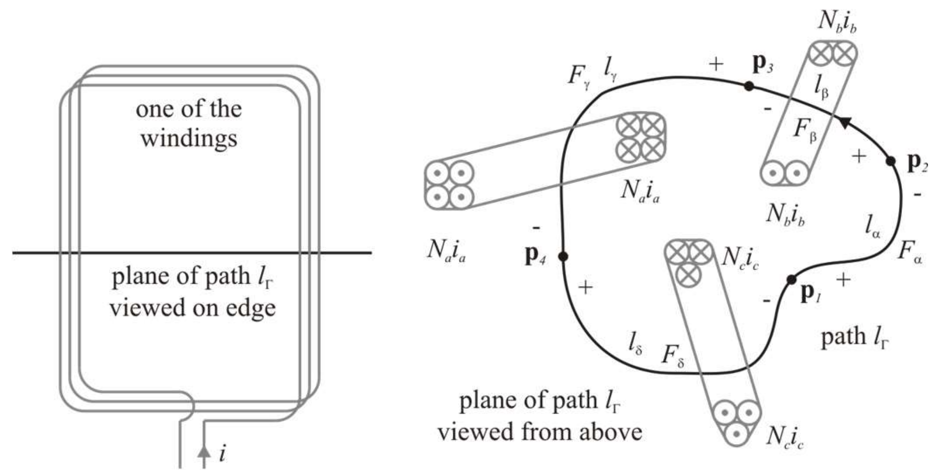
- MMF drops

$$F_y = \int_{l_y} \mathbf{H} \cdot d\mathbf{l}$$

$$\oint_{l_\Gamma} \mathbf{H} \cdot d\mathbf{l} =$$

$$\oint_{l_\Gamma} \mathbf{H} \cdot d\mathbf{l} =$$

$$\oint_{l_\Gamma} \mathbf{H} \cdot d\mathbf{l} = \sum_{d \in D_\Gamma} F_d$$



(a) winding configuration (b) path of integral

2.1 Ampere's Law, MMF, Kirchoff's MMF Law

- We have

$$\oint_{l_\Gamma} \mathbf{H} \cdot d\mathbf{l} = i_{enc,\Gamma} \qquad i_{enc,\Gamma} = \sum_{s \in S_\Gamma} F_s \qquad \oint_{l_\Gamma} \mathbf{H} \cdot d\mathbf{l} = \sum_{d \in D_\Gamma} F_d$$

- Thus

$$\sum_{d \in D_\Gamma} F_d = \sum_{s \in S_\Gamma} F_s$$

- This is Kirchoff's MMF Law: The sum of the MMF drops around a closed loop is equal to the sum of the MMF sources for that loop

2.1 Ampere's Law, MMF, Kirchoff's MMF Law

- Example 2.1A. Suppose

$$\mathbf{H} = x\mathbf{a}_x + (x^2 + y)\mathbf{a}_y$$

what is the MMF drop going from point (1,1) directly to point (5,2)

2.1 Ampere's Law, MMF, Kirchoff's MMF Law

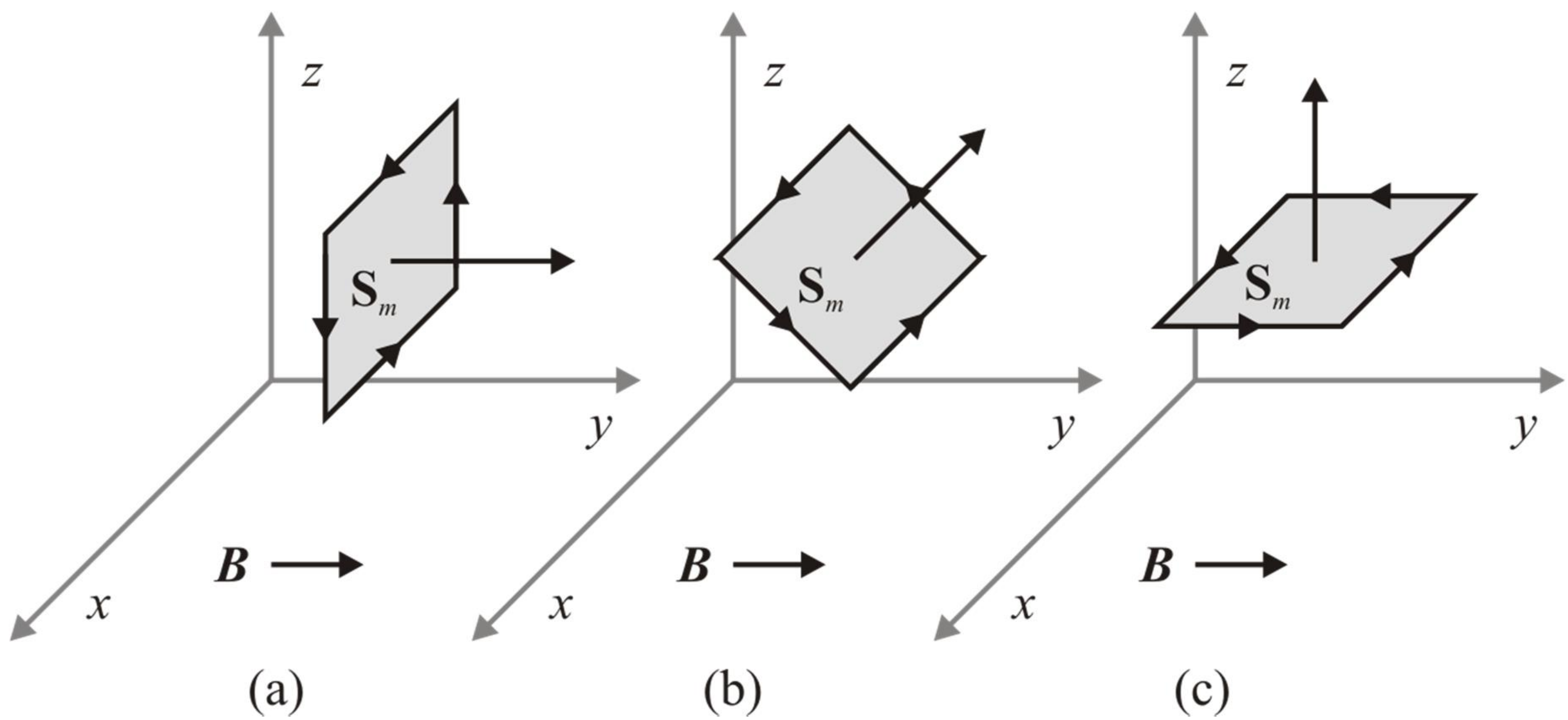
2.2 Magnetic Flux

- Flux through open surface S_m

$$\Phi_m = \int_{\mathbf{S}_m} \mathbf{B} \cdot d\mathbf{S}$$

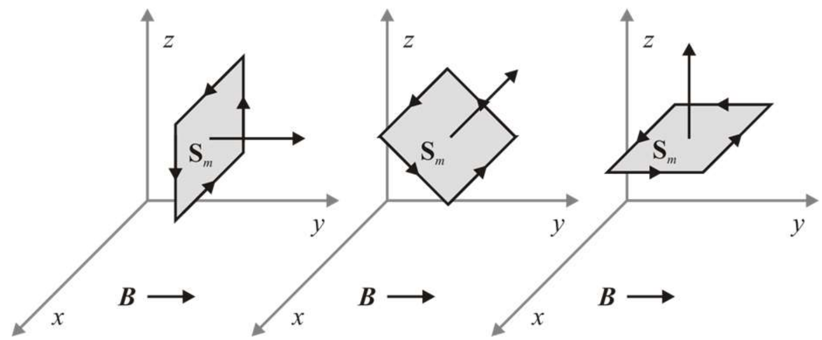
2.2 Magnetic Flux

- Example 2.2. Find the flux through the surface if $\mathbf{B} = 1.5\mathbf{a}_y$ and $|\mathbf{S}_m| = 2\ \mathrm{m}^2$



2.2 Magnetic Flux

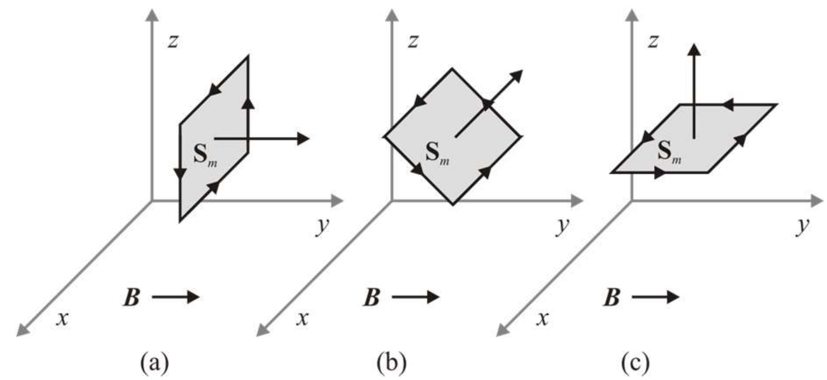
- Example 2.2 (continued)



(a) (b) (c)

2.2 Magnetic Flux

- Example 2.2 (continued)



2.2 Magnetic Flux

- Gauss's Law

$$\oint_{\mathbf{S}_\Gamma} \mathbf{B} \cdot d\mathbf{S} = 0$$

- Thus

$$\sum_{o \in \mathbf{O}_\Gamma} \int_{\mathbf{S}_o} \mathbf{B} \cdot d\mathbf{S} = 0$$

- And

$$\sum_{o \in \mathbf{O}_\Gamma} \Phi_o = 0$$

2.2 Magnetic Flux

- Kirchoff's flux law: The sum of the fluxes into (or out of) any node must be zero

2.3 Magnetically Conductive Materials

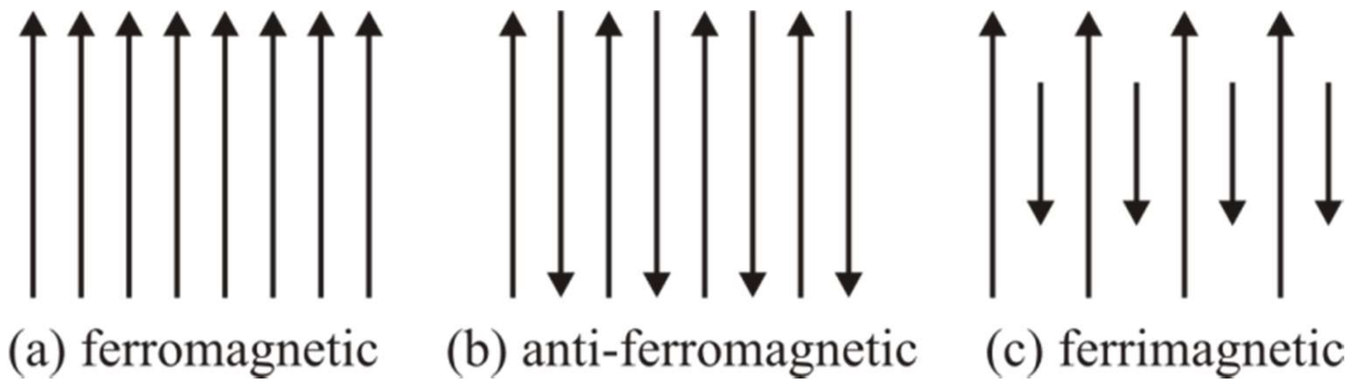
- Diamagnetic materials
  - Examples include hydrogen, helium, copper, gold, and silicon

2.3 Magnetically Conductive Materials

- Paramagnetic materials
  - Examples include oxygen and tungsten

2.3 Magnetically Conductive Materials

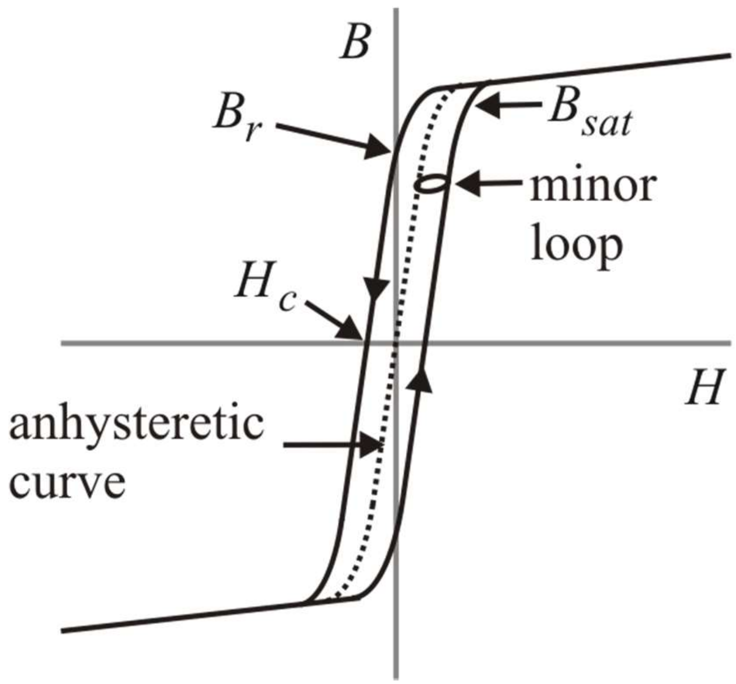
- Ferromagnetic, anti-ferromagnetic, ferrimagnetic materials



  – Ferromagnetic materials include iron, nickel, cobalt
  – Ferrimagnetic materials include iron oxide magnetite (Fe_3O_4), nickel-zinc ferrite ($Ni_{1/2}$,$Zn_{12}Fe_2O_4$), and nickel ferrite ($NiFe_2O_4$).

2.3 Magnetically Conductive Materials

- B-H characteristic of ferromagnetic and ferrimagnetic materials



2.3 Magnetically Conductive Materials

- In free space

$$B = \mu_0 H \qquad \mu_0 = 4\pi 10^{-7} \text{ H/m}$$

- In a magnetic material (anhysteretic)

$$B = \mu_0 (H + M)$$
$$M = \chi H$$

OR

$$B = \mu_0 H + M$$
$$M = \mu_0 \chi H$$

2.3 Magnetically Conductive Materials

- Relative permeability

2.3 Magnetically Conductive Materials

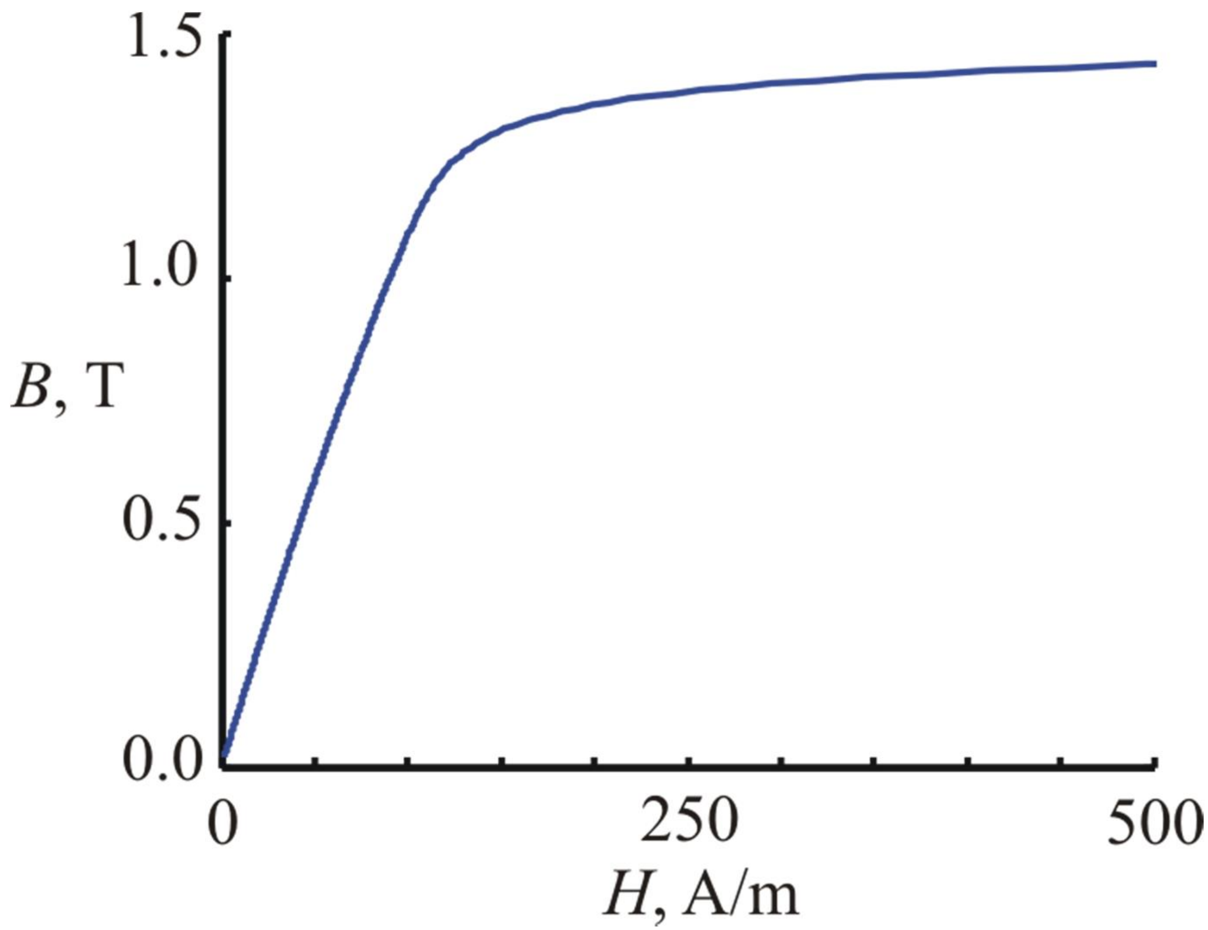
- Nonlinear anhysteretic relationships

$$B = \mu_H(H)H$$

$$B = \mu_B(B)H$$

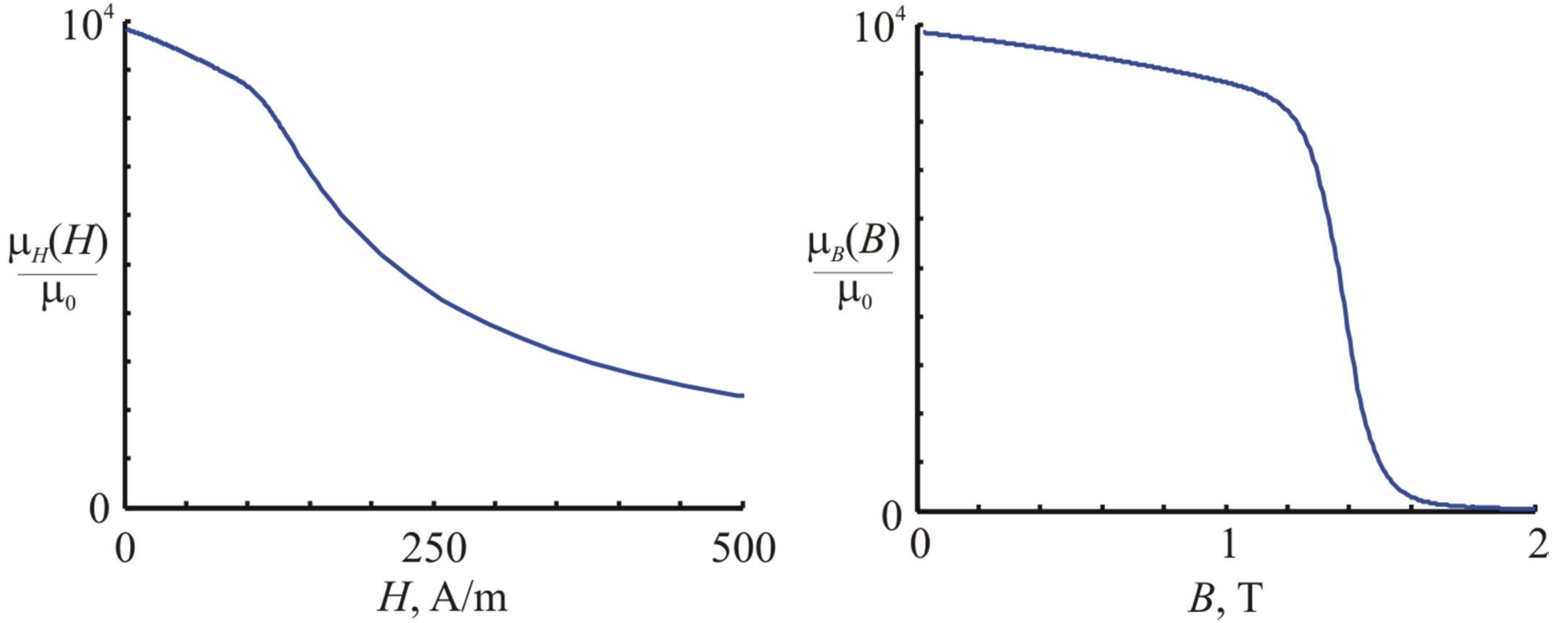
2.3 Magnetically Conductive Materials

- Consider M47 silicon steel



2.3 Magnetically Conductive Materials

- Permeability functions for M47 silicon steel



2.3 Magnetically Conductive Materials

- Permeability as a function of H

$$\mathrm{M}(H) = \operatorname{sgn}(H) \sum_{k=1}^{K} \frac{m_k \left| H / h_k \right|}{1 + \left| H / h_k \right|^{n_k}}$$

2.3 Magnetically Conductive Materials

- Permeability as a function of H

$$\mu_H(H) = \mu_0 + \sum_{k=1}^{K} \frac{m_k}{h_k} \frac{1}{1 + \left| H / h_k \right|^{n_k}}$$

- Derivative of permeability as function of H

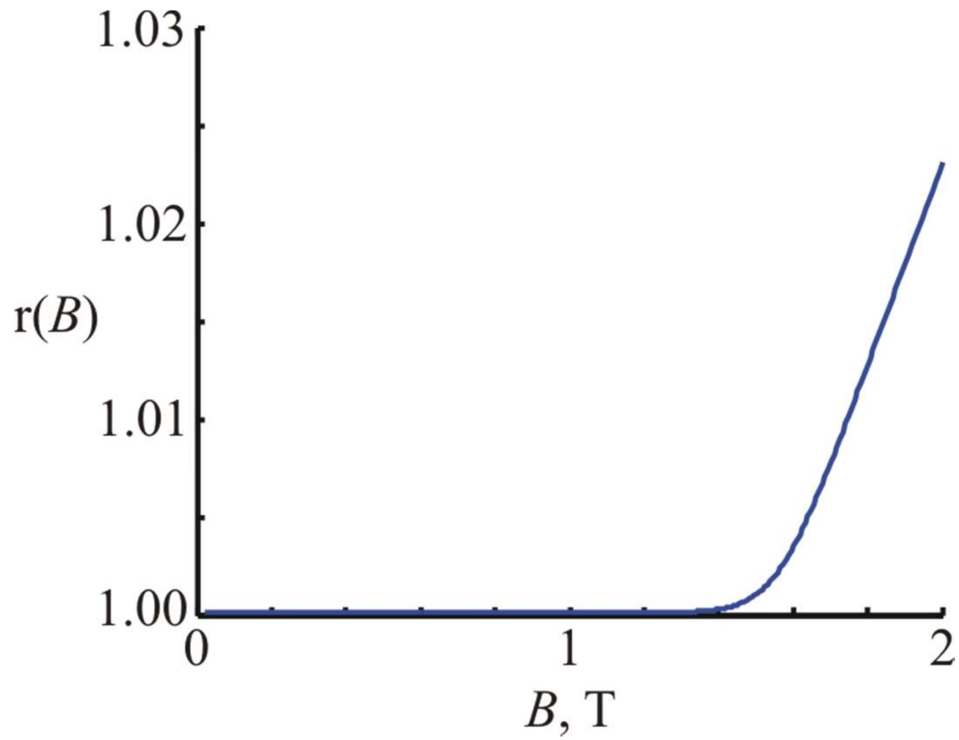
2.3 Magnetically Conductive Materials

- Derivative of permeability as a function of *H*

2.3 Magnetically Conductive Materials

- Permeability as a function of B

- Let $\mathrm{r}(B) \triangleq \dfrac{B}{\mathrm{M}(B)}$



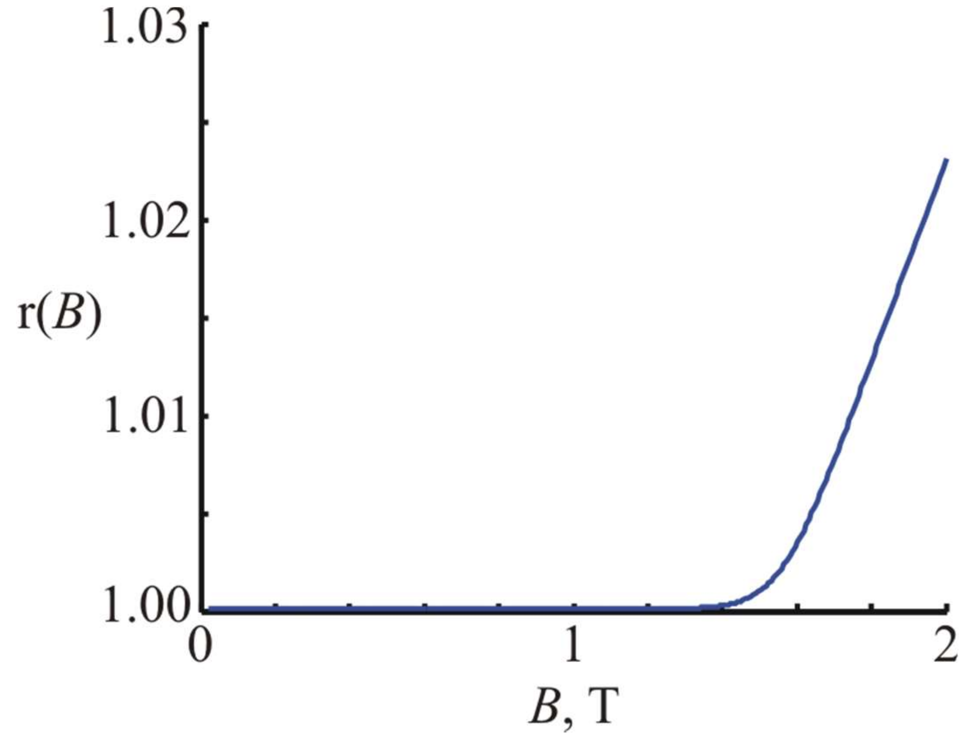
r(B) for M47 silicon steel sample

2.3 Magnetically Conductive Materials

- Permeability in terms of flux density to magnetization ratio

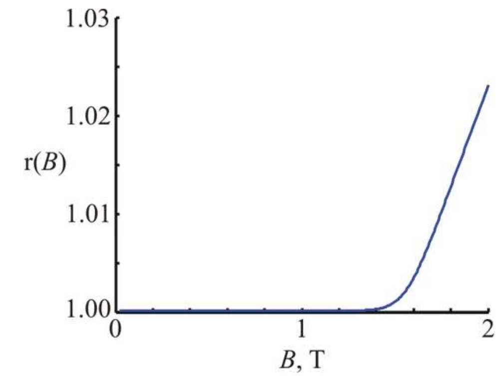
2.3 Magnetically Conductive Materials

- Representing the flux density to magnetization ratio



2.3 Magnetically Conductive Materials

- Now consider g(B). It looks like



- Thus

$$\frac{d\,\mathrm{g}(B)}{dB} = \mathrm{sgn}(B)\sum_{k=1}^{K}\frac{\alpha_k}{1+e^{-\beta_k(|B|-\gamma_k)}}$$

2.3 Magnetically Conductive Materials

- Integrating, we have

$$\mathrm{g}(B)=\sum_{k=1}^{K}\alpha_k|B|+\delta_k\ln\left(\varepsilon_k+\zeta_k e^{-\beta_k|B|}\right)$$

$$\zeta_k=\frac{1}{1+e^{-\beta_k\gamma_k}}.\quad \varepsilon_k=\frac{e^{-\beta_k\gamma_k}}{1+e^{-\beta_k\gamma_k}}\quad \delta_k=\frac{\alpha_k}{\beta_k}$$

- Also

$$\frac{d\,\mathrm{g}(B)}{dB}=\mathrm{sgn}(B)\sum_{k=1}^{K}\frac{\eta_k}{\theta_k+e^{-\beta_k|B|}}$$

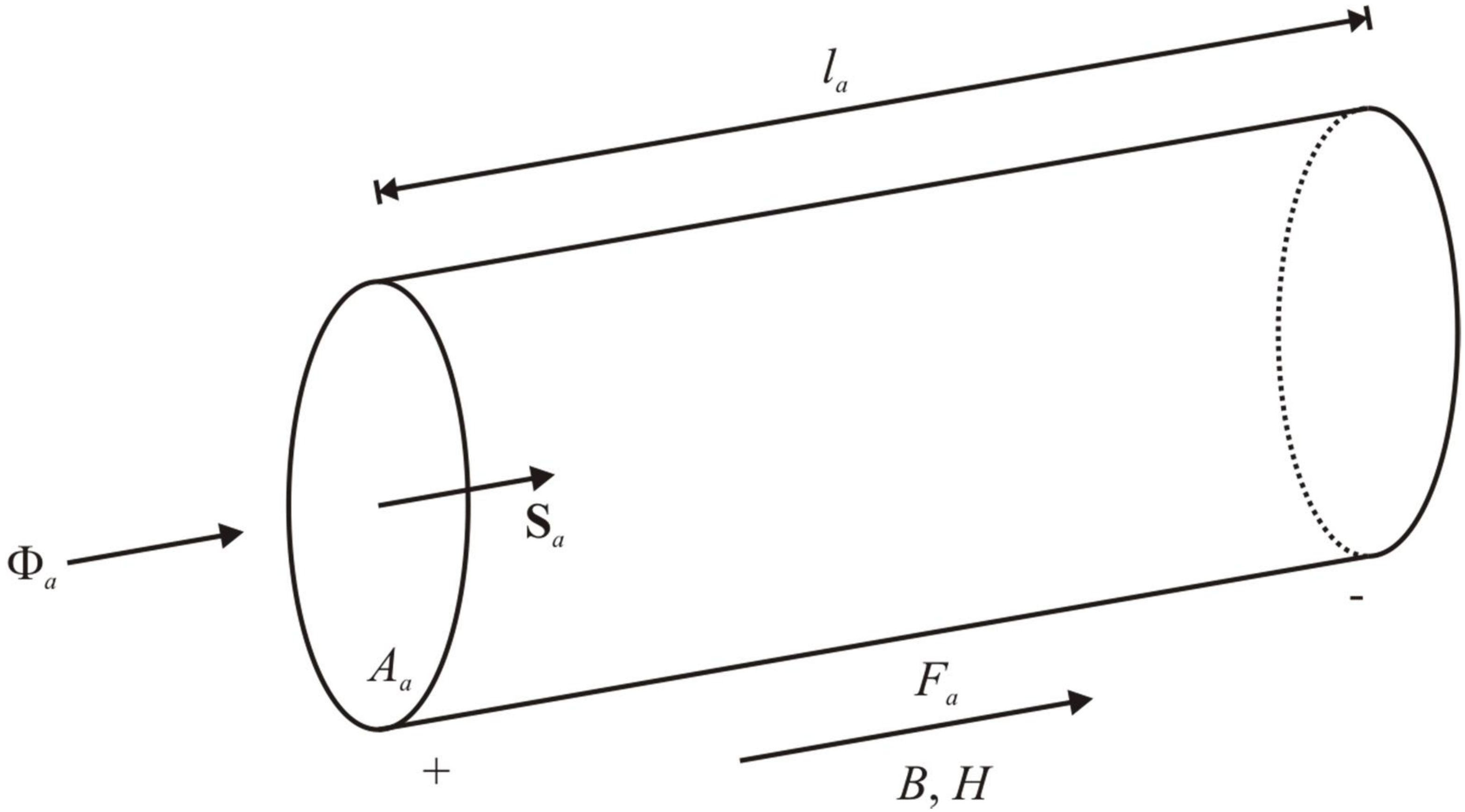
$$\eta_k=\alpha_k e^{-\beta_k\gamma_k}\qquad \theta_k=e^{-\beta_k\gamma_k}.$$

2.3 Magnetically Conductive Materials

- Derivative of permeability

2.3 Magnetically Conductive Materials

- Ohm's "Law"



2.3 Magnetically Conductive Materials

- Ohm's 'Law' (continued)

2.3 Magnetically Conductive Materials

- Ohm's 'Law' (continued)

2.3 Magnetically Conductive Materials

- Ohm's 'Law' summarized

$$F_a = \mathrm{R}_a(\xi)\Phi_a$$

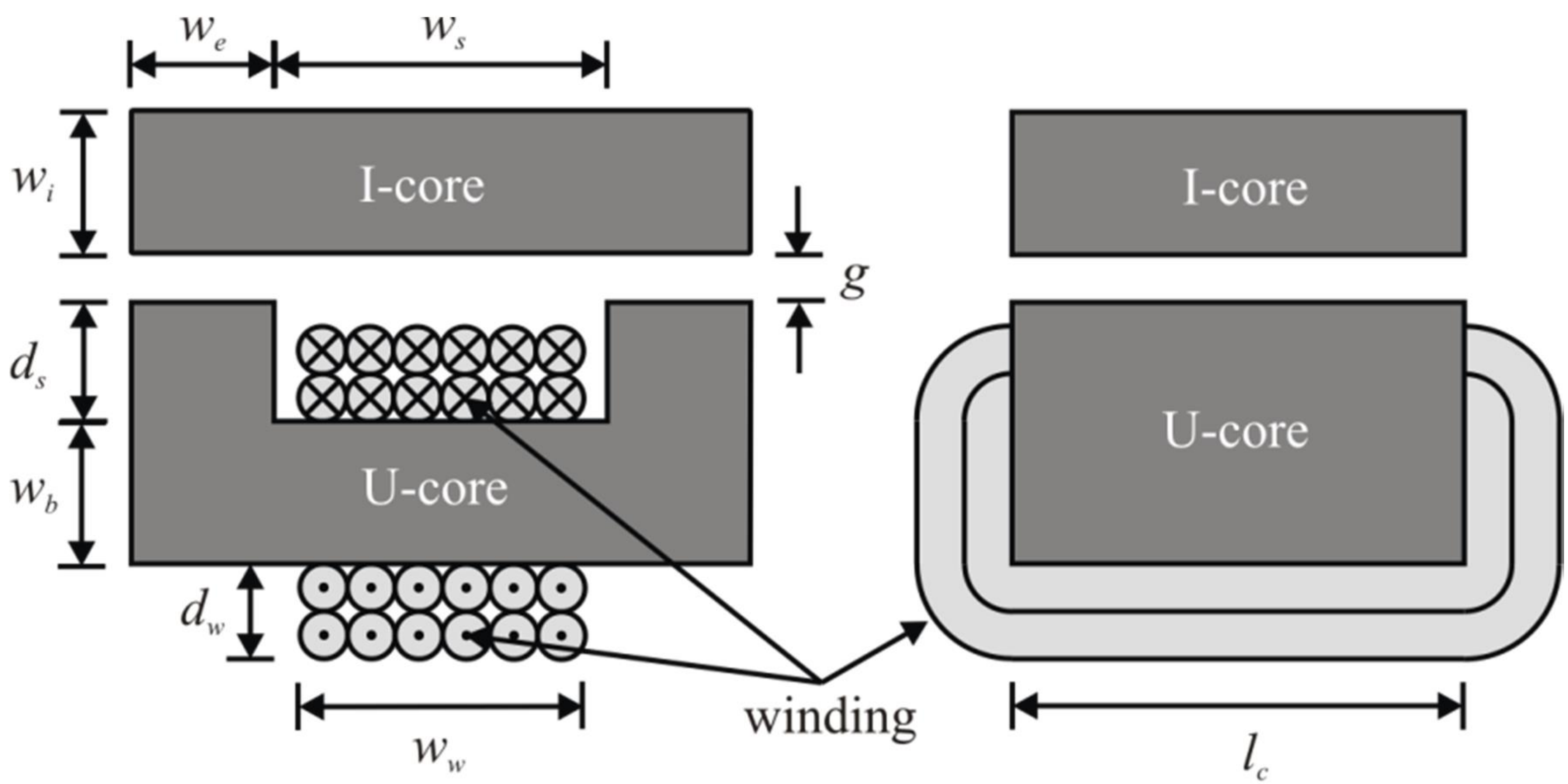
$$\mathrm{R}_a(\xi) = \begin{cases} \dfrac{l_a}{A_a \mu_H (F_a / l_a)} & \text{Form 1: } \xi = F_a \\[2ex] \dfrac{l_a}{A_a \mu_B (\Phi_a / A_a)} & \text{Form 2: } \xi = \Phi_a \end{cases}$$

$$\Phi_a = \mathrm{P}_a(\xi) F_a$$

$$\mathrm{P}_a(\xi) = \begin{cases} \dfrac{A_a \mu_H (F_a / l_a)}{l_a} & \text{Form 1: } \xi = F_a \\[2ex] \dfrac{A_a \mu_B (\Phi_a / A_a)}{l_a} & \text{Form 2: } \xi = \Phi_a \end{cases}$$

2.4 Construction of the MEC

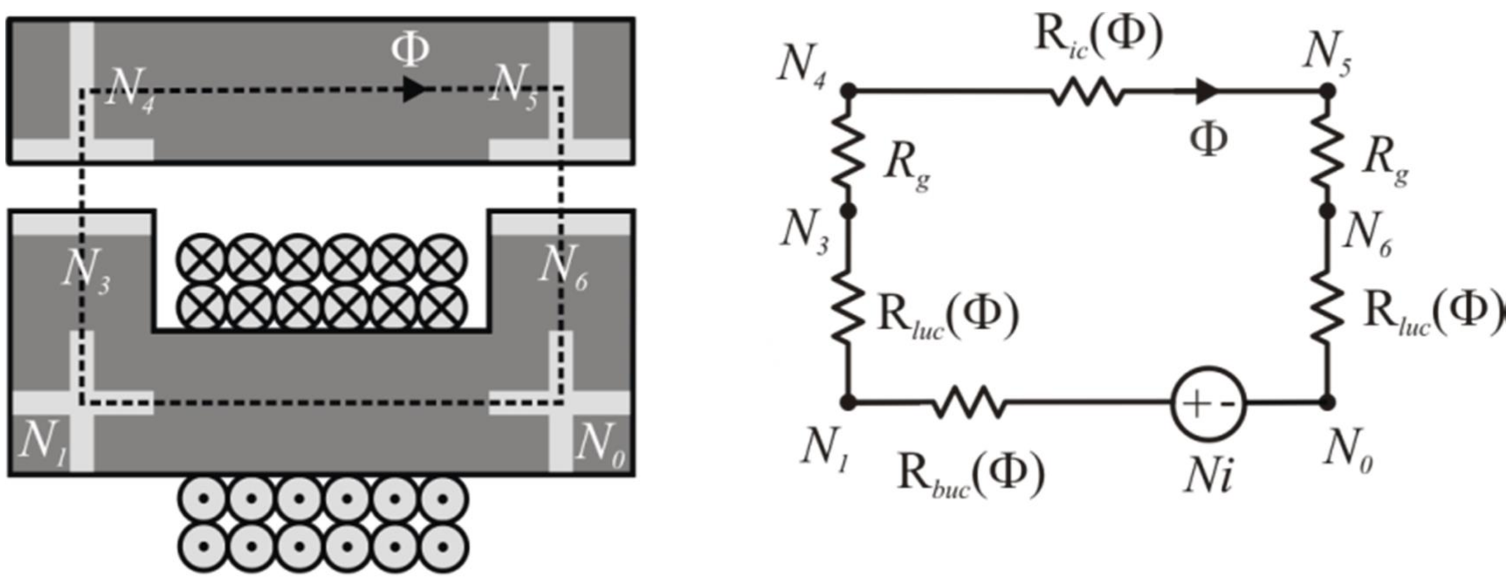
- Let's consider a UI core inductor



(a) cross section (b) side view

2.4 Construction of the MEC

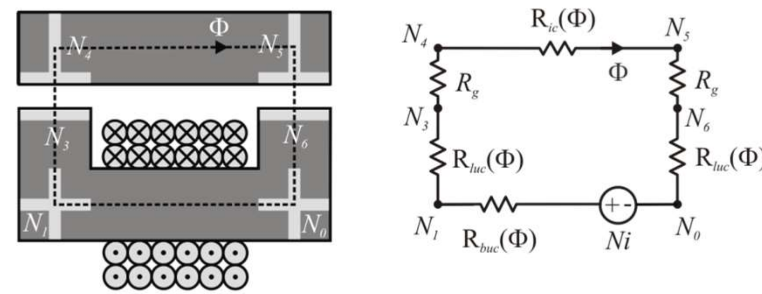
- Nodes and a possible MEC



(a) cross section showing nodes (b) magnetic equivalent circuit

2.4 Construction of the MEC

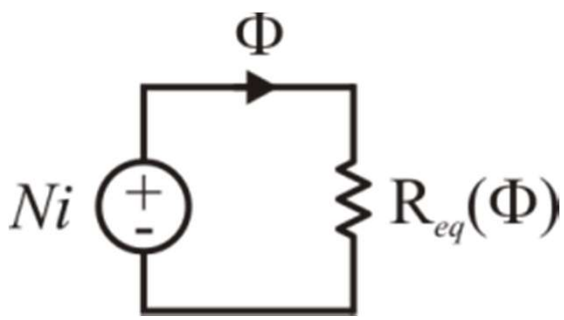
- Reluctances

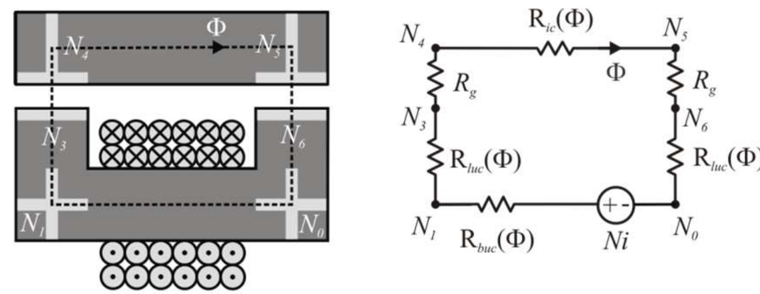


(a) cross section showing nodes (b) magnetic equivalent circuit

2.4 Construction of the MEC

- Simplified MEC





(a) cross section showing nodes (b) magnetic equivalent circuit

2.4 Construction of the MEC

- Example 2.4A. Suppose w_i = 25.1 mm, $w_b = w_e$=25.3 mm, w_s=51.2 mm, d_s=31.7 mm, l_c = 101.2 mm, g = 1.00 mm, w_w = 38.1 mm and d_w=31.7 mm. There are 35 turns with a current of 25 A. The relative permeabilty is 7700. Compute the flux and flux density in the I-core.

2.4 Construction of the MEC

- Example 2.4A (continued)

2.4 Construction of the MEC

- Example 2.4A (continued)

2.5 Translation to Electric Circuits

- Flux Linkage

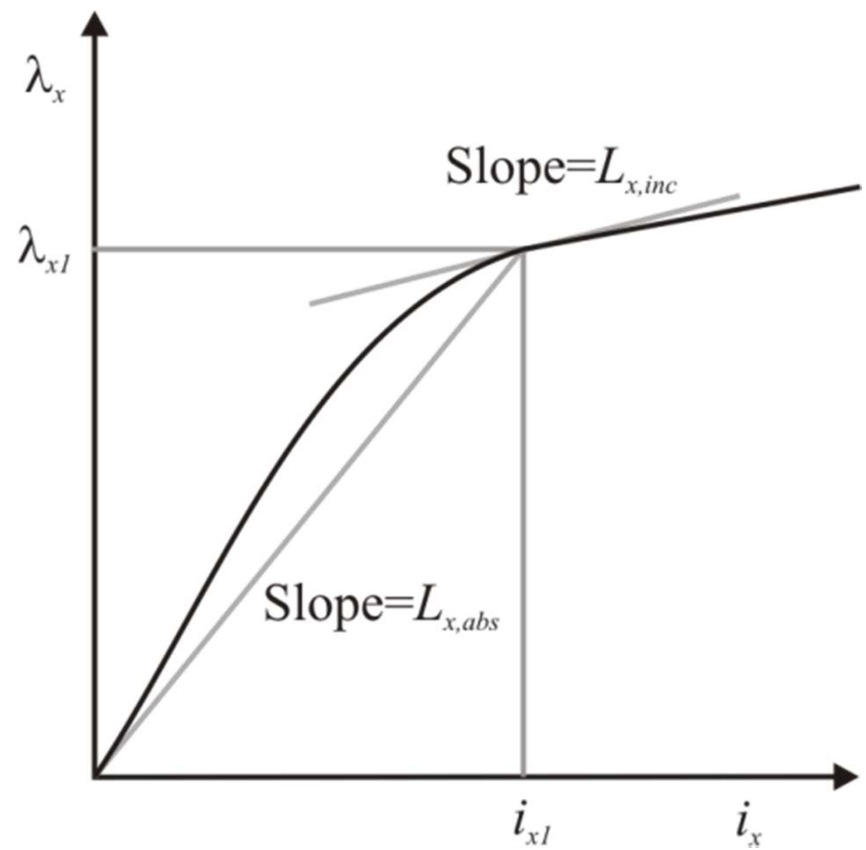
$$\lambda_x = N_x \Phi_x$$

2.5 Translation to Electric Circuits

- Inductance

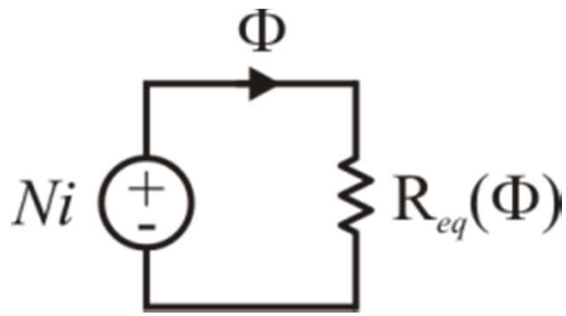
$$L_{x,abs} = \left. \frac{\lambda_x}{i_x} \right|_{i_{x1}, \lambda_{x1}}$$

$$L_{x,inc} = \left. \frac{\partial \lambda_x}{\partial i_x} \right|_{i_{x1}, \lambda_{x1}}$$



2.5 Translation to Electric Circuits

- Consider our simplified MEC



2.5 Translation to Electric Circuits

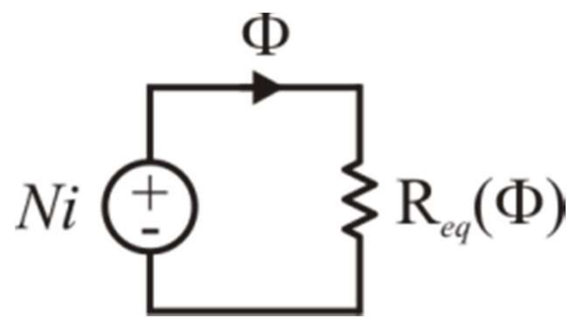
- Faraday's law

$$v_x = r_x i_x + \frac{d\lambda_x}{dt}$$

- If inductance is constant

2.5 Translation to Electric Circuits

- Example 2.5A. Consider our UI core. Suppose the material is non-linear (permeability parameters given in Table 2.5A1). Predict the λ-i characteristic.

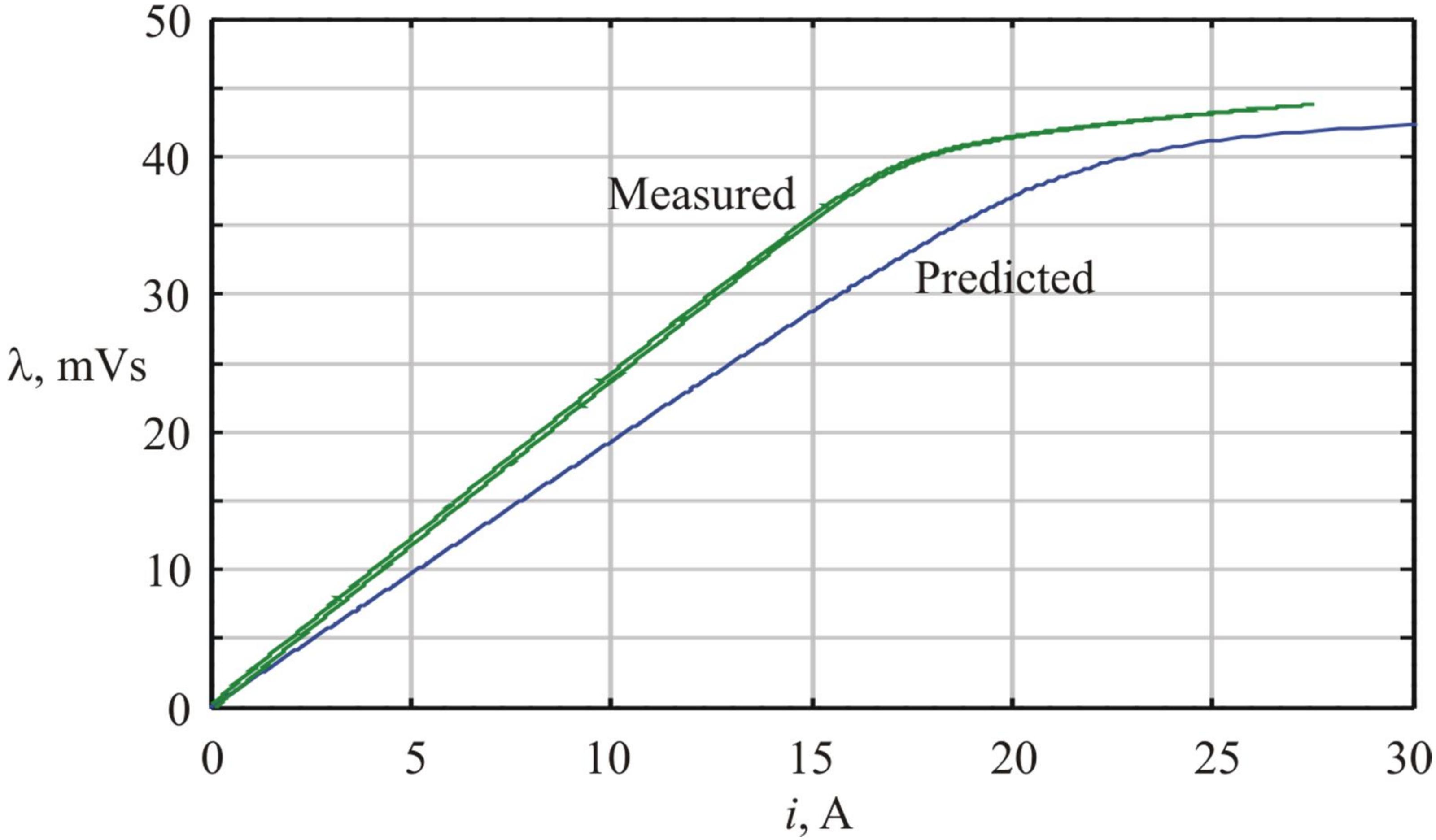


2.5 Translation to Electric Circuits

- Example 2.5A (continued)

2.5 Translation to Electric Circuits

- Example 2.5A (continued)



2.5 Translation to Electric Circuits

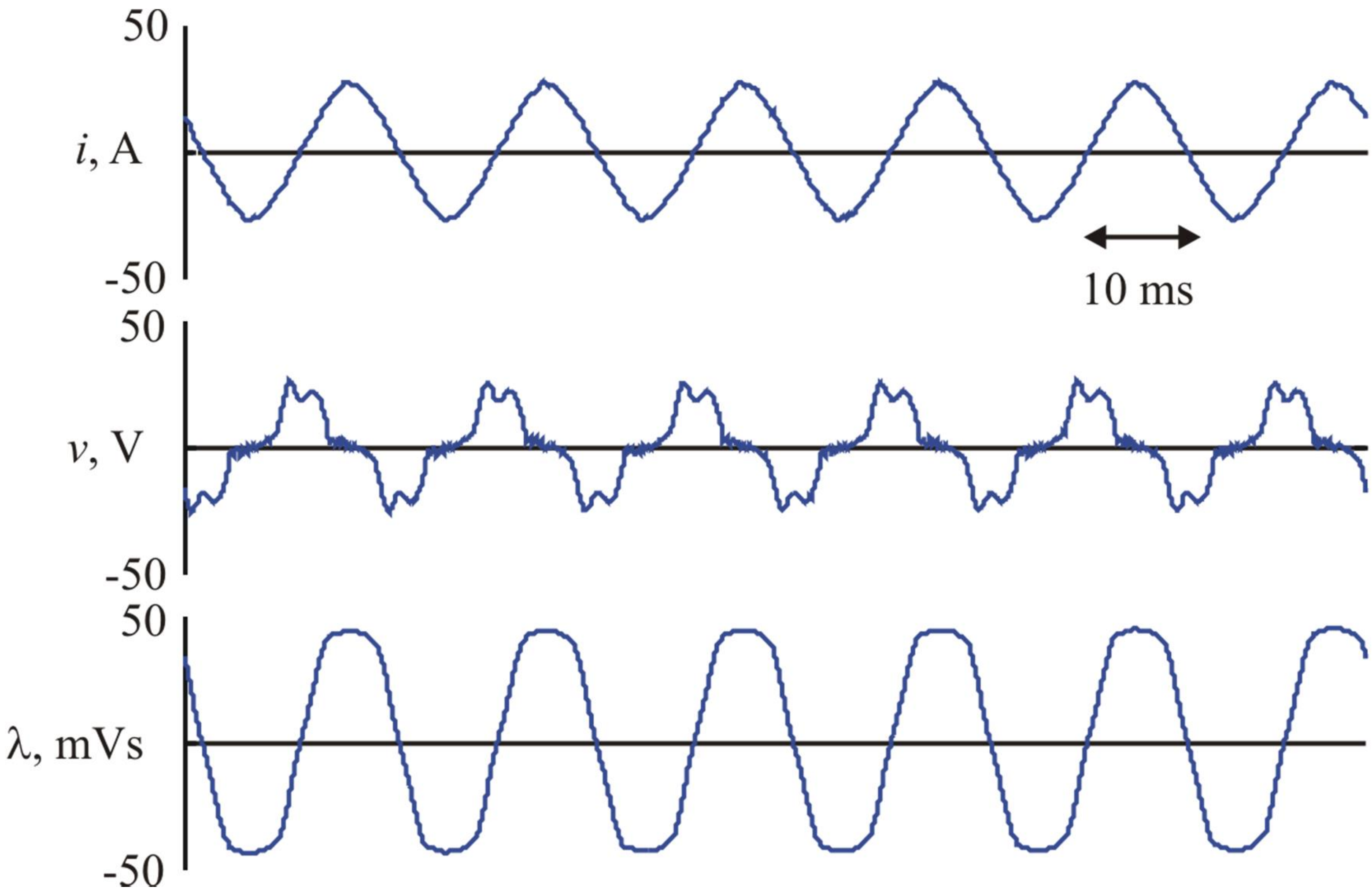
- Example 2.5B. Measure the λ-i characteristic. Idea and procedure.

2.5 Translation to Electric Circuits

- Example 2.5B (continued)

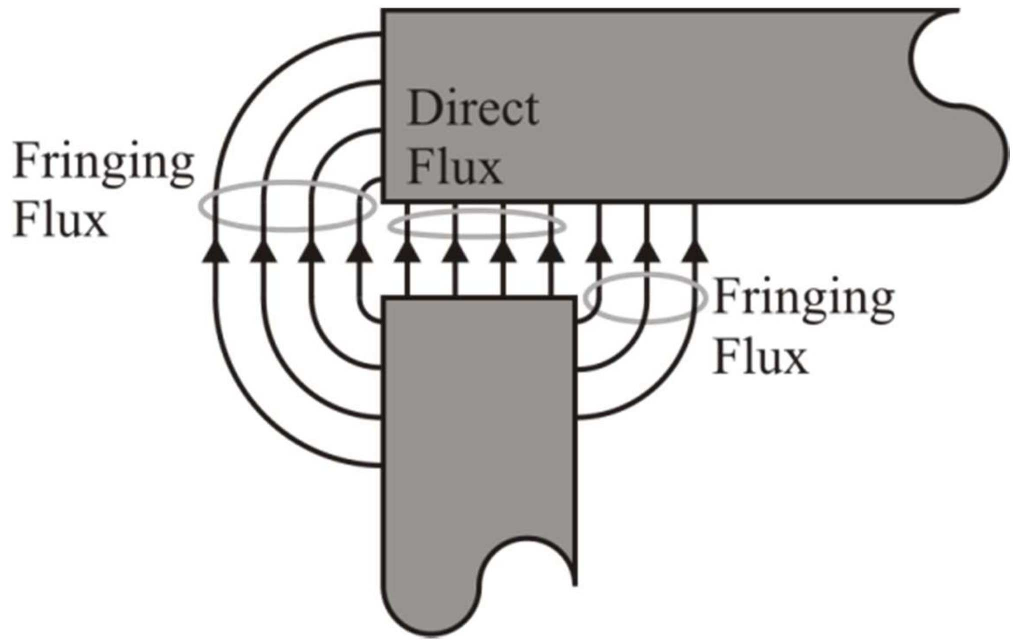
2.5 Translation to Electric Circuits

- Example 2.5B (continued)



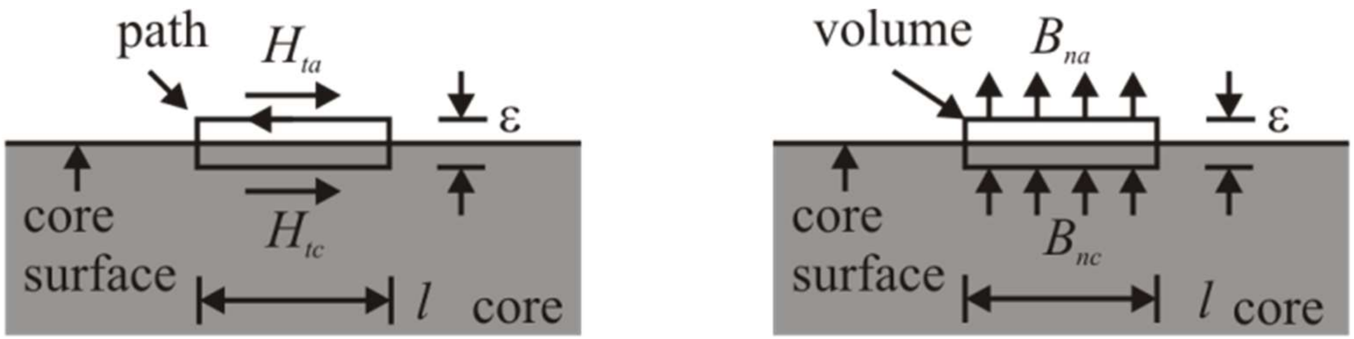
2.6 Fringing Flux

- Fringing flux paths



2.6 Fringing Flux

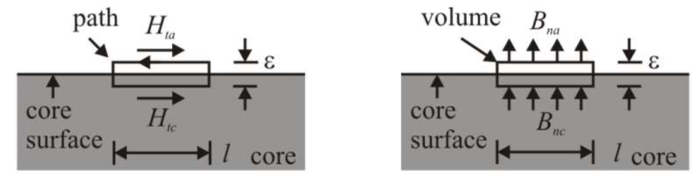
- Field components at an air core interface



(a) tangential field component (b) normal field component

2.6 Fringing Flux

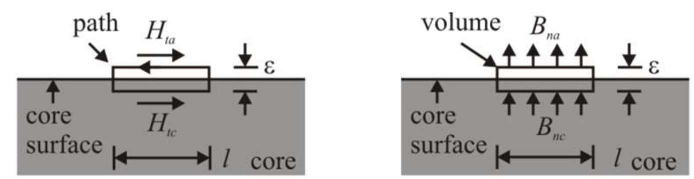
- Normal component



(a) tangential field component (b) normal field component

2.6 Fringing Flux

- Tangential component

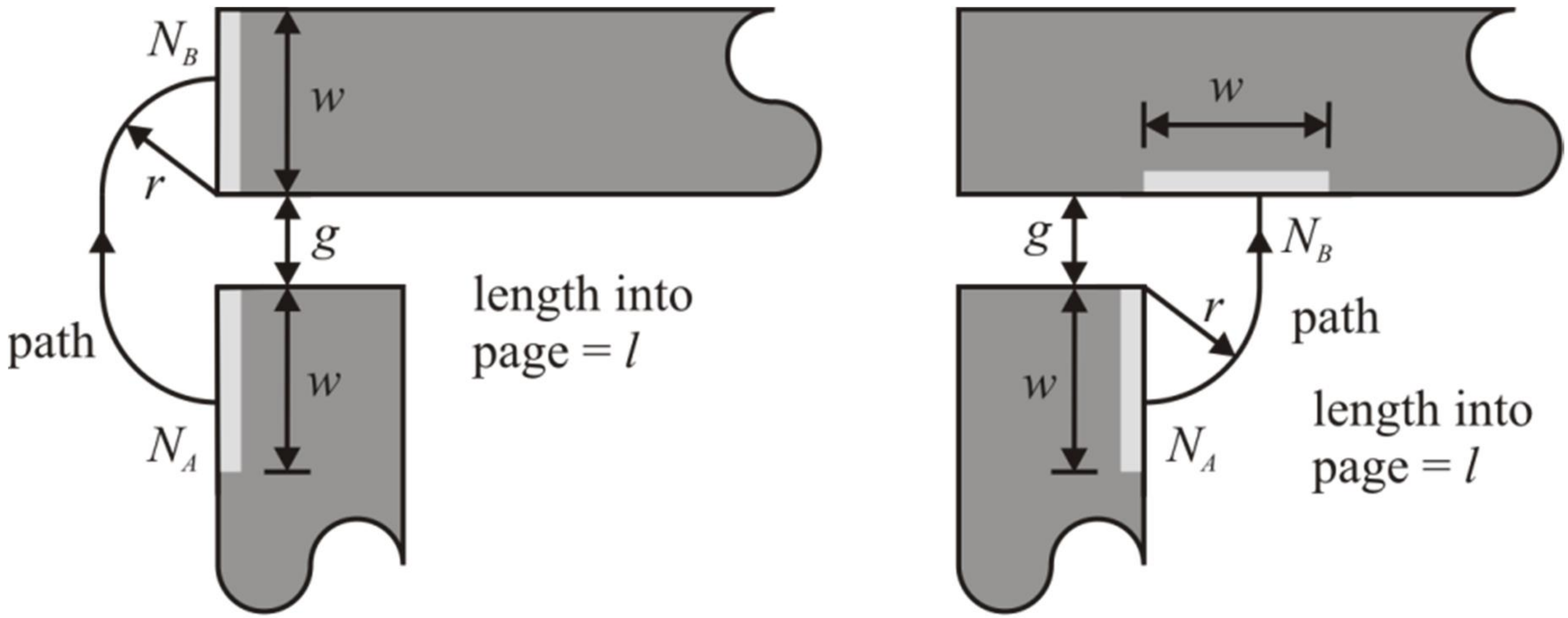


(a) tangential field component (b) normal field component

- Conclusion:

2.6 Fringing Flux

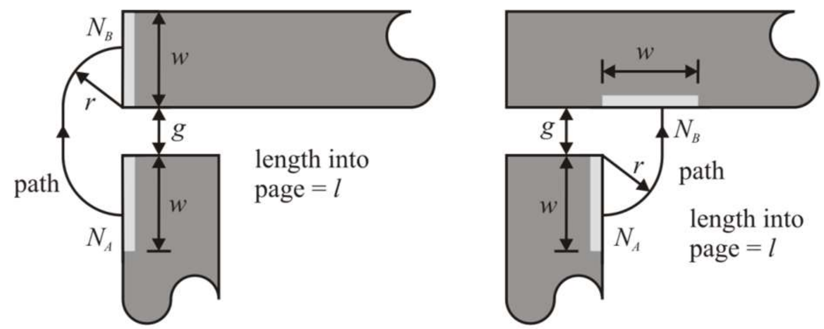
- Consider geometries (a) and (b)



(a) outer corner geometry (b) inner corner geometry

2.6 Fringing Flux

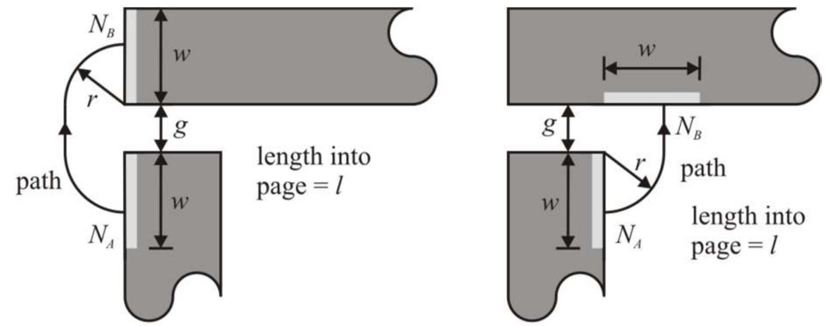
- Fringing permeance for (a)



(a) outer corner geometry (b) inner corner geometry

2.6 Fringing Flux

- Fringing permeance for (a) (continued)



(a) outer corner geometry (b) inner corner geometry

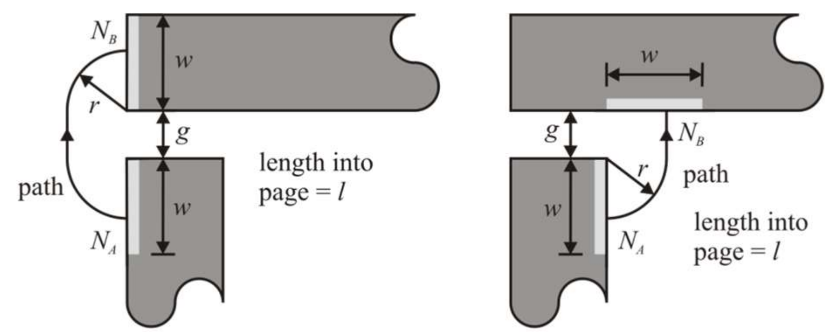
2.6 Fringing Flux

- Fringing permeance for (b)

2.6 Fringing Flux

- Fringing permeance for (b)

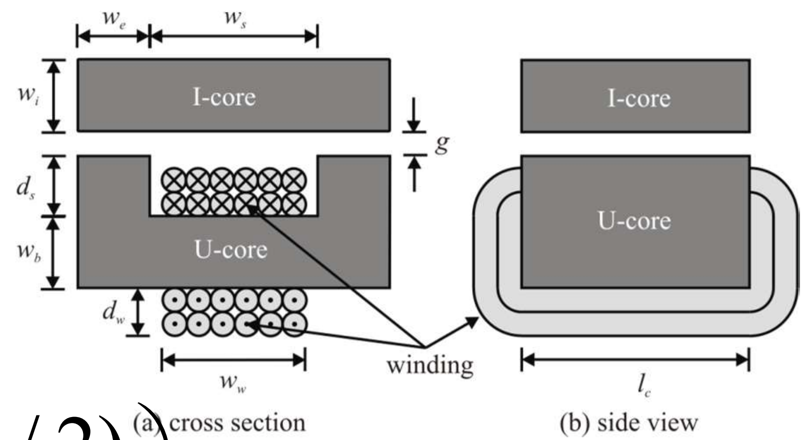
$$P_{fb} = \frac{2\mu_0 l}{\pi} \ln\left(1 + \frac{\pi}{2}\frac{w}{g}\right)$$



(a) outer corner geometry (b) inner corner geometry

2.6 Fringing Flux

- For our UI core geometry



$$P_g = P_{gd} + P_{foc} + P_{fic} + 2P_{ff}$$

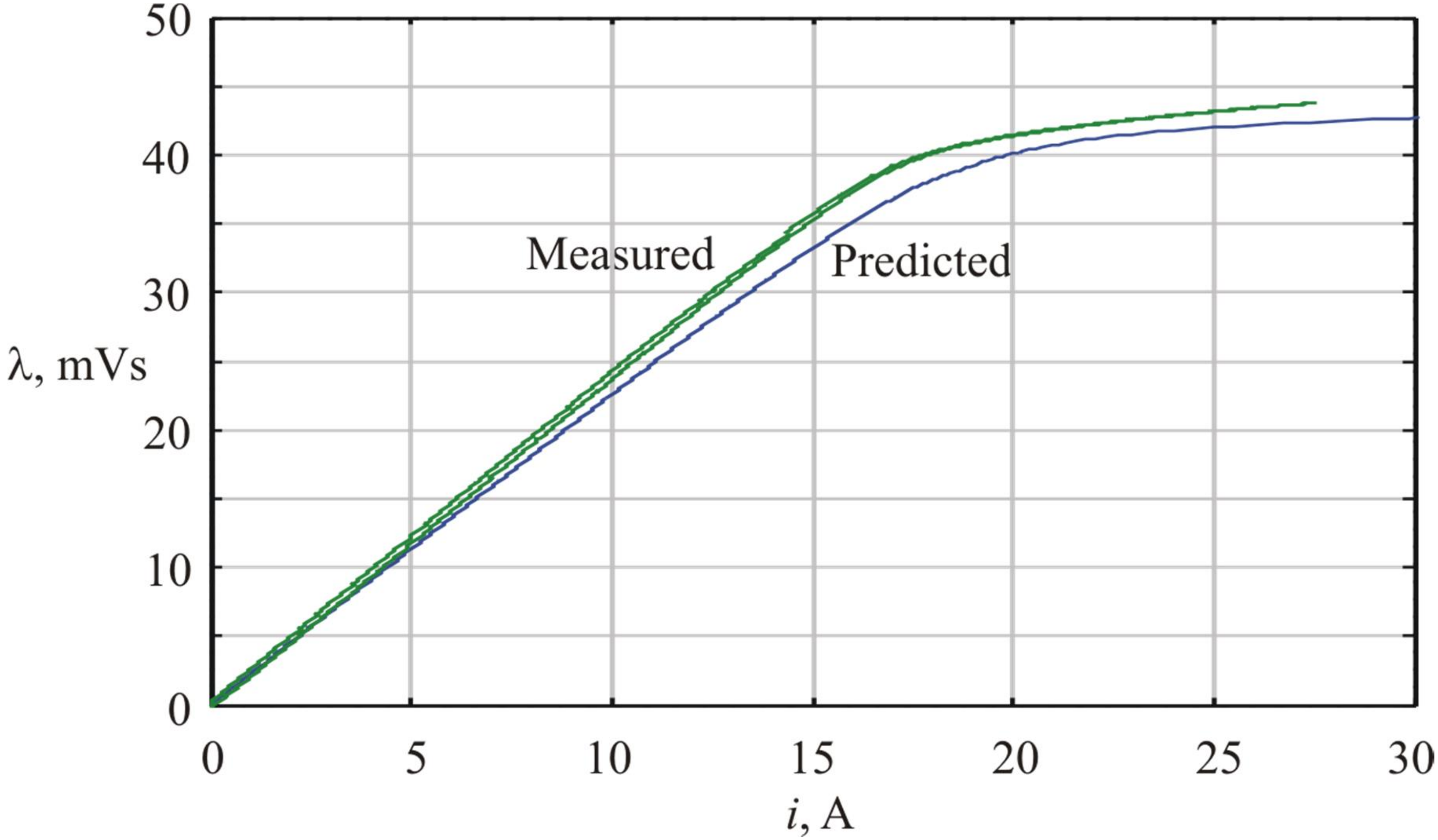
$$P_{fic} = \frac{2\mu_0 l_c}{\pi} \ln\left(1 + \frac{\pi}{2}\frac{\min(d_s, w_s/2)}{g}\right)$$

$$P_{foc} = \frac{\mu_0 l_c}{\pi} \ln\left(1 + \frac{\pi}{g}\min(w_i, d_s + w_b)\right)$$

$$P_{ff} = \frac{\mu_0 w_e}{\pi} \ln\left(1 + \pi\frac{\min(w_i, d_s + w_b)}{g}\right)$$

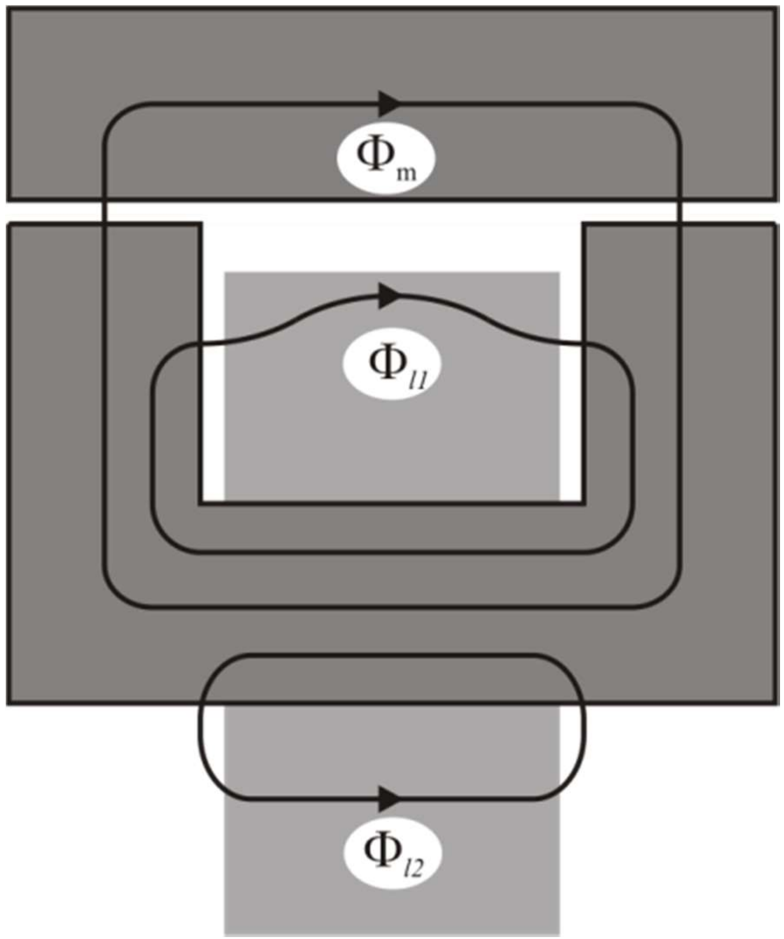
2.6 Fringing Flux

- Example 2.6A (repeating Example 2.5A)



2.7 Leakage Flux

- Leakage flux paths in a UI core inductor



2.7 Leakage Flux

- Energy stored in a magnetic linear system in terms of permeance:

$$E = \frac{1}{2} P N^2 i^2$$

- Energy stored in a magnetically linear system in terms of fields:

$$E = \frac{1}{2} \mu \int_V H^2 dV$$

2.7 Leakage Flux

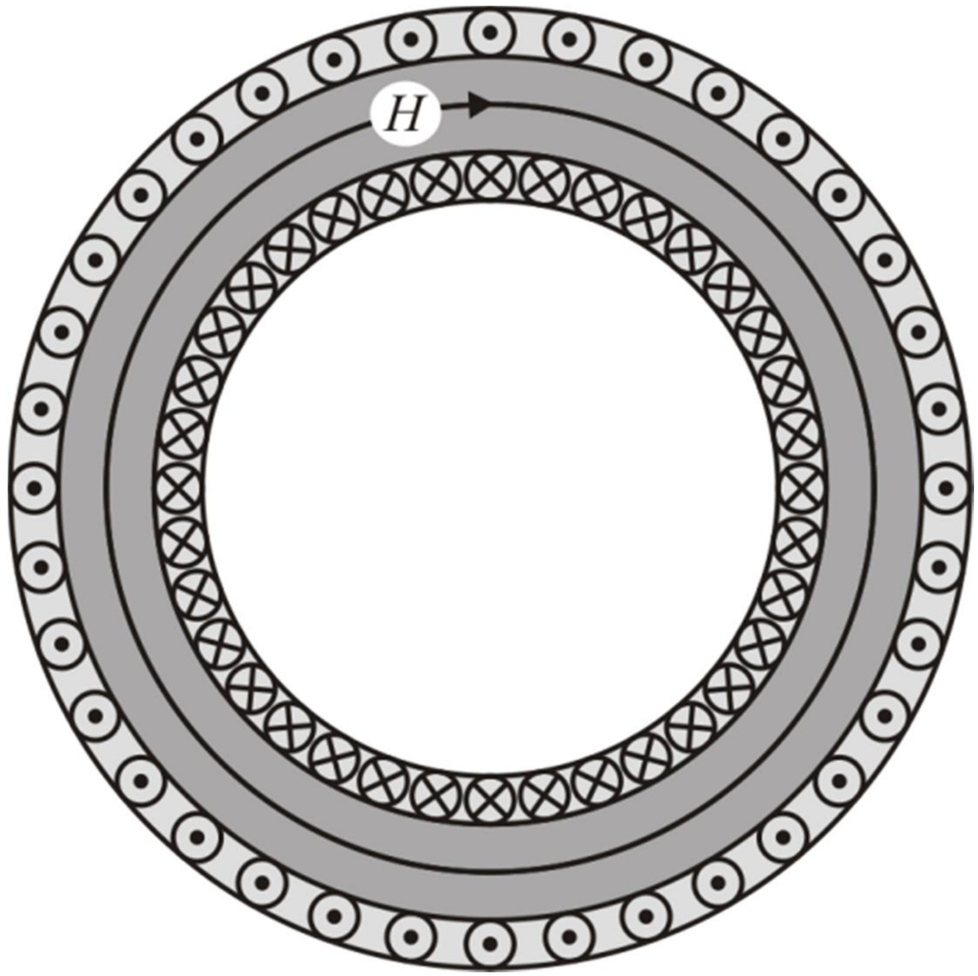
- Energy in terms of permeance

2.7 Leakage Flux

- Energy in terms of permeance (cont)

2.7 Leakage Flux

- Energy in terms of field



2.7 Leakage Flux

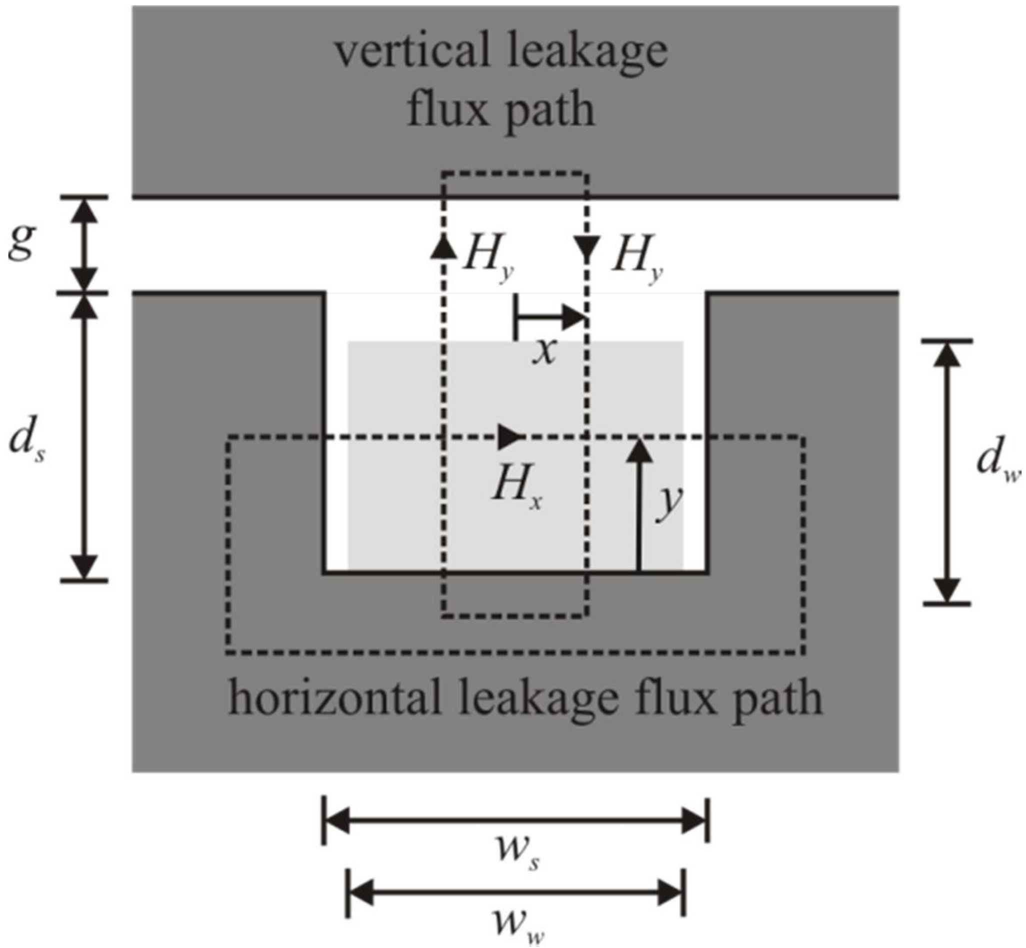
- Energy in terms of field (continued)

2.7 Leakage Flux

- Energy in terms of field (continued again)

2.7 Leakage Flux

- Horizontal slot leakage



$$P_{hsl} = \frac{\mu_0 l \left(3d_s - 2d_w\right)}{3w_s}$$

2.7 Leakage Flux

- Horizontal slot leakage (continued)

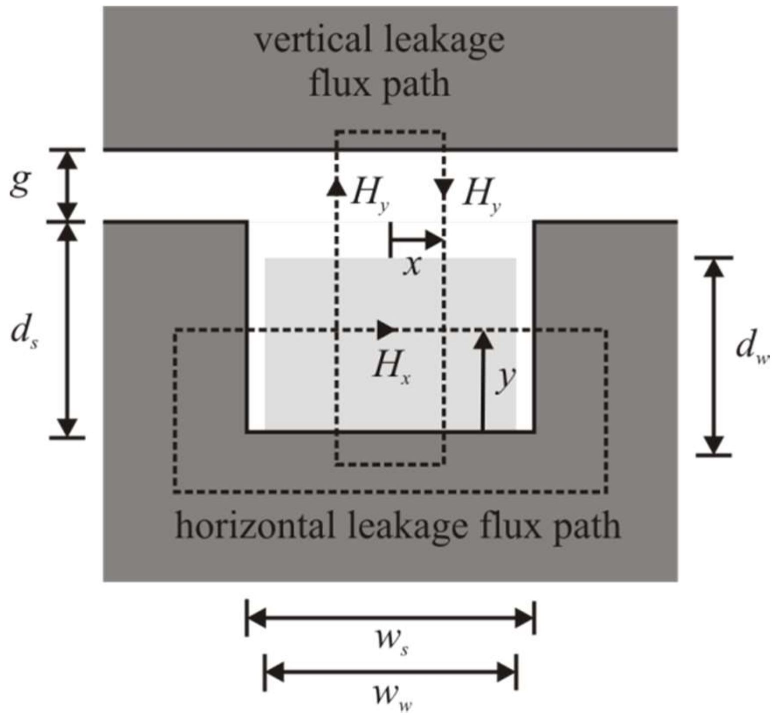
2.7 Leakage Flux

- Horizontal slot leakage (continued)

2.7 Leakage Flux

2.7 Leakage Flux

- Other expressions for horizontal slot leakage



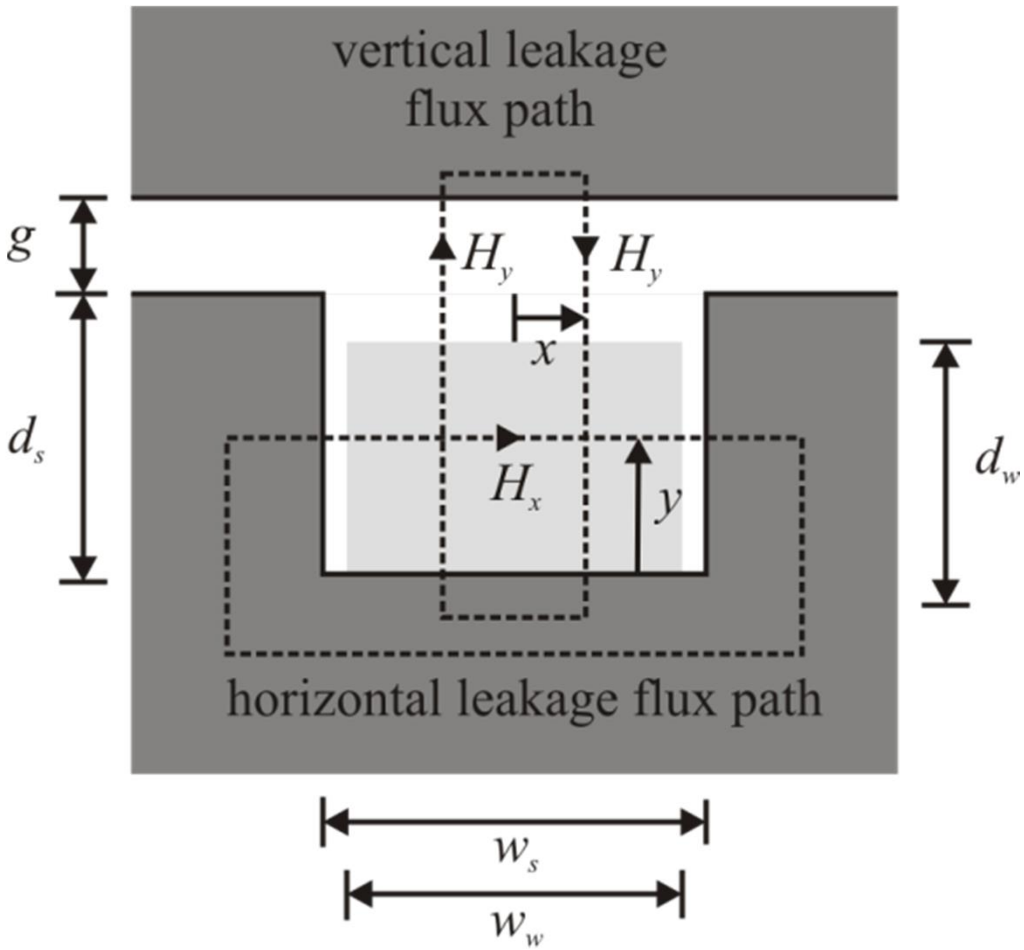
$$P'_{hsl} = \frac{\mu_0 l}{w_s}\left(d_s - \frac{1}{2}d_w\right)$$

$$d_h = \begin{cases} \sqrt{d_w(d_s - d_w/2)} & d_w \geq 2d_s/3 \\ d_s/2 + d_w/4 & d_w \leq 2d_s/3 \end{cases}$$

2.7 Leakage Flux

2.7 Leakage Flux

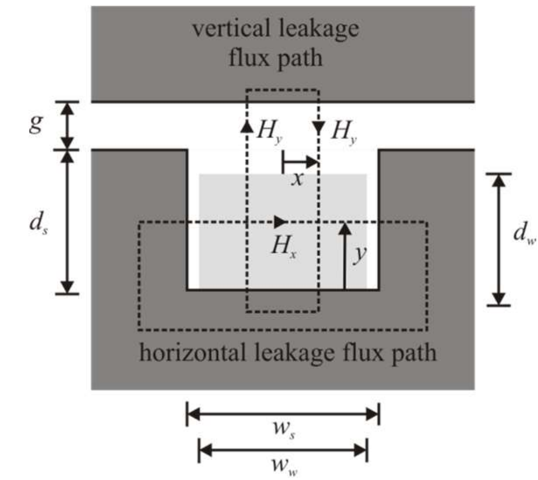
- Vertical slot leakage



$$P_{vsl} = \frac{\mu_0 l(3w_s - 2w_w)}{12\left(d_s + g\right)}$$

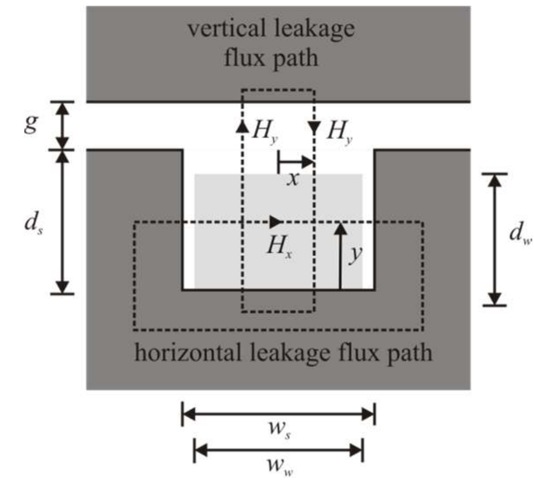
2.7 Leakage Flux

- Vertical slot leakage (continued)



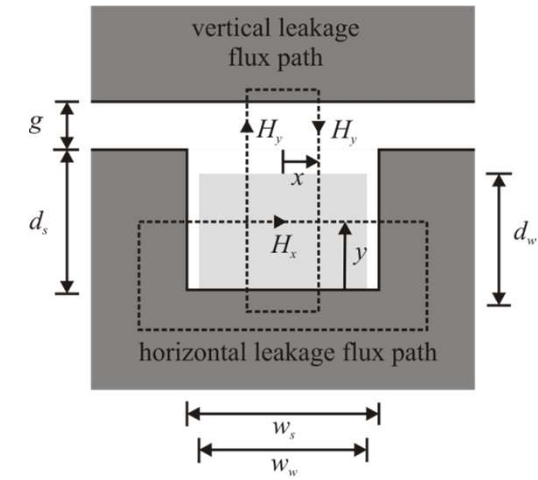
2.7 Leakage Flux

- Vertical slot leakage (continued again)



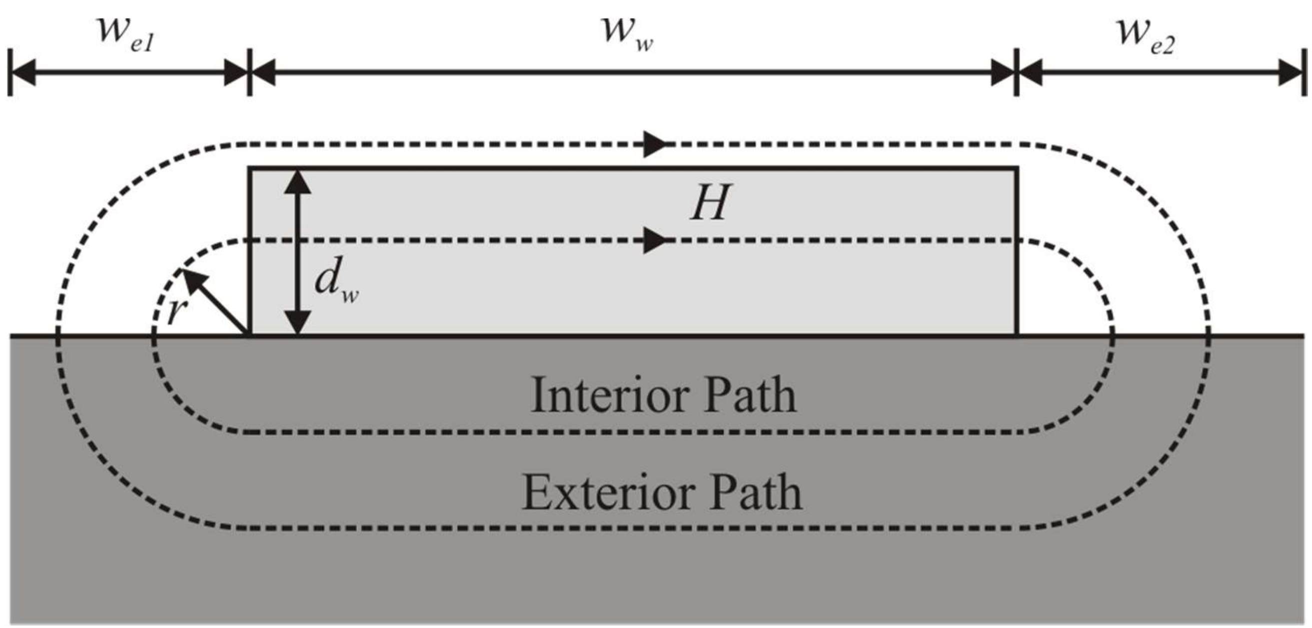
2.7 Leakage Flux

- Vertical slot leakage (continued yet again)



2.7 Leakage Flux

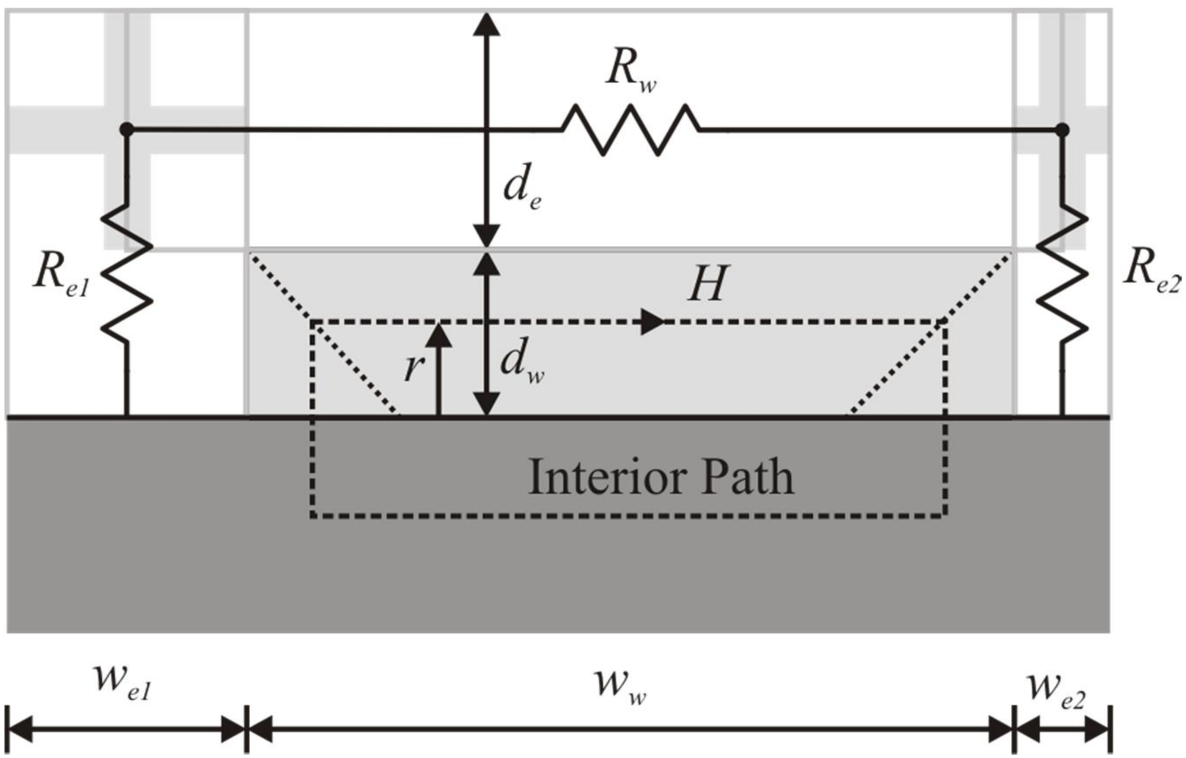
- Exterior adjacent leakage flux (approach 1)



$$P_{eal} = \frac{\mu_0 l_c}{\pi d_w^2}\left(\frac{r_1^2}{2} - \frac{w_w r_1}{\pi} + \frac{w_w^2}{\pi^2}\ln\left(\frac{w_w + \pi r_1}{w_w}\right) + d_w^2 \ln\left(\frac{w_w + \pi r_2}{w_w + \pi r_1}\right)\right)$$

2.7 Leakage Flux

- Exterior adjacent leakage flux (approach 2)



$$P_{eal} = \frac{1}{R_{eale}} + P_{eali}$$

2.7 Leakage Flux

- Exterior term

$$R_{eale} = \frac{d_w\left(w_{e1} + w_{e2}\right) + \sqrt{(2w_w + w_{e1} + w_{e2})(w_{e1} + w_{e2})w_{e1}w_{e2}}}{\mu_0 l w_{e1} w_{e2}}$$

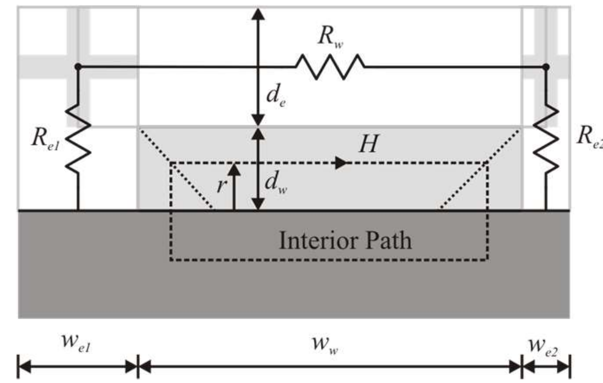
- Interior term

$$P_{eali} = \frac{\mu_0 l}{256 w_w^2 d_w^2}\left[\begin{array}{l} 16k_2^4 + 16\sqrt{2}k_1 k_2^3 + 4k_1^2 k_2^2 \\ -2\sqrt{2}k_1^3 k_2 + k_1^4 \ln\left(1 + \dfrac{2\sqrt{2}k_2}{k_1}\right) \end{array}\right]$$

$$k_1 = \left|w_w - 2d_w\right| \qquad k_2 = \frac{1}{\sqrt{2}}\min\left(2d_w, w_w\right)$$

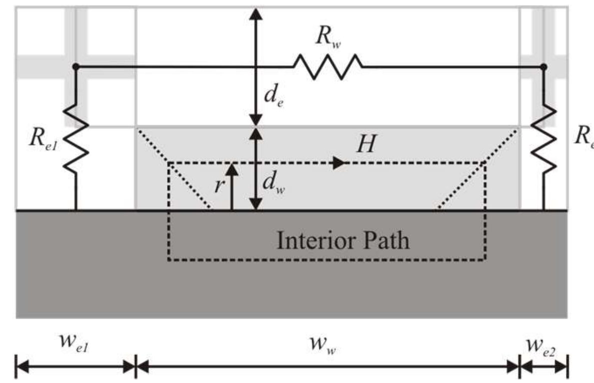
2.7 Leakage Flux

- Finding the exterior term



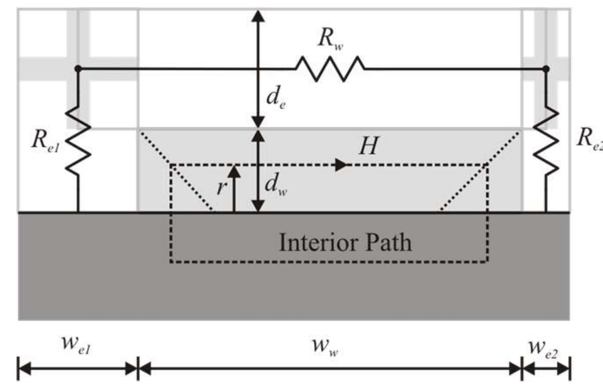
2.7 Leakage Flux

- Finding the exterior term (continued)



2.7 Leakage Flux

- Finding the interior term

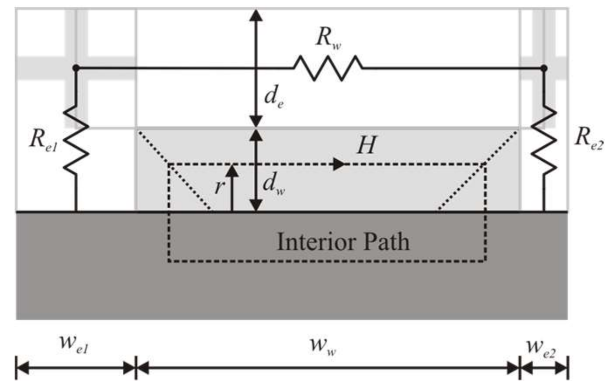


2.7 Leakage Flux

- Alternate orientation

2.7 Leakage Flux

- Finding the interior term (continued)

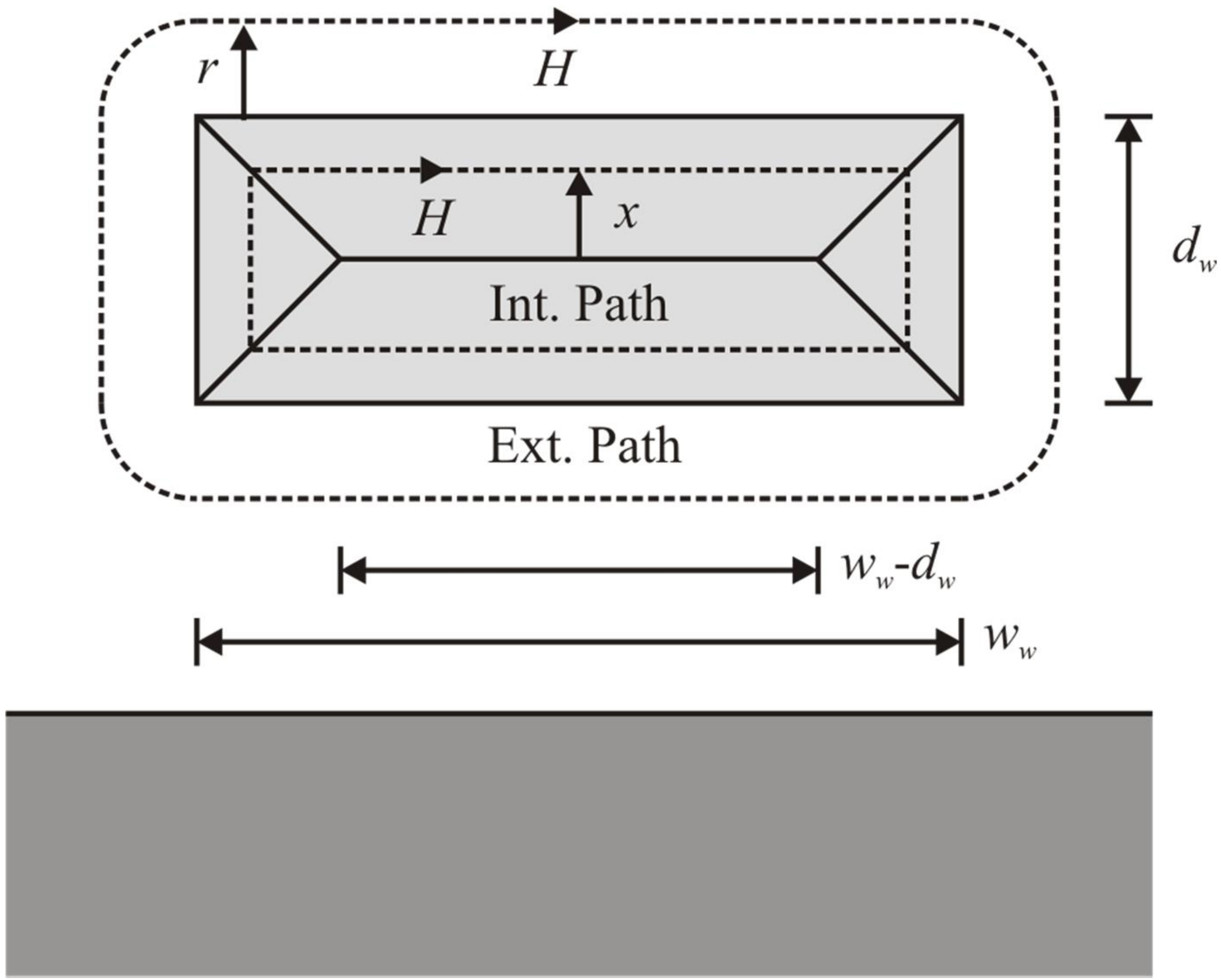


- This gets us to

$$E = \frac{1}{2}\mu_0 \frac{N_p^2 i_p^2 l}{w_w^2 d_w^2} \int_0^{r_{mx}} \frac{r^2(|w_w - 2d_w| + 2r)^2}{|w_w - 2d_w| + 4r} dr$$

2.7 Leakage Flux

- Exterior isolated leakage flux



2.7 Leakage Flux

- Exterior isolated leakage flux

$$k_1 = \left| w_w - d_w \right|$$

$$k_2 = \min(w_w, d_w)$$

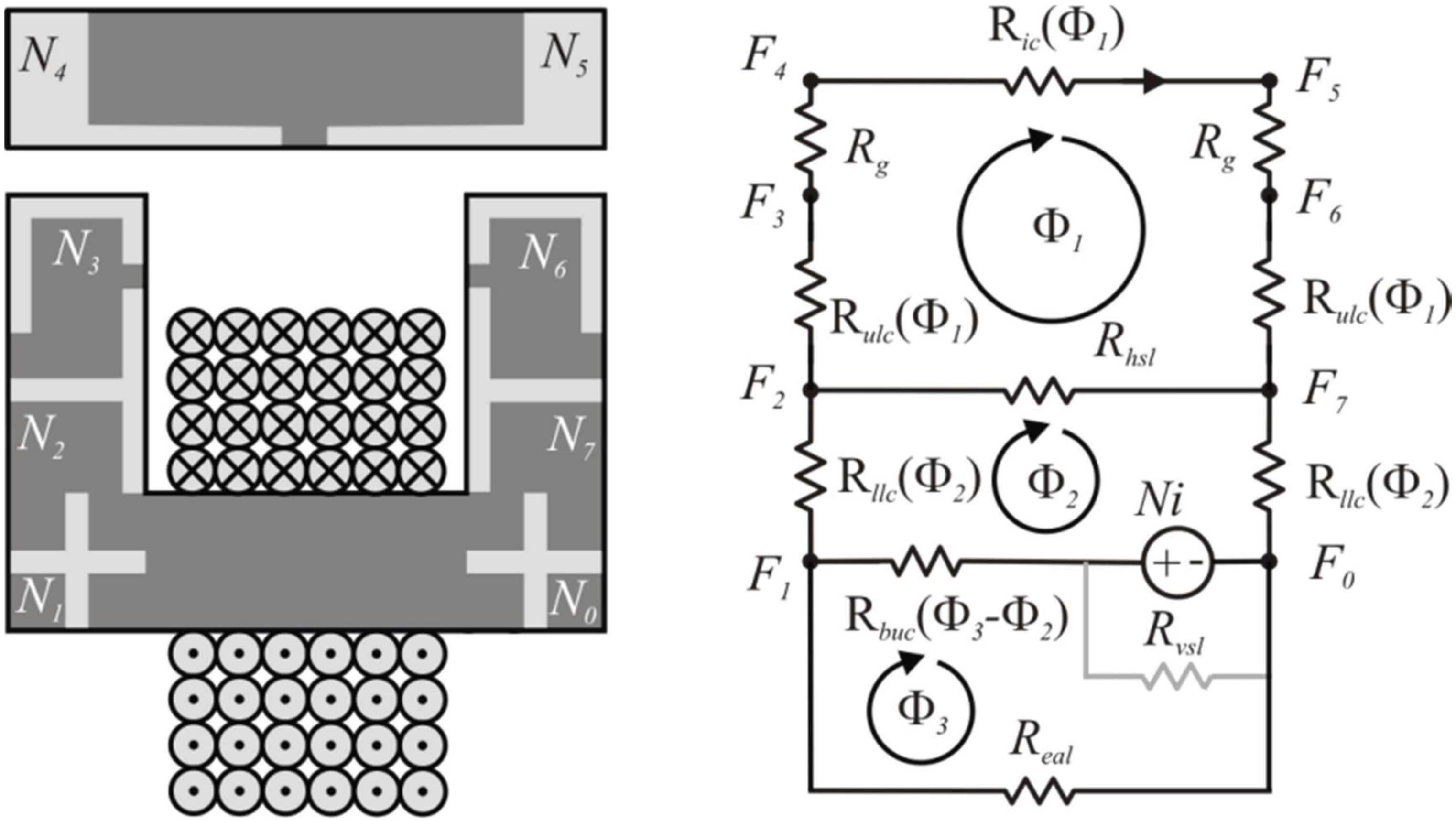
$$P_{eili} = \frac{\mu_0 l}{128 w_w^2 d_w^2}\left(4k_2^4 + 8k_1 k_2^3 + 2k_1^2 k_2^2 - 2k_1^3 k_2 + k_1^4 \ln\left(1 + \frac{2k_2}{k_1}\right)\right)$$

$$P_{eile} = \frac{\mu_0 l}{2\pi} \ln\left(1 + \frac{\pi r_{mx}}{d_w + w_w}\right)$$

$$P_{eil} = P_{eili} + P_{eile}$$

2.7 Leakage Flux

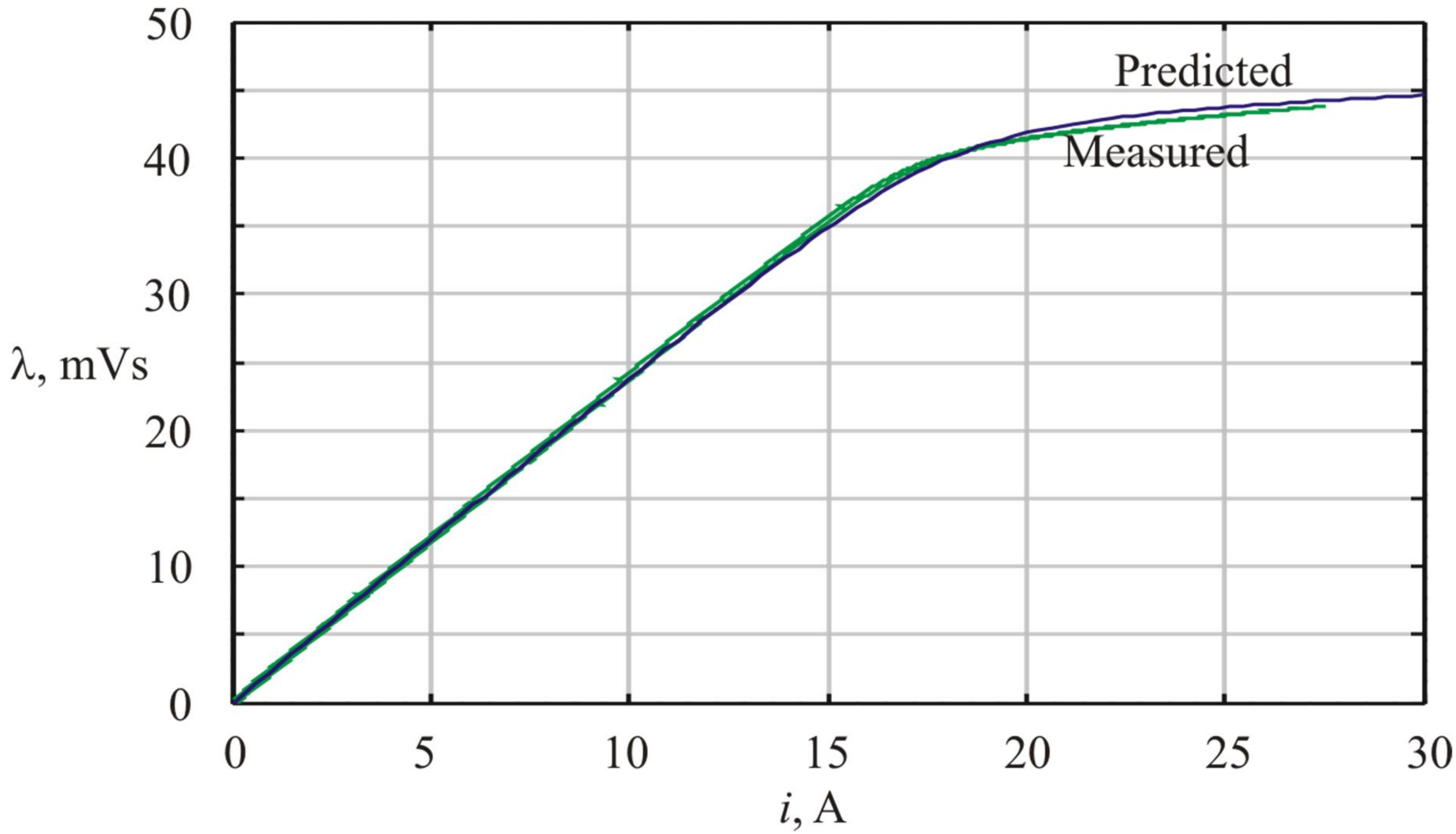
- Incorporating leakage flux into UI MEC



(a) cross section showing nodes (b) magnetic equivalent circuit

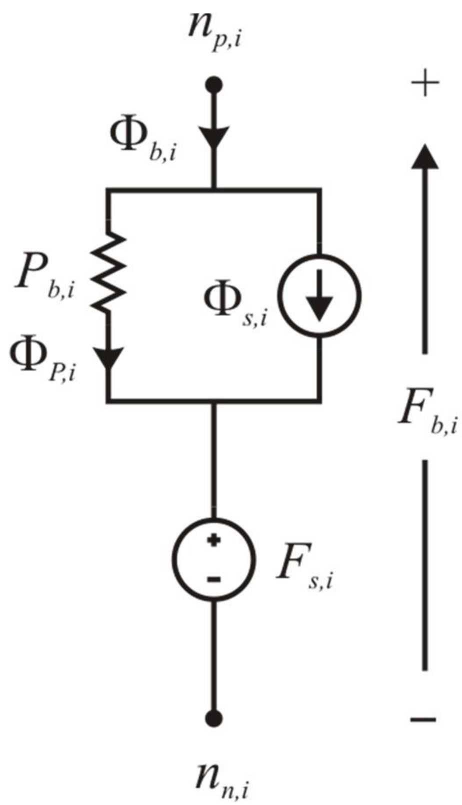
2.7 Leakage Flux

- Example 2.8D (Example 2.6A with leakage and fringing flux)

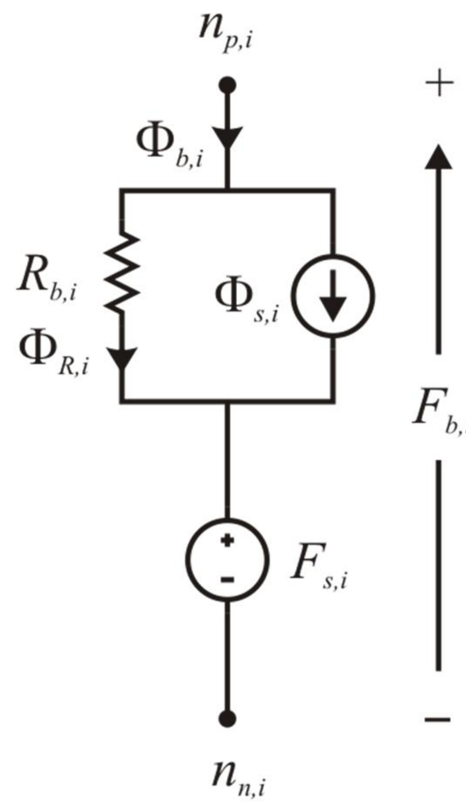


2.8 Numerical Solution of Nonlinear MECs

- Standard branch



(a) nodal branch *i*



(b) mesh branch *i*

$$\Phi_{b,i} = P_{b,i}(F_{b,i} - F_{s,i}) + \Phi_{s,i} \qquad F_{b,i} = R_{b,i}(\Phi_{b,i} - \Phi_{s,i}) + F_{s,i}$$

2.8 Numerical Solution of Nonlinear MECs

- Goal of nodal analysis

$$\mathbf{P}_N \mathbf{F}_N = \mathbf{\Phi}_N$$

- Basis of *j*'th row

$$\sum_{i \in L_{p,j}} \Phi_{b,i} - \sum_{i \in L_{n,j}} \Phi_{b,i} = 0$$

- Next

$$\Phi_{b,i} = P_{b,i}(F_{n_{p,i}} - F_{n_{n,i}} - F_{s,i}) + \Phi_{s,i}$$

2.8 Numerical Solution of Nonlinear MECs

- Example

2.8 Numerical Solution of Nonlinear MECs

- Nodal analysis – case where branch is in positive node list

2.8 Numerical Solution of Nonlinear MECs

- Nodal analysis – case where branch is in positive node list (continued)

2.8 Numerical Solution of Nonlinear MECs

- Nodal analysis – case where branch is in negative node list

2.8 Numerical Solution of Nonlinear MECs

- Nodal analysis – case where branch is in negative node list (continued)

2.8 Numerical Solution of Nonlinear MECs

- Nodal analysis formulation algorithm (NAFA)

$\mathbf{P}_N = 0$

$\mathbf{\Phi}_N = 0$

for $i = 0$ to $i = N_b$

    if $n_{p,i} > 0$

        $P_{N:n_{p,i},n_{p,i}} = P_{N:n_{p,i},n_{p,i}} + P_{b,i}$

        $\Phi_{N,n_{p,i}} = \Phi_{N,n_{p,i}} + P_{b,i}F_{s,i} - \Phi_{s,i}$

    end

    if $n_{n,i} > 0$

        $P_{N:n_{n,i},n_{n,i}} = P_{N:n_{p,i},n_{p,i}} + P_{b,i}$

        $\Phi_{N,n_{n,i}} = \Phi_{N,n_{n,i}} - P_{b,i}F_{s,i} + \Phi_{s,i}$

    end

    if $n_{p,i} > 0$ and $n_{n,i} > 0$

        $P_{N:n_{p,i},n_{n,i}} = P_{N:n_{p,i},n_{n,i}} - P_{b,i}$

        $P_{N:n_{n,i},n_{p,i}} = P_{N:n_{n,i},n_{p,i}} - P_{b,i}$

    end

end

2.8 Numerical Solution of Nonlinear MECs

- Example 2.8A

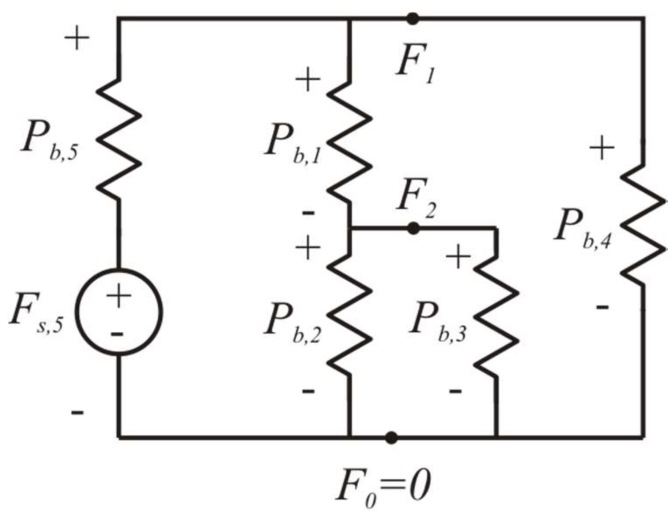


Table 2.8A-1 Branch list for nodal anlaysis.

i	$n_{p,i}$	$n_{n,i}$	$P_{b,i}$	$F_{s,i}$	$\Phi_{s,i}$
1	1	2	2	0	0
2	2	0	4	0	0
3	2	0	5	0	0
4	1	0	6	0	0
5	1	0	4	100	0

2.8 Numerical Solution of Nonlinear MECs

- Example 2.8A (continued)

2.8 Numerical Solution of Nonlinear MECs

- Example 2.8A (continued again)

2.8 Numerical Solution of Nonlinear MECs

- Example 2.8A (continued yet again)

2.8 Numerical Solution of Nonlinear MECs

- Goal of mesh analysis

$$\mathbf{R}_N \mathbf{\Phi}_N = \mathbf{F}_N$$

- Basis of *j*'th row

$$\sum_{i \in L_{p,j}} F_{b,i} - \sum_{i \in L_{n,j}} F_{b,i} = 0$$

- Flux going into *i*'th branch

$$\Phi_{b,i} = \sum_{m \in N_{p,i}} \Phi_{N,m} - \sum_{m \in N_{n,i}} \Phi_{N,m}$$

2.8 Numerical Solution of Nonlinear MECs

- Example

2.8 Numerical Solution of Nonlinear MECs

- Example (Continued)

2.8 Numerical Solution of Nonlinear MECs

- Thus

$$F_{b,k} = R_{b,k}\left(\sum_{m\in N_{p,k}} \Phi_{N,m} - \sum_{m\in N_{n,k}} \Phi_{N,m} - \Phi_{s,k}\right) + F_{s,k}$$

- Suppose *k*'th branch is in $L_{p,j}$

$$R_{b,k}\left(\sum_{m\in N_{p,k}} \Phi_{N,m} - \sum_{m\in N_{n,k}} \Phi_{N,m} - \Phi_{s,k}\right) + F_{s,k} + \sum_{i\in L_{p,j}/k} F_{b,i} - \sum_{i\in L_{n,j}} F_{b,i} = 0$$

$$R_{b,k}\left(\sum_{m\in N_{p,k}} \Phi_{N,m} - \sum_{m\in N_{n,k}} \Phi_{N,m}\right) + \sum_{i\in L_{p,j}/k} F_{b,i} - \sum_{i\in L_{n,j}} F_{b,i} = R_{b,k}\Phi_{s,k} - F_{s,k}$$

2.8 Numerical Solution of Nonlinear MECs

- Mesh analysis formulation algorithm (MAFA)

$\mathbf{R}_N = 0$

$\mathbf{F}_N = 0$

for $i = 0$ to $i = N_b$

    if $|N_{p,i}| > 0$

        $\mathbf{R}_{N:N_{p,i},N_{p,i}} = \mathbf{R}_{N:N_{p,i},N_{p,i}} + R_{b,i}$

        $\mathbf{F}_{N,N_{p,i}} = \mathbf{F}_{N,N_{p,i}} + R_{b,i}\Phi_{s,i} - F_{s,i}$

    end

    if $|N_{n,i}| > 0$

        $\mathbf{R}_{N:N_{n,i},N_{n,i}} = \mathbf{R}_{N:N_{n,i},N_{n,i}} + R_{b,i}$

        $\mathbf{F}_{N,N_{n,i}} = \mathbf{F}_{N,N_{n,i}} - R_{b,i}\Phi_{s,i} + F_{s,i}$

    end

    if $|N_{p,i}| > 0$ and $|N_{n,i}| > 0$

        $\mathbf{R}_{N:N_{p,i},N_{n,i}} = \mathbf{R}_{N:N_{p,i},N_{n,i}} - R_{b,i}$

        $\mathbf{R}_{N:N_{n,i},N_{p,i}} = \mathbf{R}_{N:N_{n,i},N_{p,i}} - R_{b,i}$

    end

end

2.8 Numerical Solution of Nonlinear MECs

- Example 2.8B

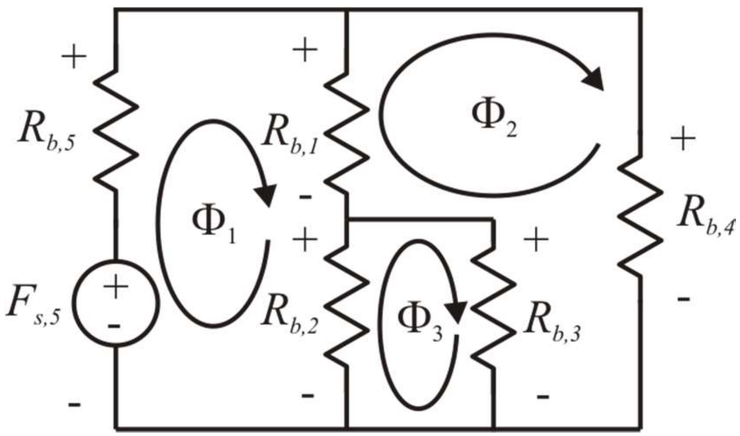


Table 2.8B-1 Branch list for mesh analysis.

i	$N_{p,i}$	$N_{n,i}$	$R_{b,i}$	$F_{s,i}$	$\Phi_{s,i}$
1	{1}	{2}	0.500	0	0
2	{1}	{3}	0.250	0	0
3	{3}	{2}	0.200	0	0
4	{2}	{}	0.167	0	0
5	{}	{1}	0.250	100	0

2.8 Numerical Solution of Nonlinear MECs

- Example 2.8B (continued)

2.8 Numerical Solution of Nonlinear MECs

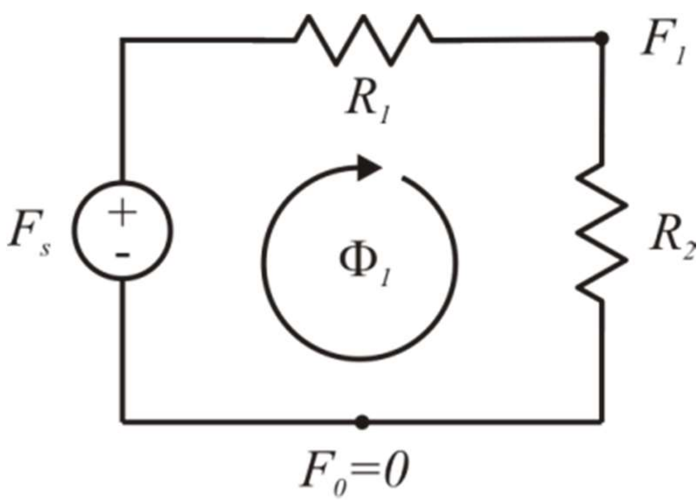
- Example 2.8B (continued again)

2.8 Numerical Solution of Nonlinear MECs

- Example 2.8B (still continued)

2.8 Numerical Solution of Nonlinear MECs

- Example 2.8C. Comparison of mesh and nodal analysis for nonlinear case



$$B = \mu_0 H + M_{sat} \frac{H}{\delta H + h}$$

$$\delta = \begin{cases} 1 & B, H \geq 0 \\ -1 & B, H < 0 \end{cases}$$

2.8 Numerical Solution of Nonlinear MECs

- Permeability as a function of H – given H find B then

$$\mu = \frac{B}{H}$$

- Permeability as a function of B. Given B we can show that the following yields H whereupon permeability may be found

$$a = \mu_0 \delta$$
$$b = \mu_0 h + M_{sat} - \delta B$$
$$c = -Bh$$

$$H = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

2.8 Numerical Solution of Nonlinear MECs

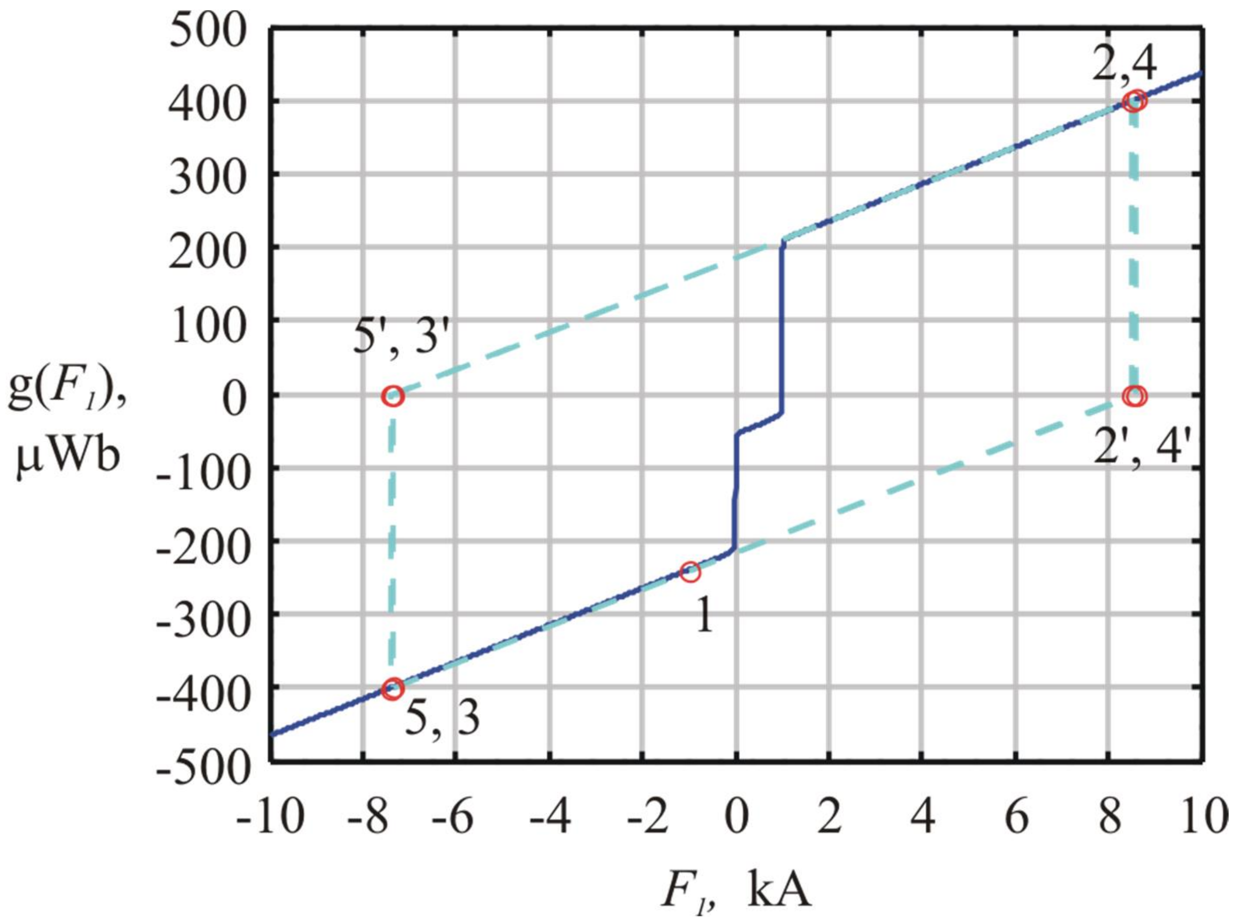
- Nodal analysis and formation of residual

2.8 Numerical Solution of Nonlinear MECs

- Nodal analysis and formation of residual (continued)

2.8 Numerical Solution of Nonlinear MECs

- Iteration of nodal equation using Newton's method



2.8 Numerical Solution of Nonlinear MECs

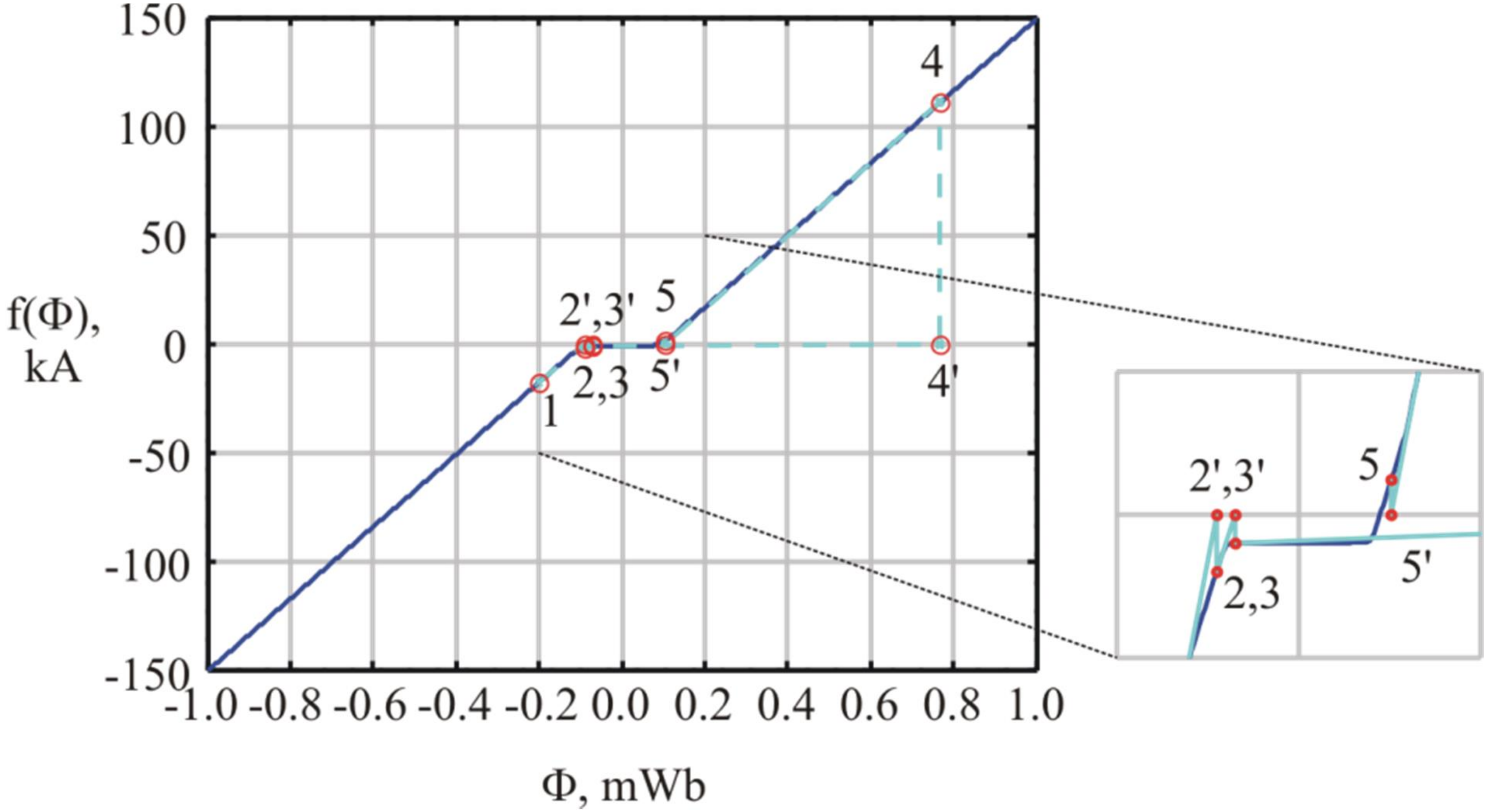
- Mesh analysis and formation of residual

2.8 Numerical Solution of Nonlinear MECs

- Mesh analysis and formation of residual (continued)

2.8 Numerical Solution of Nonlinear MECs

- Iteration of mesh equation using Newton's method



2.8 Numerical Solution of Nonlinear MECs

- Solving systems of nonlinear equations

$$f(\mathbf{x}) = 0$$

2.8 Numerical Solution of Nonlinear MECs

- Approach for MEC analysis will be to linearize each branch equation and apply MAFA
- Linearized branch equation

$$F_{b,i} \approx F_{seff,i} + R_{beff,i}\Phi_{b,i}$$

$$S_{0,i} = \frac{1}{l_{b,i}} R_{b0,i} \left.\frac{\partial \mu_B}{\partial B}\right|_{B_{0,i}} (\Phi_{b0,i} - \Phi_s)$$

$$F_{seff,i} = F_{s,i} + R_{b0,i}(S_{0,i}\Phi_{b0,i} - \Phi_{s,i})$$

$$R_{beff,i} = R_{b0,i}(1 - S_{0,i})$$

2.8 Numerical Solution of Nonlinear MECs

- Derivation of linearized branch equation

2.8 Numerical Solution of Nonlinear MECs

- Derivation of linearized branch equation (continued)

2.8 Numerical Solution of Nonlinear MECs

- Derivation of linearized branch equation (cont. 2)

2.8 Numerical Solution of Nonlinear MECs

- Derivation of linearized branch equation (cont. 3)

2.8 Numerical Solution of Nonlinear MECs

- Error criteria

$$e = \left\| \mathbf{\Phi}_b[k+1] - \mathbf{\Phi}_b[k] \right\|_2 - K_r \left\| \mathbf{\Phi}_b[k+1] + \mathbf{\Phi}_b[k] \right\|_2 - K_a$$

2.8 Numerical Solution of Nonlinear MECs

- Nonlinear MEC solution algorithm

$$\mathbf{\Phi}_b[1] = 0$$

$$k = 1$$

$$e = 0$$

while ($k = 1$)or(($k < k_{mx}$) and ($e > 0$))

    linearize branch models
    execute MAFA algorithm
    solve (2.8-11) to yield $\mathbf{\Phi}_b(k+1)$

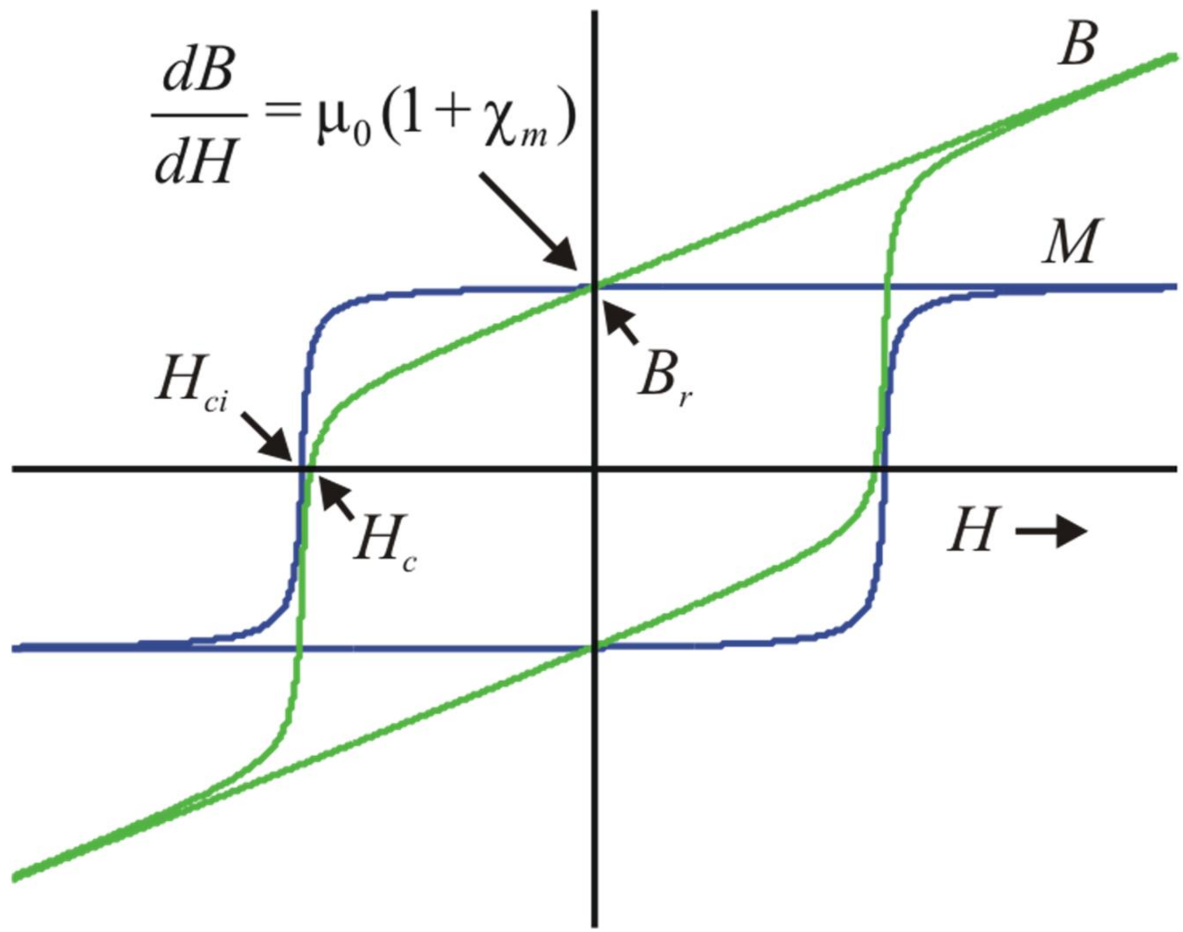
$$e = \left\| \mathbf{\Phi}_b[k+1] - \mathbf{\Phi}_b[k] \right\|_2 - K_r \left\| \mathbf{\Phi}_b[k+1] + \mathbf{\Phi}_b[k] \right\|_2 - K_a$$

    k=k+1;

end

2.9 Permanent magnet materials

- *B*-*H* and *M*-*H* characteristics



2.9 Permanent magnet materials

- To avoid demagnetization
  - $H > H_{lim}$ (positively magnetized)
  - $H < -H_{lim}$ (negatively magnetized)
  - $T < T_{cur}$

2.9 Permanent magnet materials

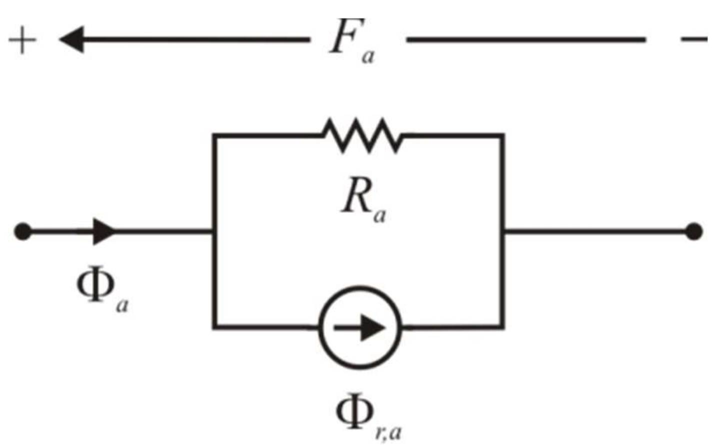
- Common materials

Table 2.9-1 Selected Permanent Magnet Properties.

Class	B_r (T)	H_{ci} (kA/m)	χ_m	ρ (g/cm^3)
Alnico	0.55 to 1.4	-38 to -150	0.60 to 6.8	6.9-7.3
SmCo	0.87 to 1.2	-1400 to -2800	0.023 to 0.096	8.3-8.4
Ferrite	0.23 to 0.41	-200 to -320	0.045 to 0.58	4.8-4.9
Bonded NdFeB	0.47 to 0.66	-470 to -1300	0.066 to 0.40	4.9 to 5.7
Sintered NdFeB	1.2 to 1.4	-960	0.092 to 0.36	7.5

2.9 Permanent magnet materials

- MEC representation of PM materials



$$F_a = R_a(\Phi_a - \Phi_{r,a})$$

$$\Phi_{r,a} = A_a B_{r,a}$$

$$R_a = \frac{l_a}{A_a \mu_0 (1 + \chi_{m,a})}$$

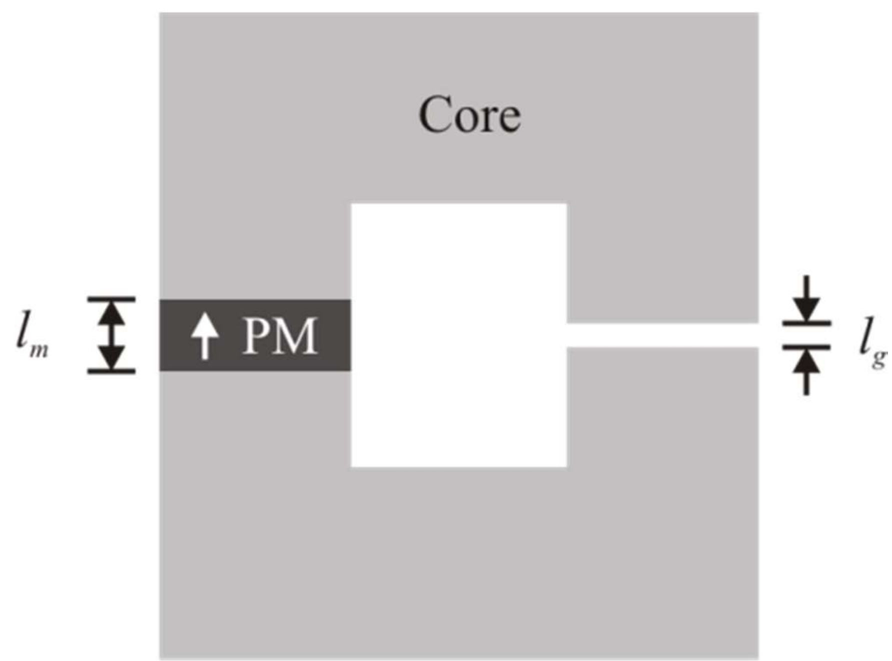
$$\Phi_a = P_a F_a + \Phi_{r,a}$$

2.9 Permanent magnet materials

- Derivation of MEC PM model

2.9 Permanent magnet materials

- Example 2.9A. Derive an expression for the field intensity in the magnet



2.9 Permanent magnet materials

- Example 2.9A

2.9 Permanent magnet materials

- Example 2.9A (continued)